Spillovers during Euro Area Sovereign Debt Crisis: a Network Analysis with Absolute Magnitude Restrictions

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Abstract

We study connectedness in European and US sovereign debt markets and find it declined steadily between 2009 and 2012, indicating increased financial fragmentation. Shocks to the Greek bond market in 2010 explain 20-30% of the variance of sovereign yields in stressed countries, while in 2011-2012 Italy (not Spain) was a key source of systemic risk. A new method that allows us to identify orthogonal country specific shocks in a panel of countries and employs restrictions on the relative size of the contemporaneous impact effect is used to derive the conclusions.

Keywords: Networks, Connectedness, Spillovers, Identification, Structural Vector Autoregression, Eurozone Debt Crisis, Sovereign bond Yields

JEL Classification: C3, G2

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1 Introduction

Spillovers and contagion were central to Eurozone’s debt crisis. World leaders and policy makers were confronted with the possibility that contagion could endanger countries with otherwise sound economic fundamentals. The fear of contagion was a main driver and a justification for bailout measures. “This is about nothing less than the future of Europe and the future of Germany in Europe,” said German leader Angela Merkel when she was defending a 120 billion euro bailout package for Greece. In this paper we propose a new method to identify country specific disturbances that allows us to identify the countries that were the source of spillovers in sovereign debt markets and measure whether connectedness and contagion have changed during the Eurozone debt crisis.

The literature has studied issues of spillovers and contagion using a variety of econometric approaches, which are summarized in Forbes [2012]. Diebold and Yılmaz [2014] suggest measuring connectedness by calculating the importance of an idiosyncratic market specific shock onto other markets using the forecast error variance decomposition (FEVD). The FEVD is typically calculated using the generalized impulse-response functions (GIRFs) of Koop et al. [1996] and Pesaran and Shin [1998] to identify shocks. GIRFs accounts for the correlation of shocks to all markets using the historically observed distribution of the shocks. However, this identification scheme does not provide a causal direction of the spillovers from one market to another. In addition, shocks extracted with this method are not orthogonal.

Because of these shortcomings, we propose a method that allows us to pin down the source of risk and provides a casual interpretation from one market to another market. The method we suggest relies on the assumption that the magnitude of the instantaneous direct effect of the shock is greater than the magnitude of the instantaneous spillover. In particular, we assume that the sovereign yield \( j \)’s shock, \( \varepsilon_j \), is the shock with a higher absolute instantaneous effect on sovereign yield \( j \). For any other sovereign yields \( i \), the effect will be smaller. Letting \( \psi_{ij} \) be the instantaneous response of variable \( i \) to shock \( j \), our identification scheme requires that \( |\psi_{ij}| < |\psi_{jj}| \forall i \neq j \). In addition, we impose orthogonality among the shocks, which provides causal direction of the spillover from one market to another.

This method is closely related to those employed by Kilian and Murphy [2012] and De Graeve.
and Karas 2014. Both studies use sign restrictions with elasticity bounds on the impulse responses to identify shocks. This approach allows them to decrease the admissible set of accepted models using external prior restrictions. We employ a similar idea, because we use bounds on the impact matrix to identify shocks, but we do not impose sign restrictions. Our approach is useful when the modeler is agnostic about the sign of the response functions and the theory provides the information on the bounds. The method seems particularly suited to study spillover/connectedness in the sovereign debt markets during the financial crisis, because sovereign yields have been influenced by both positively (i.e. contagion) and negatively (i.e. flight-to-safety and flight-to liquidity) correlated forces. Thus, sign restrictions will not be useful in identifying shocks to a particular market.

Our identification method has several advantages over GIRF. First, it provides a casual link between markets by imposing orthogonality of shocks. Second, it allows for asymmetric responses between countries. Third, the restrictions we use can be employed together with other a-priori restrictions to improve the size of the standard error of the estimates. By employing Monte Carlo simulations, we show that the median estimate of spillover and connectedness obtained with our procedure is close to the true spillover and connectedness. In general, our method produces more precise estimates of connectedness than alternative identification methods, such as GIRFs or zero restrictions.

Armed with these new tools we study the European and US sovereign debt market over the daily period January 2005-August 2014. We control for global shocks using monetary policy rates in a number of countries. We find that total connectedness declined steadily from 77% in October 2009 to 56% at the beginning of 2013 because connectedness among euro area countries fell. By means of counterfactuals we show that the lower degree of connectedness experienced in sovereign debt markets since 2010 is not due to changes in the size of shocks, but to a fall in cross-market linkages. Cross-market linkages increased after the beginning of the inter-bank credit crisis in August 2007 up to Lehman’ bankruptcy in September 2008. They declined steadily in 2009 and fell sharply since the beginning of the euro area sovereign debt crisis in the autumn of 2009. Cross-market linkages reached the lowest point at the beginning of 2013, remained weak in 2013 and improved substantially in 2014. Hence, the results support the hypothesis that during the sovereign debt crisis bond markets were highly fragmented.
We also find that sovereign debt markets are highly interconnected: none of the 12 sovereign yields we consider are insulated from spillovers coming from other markets. Interestingly enough, over the sample 2005-2014 the euro area sovereign debt market has been relatively more insulated from shocks coming from the UK and the US. However, when investors perceived a change in US monetary policy in May 2013, shocks to US sovereign yields become the most important factor influencing developments in European sovereign yields.

In the first phase of the euro area sovereign debt crisis, between 12% and 35% of the variations of Italian, Spanish, Irish and Portuguese bond yields were due to shocks originating in Greece. Instead, in 2011 and 2012, when the sovereign yields in Italy and Spain reached 6-7%, about 10% of the variations of the Spanish yields were due to shocks originated in Italy, while about 5% of the variations of the Italian yields were due to shocks originated in Spain. Furthermore, the spillovers from Italy to Belgium, Germany, the Netherlands, the UK and the US were much larger. Hence, our analysis confirms that shocks originating in Greece and Italy have contributed to developments in sovereign spreads over the last 5 years.

For comparison, we also examined the conclusions obtained by GIRFs. The results obtained with this method seem counterintuitive. For example, we find that connectedness from Greece decreases in 2009 and 2010. Our results also differ from those in [Claeys and Vašček 2014]: they find that connectedness increased and not decreased during the crisis. The difference can be tracked to the different specification of model - they analyze the spreads between sovereign yields and German Bund and include common factors obtained from those spreads into (FA)VAR model 1.

The remaining sections of the papers are structured as follows. Section 2 describes the methodology and shows the performance of the absolute magnitude restriction method relative to GIRFs using Monte Carlo simulations. Section 3 presents the data and the VAR. Section 4 describes the results and provides some robustness checks. Section 5 concludes.

1It can be shown that including principal component in a VAR may produce biased results. The problem is that the large part of contemporaneous spillovers between countries will be absorbed by the principal component.
2 Econometric methodology

To study spillover and connectedness we follow the approach of Diebold and Yılmaz [2014] and measure spillover and connectedness by means of FEVD. However, where Diebold and Yılmaz [2014] employ the so-called generalized identification scheme to identify shocks, Diebold and Yilmaz [2009] employ zero restrictions, we identify shocks by imposing constraints on the impulse response to shocks.

2.1 SVAR setup

A Structural Vector Autoregression model (SVAR) can be written as:

$$A_0 y_t = A_1 y_{t-1} + A_2 y_{t-1} + \ldots + A_N y_{t-N} + B \varepsilon_t,$$

(2.1)

where $y_t$ is a $k \times 1$ vector of endogenous variables, the structural shocks $\varepsilon_t$ are assumed to be white noise, $N(0, I_k)$ and $p \in [1, 2, \ldots, N]$ is a finite number of lags. $A_0$ describes the contemporaneous relations between the variables, while matrices $A_1, A_2, \ldots, A_N$ describe the dynamic relationships.

