Accounting for earnings and wealth inequality

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Summary: We show that a theory of earnings and wealth inequality based on the optimal choices of ex-ante identical households who face uninsured idiosyncratic shocks to their endowments of efficiency labor units can account for the earnings and wealth inequality observed in the U.S. almost exactly. Relative to previous work, we make three major changes to the way in which this basic theory is implemented: *(i)* we mix the main features of the dynastic and the life-cycle abstractions, that is we assume that our households are altruistic and that they go through the life-cycle stages of working-age and retirement; *(ii)* we model explicitly some of the quantitative properties of the U.S. social security system; and *(iii)* we calibrate our model economy to the Lorenz curves of U.S. earnings and wealth as reported by the 1992 Survey of Consumer Finances. Moreover, we succeed in doing so in spite of the disincentives created by the mildly progressive U.S. income and estate tax system that is another explicit feature of our model economy.

Keywords: Inequality; Earnings distribution; Wealth distribution; Progressive taxation.

JEL Classification: D31; E62; H23

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1 Introduction

The project. Redistribution of wealth is a central issue in the discussion of economic policy. It is also one of the arguments most frequently used to justify the intervention of the government. In spite of its importance, formal attempts to evaluate the distributional implications of policy have had little success. This is mainly because researchers have failed to come up with a quantitative theory that accounts for the observed earnings and wealth inequality in sufficient detail. The purpose of this paper is to provide such a theory.

The facts. In the U.S. economy the distributions of earnings and especially of wealth are significantly concentrated and skewed to the right. For instance, their Gini indices are 0.63 and 0.78, respectively, and the shares of earnings earned and of wealth owned by the top 1% of the distributions are 15% and 30%, respectively.[1]

The question. In this paper we ask whether we can construct a theory of earnings and wealth inequality based on the optimal choices of ex-ante identical households who face uninsured idiosyncratic shocks to their endowments of efficiency labor units that accounts for the U.S. distributions of earnings and wealth. We find that we can.

Previous answers. Quadrini and Ríos-Rull (1997) review the quantitative attempts to account for earnings and wealth inequality until that date and they show that every paper that studies the decisions of households with identical preferences has serious problems in accounting for the shares of earnings earned and of wealth owned by the households in the tails of both distributions. Later work suffers from milder versions of the same problems: it fails to account both for the extremely long and thin upper tail and for the large number of people with almost zero wealth. These results lead us to conclude that a quantitative theory of earnings and wealth inequality that can be used to evaluate the distributional implications of economic policy is still in the waits. The purpose of this paper is to construct such a theory.

This paper. Our theory of earnings and wealth inequality is based on the optimal choices of households with identical and standard preferences. These households receive an idiosyncratic

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[1] These facts and the points of the Lorenz curves of earnings and wealth reported in Table 2 below have been obtained using data from the 1992 Survey of Consumer Finances (SCF). They are reported in Díaz-Gimenez, Quadrini, and Ríos-Rull (1997) and they are confirmed by many other empirical studies (see, for example, Lillard and Willis (1978), Wolff (1987), and Hurst, Luoh, and Stafford (1998)).

random endowment of efficiency labor units, they do not have access to insurance markets and they save in part to smooth their consumption. Relative to previous work, we make three major changes to the way in which this basic theory is implemented. These changes pertain to the design of our model economy and to our calibration procedure and they are the following:

(i) We mix the main features of both the dynastic and the life-cycle abstractions. More specifically, we assume that our households are altruistic and that they go through the life-cycle stages of working-age and retirement. These features give our households two additional reasons to save —to supplement their retirement pensions and to endow their estates— and they help us to account for the upper tail of the wealth distribution.

(ii) We model explicitly some of the quantitative properties of the U.S. social security system. This feature implies that the income of many of the households in our model economy increases after retirement and it helps us to account for the lower tail of the wealth distribution.

(iii) We calibrate our model economy to the Lorenz curves of U.S. earnings and wealth as reported by the 1992 Survey of Consumer Finances (SCF). We do this instead of measuring the process on earnings directly as is standard in the literature. This feature allows us to obtain a process on earnings that is consistent with both the aggregate and the distributional data on earnings and wealth and it enables the earnings-rich households in our model economy to accumulate sufficiently large amounts of wealth sufficiently fast.

Two additional features that distinguish our model economy from those in the literature are the following: *(i)* we model the labor decision explicitly; and *(ii)* we replicate the progressivity of the U.S. income and estate tax systems. The first of these two features is important because the ultimate goal of the study of inequality is to evaluate the distributional implications of fiscal policy and to do this in models that do not study the labor decision explicitly makes virtually no sense. The second feature is important because progressive income and estate taxation distorts the labor and savings decisions significantly and it discourages the earnings-rich households both from working long hours and from accumulating large quantities of wealth. Therefore, the fact that we succeed in accounting for the observed earnings and wealth inequality in spite of the disincentives created by progressive taxation increases our confidence in the usefulness of our theory.

Finally, we also study the roles played by the life-cycle profile of earnings and by the intergenerational transmission of earnings ability in accounting for earnings and wealth inequality and we evaluate the steady-state implications of abolishing estate taxation.

Findings. We show that our model economy can be calibrated to the main U.S. macroeconomic aggregates, to the U.S. progressive income and estate tax systems, and to the Lorenz curves of both earnings and wealth and we find that there is a four-state Markov process on the endowment of efficiency labor units capable of accounting for the observed distributions of earnings and wealth almost exactly.

We also find that, even though the the roles played by the intergenerational transmission of earnings ability and the life-cycle profile of earnings are quantitatively significant, they are not the key to accounting for the earnings and wealth inequality observed in the U.S.

Finally, as far as the policy experiment of abolishing estate taxation is concerned, we find that the steady-state implications of this policy change are to increase output by 1.0% and the stock of capital by 2.7% and that its distributional implications are very small.

Sectioning. The rest of the paper is organized as follows: in Section 2 we summarize some of the previous attempts to account for earnings and wealth inequality and we justify our modeling choices; in Section 3 we describe our benchmark model economy; in Section 4 we discuss our calibration strategy; in Section 5 we report our findings; in Section 6 we evaluate the steady-state implications of abolishing estate taxation and in Section 7 we offer some concluding comments.

2 Previous literature and the rationale for our modeling choices.

In this section we summarize the findings of Aiyagari (1994), Castañeda, Díaz-Giménez, and Ríos-Rull (1998a), Huggett (1996), Quadrini (1997), Krusell and Smith (1998), De Nardi (1999) and Domeij and Klein (2000).[2] All those papers share the following features: *(i)* they attempt to account for the earnings and wealth inequality observed in the U.S.; *(ii)* they study the decisions of households who face a process on labor earnings that is random, household-specific and non-insurable; and *(iii)* these households accumulate wealth in part to smooth their consumption. We report some of their quantitative findings in Table 1.

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[2] For a detailed discussion of the contributions made in the first four of these papers see Quadrini and Ríos-Rull (1997).

Aiyagari (1994), Castañeda et al. (1998a), Quadrini (1997) and Krusell and Smith (1998) model purely dynastic households. Aiyagari (1994) measures the process on earnings using the Panel Study of Income Dynamics (PSID) and other sources and obtains distributions of earnings and wealth that are too disperse (see the third and fourth rows of Table 1). Castañeda et al. (1998a) partition the population into five household-types subject to type-specific employment processes and they find that permanent earnings differences play a very small role in accounting for wealth inequality. Quadrini (1997) explores the role played by entrepreneurship in accounting for wealth inequality and social mobility and he finds that this role is key. His model economy does not account for the earnings and wealth distributions completely, but it accounts for the fact that the wealth to income ratios of entrepreneurs are significantly higher than those of workers. Finally, Krusell and Smith (1998) use shocks to the time discount rates in their attempt to account for the observed wealth dispersion. This feature distinguishes their work from the rest of the papers discussed in this section —which study the decisions of households with identical preferences— and it allows Krusell and Smith do a fairly good job in accounting for the Gini index and for the share of wealth owned by the households in the top 5% of the wealth distribution (see the ninth and tenth rows of Table 1).

Huggett (1996) studies a purely life-cycle model. He calibrates the process on earnings using different secondary sources and he includes a social security system that pays retirees a lump-sum pension. The Gini Indices of the distributions of earnings and wealth of his model economy are larger than those in most of the other papers discussed in this section, but this is partly because of the very large number of households with negative wealth. Moreover, he also falls short of accounting for the share of wealth owned by the households in the top 5% of the wealth distribution (see the eleventh and twelfth rows of Table 1).

In a recent working paper De Nardi (1999) studies a life-cycle model economy with intergenerational transmission of genes and joy-of-giving bequests. This is a somewhat *ad hoc* way of modeling altruism and it makes her results hard to evaluate. It is hard to tell how much joy-of-giving is appropriate and it is not clear whether her parametrization implies that the agents care more, less or the same for their children than for themselves. With the significant exception of the top 1% of the wealth distribution, she comes reasonably close to accounting for the wealth inequality observed in the U.S. (see the last two rows of Table 1).