The diagonal matrix $B$ contains the standard errors of the structural shocks. The system (2.1) implies the following structural moving average representation:

$$y_t = B(L) \varepsilon_t,$$

where $B(L)$ is a polynomial in the lag operator. The system in (2.1) cannot be estimated directly, but needs to be estimated in its reduced form:

$$y_t = A^*_1 y_{t-1} + A^*_2 y_{t-1} + \ldots + A^*_N y_{t-N} + u_t,$$

(2.2)

where $u_t = A_0^{-1} B \varepsilon_t$ and $A^*_p = A_0^{-1} A_p$ for $p \in [1, 2, \ldots, N]$.

The moving average representation of (2.2) is $y_t = C(L) u_t$. Therefore, the non-structural responses function, $C(L)$, is related to the structural impulse responses function by $B(L) = A_0 C(L)$.

Defining $S = A_0^{-1} B$ and $\Sigma_\varepsilon = I$, it must be the case that $\Sigma_u = SS'$, where $\Sigma_u$ is the variance-covariance matrix of the reduced form errors. The decomposition $\Sigma_u = SS'$ is not unique. For some arbitrary $\hat{S}$, (e.g. a Choleski decomposition), an alternative decomposition can be obtained by post-multiplying $\hat{S}$ by a matrix $H$ such that $HH' = I$. In this case, $\hat{S}H(\hat{S}H)' = \hat{S}\hat{S}'$, and
condition $\Sigma_{u} = SS'$ is still satisfied.

Once a $H$ matrix is selected and shocks identified, one can compute the h-step ahead forecast error:

$$y_{t+h} - E_{t}y_{t+h} = \sum_{\tau=0}^{h-1} C_{\tau} \tilde{S}H \xi_{t+h-\tau}.$$  
(2.3)

Denoting by $\psi_{i,j,h}$ the $i,j$-th element of the orthogonalized impulse response coefficient matrix $C(L)\tilde{S}H$ at horizon $h$, the h-step ahead forecast error variance of variable $i$ is:

$$\varsigma_{i}^{2}(h) = \sum_{\tau=0}^{h-1} (\psi_{i,1,\tau}^{2} + \psi_{i,2,\tau}^{2} + ... + \psi_{i,N,\tau}^{2})$$  
(2.4)

while $(\psi_{i,j,0}^{2} + \psi_{i,j,1}^{2} + ... + \psi_{i,j,h-1}^{2})$ provides the contribution of shock $j$ to the h-step forecast error variance of variable $i$. Hence, the percentage contribution of shock $j$ to the h-step forecast error variance of variable $i$ is:

$$\omega_{i,j}^{2}(h) = \frac{(\psi_{i,j,0}^{2} + \psi_{i,j,1}^{2} + ... + \psi_{i,j,h-1}^{2})}{\varsigma_{i}^{2}(h)}.$$  
(2.5)

### 2.2 Diebold-Yilmaz’s connectedness measures

Diebold and Yılmaz [2014] propose to capture connectedness via FEVD. FEVD is an adjacency matrix that defines a directed weighted network. The relation between adjacency matrix and FEVD is best understood by using a connectedness table (see Table 1).

The upper-left $N \times N$ block contains the FEVD at horizon $h$. The off-diagonal elements describe pairwise directional connectedness measures from market $j$ to market $i$, $C_{i \leftarrow j}^{H} = \omega_{i,j}^{2}(h)$: they are weighted, $\omega_{i,j}^{2}(h) \in [0, 1]$, and directed, $C_{i \leftarrow j}^{H} \neq C_{i \rightarrow j}^{H}$ The diagonal elements define market’s own connectedness, $C_{i \leftrightarrow j}^{H} = \omega_{i,j}^{2}(h)$.

The aggregate connectedness statistics are obtained by taking row and column sums of off-diagonal elements. Total directional connectedness from market $j$ to others is defined as:

$$C_{\bullet \leftarrow j}^{H} = \frac{1}{N-1} \sum_{i=1}^{N} \omega_{i,j}^{2}(h).$$  
(2.6)

2 For more details see Diebold and Yılmaz [2014].

3 In unweighted network $\omega_{i,j}$ is either 1 or 0 - adjacency matrix only specifies whether relation exists or not, but does not specifies strength of relation. In undirected network relations are symmetric, $C_{i \leftrightarrow j}^{H} = C_{j \leftrightarrow i}^{H}$. 

6
Table 1: Connectedness Table

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\cdots$</th>
<th>$\varepsilon_N$</th>
<th>From Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>$\omega_{1,1}^2(h)$</td>
<td>$\omega_{1,2}^2(h)$</td>
<td>$\cdots$</td>
<td>$\omega_{1,N}^2(h)$</td>
<td>$\frac{1}{N-1} \sum_{j=1}^{N} \omega_{1,j}^2(h), j \neq 1$</td>
</tr>
<tr>
<td>$y_2$</td>
<td>$\omega_{2,1}^2(h)$</td>
<td>$\omega_{2,2}^2(h)$</td>
<td>$\cdots$</td>
<td>$\omega_{2,N}^2(h)$</td>
<td>$\frac{1}{N-1} \sum_{j=1}^{N} \omega_{2,j}^2(h), j \neq 2$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\cdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$y_N$</td>
<td>$\omega_{N,1}^2(h)$</td>
<td>$\omega_{N,2}^2(h)$</td>
<td>$\cdots$</td>
<td>$\omega_{N,N}^2(h)$</td>
<td>$\frac{1}{N-1} \sum_{j=1}^{N} \omega_{N,j}^2(h), j \neq N$</td>
</tr>
</tbody>
</table>

| To Others | $\frac{1}{N-1} \sum_{i=1}^{N} \omega_{i,1}^2(h)$ | $\frac{1}{N-1} \sum_{i=1}^{N} \omega_{i,2}^2(h)$ | $\cdots$ | $\frac{1}{N-1} \sum_{i=1}^{N} \omega_{i,N}^2(h)$ | $\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{i,j}^2(h)$ |

i.e $C_{i\to j}^H$ is the sum of the $j$-th column elements of the FEVD except its own share, $\omega_{j,j}^2(h)$. Connectedness ‘to others’ provides the average share of the h-step forecast-error variance explained by shock $j$ and, therefore, it summarizes how important are specific shocks in inducing fluctuations in all other markets. Similarly, by taking sums over rows a total directional connectedness to market $i$ from all shocks can be constructed:

$$C_{i\to \bullet}^H = \frac{1}{N-1} \sum_{j=1}^{N} \omega_{i,j}^2(h), \quad (2.7)$$

$C_{i\to \bullet}^H$ gives the average share of the h-step forecast-error variance of market $i$ coming from shocks originating arising in all other markets. Finally, total connectedness is:

$$C^H = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{i,j}^2(h), \text{ for } i \neq j \quad (2.8)$$

Similarly

$$C_{i\in S}^H = \frac{1}{NS} \sum_{i\in S} \sum_{j\in S} \omega_{i,j}^2(h), \text{ for } i \neq j \quad (2.9)$$

measures total connectedness among a subset $S$ of markets (for example only euro area countries).

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4Compared to Diebold and Yılmaz [2014], we prefer to scale ‘to others’ connectedness by $N - 1$, because this statistic is bounded in the interval $[0, 1]$ and it is easier to interpret. We adopt the same scaling for ‘from others’ connectedness.
2.3 Magnitude restrictions

To compute connectedness, spillovers and contagion measures, it is necessary to identify structural shocks. In order to obtain partial identification of the system, we impose restrictions on the size of contemporaneous spillovers.