Finally, in a very recent working paper Domeij and Klein (2000) study an overlapping generations model without leisure that follows people well into their old age. They find that a generous pension scheme is essential to accounting for the high concentration of wealth.[3] In accordance with Huggett (1996) and with the pure life-cycle tradition, Domeij and Klein also find that the share of wealth owned by the very wealthy households in their model economy is much smaller than in the data. This is because in model economies that abstract from altruism the old have no reasons to save and, consequently, they end up consuming most of their wealth before they die. This brief literature review shows that both purely dynastic and purely life-cycle model economies fail to generate enough savings to account for the wealth inequality observed in the U.S. In purely dynastic models this is mainly because the wealth to earnings ratios of the earnings-rich are too low and those of the earnings-poor are too high.

Table 1: The earnings and wealth distributions of the U.S. and of selected model economies.

	Gini	*Bottom 40%*	*Top 5%*	*Top 1%*
U.S. Economy				
Earnings	0.63	3.2	31.2	14.8
Wealth	0.78	1.7	54.0	29.6
Aiyagari (1994)				
Earnings	0.10	32.5	7.5	6.8
Wealth	0.38	14.9	13.1	3.2
Castañeda et al. (1998)				
Earnings	0.30	20.6	10.1	2.0
Wealth	0.13	32.0	7.9	1.7
Quadrini (1998)				
Earnings	n/a	n/a	n/a	n/a
Wealth	0.74	n/a	45.8	24.9
Krusell and Smith (1998)				
Earnings	n/a	n/a	n/a	n/a
Wealth	0.82	n/a	55.0	24.0
Huggett (1996)				
Earnings	0.42	9.8	22.6	13.6
Wealth	0.74	0.0	33.8	11.1
De Nardi (1999)				
Earnings	n/a	n/a	n/a	n/a
Wealth	0.61	1.0	38.0	15.0

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[3] Unlike the rest of the papers discussed in this section, Domeij and Klein attempt to account for income and wealth inequality in Sweden. Like their U.S. counterparts, the distributions of income and wealth in Sweden are significantly concentrated and skewed to the right.

In purely life-cycle models this is mainly because households have neither the incentives nor the time to accumulate sufficiently large amounts of wealth. To overcome these problems, our model economy includes the main features of both abstractions —namely, retirement and bequests.

Our review of the literature also shows that theories that abstract from social security result in wealth to earnings ratios of the households in the lower tails of the distributions that are too high. To overcome this problem, our model economy includes an explicit pension system that reduces the life-cycle savings of the earnings-poorest.

Another important conclusion that arises from our review of the literature is that attempts to measure the process on earnings directly using sources that are subject to top-coding and other problems misrepresent the income of the highest earners and fail to deliver the distribution of earnings as measured by the SCF. Since in those theories the earnings of highly-productive households are much too small, it is hardly surprising that those households fail to accumulate enough wealth. To overcome this problem, in this paper we use the Lorenz curves of both earnings and wealth to calibrate the process on the endowment of efficiency labor units faced by our model economy households and we find that this procedure allows us to account for the U.S. distributions of earnings and wealth almost exactly.

Finally, in a previous version of this paper (see Castañeda, Díaz-Giménez, and Ríos-Rull (1998b)) we found that progressive income taxes play an important role in accounting for the observed earnings and wealth inequality. Specifically, in that paper we studied two calibrated economies that differed only in the progressivity of their income taxes —in one of them they were constant and in the other one they mimicked the progressivity of U.S. effective rates— and we found that their distributions of wealth differed significantly.[4] We concluded that theories that abstract from the labor decision and from progressive income taxes make it significantly easier for earnings-rich households to accumulate large quantities of wealth. This is because both the after-tax wage and the after-tax rate of return are significantly larger than those observed, and this disparity exaggerates the ability of those models to account for the observed wealth inequality very significantly. To overcome this problem, in our model economy we study the labor decision explicitly and we replicate the progressivity of the U.S. income and estate tax systems.

Summarizing, our literature review leads us to conclude that previous attempts to account

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[4] For example, the steady-state share of wealth owned by the households in the top 1% of the wealth distribution changed from 29.5% to 39.0%; the share owned by those in the bottom 60%, from 3.8% to 0.1%; and the Gini Index, from 0.79 to a startling 0.87.

for the observed earnings and wealth inequality have failed to provide us with a theory in which: *(i)* households have identical and standard preferences; *(ii)* the earnings process is consistent both with the U.S. earnings distribution and with aggregate earnings; and *(iii)* the tax system resembles the U.S. tax system. The main purpose of this paper is to provide such a theory.

3 The model economy

The model economy analyzed in this paper is a modified version of the stochastic neoclassical growth model with uninsured idiosyncratic risk and no aggregate uncertainty. The key features of our model economy are the following: *(i)* it includes a large number of households with identical preferences; *(ii)* the households face an uninsured, household-specific shock to their endowments of efficiency labor units; *(iii)* the households go through the life-cycle stages of working-age and retirement; *(iv)* retired households face a positive probability of dying, and when they die they are replaced by a working-age descendant; and *(v)* the households have altruistic feelings towards their descendants.

3.1 The private sector

3.1.1 Population dynamics and information

We assume that our model economy is inhabited by a continuum of households. The households can either be of working-age or they can be retired. Working-age households face an uninsured idiosyncratic stochastic process that determines the value of their endowment of efficiency labor units. They also face an exogenous and positive probability of retiring. Retired households are endowed with zero efficiency labor units and they face an exogenous and positive probability of dying. When a household dies it is replaced by a working-age descendant who inherits the retired household estate if any and, possibly, some of the deceased household's earning abilities. We use the one-dimensional shock s_t to denote the household's random age and random endowment of efficiency labor units jointly (see Section 3.1.2 below). We assume that the s_t are independent and identically distributed across households and that they follow a finite state Markov chain with conditional transition probabilities given by $\Gamma_{s,s'} = \Gamma(s' \mid s) = Pr\{s_{t+1} = s' \mid s_t = s\}$, where $s, s' \in S = \{1, 2, \ldots, n_s\}$.

3.1.2 Employment opportunities

We assume that every household is endowed with ℓ units of disposable time and that the joint age and endowment shock takes values in one of two possible J–dimensional sets $s \in S = \mathcal{E} \cup \mathcal{R} = \{e_1, \ldots, e_J, \varrho_1, \ldots, \varrho_J\}$. When a household draws shock $e_j \in \mathcal{E}$ we say that it is of working-age and we assume that it is endowed with $e_j > 0$ efficiency labor units. When a household draws shock $\varrho_j \in \mathcal{R}$ we say that it is retired and we assume that is is endowed with zero efficiency labor units. We use the ϱ's to keep track of the realization of e that each household faced during the last period of its working-life. This knowledge is essential to analyze the role played in this class of economies by the intergenerational transmission of earnings ability.

The notation described above allows us to represent every demographic change in our model economy as a transition between the sets $\mathcal{E}$ and $\mathcal{R}$. When a household's shock changes from $s \in \mathcal{E}$ to $s' \in \mathcal{R}$ we say that it has retired and when it changes from $s \in \mathcal{R}$ to $s' \in \mathcal{E}$ we say that it has died and has been replaced by a working-age descendant. Moreover, this specification of the joint age and endowment process implies that matrix Γ controls *(i)* the demographics of the model economy, by determining the expected durations of the households' working-lives and retirements; *(ii)* the life-time persistence of earnings, by determining the mobility of households between the states in $\mathcal{E}$; *(iii)* the life-cycle pattern of earnings, by determining how the endowments of efficiency labor units of new entrants differ from those of senior working-age households; and *(iv)* the intergenerational persistence of earnings, by determining the correlation between the states in $\mathcal{E}$ for consecutive members of the same dynasty. In Section 4 we discuss these issues in detail.

3.1.3 Preferences

We assume that households value their consumption and leisure and that they care about the utility of their descendents as much as they care about their own utility. Consequently, the households' preferences can be described by the following standard expected utility function:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \;\; u(c_t, \ell - l_t) \mid s_0 \right\} \tag{1}$$

where u is continuous and strictly concave; $0 < \beta < 1$ is the time-discount factor; $c_t \geq 0$ is the households' consumption; ℓ is the households' endowment of productive time and l_t is labor. Consequently, $\ell - l_t$ is time allocated by the household to non-market activities.

3.1.4 Production possibilities

We assume that aggregate output, Y_t, depends on aggregate capital, K_t, and on the aggregate labor input, L_t, through a constant returns to scale aggregate production function, $Y_t = f(K_t, L_t)$. Aggregate capital is obtained aggregating the wealth of every household and the aggregate labor input is obtained aggregating the efficiency labor units supplied by every household. We assume that capital depreciates geometrically at a constant rate δ.

3.1.5 Transmission and liquidation of wealth

We assume that in the first period of its working-life every household inherits the estate of the previous member of its dynasty. Specifically, we assume that when a retired household dies, it does so at the end of the period after consumption and savings have taken place. At the beginning of the following period the deceased household's estate is liquidated and a fraction $1 - \tau_E(z_t)$ is inherited by the deceased household's descendant. The remaining wealth is instantaneously and costlessly transformed into the current period consumption good and it is taxed away by the government. Note that variable z_t denotes the value of the household's stock of wealth at the end of period t.

3.2 The government sector

We assume that the government in our model economies taxes households' income and estates and that it uses the proceeds of taxation to make real transfers to retired households and to finance government consumption. We assume that income taxes are described by function $\tau(y_t)$, where y_t denotes household income; that estate taxes are described by function $\tau_E(z_t)$; and that public transfers are described by function $\omega(s_t)$. Therefore, in our model economies a government policy rule is a specification of $\{\tau(y_t),\ \tau_E(z_t),\ \omega(s_t)\}$ and of a process on government consumption, $\{G_t\}$. Since we also assume that the government must balance its budget every period, these policies must satisfy the following restriction:

$$G_t + Tr_t = T_t \tag{2}$$

where Tr_t and T_t denote aggregate transfers and aggregate tax revenues, respectively.[5]

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[5] Note that social security in our model economy takes the form of transfers to retired households and that these transfers do not depend on past contributions made by these households. We make this assumption in part for technical reasons. Discriminating between the households according to their past contributions to a social security system requires the inclusion of a second asset-type state variable in the household decision

3.3 Market arrangements

We assume that there are no insurance markets for the household-specific shock, s_t. Moreover, we assume that the households in our model economy cannot borrow.[6] To buffer their streams of consumption against these shocks, the households can accumulate wealth in the form of real capital, a_t.[7] We assume that these wealth holdings belong to a compact set $\mathcal{A}$. The lower bound of this set can be interpreted as a form of liquidity constraints or, alternatively, as a solvency requirement. The existence of an upper bound for the asset holdings is guaranteed as long as the after-tax rate of return to savings is smaller than the households' common rate of time preference.[8] This condition is satisfied in every model economy that we study. Finally, we assume that firms rent factors of production from households in competitive spot markets. This assumption implies that factor prices are given by the corresponding marginal productivities.

3.4 Equilibrium

Each period the economy-wide state is a measure of households, x_t, defined over $\mathcal{B}$, an appropriate family of subsets of $\{S \times \mathcal{A}\}$. As far as each individual household is concerned, the state variables are the realization of the household-specific shock, s_t, its stock of wealth, a_t, and the aggregate state variable, x_t. However, for the purposes of this paper it suffices to consider only the steady-states of the market structure described above. These steady-states have the property that the measure of households remains invariant even though both the state variables and the actions of the individual households change from period one to period the next. This implies that in a steady-state the individual households' state variable is simply the pair (s_t, a_t). Since the structure of the households' problem is recursive, henceforth, to simplify the notation, we drop the time subscript from all the current-period variables and

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problem and this increases the computational costs significantly.

[6] Given that leisure is an argument in the households' utility function, this borrowing constraint can be interpreted as a solvency constraint that prevents the households from going bankrupt in every state of the world.

[7] This is a key feature of this class of model worlds. When insurance markets are allowed to operate, our model economies collapse to a standard representative household model, as long as the right initial conditions hold. In a recent paper Cole and Kocherlakota (1997) have studied economies of this type with the additional characteristic that private storage is unobservable. They conclude that the best achievable allocation is the equilibrium allocation that obtains when households have access to the market structure assumed in this paper. We interpret this finding to imply that the market structure that we use could arise endogenously from certain unobservability features of the environment, namely of the realization of the shock, s_t, and of the amount of wealth, a_t.

[8] See Huggett (1993), Aiyagari (1994) and Ríos-Rull (1998) for details.

we use primes to denote the values of variables one period ahead.

3.4.1 The households' decision problem

The dynamic program solved by a household in state (s, a) is the following:

$$\begin{aligned} v(s,a) &= \max_{\substack{c \geq 0 \\ z \in \mathcal{A} \\ 0 \leq l \leq \ell}} \; u(c, \ell - l) \; + \; \beta \sum_{s' \in S} \Gamma_{s,s'} \; v[s', a'(z)] && (3) \\ \text{s.t.} \quad & c + z = y - \tau(y) + a && (4) \\ & y = a\, r + w\, e_s\, l(s,a) + \omega(s) && (5) \\ & a'(z) = \begin{cases} z - \tau_E(z) \text{ if } s \in \mathcal{R} \text{ and } s' \in \mathcal{E} \\ z \text{ otherwise} \end{cases} && (6) \end{aligned}$$

where v denotes the households' value function, r denotes the rental price of capital and w denotes the wage rate. Note that the definition of income, y, includes three terms: capital income that can be earned by every household, labor income that can only be earned by working-age households —recall that $e_s = 0$ when $s \in \mathcal{R}$— and social security income which can only be earned by retired households —recall that $\omega(s) = 0$ when $s \in \mathcal{E}$. The household policy that solves this problem is a set of functions that map the individual state into choices for consumption, for gross savings and for hours worked. We denote this policy by $\{c(s,a)$, $z(s,a)$, $l(s,\, a)\}$.

3.4.2 Definition of equilibrium

A steady state equilibrium for this economy is a household policy and value function, $\{c(s,a)$, $z(s,a)$, $l(s,\, a)$, $v(s,a), \}$; a government policy, $\{\tau(y)$, $\tau_E(z)$, $\omega(s)$, $G\}$; a stationary probability measure of households, x; factor prices, (r, w); and macroeconomic aggregates, $\{K, L, T, Tr\}$, such that:

(i) Factor inputs, tax revenues and total transfers are obtained aggregating over households:

$$\begin{aligned} K &= \int a \, dx && (7) \\ L &= \int l(s,a)\, e_s \, dx && (8) \\ T &= \int \tau(y)\, dx + \int \xi_{s \in \mathcal{R}} \; \gamma_{s,\mathcal{E}} \; \tau_E(z)\, z(s,a) \; dx && (9) \end{aligned}$$

$$Tr \;=\; \int \omega(s)\ dx. \tag{10}$$

where household income, $y(s,a)$, is defined in equation (6), ξ denotes the indicator function, $\gamma_{s,\mathcal{E}} \equiv \sum_{s' \in \mathcal{E}} \Gamma_{s,s'}$ and, consequently, $(\xi_{s \in \mathcal{R}}\, \gamma_{s,\mathcal{E}})$ is the probability that a household of type s dies —recall that this probability is 0 when $s \in \mathcal{E}$ since we have assumed that working-age households do not die. All integrals are defined over the state space $S \times \mathcal{A}$.

(ii) Given x, K, L, r and w, the household policy solves the households' decision problem described in (3), and factor prices are factor marginal productivities:

$$r = f_1(K,L) - \delta \quad \text{and} \quad w = f_2(K,L). \tag{11}$$

(iii) The goods market clears:

$$\int [c(s,a) + z(s,a)]\ dx + G = f(K,L) + (1-\delta)\ K \tag{12}$$

(iv) The government budget constraint is satisfied:

$$G + Tr = T \tag{13}$$

(v) The measure of households is stationary:

$$x(B) = \int_B \left\{ \int_{S,\mathcal{A}} \left[\xi_{z(s,a)}\ \xi_{s \notin \mathcal{R} \vee s' \notin \mathcal{E}} + \xi_{[1-\tau_E(z)]z(s,a)}\ \xi_{s \in \mathcal{R} \wedge s' \in \mathcal{E}} \right]\ \Gamma_{s,s'}\ dx \right\} dz\ ds' \tag{14}$$

for all $B \in \mathcal{B}$, where $\vee$ and $\wedge$ are the logical operators "or" and "and". Equation (14) just counts the households, and the cumbersome indicator functions and logical operators are used to account for estate taxation. We describe the procedure that we use to compute this equilibrium in Section C of the Appendix.

4 Calibration

In this paper we use the following calibration strategy: *(i)* we target key ratios of the U.S. national income and product accounts, some features of the current U.S. income and estate taxes, some descriptive statistics of U.S. demographics, and some features of the life-cycle profile and of the intergenerational persistence of U.S labor earnings;[9] and *(ii)* we target the Lorenz curves of the U.S. distributions of earnings and wealth reported in Table 2. This last feature is a crucial step in our calibration strategy and we feel that it should be discussed in some detail.

Recall that in Section 2 we have highlighted that the tradition in the literature is to model the process on earnings using direct measurements from some source of earnings data —typically the Panel Study of Income Dynamics (PSID), the Current Population Survey (CPS) or even the Consumption Expenditure Survey (CEX). However, all these data sources suffer from an important shortcoming: since they are not specifically concerned with obtaining a careful measurement of the earnings of the households that belong to the very top of the earnings distribution, they all use top-coding —a procedure that groups every household whose earnings are above a certain level. This important shortcoming has the following implications: *(i)* the measure of aggregate earnings obtained using those databases is inconsistent with the measure obtained from National Income and Product Accounts (NIPA) data; and *(ii)* the distributions of earnings generated by those processes are significantly more disperse than the distribution of U.S. earnings as obtained from SCF data —to verify this fact, simply compare the distributions of earnings of the U.S. and of the model economies reported in Table 1.[10] Furthermore, the methods used to estimate the persistence of the earnings from direct data are rather controversial.[11] To get around these problems, instead of using direct estimates from earnings data, we use our own model economy to obtain a process on earnings that is consistent with the U.S. distributions of earnings and wealth as measured by the SCF. Specifically, as we discuss in detail below, our calibration procedure uses the Gini indices and a small set of points of the Lorenz curves of both earnings and wealth as part of our calibration targets. With this calibration procedure we are looking for a parsimonious process on the endowment of efficiency labor units, which, together with the remaining functions and parameter values, allows us to account for the earnings and wealth inequality and for all of our aggregate targets simultaneously.