Let $\psi_{ij}$ be the instantaneous response of variable $i$ to shock $j$ and $\hat{\psi}_{jj}$ the instantaneous response of variable $j$ to the structural shock $j$. Let

$$\hat{a}_{ij} = \frac{\hat{\psi}_{ij}}{\hat{\psi}_{jj}}$$

(2.10)

Then, we identify the orthogonal structural shock in market $j$, $\varepsilon_j$, by assuming $|\hat{a}_{ij}| < 1 \forall i \neq j$. Intuitively, assumption implies that contemporaneous spillovers to other markets from shock $\varepsilon_j$ are smaller than the direct effect of shock $\varepsilon_j$ on market $j$. Shock $\varepsilon_j$ of size $\hat{\psi}_{jj}$ increases asset price $j$ by $\hat{\psi}_{jj}$ and asset prices $i$ by $|\hat{\psi}_{ij}|$ such that $|\hat{\psi}_{ij}| < |\hat{\psi}_{jj}|$. Hence, all non-diagonal elements of the impact matrix should be smaller than one in absolute value, $|\hat{a}_{ij}| < 1$ for $i \neq j$.

It is very easy to implement such restrictions in principle. Consider the estimate of impulse responses in the first period:

$$\hat{\Psi}(1) = \hat{A}^{-1}_0 \hat{B} = \tilde{S}H,$$

(2.11)

where $\tilde{S}$ is the Cholesky decomposition of the estimated variance-covariance matrix, $\hat{\Sigma}_u$, and $H$ is a orthonormal matrix. For a given $H$ we obtain an estimate of $\hat{A}^{-1}_0$. Then we keep the corresponding estimate of IRFs and FEVD, if the restrictions on the size of spillover effects are satisfied.

2.4 Estimation algorithm

The estimation procedure consists of three steps. In the first step, the reduced form VAR model is estimated. In the second step, the structural shocks are identified taking into account identification uncertainty. In the third step estimation uncertainty is taken into account. The steps are:

1. Estimate reduced-form VAR: Given the number of lags, $\hat{p}$, a $VAR(\hat{p})$ is estimated by Ordinary Least Squares (OLS) to obtain an estimate of autoregressive coefficients $A(L)$.
and of the variance-covariance of reduced form errors, $\hat{\Sigma}_u$.

2. **Identification restrictions:** The non-structural impulse responses function, $C(L)$, is related to the structural impulse responses function via $B(L) = A_0 C(L)$ and reduced form errors, $u_t$, are related to structural errors as $u_t = A_0^{-1} B \tilde{\varepsilon}_t$. The impact matrix, $S = A_0^{-1} B$, must satisfy:

$$\Sigma_u = SS'$$

(2.12)

We distinguish the approach used in small systems and in large ones.

**Small systems:**

(a) The initial estimate of $\hat{S}$ is obtained by a Cholesky decomposition of the variance-covariance matrix of reduced form errors, $\hat{S} = \text{chol}(\hat{\Sigma}_u)$, and the initial estimate of impulse response function is $\hat{B}(L) = \hat{C}(L) \hat{S}$.

(b) The $q \times q$ matrix $P$ drawn from standard normal distribution, $\mathcal{N}(0, 1)$ and the QR decomposition of $P$ is derived. Note that $P = QR$ and $QQ' = I$.

(c) The initial estimate of impulse responses function is post-multiplied by $Q$, $\hat{B}^*(L) = \hat{C}(L) \hat{S}Q$. The candidate impulse responses function $\hat{B}^*(L)$ is retained if it satisfies the restrictions.

(d) The steps 2b-2c are repeated until one candidate impulse responses is retained.

**Large systems:**

(a) The problem is initialized from a random matrix $Q$ that is obtained as explained in steps 2b-2c above.

(b) Consider the following minimization problem:
\[
Q^* = \arg\min_{Q^*} (\hat{A}_0^{-1} - A_0^{*-1})^2
\]
subject to
\[
\hat{A}_0^{-1} = \hat{S}_{ij}/\hat{S}_{jj} \quad \forall \quad j
\]
\[
\hat{S} = \hat{SQ}^*
\]
\[
Q^*Q'^* = I
\]
\[
c(\hat{A}_0^{-1}) \geq 0
\]
\[
c(A_0^{*-1}) \geq 0
\]
\[
\hat{S}S' = \Sigma_u
\]
where \(\hat{A}_0^{-1}\) is estimated matrix of contemporaneous effects. \(c(\hat{A}_0^{-1})\) is a constraint function. In the baseline estimation we use \(c\) such that \(\hat{A}_{0ij}^{-1} < 1 \quad \forall \quad i \neq j\). \(A_0^{*-1}\) is a random matrix of contemporaneous effects that satisfies the constraints, \(\hat{S} = \text{chol}(\Sigma_u)\) and \(Q^*\) is orthonormal matrix.

(c) Candidate impulse responses function is calculated as \(\hat{B}^*(L) = \hat{C}(L)\hat{SQ}^*\) and retained whenever if it satisfies the restrictions.

(d) In case constraints are not satisfied - algorithm does not converge to the feasible solution -, repeat the steps 2a-2c until candidate impulse responses that satisfies the restrictions is found.

Different approaches for small and larger systems are used only because of computational convenience. While random drawing an orthonormal matrix \(Q\) is faster when only small amount of variables is used, numerical optimization becomes necessary when number of variables is large because the probability of obtaining a successful draw by randomly drawing is decreasing with the size of the system.

3. **Estimation uncertainty:** to account for estimation uncertainty, we repeat steps 1-2 1000 times, each time with a new artificially constructed data sample, \(Y^*\). To construct data samples, we use re-sampling of errors. New data sample is constructed recursively as \(y_t^* = \hat{A}_1^*y_{t-1}^* + ... + \hat{A}_N^*y_{t-N}^* + \hat{u}_t^*\), starting from initial values \([y_0, ..., y_{N-1}]\). \(\hat{A}_n^*\) are estimated reduced form autoregressive coefficients. \(\hat{u}_t^*\) are drawn randomly with replacement from estimated reduced form errors, \(\hat{u}_t\).
Point estimates and confidence bands are constructed by retaining the relevant percentiles and the median of a distribution of retained statistics.

2.5 Comparison with alternative identification methods

The advantage of using magnitude restrictions over alternative methods can be appreciated with a simple example. Consider a system with two variables, and let:

\[ A_{0}^{-1} = \begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix} \quad B = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \]

so that

\[ A_{0}^{-1}B = \begin{bmatrix} \sigma_1 & b\sigma_2 \\ c\sigma_1 & \sigma_2 \end{bmatrix} \quad \Sigma_u = \begin{bmatrix} \sigma_1^2 + b^2\sigma_2^2 & c\sigma_1\sigma_2 \\ c\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \]

In case of zero restrictions the estimated impact matrix obtained via cholesky decomposition of variance-covariance matrix when the variables are ordered as \([y_1 \ y_2]\) is:

\[ \hat{A}_{0}^{-1}B = \begin{bmatrix} \sqrt{\sigma_1^2 + b^2\sigma_2^2} & 0 \\ \frac{c\sigma_1\sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} & \sqrt{\sigma_1^2 + \sigma_2^2} - \frac{(c\sigma_1\sigma_2)^2}{\sigma_1^2 + \sigma_2^2} \end{bmatrix} \]

Clearly \(\hat{A}_{0}^{-1}B\) will coincide with the true \(A_{0}^{-1}B\) only when \(b = 0\), namely when there is no instantaneous spillover from market 2 to market 1. If the variables are ordered as \([y_2 \ y_1]\) it can be similarly shown that estimated and true impact matrix will coincide when \(c = 0\).

The GIRFs for the first period are:

\[ \hat{A}_{0}^{-1}B^g = \begin{bmatrix} \frac{\sigma_1^2 + b^2\sigma_2^2}{\sqrt{\sigma_1^2 + \sigma_2^2}} & \frac{c\sigma_1\sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \\ \frac{c\sigma_1\sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} & \frac{\sigma_1^2 + \sigma_2^2 - \frac{(c\sigma_1\sigma_2)^2}{\sigma_1^2 + \sigma_2^2}}{\sigma_1^2 + \sigma_2^2} \end{bmatrix} \]

In this case \(\hat{A}_{0}^{-1}B^g\) will coincide with the true \(A_{0}^{-1}B\) only when \(b = 0\) and \(c = 0\), namely only when there are no instantaneous spillovers between markets. Since in general the off-diagonal elements of \(\hat{A}_{0}^{-1}B^g\) are non-zero, shocks identified with GIRF are not orthogonal.