In the subsections that follow we first discuss our choices for the model economy's functional forms and we identify their parameters, we then describe our calibration targets and

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[9] Note that throughout this paper our definition of earnings both for the U.S. and for the model economies includes only before-tax labor income. It does not include either capital income or government transfers. The sources for the data and the definitions of all the distributional variables used in this paper can be found in Díaz-Gimenez et al. (1997).

[10] Note that the earnings distributions summarized in Table 1 have been generated using processes that match the main features of data sources other than the SCF.

[11] See Storesletten, Telmer, and Yaron (1999) for a discussion of these issues.

we finally describe the procedure that allows us to reach our parameter choices. The numerical values of the model economies' parameters that implement these targets are reported in Section A of the Appendix.

4.1 Functional forms and parameters

4.1.1 Preferences

Our choice for the model economy's utility function is[12]

$$u(c, l) = \frac{c^{1-\sigma_1}}{1-\sigma_1} + \chi \frac{(\ell - l)^{1-\sigma_2}}{1-\sigma_2} \tag{15}$$

We make this choice because standard, non-separable preferences of the type used in real business cycle models result in large variations in hours in response to small changes in productivity. Moreover, due to the nature of the earnings process used in this paper, the households in our model economies face very large changes in productivity which, under non-separable preferences, would result in extremely large variations in hours. This reasoning lead us to choose a more flexible functional form with different curvatures of consumption and leisure. Consequently, in order to characterize the households' preferences completely, we must choose the numerical values of 5 parameters.

4.1.2 The joint age and endowment of efficiency labor units process

In Section 3 we have assumed that the joint age and endowment of efficiency labor units process, $\{s\}$, takes values in $S = \{\mathcal{E} \cup \mathcal{R}\}$ where $\mathcal{E}$ and $\mathcal{R}$ are two J-dimensional sets. Hence, the number of realizations of the process is $2J$. Therefore, to specify this process we must choose a total of $(2J)^2 + J$ parameters. Of these parameters, $(2J)^2$ correspond to the transition probability matrix on s and J to the endowments of efficiency labor units, e_s —recall that we have assumed that $e_s = 0$ for $s \in \mathcal{R}$.

However, our assumptions about the nature of the joint age and endowment of efficiency labor units process impose some additional structure on the transition probability matrix. To understand this feature of our model economy, it helps to consider the following partition of this matrix:

$$\Gamma_{SS} = \begin{bmatrix} \Gamma_{\mathcal{E}\mathcal{E}} & \Gamma_{\mathcal{E}\mathcal{R}} \\ \Gamma_{\mathcal{R}\mathcal{E}} & \Gamma_{\mathcal{R}\mathcal{R}} \end{bmatrix} \tag{16}$$

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[12] Note that we have assumed that retired households do not work and, consequently, the second term in expression (15) becomes an irrelevant constant for these households.

In expression (16) submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ describes the changes in the endowments of efficiency labor units of working-age households that remain of working-age one period later; submatrix $\Gamma_{\mathcal{E}\mathcal{R}}$ describes the transitions from the working-age states into the retirement states; submatrix $\Gamma_{\mathcal{R}\mathcal{E}}$ describes the transitions from the retirement states into the working-age states that take place when a retired household dies and is replaced by its working-age descendant; and, finally, submatrix $\Gamma_{\mathcal{R}\mathcal{R}}$ describes the changes in retirement states of retired households that remain retired one period later. In the paragraphs that follow we describe our assumptions with respect to these four submatrices.

In order to represent the earnings ability of retired households parsimoniously, we choose to keep track of the endowment of efficiency labor units in only the last period of each household's working-life and we assume that every working-age household faces the same probability of retiring, $p_{e,\varrho}$. These two assumptions imply that $\Gamma_{\mathcal{E}\mathcal{R}} = p_{e,\varrho}I$, where I is the identity matrix. Therefore, to determine $\Gamma_{\mathcal{E}\mathcal{R}}$ we are free to choose the value of only 1 parameter. Similarly, we assume that every retired household faces the same probability of dying, $(1 - p_{\varrho,\varrho})$. This assumption implies that $\Gamma_{\mathcal{R}\mathcal{R}} = p_{\varrho,\varrho}I$ and, once again, to determine $\Gamma_{\mathcal{R}\mathcal{R}}$ we are free to choose the value of only 1 parameter. As far as $\Gamma_{\mathcal{E}\mathcal{E}}$ is concerned, we impose no restrictions on the transitions between the working-age states and, therefore, to determine $\Gamma_{\mathcal{E}\mathcal{E}}$ we must choose the values of J^2 parameters. Finally, our assumptions with respect to $\Gamma_{\mathcal{R}\mathcal{E}}$ are dictated by the secondary purpose of this paper which is to evaluate the roles played in this class of economies by the life-cycle profile of earnings and by the intergenerational transmission of earnings ability in accounting for earnings and wealth inequality. As we show below, these two roles can be modeled very parsimoniously with the aid of only two parameters.

As far as the intergenerational persistence of earnings is concerned, if we assumed that working-age households drew their first shock from the stationary distribution implied by $\Gamma_{\mathcal{E}\mathcal{E}}$, which we denote $\gamma^*_{\mathcal{E}}$, then the intergenerational correlation of earnings would be very small. On the other hand, if we assumed that every working-age household inherited the endowment of efficiency labor units that its predecessor had upon retirement, then the intergenerational correlation of earnings would be relatively large. Since our target value for this correlation, 0.4, is between these two extremes, we use 1 additional parameter, ϕ_1, that acts as a weight to average between $\gamma^*_{\mathcal{E}}$ and the identity matrix. Essentially what ϕ_1 does is to shift the probability mass of $\Gamma_{\mathcal{R}\mathcal{E}}$ towards its diagonal.

To represent the life-cycle profile of earnings parsimoniously, we use as a calibration target the ratio of the average earnings of households between ages 60 and 41 to that of households between ages 40 and 21. If we assumed that households drew their first working-life shock from $\gamma_{\mathcal{E}}^*$, then household earnings would be essentially independent of household age — except for the different wealth effects that result from the household-specific bequests. On the other hand, if we assumed that every household started its working-life with the smallest endowment of efficiency labor units, then household earnings would grow significantly with household age. Again, since we target an intermediate value for our chosen earnings ratio, we need 1 additional parameter, ϕ_2, in order to shift the probability mass of $\Gamma_{\mathcal{RE}}$ towards its first column and, thereby, to attain the appropriate value between these two extremes.

Note that the effects of parameters ϕ_1 and ϕ_2 on the two statistics that interest us work in different directions. Our starting point for $\Gamma_{\mathcal{RE}}$ is a matrix with $\gamma_{\mathcal{E}}^*$ in every row. Then, while parameter ϕ_1 attempts to displace the probability mass from the extremes of that matrix towards its diagonal, parameter ϕ_2 attempts to displace the mass towards its first column.[13] Consequently, as it turns out to be the case, this very parsimonious modeling strategy might not be flexible enough to allow us to attain every desired pair of values for our targeted statistics.[14] All these assumptions imply that of the $(2J)^2 + J$ parameters needed in principle to completely determine the process on s we are left with only $J^2 + J + 4$ parameters. To keep the process on J as parsimonious as possible we choose $J = 4$. This choice implies that to specify the process on s completely we must choose the values of 24 parameters.[15]

4.1.3 Technology

In the U.S. after World War II, the real wage has increased at an approximately constant rate —at least until 1973— and factor income shares have displayed no trend. To account for these two properties, we choose a standard Cobb-Douglas aggregate production function in capital and efficiency labor units. Therefore, in order to specify the aggregate technology fully, we must choose the value of 2 additional parameters: the capital share of income, θ, and the depreciation rate of capital, δ.

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[13] See Section B in the Appendix for the formula that we use to compute $\Gamma_{\mathcal{RE}}$ from ϕ_1, ϕ_2 and $\gamma_{\mathcal{E}}^*$.

[14] We discuss this property of our model economy in the first paragraph of Section 5 and in the fourth paragraph of Section 5.1 below.

[15] Note that when counting the number of parameters for the age and employment process we have not imposed the requirement that Γ must be a Markov matrix.

4.1.4 Government Policy

To describe the government policy in our model economies we must specify the values of the steady-state government consumption, G, of the transfers to the retirees, $\omega(s)$, and of the functions that describe the income and estate taxes.

Income taxes: Our choice for the model economy's income tax function is

$$\tau(y) = a_0(y - (y^{-a_1} + a_2)^{-1/a_1}) + a_3 y. \tag{17}$$

The reasons that justify this choice are the following: *(i)* the first term of expression (17) is the function chosen by Gouveia and Strauss (1994) to characterize the 1989 U.S. effective household income taxes; and *(ii)* we add constant a_3 to this function to deal with the facts that in our model economy we abstract from property, consumption and excise taxes and that the U.S. government obtains tax revenues also from these sources.[16] Therefore, to specify the model economy income tax function completely we must choose the values of 4 parameters.