How big are the errors when measuring spillovers via GIRF? To simplify, assume that \(\sigma_1 = x\sigma_2\), where \(x > 0\). Then the GIRFs is

\[ \hat{A}_{0}^{-1}B^g = \begin{bmatrix} \sigma_1\sqrt{1 + c^2x^2} & \sigma_1b(1 + x^2)\left[\sqrt{b^2 + x^2}\right]^{-1} \\ \sigma_1c(1 + x^2)\left[\sqrt{1 + c^2x^2}\right]^{-1} & \sigma_1\sqrt{b^2 + x^2} \end{bmatrix} \quad (2.14) \]
The true instantaneous impact of $y_2$ on $y_1$ is $b$. The estimated instantaneous impact is $\hat{b} = b(1 + x^2) \left[ b^2 + x^2 \right]^{-1}$. Figure shows the relationship between $b$ and $\hat{b}$. The curved area represents the instantaneous impact obtained by using GIRFs, the transparent grey areas represents the bounds implied by the absolute magnitude restrictions, while the slightly transparent red area represents the true $b$. We can notice that when $b = 0$, also $\hat{b} = 0$.

However, when the standard deviations of the shocks differ substantially, for example when standard deviation of the shock from $y_2$ is ten times that from $y_1$, $x = 0.1$, the estimated instantaneous impact obtained using GIRFs can reach 4, while the true instantaneous impact is close to zero. On the other hand, absolute magnitude restrictions are designed such that the estimated instantaneous impact cannot exceed one in absolute value (see transparent grey area in Figure). Nevertheless, $\hat{b}$ is inside the transparent grey areas for certain parameters, which implies that the results obtained with the two methodologies in some specific cases may not differ considerably.

Figure shows the root square error of the estimated instantaneous impact when using GIRFs (red area) and when using absolute magnitude restrictions (blue area). The difference is most pronounced when the standard deviations of shocks differ substantially.

It is useful to anticipate that in the empirical section we find that the spillover from Italian to Greek sovereign yields is more than one when using GIRFs. The numerical method would suggest that this may be due to the standard deviation of shocks on Greek sovereign yields being considerably greater.

2.6 Monte Carlo simulation

To strengthen the results if the previous subsection, we perform a Monte Carlo simulation. Data for four markets are simulated using the following SVAR:

$$A_m y_t^m = A_m y_{t-1}^m + B_m\varepsilon_t^m,$$

$$y_t^m = (A_0^m)^{-1}A_1^m y_{t-1}^m + (A_0^m)^{-1}B_m\varepsilon_t^m,$$  

(2.15)
where subscript $m$ stands for the fact that we generate multiple models, $m = 1, 2, ..., M$. We use multiple models to simulate a time-varying model, given that a rolling window estimation is carried out in the empirical section.

We model time-varying data generating process by assuming that impact matrices, $A^m_0$, and the standard errors of the shocks, $B^m$, are not constant. In the baseline exercise the non-diagonal entries of $(A^m_0)^{-1}$ are randomly selected from the interval $[-0.99, 0.99]$, while diagonal entries equal one. The diagonal entries of matrix $B^m$ - the standard errors of structural shocks - are also selected randomly from a uniform distribution $U[0.05, 0.45]$. We simulate the data across models assuming that $A^m_1$ remains constant, $A^m_1 = A_1 \forall m$.

We compare the results from our identification scheme with those obtained using GIRFs. The auto-regressive parameters in $A^m_1$ and the variance-covariance matrix, $\Sigma^m$, are estimated by using OLS and the small sample distribution of parameters is obtained by using bootstrap methods.

The performance of identification schemes can be compared by using the root mean squared error (RMSE) of different statistics. The RMSE of IRFs’ estimates is:

$$RMSE(\text{irf}) = \sqrt{\frac{1}{N^2} \frac{1}{1} \sum_{h=1}^{H} \sum_{m=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \Psi_{i,j,h,m} - \bar{\Psi}_{i,j,h,m} \right)^2}$$

(2.16)

where $\Psi_{i,j,h,m}$ is the true IRF of variable $i$ to the shock $j$ for horizon $h$ for the sample $m$ and $\bar{\Psi}_{i,j,h,m}$ is its median estimated counterpart. The RMSE of estimated ‘to others’ connectedness is:

$$RMSE(\text{to others}) = \sqrt{\frac{1}{N} \frac{1}{H} \frac{1}{M} \sum_{j=1}^{N} \sum_{h=1}^{H} \sum_{m=1}^{M} \left( \tilde{C}_{h,m} - \bar{C}_{h,m} \right)^2}$$

(2.17)

where $C_{h,m}$ is to others connectedness for shock $j$ for horizon $h$ obtained from the sample $m$. $\tilde{C}_{h,m}$ is its median estimated counterpart. Finally, the MSE of estimated grand average

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5 In order to better replicate empirical regularities with simulated data, we model the non-diagonal entries of $(A^m_0)^{-1}$ in the following way. Every element $a_{i,j}$ is a sum of three components. The first component serves to replicate the fact that total connectedness is changing over time, therefore we model it as a process going from 0 to 0.49. The second component serves to capture the fact that ‘TO others’ connectedness is volatile over time - we model this by adding country specific shocks drawn from the uniform distribution $U[-0.35, 0.35]$ to every column of matrix $(A^m_0)^{-1}$. The last component is drawn from the uniform distribution $U[-0.15, 0.15]$ and added to all non-diagonal elements and serves as idiosyncratic component.

6 To calculate RMSE we simulate 1000 different models and each model is estimated from 200 samples.
connectedness is:

$$\text{RMSE(\text{grand average})} = \sqrt{\frac{1}{H M} \sum_{h=1}^{H} \sum_{m=1}^{M} \left( \bar{\tilde{C}}_{h,m} - C_{h,m} \right)^2}$$  \hfill (2.18)

where $C_{h,m}$ is total connectedness for horizon $h$ obtained from the sample $m$. $\bar{\tilde{C}}_{h,m}$ is its median estimated counterpart.

Table 2 reports RMSE of magnitude restrictions, GIRF and zero restrictions with random ordering. In the first 10 periods, the RMSE of the IRFs’ estimates is almost 2-times smaller when using magnitude restrictions relative to GIRFs. Magnitude restrictions are also better than zero restrictions using a random ordering that were also employed by Diebold and Yılmaz [2014].

Table 2: The relative root mean squared error (RMSE) of the estimates for different identification methodologies

<table>
<thead>
<tr>
<th>horizon</th>
<th>Magnitude res.</th>
<th>GIRF</th>
<th>Zero res. (random order)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-10</td>
<td>1-10</td>
<td>1-10</td>
</tr>
<tr>
<td>IRFs</td>
<td>0.06 0.05</td>
<td>0.11 0.09</td>
<td>0.09 0.07</td>
</tr>
<tr>
<td>TO connectedness</td>
<td>0.23 0.12</td>
<td>0.42 0.18</td>
<td>0.39 0.21</td>
</tr>
<tr>
<td>Grand Average</td>
<td>0.07 0.09</td>
<td>0.08 0.11</td>
<td>0.08 0.11</td>
</tr>
</tbody>
</table>

Figure 3 shows directional connectedness ‘to’ others from all four markets for first 10 models. Our methodology is performing reasonably well, as the true directional connectedness ‘to’ others is mostly included in the two-standard error bands. The same cannot be said for the estimates obtained using GIRFs: the true directional connectedness ‘to’ others is mostly outside the two-standard error bands (see Figure 4). The RMSE of estimated directional connectedness ‘to’ others is around 50 percent smaller when using magnitude restrictions as compared to GIRFs (see Table 2).

Estimates of total connectedness differ little across methodologies: estimates obtained using magnitude restrictions are still closer to the true total connectedness (see Figures 5 and 6), but in terms of RMSE, they are now less than 20 percent more precise.

It is also interesting to explore how magnitude restrictions would perform when estimates

\footnotetext[7]{We show only first 10 models due to presentational reasons. On request we can provide same figures for other models.}
obtained via GIRF and the true model coincide. In Section 2.5 we show that theoretically they coincide only when there are no contemporaneous spillovers between markets. To see how magnitude restrictions perform in case of no spillovers, we repeat the baseline exercise but now the non-diagonal entries of \((A_m^n)^{-1}\) are set to zero, while diagonal entries still equal one. Table 3 reports RMSE of magnitude restrictions, GIRF and zero restrictions with random ordering. Clearly, estimates obtained by GIRF and zero restrictions are more precise as the errors are only due to small sample estimation errors while identification restrictions coincide with the theoretical model. Magnitude restrictions perform reasonably well. The RMSE of the IRFs’ estimates (0.01) and of the estimated directional connectedness ‘to’ others (0.06) is smaller than in general case. The estimates of total connectedness obtained with magnitude restrictions are however biased when actual total connectedness is zero.

Table 3: The root mean squared error (RMSE) of the estimates for different identification methodologies - model with no spillovers

<table>
<thead>
<tr>
<th>horizon</th>
<th>Magnitude res.</th>
<th>GIRF</th>
<th>Zero res. (random order)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>1-10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>TO connectedness</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>Grand Average</td>
<td>0.21</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>

To summarize, whenever we care only about total connectedness it does not really matter which identification approach is used. However, whenever we are interested in less aggregate statistics, such as bilateral relations between countries or the importance of specific shock, using magnitude restrictions produces more precise estimates.

3 Data and specification of the VAR

The measures of spillovers and connectedness we consider depend on the set of variables \(x\), whose connectedness is to be examined, on the predictive horizon \(H\), on the dynamics \(A(L)\) and on the VAR model adopted.

*To understand this results, set \(\sigma_1 = \sigma_2 = 1\) in example from Section 2.5. It can be shown that when \(b = c = 0\), there are at least two candidate impact matrices that are consistent with magnitude restrictions. First, when \(\hat{b} = \hat{c} = 0\) and second when \(\hat{b} = -\hat{c}\). In the second case, the estimate of connectedness is not equal to zero implying the estimate of connectedness is biased when true model is such that \(b = c = 0\).
Because of tensions emerging during the euro area sovereign debt crisis and the political discussion on whether sovereign yields were affected primarily by credit risk or whether spillover and contagion has also played a key role we apply our method to study the transmission of the shock in the sovereign debt market in the main ten euro area countries (Austria, Belgium, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain), the United Kingdom (UK) and United States (US). Because monetary policy has an important role in shaping the yield curves in all countries, we also include in the analysis the policy rates of the euro area, the UK and the US. This way a sovereign yield shock can be distinguished from a monetary policy shock.

The sovereign yields are the 10-year benchmark rates provided by Reuters. The monetary policy rates are the 3-month Overnight Indexed Swap (OIS) rates provided by Reuters for the Euro area and the US and Thomson DataStream for the UK. An OIS uses an overnight rate index, such as the EONIA for euro-denominated products, the SONIA for sterling-denominated products or the Federal Funds Rate for US dollar-denominated products. At short maturity, the OIS rates are good indicators of the monetary policy stance. The frequency is daily from January 2005 to August 2014. Policy rates are identified with zero restrictions and are treated as a ‘fast moving’ variables to absorb common effects on bond markets.

The predictive horizon \( H \) is important because spillovers and connectedness measures are time dependent. Following Diebold and Yılmaz [2014] we focus on a medium-run horizon of \( H = 12 \) days, although in online appendix we also present results on a short-run horizon of \( H = 2 \) days.

To provide a sense of the time variation present in the data, we follow Diebold and Yılmaz [2014] and estimate the VAR using a rolling window of 200 days. VAR is estimated in levels with a constant and the optimal lag length of the VAR is selected using the Bayesian information criterion (BIC).

4 Results

To identify the shocks, which are specific to each market, we assume that (i) the instantaneous response of the sovereign yield in country \( j \) is larger in absolute value than the instantaneous
response of sovereign yields in country \( i \) and (ii) monetary policy rates do not react contemporaneously to sovereign yield shocks, while monetary policy shocks affect contemporaneously sovereign yields. Therefore, we assume that short-term monetary policy rates are one of the driving forces of sovereign yields.

We proceed in two steps. In Section 4.1, we show the full-sample (static) connectedness analysis, which sets the stage for Section 4.2, where we carry out the rolling-sample (dynamic) analysis. Appendix B provides the detailed results for all sovereign yields and monetary policy rates under investigation.

### 4.1 Full-sample analysis

The connectedness table for the full-sample appears as Table 4. The first row of each market denotes the pairwise directional connectedness obtained using the magnitude restriction method. The second row shows the pairwise directional connectedness obtained using GIRFs.

The diagonal element (own connectedness) tends to be the largest individual elements of the table. Total connectedness amounts to 51%. This means that on average, 51% of the variance of sovereign yields in the sample can be explained by shocks to foreign sovereign yields and monetary policy rates.

Some blocks of high pairwise directional connectedness exist, especially from German yields to sovereign yields of Austria (15%), the Netherlands (27%), France (17%), the UK (24%) and the US (17%). Note that Austrian (14%) and Dutch (14%) sovereign yields do have an important influence on the German Bund. In general, the sovereign markets of Germany, Austria and the Netherlands are highly inter-connected, and the same is true for the markets of Italy and Spain and France and Belgium.

It is useful to point out that UK and US sovereign yields have limited explanatory power in the developments of the euro area sovereign yields. Moreover, the column average of all pairwise connectedness measures corresponding to UK and US sovereign yields indicates that total directional connectedness to others is only 2%. The sovereign yield with the largest influence on other markets is the German Bund (7% on average).
4.2 Rolling-sample analysis

We start the discussion by presenting in Figure 7 the estimated standard deviation of the shocks to sovereign yields for each 200-day rolling-sample windows. According to our method the shocks to yields on sovereign bonds issued by Germany, Austria, the Netherlands, Belgium, France, the UK and the US were rather stable and relatively marginal. Conversely, the shocks originating in Greece, Portugal, Ireland, Italy and Spain were at times very large.

[Insert Figure 7 here]

Overall, the dynamics of shocks obtained with our method are in line with anecdotal evidence. For example, at their peak, the size of the shocks to sovereign yields is very large in Greece, followed by Portugal, Ireland, Italy and Spain. In addition, the dynamics of the shocks...
matches the conventional wisdom of shocks being large in Greece in 2010 (resulting in the EU Commission/ECB/IMF program) and 2012 (due to the credit event on Greek sovereign debt) and in Italy and Spain in 2011 and 2012 (due to fears of euro are break-up). However, our method also shows a rising trend for all euro area shocks initiated with the crisis in inter-bank market in August 2007, and followed by shocks to the UK and US sovereign yields at the end of 2007.

4.2.1 Total connectedness

Total connectedness is plotted in Figures 8 and 9. Figure 8 is constructed using all shocks to sovereign yields and monetary policy rates and, hence, shows the overall degree of connectedness among short-term money market rates and long-term sovereign yields. Figure 9 is constructed using all shocks to sovereign yields only; therefore, it measures the degree of connectedness among long-term sovereign yields.

Over the period January 2005 and September 2009, total connectedness among sovereign yields was very volatile and ranged between 60% and 78% according to our suggested method. Total connectedness declined steadily from 77% in October 2009 to 56% at the beginning of 2013. Then, it steadily increased reaching the pre-crisis levels in 2014.