Estate Taxes: Our choice for the model economy's estate tax function is

$$\tau_E(z) = \begin{cases} 0 & \text{for } z < \underline{z} \\ \tau_E(z - \underline{z}) & \text{for } z > \underline{z} \end{cases} \tag{18}$$

The rationale for this choice is the following: the current U.S. estate tax code establishes a tax exempt level and a progressive marginal tax rate thereafter. However, because of the many legal loop-holes, the effective marginal tax rates faced by the households have been estimated to be significantly lower than the nominal tax rates.[17] This fact leads us to consider the importance of the progressivity of effective estate taxes in the U.S. to be of second order, and we choose to approximate the U.S. effective estate taxes with a tax function that specifies a tax exempt level, $\underline{z}$, and a single marginal tax rate, τ_E. These choices imply that to specify the model economy estate tax function we must choose the values of 2 additional parameters.

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[16] Note that this implies that we are effectively assuming that all sources of tax revenues are proportional to income. This assumption is equivalent to assuming that the government in the model economy uses a proportional income tax to obtain all the non-income-tax revenues levied by the U.S. government.

[17] See Footnote 21.

4.1.5 Adding Up

Our modeling choices and our calibration strategy imply that to compute the equilibrium of our model economy we must choose the values of a total of 39 parameters. Of these 35 parameters, 5 describe household preferences, 2 describe the aggregate technology, 8 describe the government policy, and 24 describe the joint age and employment process.

4.2 Targets

In order to determine the values of the 39 model economy parameters we do the following: *(i)* we target a set of U.S. economy statistics and ratios that we would like our economy mimic; *(ii)* in one case —that of the intertemporal elasticity of substitution for consumption— we use an off-the-shelf, ready-to-use value; and *(iii)* we impose five normalization conditions. In the subsections below we describe these target values and normalization conditions.

4.2.1 Model period

Time aggregation matters for the cross-sectional distribution of flow variables such as earnings. Short model periods imply high wealth to income ratios and are, therefore, computationally costly. Hence, computational considerations lead us to prefer long model periods. Since our main data source is the 1992 Survey of Consumer Finances, and since the longest model period that is consistent with the data collection procedures used in that dataset is a year, the duration of each time period in our model economy is also one year.

4.2.2 Macroeconomic aggregates

We want our model economy's macroeconomic aggregates to mimic the macroeconomic aggregates of the U.S. economy. Therefore, we target a capital to output ratio, K/Y, of 3.13, a capital income share of 0.376, an investment to output ratio, I/Y, of 18.6%, a government expenditures to output ratio, G/Y, of 20.2%, and a transfers to output ratio, Tr/Y, of 6.6%.

The rationale for these choices is the following: According to the 1992 SCF, average household wealth was $184,000. According to the Economic Report of the President (1998), in the U.S. in 1992 per household GDP was $58,916.[18] Dividing these two numbers, we obtain 3.13 which is our target value for the capital output ratio. The capital income share is the value

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[18] This number was obtained using the U.S. population quoted for 1992 in Table B-31 of the Economic Report of the President (1998) and an average 1992 SCF household size of 2.41 as reported in Díaz-Gimenez et al. (1997).

that obtains when we use the methods described in Cooley and Prescott (1995) excluding the public sector from the computations.[19] The investment, government expenditures and transfers to output ratios are obtained using data for 1992 from the Economic Report of the President (1998). The value for investment is calculated as the sum of gross private domestic investment, change in business inventories and 75% of the private consumption expenditures in durables goods. This definition of investment is approximately consistent with the 1992 SCF which includes the value of vehicles in the definition of household wealth. The value for government expenditures is the figure quoted for government consumption expenditures and gross government investment. Finally, the value for transfers is the figure quoted for the old age, survivors, disability and health insurance benefits —which are the components of U.S. transfers that are essentially lump-sum. These choices give us a total of 5 aggregate targets.

4.2.3 Allocation of time and consumption

First, for the endowment of disposable time we choose a value of $\ell = 3.2$. The rationale for this choice is that it is the value that makes the aggregate labor input approximately equal to one. Given this choice, we target the share of disposable time allocated to working in the market to be the standard 33%.[20] Next, for the curvature of consumption we choose a value of $\sigma_1 = 1.5$ which falls within the range (1–3) that is standard in the literature. Finally, we want our model economy to mimic some characteristics of the U.S. cross-sectional distributions of consumption and hours. To this purpose, we focus on the ratio of the cross-sectional coefficients of variation of consumption and hours and we target a value of 3.5 for this ratio. These choices give us 4 additional targets.

4.2.4 The age structure of the population

We want our model economy to mimic some features of the age structure of the U.S. population. Since in our model economy there are only working-age and retired households and since the model economy households age stochastically, we target the expected durations of their working-lives and retirements to be of 45 and 18 years, respectively. These choices give us 2 additional targets.

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[19] See Castañeda et al. (1998a) for details about this number.

[20] This choice is based on the findings of Ghez and Becker (1975) and Juster and Stafford (1991) who conclude that U.S. households allocate 1/3 of their discretionary time to the market.

4.2.5 The life-cycle profile of earnings

We want our model economy to mimic a stylized characterization of the life-cycle profile of U.S. earnings. As we have already mentioned, to measure this profile we use the ratio of the average earnings of households between ages 60 and 41 to that of households between ages 40 and 21. According to the PSID, in the 1972–1991 period the average value of this statistic was 1.303. This choice gives us 1 additional target.

4.2.6 The intergenerational transmission of earnings ability

We want our model economy to mimic the intergenerational transmission of earnings ability in the U.S. economy. As we have already mentioned, to measure this feature we use the cross-sectional correlation between the average life-time earnings of one generation of households and the average life-time earnings of their immediate descendants. Solon (1992) and Zimmerman (1992) have measured this statistic for fathers and sons in the U.S. economy and they have found its value to be of of approximately 0.4. This choice gives us 1 additional target.

4.2.7 Income taxation

We want our model economy's income tax function to mimic the progressivity of U.S. effective income taxes as measured by Gouveia and Strauss (1994). Therefore we choose our model economy's income tax function from the family of functions described by expression (17). To identify our function fully, we must choose numerical values for parameters a_0, a_1, a_2 and a_3. Since a_0 and a_1 are unit independent, we use the values reported by Gouveia and Strauss (1994) for these parameters, namely, $a_0 = 0.258$ and $a_1 = 0.768$. The two additional targets result, *(i)* from imposing that the shape of the model economy tax function must resemble the shape of the function estimated by Gouveia and Strauss (1994) in spite of the change in units and, *(ii)* from assuming that all revenues levied from sources other than the federal income tax are proportional to income. The first target is achieved by choosing a_2 so that the value of the tax rate levied on mean income in our benchmark model economy replicates the effective tax rate on mean income in the U.S. economy. The second target is achieved by choosing a_3 so that the government in our model economy balances its budget, that is by choosing the value of a_3 so that the steady-state values of government spending, G, aggregate transfers, Tr, and total tax revenues, T, satisfy the condition described in expression (2). These choices give us 4 additional targets.

4.2.8 Estate taxation

We want our model economy to mimic the tax exempt level specified in the U.S. estate tax code which during the 1987–1997 period was \$600,000. Since average per household income, $\bar{y}$, during that period was, approximately, \$60,000, our target value for the size of estates that are tax exempt in our model economy is $\underline{z} = 10\bar{y}$. We also want our model economy's estate taxes to mimic the revenue levied in the U.S. through estate taxation. During the 1985–1997 period this revenue was only 0.2% of GDP.[21] These choices give us 2 additional targets.

4.2.9 Normalization

We have one degree of freedom to determine the units in which labor is measured. This allows us to normalize the endowment of efficiency labor units of the least productive households to be $e_1 = 1.0$. Moreover, since matrix Γ is a Markov matrix, its rows must add up to 1. This property imposes 4 additional normalization conditions on the rows of $\Gamma_{\mathcal{E}\mathcal{E}}$.[22] Therefore, normalization provides us with a total of 5 additional targets.

4.2.10 The distributions of earnings and wealth

The conditions that we have described so far specify a total of 24 targets. Since to solve our model economy we have to determine the values of 39 parameters, we need 15 additional targets. Given our calibration strategy, these 15 targets in principle would be the Gini indices and 13 additional points form the Lorenz curves of U.S. earnings and wealth reported in Table 2. However, there are some additional restrictions that our parameter choices have to satisfy and that we have yet to discuss. These restrictions arise from imposing that Γ must be a Markov matrix and, hence, that all its elements must be non-negative.

In order to do this, we equated to zero the transition probabilities that the non-linear equation solver attempted to make negative. In this way, it turned out that in our final calibration exercise 4 of the transition probability parameters of submatrix $\Gamma_{\mathcal{E},\mathcal{E}}$ were equated to zero (see Table 13 in the Appendix). This gave us 4 additional targets and, consequently, it reduced the number of target points of the Lorenz curves from 13 to 9. Note that the number of points that we target is relatively small: about half of the number of points that

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[21] See, for example, Aaron and Munnell (1992).

[22] Note that our assumptions about the structure of Γ imply that, once $\Gamma_{\mathcal{E}\mathcal{E}}$ has been normalized, every row of Γ can be made to add up to one without imposing any further restrictions.

we report in Tables 2, 4, 7 and 10 for the U.S. and for the model economies. In practice, rather than targeting 9 specific points, we searched for a set of values for the parameters such that all the points of the Lorenz curves reported for the model economies were similar to the corresponding points in the data.

Table 2: The distributions of earnings and wealth in the U.S.

The Distribution of Earnings (%)

Gini	*Quintiles*					*Top Groups (%)*		
	1st	2nd	3rd	4th	5th	90–95	95–99	99–100
0.63	–0.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76

The Distribution of Wealth (%)

Gini	*Quintiles*					*Top Groups (%)*		
	1st	2nd	3rd	4th	5th	90–95	95–99	99–100
0.78	–0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55

4.3 Choices

The values of some of the parameters can be obtained very simply because they are uniquely determined by the value of one of our targets. In this fashion, we set $\sigma_1 = 1.5$ and $\theta = 0.376$.[23] Similarly, the values of $p_{e,\varrho}$ and $p_{\varrho,\varrho}$ were obtained directly from our targets for the durations of the working-life and of retirement, respectively. The values for two of the parameters of the income tax function, a_0 and a_1 were also taken directly from the values estimated by Gouveia and Strauss (1994) for the U.S. economy. Finally, our choice for the value of the endowment of time implies that $\ell = 3.2$ and the normalization of the endowment of efficiency labor units implies that $e_1 = 1.0$.

The values of the remaining 31 parameters were determined solving a system of non-linear equations obtained by imposing that the relevant statistics of the model economy should be equal the corresponding targets and that the model economy should be in a steady-state equilibrium. This last condition adds one additional unknown and one additional equation to our tally. The unknown is the capital-labor ratio, and the equation is the requirement that the value that the households take as given for this ratio should be equal to the value

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[23] Note that given our choice for the aggregate production function, the value of the capital income share is exactly θ.

implied by their decisions. Therefore, the calibration of this model economy amounted to solving a system of 32 non-linear equations in 32 unknowns.[24] Unfortunately, solutions for these systems are not guaranteed to exist and, when they do exist, they are not guaranteed to be unique. Consequently, we tried many different initial values in order to find the best parametrization possible. We report the numerical choices for the 39 benchmark model economy parameters in Tables 12 and 13 and in the first row of Table 14 and we discuss the results of our calibration exercise in Section 5.1 below.

5 Findings

In this section we report our findings. We do this in two stages. In Section 5.1 we report the behavior of our benchmark model economy which we have calibrated to the targets described in Section 4 above. As we have already mentioned, we find that the parsimonious way in which we model the life-cycle prevents our benchmark model economy from matching the targeted values for the intergenerational correlation of earnings and for the life-cycle earnings profile simultaneously. This finding lead us to carry out two additional computational experiments which we report in Section 5.2. The purpose of these experiments is to find out whether or not our model economy can match those two targets separately. More specifically, in the first additional experiment we mimic the intergenerational correlation of earnings observed in the data in a model economy with a flat life-cycle earnings profile and in the second additional experiment we mimic the life-cycle profile of earnings observed in the data in a model economy in which earnings are uncorrelated across generations.

5.1 The benchmark model economy: a theory of inequality

In this section we report the calibration results, we discuss the reasons that allow us to account for the U.S. earnings and wealth distributions almost exactly, and we assess our benchmark model economy as a theory of inequality.

Macroeconomic aggregates and the allocation of time and consumption: We report the values of our aggregate targets for the U.S. and the benchmark model economies in

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[24] Actually we solved a smaller system of 26 equations and 26 unknowns because our guess for the value of aggregate output uniquely determines the value of parameters a_2 and $\underline{z}$, because the value of G is determined residually from the government budget constraint and because the normalization of $\Gamma_{\mathcal{E}\mathcal{E}}$ allows us to determine the values of 4 of the transition probabilities directly.

Table 3: The values of the targeted ratios and aggregates of the U.S. and in the benchmark model economies

Economy	I/Y	G/Y	Tr/Y	K/Y	h^a	cv_c/cv_l^b	T_E/Y	$e_{40/20}$	$\rho(f,s)$
Target (U.S.)	18.6%	20.2%	6.6%	3.13	33.3%	3.50	0.20%	1.30	0.40
Benchmark	18.7%	20.1%	6.9%	3.10	32.9%	3.51	0.19%	1.10	0.25

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[a] Variable h denotes the average share of disposable time allocated to the market.

[b] This statistic is the ratio of the coefficients of variation of consumption and of hours worked.

the first two columns of Table 3 and the ratio of the coefficients of variation of consumption and hours for the U.S. and the benchmark model economies in the next two columns of that same table. We find that all these statistics are very similar in both economies.

The age structure of the population: Our specification of the joint age and endowment process allows us to match the targeted expected durations of the working-life and retirement exactly. Hence in the every model economy analyzed in this paper the expected duration of the working-life is 45 years and the expected duration of retirement is 18 years.

The life-cycle profile of earnings and the intergenerational transmission of earnings ability: As we have already mentioned, we find that our parsimonious modeling of the life-cycle does not allow us to match the targeted values for the intergenerational correlation of earnings and the life-cycle earnings profile simultaneously. Given this limitation, we decided to go part of the way and we targeted a value of 1.1 for the age-dependent earnings ratio, and a value of 0.27 for the intergenerational correlation of earnings. These values are, respectively, 1/3 and 2/3 of the corresponding statistics for the U.S. economy. The rationale for these choices is that we feel that the intergenerational transmission of earnings is more closely related to inequality than the life-cycle profile of earnings. We find that our benchmark model economy comes very close to matching those targets (see the last two columns of Table 3). Recall that to ensure that our findings are not dependent on the specific choices for these two targets, in Section 5.2 below we carry out two robustness exercises designed to explore the ability of our model economy to account for each one of these two features of the data separately.

Income taxes: As we have already discussed in Section 4.2.7, the tax function proposed by Gouveia and Strauss (1994) has to be normalized before it can be used in the model economy. As it can be seen from Figure 1, the average income tax rates in our benchmark model economy are very similar to those reported by Gouveia and Strauss (1994) for the U.S. economy.

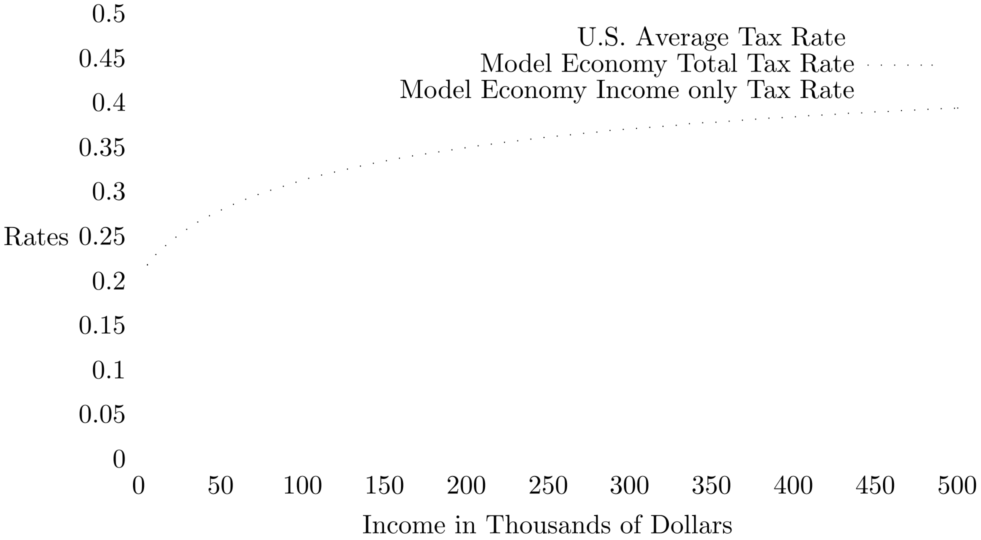

Figure 1: The effective income tax functions in U.S. and in the benchmark model economies

Estate taxes: We report the estate tax revenue to income ratios in the U.S. and in the benchmark model economies in the second to last column of Table 3. We find that these ratios are very similar in both economies.

The distribution of earnings: We report the Gini indices and selected points of the Lorenz curves of earnings in the U.S. and in the benchmark model economies in the top half of Table 4. We find that the distributions of earnings are very similar in both economies. Moreover, our benchmark model economy does a significantly better job in accounting for the observed distribution of earnings than any of the previous attempts in the literature reported in Table 1.

If we look at the fine print, we find that the main differences between the model and the data are that the share earned by the fourth quintile is lower in the model economy than in the data and that this is compensated by the shares earned by the other quantiles which are slightly higher in the model economy than in the data. During the course of this research

Table 4: The distributions of earnings and wealth in the U.S. and in the benchmark model economies

The Distributions of Earnings (%)

	Gini	*Quintiles*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
U.S.	0.63	–0.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Benchmark	0.60	0.00	6.27	14.66	15.28	63.80	16.73	16.12	15.50

The Distributions of Wealth (%)

	Gini	*Quintiles*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
U.S.	0.78	–0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Benchmark	0.79	0.01	0.41	3.44	15.16	80.98	16.87	15.09	29.46

we tried different parameterizations of our model economy increasing the accuracy of these statistics at the expense of the accuracy of other calibration targets and it turned out that these changes made little difference for our overall findings. Our results lead us to conjecture that these differences would have been reduced if we had chosen a process on s of a higher dimension.