This dynamics reflects developments in the euro area sovereign debt market. In fact, as Figure 10 shows, total connectedness among euro area sovereign yields is due to shocks to euro area sovereign yields only. Total cross-border connectedness among euro area countries declined steadily since October 2009. The negative trend in connectedness rose from the beginning of 2011 to the summer of 2012, when the fears of the break-up of the euro area exacerbated the financial crisis and the policymakers pointed to financial fragmentation as one of the main cause for the impairment of the transmission mechanism of the monetary policy. Connectedness among euro area countries steadily increased since the beginning of 2013. The degree of connectedness in 2014 is similar to the level recorded before the financial crisis, suggesting that financial fragmentation in the euro area sovereign debt market is no longer a key issue.

[Insert Figures 8, 9 here]
4.2.2 Total directional connectedness

Total directional connectedness among the Euro area, the UK and US is plotted in Figures 11 and Figure 12. The former figure combines region specific shocks to sovereign yields and monetary policy rates, the latter figure focuses on the effects of shocks to sovereign yields only. The plots on the diagonal provide the effects of its own shock on the domestic markets. This explains why the results from Euro area to itself reported in Figure 14 differ from those reported in Figure 12, where only the cross-border effects are taken into consideration.

The results are revealing. First of all, during the sample period 2005-2014, euro area as a whole is relatively insulated from foreign developments, given that about 80% of the variance of the euro area yields can be explained by shocks originated within the euro area. The US has been affecting the euro area relatively more before the start of the inter-bank market crisis in 2006, in the first half of 2007 and soon after at the end of 2007; during this period shocks from the US explained about 20-25% of euro area sovereign yields. Shocks from the UK can explain 5 to 10% of the variance of the euro area sovereign yields, except in the period just after Lehman bankruptcy, when they explain 20% of the variance of euro area sovereign yields. Second, UK and US domestic shocks affected their own economies in a range between 10 and 40% over all samples, while sovereign yields of both countries are highly affected by shocks originated in the euro area. Interestingly, Figure 12 suggests that shocks to euro area sovereign yields, rather than monetary policy shocks, are the driving force of shocks to all economies sovereign yields.

Figure 13 shows the estimated total directional connectedness among countries sovereign yields due to shocks to sovereign yields stemming from specific countries. Each country’s bond yield contributes to the variance of foreign sovereign yields by about 4-5% on average. However, there are specific developments in some periods with larger effects that are in line with conventional wisdom. For example, connectedness from Greece in the spring of 2010 doubled from 5% to 11%. Or after the Deauville agreement upon Private Sector Involvement on 18 October 2010, when it was agreed that private investors would share the burden of future defaults with the taxpayer, the sovereign yields in the stressed euro area countries started to increase and the shock identification reveals Ireland to be the source of spillover, as connectedness from this
market rose on average from 4% to 10%. Similarly, when the risk of euro area break-up unfolded in 2011 and 2012, connectedness from Italy rose from 3% in the spring 2011 to 7% in the first half of 2012. Connectedness from Italy increased further to 8% in the autumn of 2012 after the whatever it takes speech by Mario Draghi on 26 July 2012 and the launch in September of the Eurosystem’s Outright Monetary Transactions (OMTs) in secondary sovereign bond markets to protect the euro area from collapse. Connectedness after Draghi’s speech was a desired outcome as sovereign yields started a steady decline. On the other hand, connectedness from Spain has been rather stable rising only marginally since the beginning of the euro area sovereign debt crisis. This suggests that Italy and not Spain is a key source of systemic risk in the sovereign debt markets.

Finally, it is useful to point out the increase in connectedness from the US in March 2009 and May 2013. In March 2009, the Federal Open Market Committee (FOMC) announced it would purchase USD 300 billion in long-term Treasuries, which was subsequently expanded. The average spillover effect from the US rose from 4% in the spring to 8% in the summer 2010. Since the May 2013 FOMC announcement released on 22 May 2013, which financial markets perceived as the beginning of the end of accommodative monetary policies in the US, connectedness from the US rose sharply from 3% in May 2013 to 10% in March 2014. The time variation of shocks is also in this case in line with conventional wisdom. The next section address these developments more in detail looking at pairwise directional connectedness.

4.2.3 Pairwise directional connectedness

Pairwise directional connectedness measures the effect of a specific shock on a specific market. Given that we consider 15 markets, presenting such plots for each of the 210 pairwise directional measures is not feasible. Therefore, we present some relevant case studies and look at shocks originated in the US, Greece, Italy and Spain.

Figure 14 shows connectedness from the US sovereign yields to the sovereign yields of other countries. Before Lehman’s bankruptcy about 10% of the variance of the US Treasury yields is explained by own shocks; thereafter, the share rose to 17%. As discussed in the previous
section, there is an increase in connectedness in March 2009 and May 2013. The sovereign yields most affected are those of the UK and Germany, followed by Austria, the Netherlands, Belgium and France. The stressed countries (Greece, Ireland, Portugal, Italy and Spain) were mildly influenced. Therefore, pairwise directional connectedness helps us to understand the degree of interlinkages between the groups of stressed and non-stressed countries.

A key case study is Greece. Figure 15 shows connectedness from the Greek sovereign yields to the sovereign yields of all other markets. Before the sovereign debt crisis, a Greek shock affected its own sovereign yield marginally, because the shock was relatively small (see Figure 7). Thereafter, the shocks become large and explain 40% of the variance of the Greek sovereign yields in 2010 and about 70% in 2013. The sovereign yields most affected by the developments in Greece are in order of magnitude Portugal, Ireland, Italy and Spain and this is also in line with conventional wisdom. Shocks stemming from Greek sovereign yields in 2010 explain 30% of the variance of Portuguese yields, 25% of the variance of Irish yields, 15% of the variance of Italian yields and 10% of the variance of Spanish yields. Interestingly, also the German Bund was affected: with the intensification of the crisis, international investors become more risk adverse and demanded more liquid and relatively saver assets. After the 2010 peak, Greek shocks did not spill over the sovereign yields of the other stressed countries, except in March 2012 when Greece declared a credit event. In 2013, sovereign yields in Greece declined sharply. Again, the countries positively affected by these developments were the stressed countries: Portugal (30%), Italy (20%), Spain (15%) and Ireland (15%). There is a specific period in the first half of 2013 with the peak in March 2013 when the Greek shocks influenced also non-stressed economies, such as the UK (23%) and the US (19%). During that period, the Greek sovereign yields declined from 23% in September 2012 to 10% in March 2013. This large shock contributed to the decline in yields in the stress countries as well as in France, Belgium and Austria and to an increase in yields by few basis points in Germany, the Netherlands, the UK and the US, due to portfolio reallocation as international investors risk aversion receded.

Other interesting case studies are Italy and Spain. In 2011 and 2012 the shocks from Italy (see Figure 16) spilled over the sovereign yields in other countries, contributing to their rise in Spain (16% at peak) and due to portfolio reallocation their decline in Belgium (9% at peak), Germany (8% at peak), the Netherlands (7% at peak), the UK (9% at peak) and the US (7% at peak).
The whatever-it-takes speech reversed the dynamics of Italian sovereign yields. They started to
decline fast contributing to the decline in Spanish sovereign yields, but with the opposite effect on
the UK and the US Treasuries. Conversely, the shocks stemming from Spanish sovereign yields
have affected its own sovereign market and, only to a limited extent, the developments in Italian
sovereign yields (see Figure 17). All other sovereigns did not record a significant change in the
contribution of Spanish sovereign yields shocks. Thus, Italian and not the Spanish sovereign
debt markets are the source of systemic risk in euro area bond markets.

[Insert Figures 14-17 here]

4.2.4 Spillover effects

The key input for the analysis carried out in the previous sections are the IRFs. Given that we
consider 15 markets and 46 rolling windows, presenting and discussing 9660 IRFs is impractical.
Here, we present a sub-set of IRFs between the sovereign yields of the key countries (Greece,
Italy, Germany, UK and US) over the two specific sovereign crisis periods: the October 2009-July
2010 period and the May 2011- April 2012 period.