The distribution of wealth: We report the Gini indices and selected points of the Lorenz curves of wealth in the U.S. and in the benchmark model economies in the bottom half of Table 4. We find that the benchmark model economy accounts for the U.S. distribution of wealth almost exactly and that it does a particularly good job in accounting for the top tail of the distribution. Again, we find that, overall, we do a significantly better job in accounting for the observed wealth inequality than any of the previous attempts in the literature reported in Table 1.

If we look at the fine print, we find that the main differences between the model and the data are that the share of wealth owned by the 90–95 quantile is higher in the model economy than in the data and that this is compensated by the shares owned by the third quintile and by the 95–99 quantile being lower. We contend that the conjecture about the dimension of s discussed above is valid also in this case. We conclude that our choice of four realizations for the employment process is a good compromise between the resulting number of degrees of freedom and the accuracy in accounting for the earnings and wealth distributions.

Table 5: Earnings and wealth persistence in the U.S. and in the benchmark model economies: fractions of households that remain in the same quintile after 5 years

Earnings Persistence					
Economy	1st	2nd	3rd	4th	5th
U.S.	0.86	0.41	0.47	0.46	0.66
Benchmark	0.70	0.50	0.59	0.77	0.78
Wealth Persistence					
Economy	1st	2nd	3rd	4th	5th
U.S.	0.67	0.47	0.45	0.50	0.71
Benchmark	0.96	0.92	0.87	0.82	0.89

Mobility: People do not stay in the same earnings and wealth groups forever. Consequently, a convincing theory of earnings and wealth inequality should also be able to account for at least some of the features of the observed earnings and wealth mobility of households. One way to summarize this economic mobility is to compute the fractions of households that remain in the same earnings and wealth quintiles after a certain period of time, say five years. We call these fractions the persistence statistics. Note that in our calibration exercise we have not targeted any of these statistics. Therefore, they can be considered to be over-identifying restrictions for our theory.

In Table 5 we report the persistence statistics for the earnings and wealth quintiles of the U.S. and of the benchmark model economies.[25] We interpret these results to be an additional success for our theory. This is because there is nothing in our theory that would have made us predict that our model economy was going to match any of these statistics. In particular, our parsimonious way of modeling the life-cycle makes it very difficult for our model economy to mimic this feature of the data, specially if we take into account the large role played by the life-cycle in shaping persistence.[26] This notwithstanding, both our

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[25] The U.S. persistence statistics reported in Table 5 are the same as those reported in Díaz-Gimenez et al. (1997). The source for their raw data was the PSID. The period considered was the five years between 1984 and 1989. To construct the quintiles they took into account only the households that belonged to both the 1984 and the 1989 PSID samples.

[26] For instance, Auerbach and Kotlikoff (1987), Ríos-Rull (1996) and others find that the age-earnings profile of the PSID sample displays a clear inverted-U shape.

benchmark model economy and the data display a large earnings and wealth persistence, and both in our benchmark model economy and in the data the top and the bottom quintiles are more persistent than the middle quintiles. We also find that earnings and wealth are uniformly more persistent in the benchmark model economy than in the U.S. economy. This was to be expected from the parsimonious modeling of the life-cycle in the model and from the fact that much of the mobility in the data is linked to the earnings and wealth life-cycles.

Overall, we consider our mobility findings to be particularly encouraging and we conjecture that versions of our model economy that included a more detailed specification of the age-earnings profile of households would allow us mimic the U.S. persistence statistics significantly better.

An assessment: We find that our benchmark model economy does an extremely good job in accounting for the U.S. earnings and wealth inequality and that it improves previous results reported in the literature significantly. We think that our result is particularly creditworthy if we take into account our parsimonious model design and the many computational difficulties solved in this research. As we have already mentioned, we are convinced that a more sophisticated implementation of the age-earnings profile of households would greatly enhance the ability of this class of model economies to address the life-cycle profile of earnings, the intergenerational transmission of earnings and the economic mobility of households simultaneously. Those enhanced models should be able to capture with enough detail the features of earnings and wealth inequality that are due to the life-cycle and to other reasons, such as those represented by the idiosyncratic shocks that are the gist of this paper. Finally, we are convinced that this class of models will soon prove to be very useful to evaluate the distributional implications of policy.

Table 6: The targeted macroeconomic ratios and aggregates of the model economies

	I/Y	G/Y	Tr/Y	K/Y	h	cv_c/cv_l	T_E/Y	$e_{40/20}$	$\rho(f,s)$
Benchmark	18.7%	20.1%	6.9%	3.10	32.9%	3.51	0.19%	1.10	0.25
Match Autocorrelation	18.8%	20.1%	7.0%	3.16	33.2%	3.66	0.22%	1.01	0.40
Match Life-Cycle	19.2%	19.9%	6.9%	3.19	34.4%	3.66	0.24%	1.29	–0.03

5.2 Two robustness exercises

As we have already mentioned, our parsimonious modeling of the life-cycle does not allow us to match the intergenerational correlation of earnings and the ratio of the average earnings of households between ages 60 and 41 to that of households between ages 40 and 21 simultaneously. To find out whether or not this is an important shortcoming of our model economy, we carry out two robustness exercises. First, we attempt to mimic the observed intergenerational correlation of earnings while allowing earnings to display no life-cycle and then we attempt to mimic the observed life-cycle earnings ratio while allowing earnings to display no intergenerational correlation. We find that, even though the parameters that implement the calibration differ somewhat, the steady state equilibrium allocations of these two model economies are very similar to those that obtain in the benchmark model economy. These findings lead us to conclude that, even though the the roles played by the intergenerational transmission of earnings ability and the life-cycle profile of earnings are quantitatively significant, they are not the key to accounting for the earnings and wealth inequality observed in the U.S.

Table 7: The distributions of earnings and wealth in the model economies

The Distributions of Earnings (%)

	Gini	*Quintiles*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
Benchmark	0.60	0.00	6.27	14.66	15.28	63.80	16.73	16.12	15.50
Match Autocorrelation	0.60	0.00	6.68	14.73	15.22	63.37	15.59	15.01	15.51
Match Life-Cycle	0.61	0.00	6.42	14.41	14.95	64.22	16.16	18.37	16.50

The Distributions of Wealth (%)

	Gini	*Quintiles*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
Benchmark	0.79	0.01	0.41	3.44	15.16	80.98	16.87	15.09	29.46
Match Autocorrelation	0.79	0.01	0.33	2.96	14.19	82.52	17.17	14.72	29.72
Match Life-Cycle	0.80	0.01	0.39	3.24	14.27	82.09	16.56	18.58	29.06

5.2.1 Accurate intergenerational transmission of earnings ability at the expense of the life-cycle profile of earnings

The households in this model economy draw their first working-life shock from four different conditional distributions in which the last shocks of their predecessors are significantly more likely than any of the other shocks. This feature allows us to match our targeted intergenerational correlation of earnings almost exactly —the value that we obtain for this statistic in this model economy is 0.40, which is the value of this statistic reported for the U.S. economy. However, this feature also implies that the distribution of the new-entrants in this economy is very similar to the distribution of the households who have been working for some time and, consequently, in this economy the earnings process does not display any life-cycle pattern —the value that we obtain for the age-dependent earnings ratio in this model economy is 1.0.

We solve this model economy using the calibration procedure described in Section 4 above and we report our findings in the rows labeled *"Match Autocorrelation"* of Tables 6, 7 and 8. A quick glance at those three tables shows that the differences between the *Match Autocorrelation* and the benchmark model economies are very small.

Table 8: Earnings and wealth persistence in the model economies: fractions of households that remain in the same quintile after 5 years

	Earnings Persistence				
Economy	1st	2nd	3rd	4th	5th
Benchmark	0.70	0.50	0.59	0.77	0.78
Match Autocorrelation	0.70	0.49	0.57	0.74	0.77
Match Life-Cycle	0.70	0.53	0.63	0.80	0.81

	Wealth Persistence				
Economy	1st	2nd	3rd	4th	5th
Benchmark	0.96	0.92	0.87	0.82	0.89
Match Autocorrelation	0.96	0.92	0.89	0.82	0.90
Match Life-Cycle	0.97	0.94	0.85	0.77	0.87

5.2.2 Accurate life-cycle profile of earnings at the expense of the intergenerational transmission of earnings ability

In this model economy every household draws its first working-life shock from a distribution in which the low-productivity shocks are more likely than the high-productivity shocks. Consequently, labor earnings tend to improve significantly with household age and this allows us to match the age-dependent earnings ratio that we have chosen to measure the earnings life-cycle almost exactly —the value that we obtain for this statistic in this model economy is 1.29 and in the U.S. economy it is 1.30. Moreover, since every household draws its first working-life shock from the same distribution, there is no intergenerational transmission of earnings ability and, consequently the intergenerational correlation of earnings ability is very close to zero.

We solve this model economy using the calibration procedure described in Section 4 above and we report our findings in the rows labeled *"Match Life-Cycle"* of Tables 6, 7 and 8. Again, a quick glance at those three tables shows that the differences between the *Match Life-Cycle* and the benchmark model economies are very small.