The results based on our method suggest that Greece plaid a key role in 2010 driving up
the sovereign yields in stress countries and down those of non-stressed countries including the
United States (see Figure xxx)). Instead Italy became a source of risk in the second phase of the
crisis, when financial markets were pricing the risk of euro area break-up. For example, during
the May 2011- April 2012 period, according to our method, a shock amounting to 100 basis
points in Italian sovereign yields implied an increase in Spanish sovereign yields equal to 55 basis
points (non reported), and at peak after 20 business days an increase in Greek (Portuguese)
sovereign yields by 300 (40, non reported) basis points. At the same time, flight-to-liquidity and
flight-to-safety phenomena drove down by 15 basis points the German Bund, the British Guilt
and the US Treasury.

It is also interesting to note that US shocks have clearly spilled over to Germany in 2010,
but not in 2011-2012, when the Euro area sovereign debt crisis exacerbated, suggesting that the
time-varying analysis of such shocks is relevant.

[Insert Figure 18 here]
4.3 Network concepts

Network connectedness, spillover and contagion are sometimes used interchangeably. For this reason, we provide precise definitions of these concepts in this subsection.

- **Connectedness:**
  \[
  \Omega(h) = f(\Phi(A_1, ..., A_N), A_{0}^{-1}, B)
  \]
  Connectedness is constructed using the Forecast Error Variance Decomposition (FEVD), which is a non-linear function of dynamic effects of shocks, \( \Phi(A_1, ..., A_N) \), of the matrix of contemporaneous effects, \( A_{0}^{-1} \), and of the size of the shocks, \( B \).

- **Spillovers:**
  \[
  \Psi(h) = \Phi(A_1, ..., A_N)A_{0}^{-1}B
  \]
  Spillovers are measures using the impulse response function (IRF), which is a non-linear function of dynamic effects, \( \Phi(A_1, ..., A_N) \), of the matrix of contemporaneous effects, \( A_{0}^{-1} \), and of the size of the shocks, \( B \). While connectedness is always positive, spillovers can be positive or negative.

- **Contagion:**
  \[
  \Upsilon(h) = \Phi(A_1, ..., A_N)A_{0}^{-1}
  \]
  As in Forbes and Rigobon [2002], contagion is related to the structure of economy and thus is captured using the dynamic effects, \( \Phi(A_1, ..., A_N) \), and the matrix of contemporaneous effects, \( A_{0}^{-1} \). Note that contagion is nothing more than spillovers when the size of each of the shocks is normalized to 1.

4.4 What drives changes in connectedness

In subsection 4.3 we have defined three different network concepts - spillover, contagion and connectedness. We can decompose connectedness, \( \Omega(h) = f((A_1, ..., A_N), A_{0}^{-1}, B) \), on two parts:

- Due to the size of the shocks: \( B \)
- Due to contagion: \( \Upsilon(h) = \Phi(A_1, ..., A_N)A_{0}^{-1} \)
Contagion changes whenever the matrix of contemporaneous effects, $A_0^{-1}$, or dynamic effects, $\Phi(A_1, \ldots, A_N)$, change. Consider example when Portuguese bond markets become contemporaneously more vulnerable to contagion from Greece - this implies the element of matrix $A_0^{-1}$ that describes the transmission of Greek shocks to Portuguese bond yields increases. Clearly, connectedness from Greece to Portugal increases. Connectedness can also increase due to increase of the size of the shock. Taking previous example, connectedness from Greece to Portugal increases whenever the standard deviation of Greek shocks increases, keeping contagion and other shocks fixed.

In this subsection we ask how much of the time variations in connectedness are due to the changes in contagion and how much due to changes in the distribution of the shocks. Define $\Omega(h)^w$ to be the estimate of total connectedness in sample $w$. Take pre-crisis estimates of dynamic effects, $\Phi(A_1^P, \ldots, A_N^P)$, the matrix of contemporaneous effects, $A_0^{-1P}$, and the size of the shocks, $B^P$, where superscript $P$ stands for the estimates from pre-crisis sample. We can then construct the three measures of connectedness:

- Fixed shocks: $\Omega(h)^w_{B^P} = f(\Upsilon^w, B^P)$
- Fixed contagion: $\Omega(h)^w_{\Upsilon^P} = f(\Upsilon^P, B^w)$
- Final estimate: $\Omega(h)^w = f(\Upsilon^w, B^w)$

To construct $\Omega(h)^w_{B^P}$ we fix the distribution of the shocks to the estimate from the base period $P$ (pre-crisis), $B^w = B^P \forall w$. More specifically, we fix $B^w$ to the estimate obtained in the first sample in rolling window estimation that uses data from 3rd January 2005 to 7th October 2005, $B^w = B^1 \forall w$. The change of connectedness in this case is only due to the change in contagion. To construct $\Omega(h)^w_{\Upsilon^P}$ we fix contagion to the estimate from the base period $P$, $\Upsilon^w = \Upsilon^P \forall w$. The change of connectedness in this case is only due to the change in the distribution of the shocks. Finally, we can compare those estimates with baseline connectedness measures that takes into account both changes, in contagion and in the distribution of shocks, $\Omega(h)^w$.

It can be shown formally that connectedness from country $j$ to country $i$ increases if contagion from country $j$ to country $i$ increases and if the standard deviation of shock from country $j$ increases.

[Insert Figure 19 here]
Figure 19 applies the decomposition to total connectedness among sovereign yields due to shocks to sovereign yields. The black line is median estimate of total connectedness and therefore is the same as in Figure 9.

The red line is the median estimate of total connectedness when the distribution of shocks is fixed to pre-crisis level. The drop in total connectedness that we can observe implies changes in contagion were a driving force for a fall in baseline total connectedness. Since total connectedness declines considerably there are large changes in cross-market linkages - market fragmentation is especially evident at the start of Eurozone debt crisis in 2009 and in 2012 with the intensification of debt crisis.

The green line is the median estimate of total connectedness when contagion is fixed to pre-crisis level. Note that with fixed contagion total connectedness increases after 2009 and not decreases as baseline total connectedness. The changes in the size of the shocks would therefore imply large increase in connectedness if the cross-market linkages were the same as in pre-crisis time. For example, with fixed transmission mechanism from Greece to other countries other sovereign yields would be greatly affected by Greek shocks given that the standard deviation of Greek shocks increased by a factor of 161.

In conclusion, the lower degree of connectedness in sovereign debt markets since 2010 is not due to changes in the size of shocks, but to the fall in cross-market linkages.

In conclusion, the lower degree of connectedness in sovereign debt markets since 2010 is not due to changes in the size of shocks, but to the fall in cross-market linkages.

4.5 What does traditional identification tell us about sovereign bond markets linkages?

The key issue of using GIRFs is that shocks are not orthogonal. Therefore, they might be incorrect. For example, during the October 2009-July 2010 period, sovereign yield shocks stemming from Italy on Greek sovereign yields turn out to be positive using GIRFs and negative using our method. The impact of GIRFs is sometime even larger than unity: during the October 2009-July 2010 period, an Italian shock amounting to 100 basis points identified using GIRFs implies an average increase by 400 basis points in Greek sovereign yields, by 210 basis points in Portuguese sovereign yields and by 150 basis points in Irish sovereign yields. This is not realistic given that
Greece is believed to be the source of the crisis during this period.

The shocks estimated using GIRFs always overestimate the size of the shocks using our method. Moreover, in the period before the start of the crisis in the inter-bank market in August 2007, the size of the shocks identified using GIRFs exhibited a clear declining trend in Germany, Austria, the Netherlands, Belgium, France, Italy and Spain. Also during the periods preceding both the Lehman’s bankruptcy in September 2008 and the euro area sovereign debt crisis in October 2009, the estimated shocks on the mentioned benchmark sovereign yields presented trends that were steeper than those suggested by our method.

This mistake is then reflected in all the relevant measures of connectedness.

The results obtained using absolute magnitude restrictions and GIRFs differ substantially if connectedness is generated by shocks to long-term sovereign yields only, before the beginning of the sovereign debt crisis in October 2009 due to the sizable influence of monetary policy rates on sovereign yields that we are able with our method to disentangle from sovereign yield shocks (see Figure 9). Since October 2009, the two methods produced very similar results owing to the relatively marginal impact that the short-term monetary policy rates have had on sovereign yields, given that they reached very low levels.

The results obtained using absolute magnitude restrictions and GIRFs are generally different when looking at total directional connectedness (see Figure 13). For example, connectedness estimated using GIRFs from Greece declined in 2009 and 2010, in contrast with conventional wisdom.

5 Conclusion

Structural shock identification is a key issue in macroeconomics and a challenging topic in finance, because asset prices move very sharply and simultaneously. Diebold and Yılmaz [2014] use the GIRF method to address spillover/connectedness. However, it is a well known fact that shocks identified using GIRFs are not orthogonal, because contemporaneous correlations remain an unaddressed issue. This is very relevant if one wishes to study the source of the spillover.

We propose a new method, which identifies structural shocks in asset prices, based on the relative size of its average contemporaneous impact over the sample period. The spillover from
variable $j$ to variable $i$ is identified, if the average impact response over the sample period is in absolute value lower than the average impact response to variable $j$. The method imposes bounds on the impact matrix, but it remains agnostic about the sign.

First we show analytically that the GIRF method does not provide a structural identification and using Monte Carlo we can show that the mistake can be relevant, when using GIRF. Conversely, the absolute magnitude restriction method is much closer to the true value.

Second, we apply the method to the US and European sovereign yield markets. We find that (i) the shocks estimated using GIRFs always overestimate the size of the shocks using our method, (ii) the shocks estimated using GIRFs may present different trends that those suggested by our method in normal times, (iii) the shocks estimated using GIRFs may present more accelerated trends than those suggested by our method in crisis times.

We find that total cross-border connectedness among euro area countries declined steadily between October 2009 and December 2012 in line with the policymakers view that financial fragmentation was one of the main causes for the impairment of the transmission mechanism of the monetary policy. Connectedness among euro area countries improved in 2013 and reached in 2014 the level recorded before the financial crisis, suggesting that financial fragmentation in the euro area sovereign debt market has been no longer the key issue.

We also find that, during the sample period 2005-2014, euro area as a whole was relatively much more insulated from foreign developments, given that about 80% of the variance of the euro area yields can be explained by shocks originated within the euro area. However, the spillover is a dynamic phenomenon. For example, in May 2013 when financial markets perceived a potential change in the US monetary policy, US sovereign yields rose influencing the developments in sovereign yields in other countries.

The absolute magnitude restriction method can pin down the source of the shock. For example, shocks stemming from Greek sovereign yields in the first half of 2010 could explain 30% of the variance of Portuguese yields, 25% of the variance of Irish yields, 15% of the variance of Italian yields and 10% of the variance of Spanish yields. Similarly, focusing on the 2011-2012 period when financial markets were pricing the risk of break-up of the euro area, our method suggests that Italy and not Spain was a key source of systemic risk in the sovereign debt markets.
References


A Appendix - Absolute magnitude restrictions vs. GIRF

Figure 1: The estimated and the true spillover

The colored area present the estimated spillover, \( \hat{b} \), obtained by GIRF. The red area area is the true spillover coefficient, \( b \). The gray bands represent the absolute magnitude restrictions on the interval \((-1, 1)\).

Figure 2: The root squared error of estimated spillover by GIRF (red) and magnitude res. (blue)

The red area present the graph presents the root squared error of estimated spillover (RSE) by GIRF, while blue area presents the RSE of estimated spillover by absolute magnitude restrictions.
B Monte Carlo results

Figure 3: Estimated directional connectedness ‘to’ others by using absolute magnitude restrictions

The graph presents the estimated directional connectedness ‘to’ others for simulated markets. The red line is median estimate and the gray bands are 95% error bands. Black line with circles is the true directional connectedness ‘to’ others.

Figure 4: Estimated directional connectedness ‘to’ others by using generalized IRFs

The graph presents the estimated directional connectedness ‘to’ others for simulated markets. The red line is median estimate and the gray bands are 95% error bands. Black line with circles is the true directional connectedness ‘to’ others.
Figure 5: Estimated total connectedness by using absolute magnitude restrictions

The graph presents the estimated total connectedness for simulated countries. The red line is median estimate and the gray bands are 95% error bands. Black line with circles is the true directional connectedness 'to' others.

Figure 6: Estimated total connectedness by using generalized IRFs

The graph presents the estimated total connectedness for simulated countries. The red line is median estimate and the gray bands are 95% error bands. Black line with circles is the true directional connectedness 'to' others.
C Empirical results

Figure 7: The estimated standard deviation of the shocks to sovereign yields

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. The vertical axis shows the standard deviation in percentage points. The first vertical bar denotes the beginning of the crisis in the interbank market on 9 August 2007. The second vertical bar denotes Lehman’s bankruptcy on 15 September 2008. The third vertical bar denotes the parliamentary speech by the new Greek Prime Minister disclosing the budget situation in Greece on 16 October 2009. The fourth vertical bar denotes the Deauville agreement upon Private Sector Involvement on 18 October 2010. The fifth vertical line denotes the launch of the 3-year LTROs on 8 December 2011. The sixth vertical line denotes the Draghi speech in London on 26 July 2012.
Figure 8: Estimated total connectedness among sovereign yields and monetary policy rates

![Chart](image1.png)

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.

Figure 9: Estimated total connectedness among sovereign yields due to shocks to sovereign yields

![Chart](image2.png)

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.

Figure 10: Estimated total connectedness among euro area sovereign yields due to shocks to euro area sovereign yields

![Chart](image3.png)

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.
Figure 11: Total connectedness among regions sovereign yields due to shocks to sovereign yields and monetary policy rates

![Graphs showing total connectedness among regions sovereign yields due to shocks to sovereign yields and monetary policy rates. The blue line is the median estimate and the shaded area are 68% error bands. The red line is the median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.](image)

Figure 12: Total connectedness among regions sovereign yields due to shocks to sovereign yields

![Graphs showing total connectedness among regions sovereign yields due to shocks to sovereign yields. The blue line is the median estimate and the shaded area are 68% error bands. The red line is the median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.](image)
Figure 13: Total connectedness among countries sovereign yields due to shocks to sovereign yields

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.

Figure 14: Estimated pairwise directional connectedness to countries sovereign yields due to shocks to US sovereign yields

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.
Figure 15: Estimated pairwise directional connectedness to countries sovereign yields due to shocks to Greek sovereign yields

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.

Figure 16: Estimated pairwise directional connectedness to countries sovereign yields due to shocks to Italian sovereign yields

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.
Figure 17: Estimated pairwise directional connectedness to countries sovereign yields due to shocks to Spanish sovereign yields

The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF. For the description of dates marked by vertical bars see footnote in Figure 7.
The blue line is median estimate and the shaded area are 68% error bands. The red line is median estimate obtained by GIRF.


**D  Contagion vs. the size of shocks**

Figure 19: Decomposition of total connectedness among sovereign yields

The black line is median estimate of total connectedness (grand average) for Eurozone countries and the shaded area are 68% error bands. The red line is median estimate of total connectedness (grand average) and red shaded area corresponding 68% error bands when the distribution of shocks is fixed to pre-crisis level. The green line is median estimate of total connectedness (grand average) and green shaded area corresponding 68% error bands when the contagion is fixed to pre-crisis level. We fix the distribution of shocks and contagion for all sovereign yields.