6 A Policy Experiment

In this section we evaluate the steady-state implications of abolishing estate taxation. To this purpose, we study the aggregate, distributional and mobility properties of a model economy that has exactly the same fundamentals as the benchmark economy with the only exception that estates are not taxed. More specifically, the process on the joint age and endowment of efficiency labor units process, the preference and technology parameters, the sizes of government expenditures and transfers and the progressivity of income taxes are identical across both economies. The only difference between the two model economies lies in the proportional part of the income tax that adjusts to keep the government budget constraint balanced.

Table 9: The targeted macroeconomic ratios and aggregates of the model economies

	I/Y	G/Y	Tr/Y	K/Y	h	cv_c/cv_l	T_E/Y	$e_{40/20}$	$\rho(f,s)$
Benchmark	18.7%	20.1%	6.9%	3.10	32.9%	3.51	0.19%	1.10	0.25
No Estate Tax	19.0%	19.9%	6.8%	3.15	32.8%	3.51	0.00%	1.10	0.24

Once we have solved the benchmark model economy, computing the solution to this economy amounts to solving a much simpler system of three non-linear equations in three unknowns —the capital-labor ratio, aggregate output and the proportional part of the income tax. We report the statistics of this model economy in the rows labeled "*No Estate Tax*" of Tables 9, 10 and 11.

Table 10: The distributions of earnings and wealth in the model economies

The Distributions of Earnings (%)

	Gini	*Quintiles*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
Benchmark	0.60	0.00	6.27	14.66	15.28	63.80	16.73	16.12	15.50
No Estate Tax	0.60	0.00	6.21	14.63	15.28	63.88	16.80	16.16	15.51

The Distributions of Wealth (%)

	Gini	*Quintiles*					*Top Groups (%)*		
Economy		1st	2nd	3rd	4th	5th	90–95	95–99	99–100
Benchmark	0.79	0.01	0.41	3.44	15.16	80.98	16.87	15.09	29.46
No Estate Tax	0.78	0.01	0.43	3.54	15.28	80.74	16.81	14.98	29.36

We find that abolishing estate taxation brings about an increase in steady-state output of 1.0%, an increase in the steady-state stock of capital of 2.7% and a reduction of steady-state hours of 0.3%. Along every other dimension the differences between the benchmark and the *No Estate Tax* model economies are very small. In particular, we find that the distributional implications of this policy change are negligible. We conjecture that the main reasons that justify this finding are, first, that given the demographics of our model economy, the role played by the estate tax rate in determining the after tax rate of return of the economy is quantitatively very small and, second, that the effective tax rate chosen during the calibration process is significantly lower than its nominal value in the U.S. and, therefore, abolishing this rather small effective estate tax brings about very little change.

7 Concluding comments

In this paper we provide a theory of earnings and wealth inequality based on the optimal choices of households with identical and standard preferences that accounts for the earnings and wealth inequality observed in the U.S. economy almost exactly. We show that uninsured

Table 11: Earnings and wealth persistence in the model economies: fractions of households that remain in the same quintile after 5 years

	Earnings Persistence				
Economy	1st	2nd	3rd	4th	5th
Benchmark	0.70	0.50	0.59	0.77	0.78
No Estate Tax	0.70	0.50	0.59	0.77	0.78
	Wealth Persistence				
Economy	1st	2nd	3rd	4th	5th
Benchmark	0.96	0.91	0.87	0.81	0.88
No Estate Tax	0.96	0.92	0.87	0.82	0.89

idiosyncratic earnings risk, retirement, altruism and a sufficiently large system of government transfers are essential ingredients for any such theory, since they are needed to mimic the observed earnings to wealth ratios of rich and poor households simultaneously. We also show that calibrating the earnings process directly is a must if we want our model economies to mimic the observed distributions of earnings and wealth in sufficient detail.

Our findings also indicate that we can account for the earnings and wealth inequality without having to model the rich and the poor as being different. They can be thought of as being essentially the same type of people that have been subject to a different set of circumstances. We are convinced that these findings will have important implications for future research.

We consider this paper to be a necessary first step in the formal attempt to evaluate the distributional implications of fiscal policy, of which the abolition of estate taxation discussed in Section 6 is only a preview. We intend to take the next step in a companion paper where we plan to use the model economy described in this paper to quantify the trade-offs implied by different income tax policies.

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Appendix

A Parameter values

Table 12: Parameter Values for the Benchmark Model Economy

Preferences		
Time discount factor	β	0.921
Curvature of consumption	σ_1	1.500
Curvature of leisure	σ_2	1.251
Relative share of consumption and leisure	χ	1.334
Productive time	ℓ	3.200
Age and employment process		
Common probability of retiring	$p_{e,\varrho}$	0.022
Common probability of dying	$1 - p_{\varrho,\varrho}$	0.066
Earnings life-cycle controller	ϕ_1	0.955
Intergenerational earnings persistence controller	ϕ_2	0.601
Technology		
Capital share	θ	0.376
Capital depreciation rate	δ	0.060
Government policy		
Government expenditures	G	0.292
Transfers	ω	0.348
Income tax function parameters		
	a_0	0.258
	a_1	0.768
	a_2	0.255
	a_3	0.193
Estate tax function parameters		
Tax exempt level	$\underline{z}$	14.144
Marginal tax rate	τ_E	0.211

Table 13: The transition probabilities of the household-specific process for working-age households that remain of working-age one period later ($\Gamma_{\mathcal{E}\mathcal{E}}$)

	To e'			
From e	e_1	e_2	e_3	e_4
e_1	0.965	0.936E-2	0.375E-2	0.536E-4
e_2	0.837E-1	0.888	0.579E-02	0.000
e_3	0.873E-2	0.140E-2	0.967	0.172E-3
e_4	0.000	0.978	0.000	0.000

Table 14: The relative endowments of efficiency labor units, e_s, and the stationary distribution of working-age households, $\gamma^*_{\mathcal{E}}$ (%)

	$s = e_1$	$s = e_2$	$s = e_3$	$s = e_4$
e_s	1.000	3.309	8.190	2304.445
$\gamma^*_{\mathcal{E}}$ (%)	64.9	7.3	27.7	0.0084

B The formal definition of parameters ϕ_1 and ϕ_2

Let p_{ij} denote the transition probability from $s = \rho_i$ to $s' = e_j$, let π^*_i be the invariant measure of households that receive shock $s = e_i$ and let ϕ_1 and ϕ_2 be the two parameters whose roles are described in Section 4.1.2, then the recursive procedure that we use to compute the p_{ij} is the following:

- *Step 1:* First, we use parameter ϕ_1 to displace the probability mass from a matrix with vector $\gamma^*_{\mathcal{E}} = [\gamma^*_1\ \gamma^*_2\ \gamma^*_3\ \gamma^*_4]$ in every row towards its diagonal as follows:

$$\begin{aligned}
p_{11} &= \gamma^*_1 + \phi_1\gamma^*_2 + \phi_1^2\gamma^*_3 + \phi_1^3\gamma^*_4 \\
p_{12} &= (1-\phi_1)[\gamma^*_2 + \phi_1\gamma^*_3 + \phi_1^2\gamma^*_4] \\
p_{13} &= (1-\phi_1)[\gamma^*_3 + \phi_1\gamma^*_4] \\
p_{14} &= (1-\phi_1)\gamma^*_4 \\
p_{21} &= (1-\phi_1)\gamma^*_1
\end{aligned}$$

$$
\begin{aligned}
p_{22} &= \phi_1\gamma_1^* + \gamma_2^* + \phi_1\gamma_3^* + \phi_1^2\gamma_4^* \\
p_{23} &= (1-\phi_1)[\gamma_3^* + \phi_1\gamma_4^*] \\
p_{24} &= (1-\phi_1)\gamma_4^* \\
p_{31} &= (1-\phi_1)\gamma_1^* \\
p_{32} &= (1-\phi_1)[\phi_1\gamma_1^* + \gamma_2^*] \\
p_{33} &= \phi_1^2\gamma_1^* + \phi_1\gamma_2^* + \gamma_3^* + \phi_1\gamma_4^* \\
p_{34} &= (1-\phi_1)\gamma_4^* \\
p_{41} &= (1-\phi_1)\gamma_1^* \\
p_{42} &= (1-\phi_1)[\phi_1\gamma_1^* + \gamma_2^*] \\
p_{43} &= (1-\phi_1)[\phi_1^2\gamma_1^* + \phi_1\gamma_2^* + \gamma_3^*] \\
p_{44} &= \phi_1^3\gamma_1^* + \phi_1^2\gamma_2^* + \phi_1\gamma_3^* + \gamma_4^*
\end{aligned}
$$

- *Step 2:* Then we use parameter ϕ_2 to displace the resulting probability mass towards the first column as follows:

$$
\begin{aligned}
p_{i1} &= p_{i1} + \phi_2 p_{i2} + \phi_2^2 p_{i3} + \phi_2^3 p_{i4} \\
p_{i2} &= (1-\phi_2)[p_{i2} + \phi_2 p_{i1} + \phi_2^2 p_{i4}] \\
p_{i3} &= (1-\phi_2)[p_{i3} + \phi_2 p_{i4}] \\
p_{i4} &= (1-\phi_2)p_{i4}
\end{aligned}
$$

for $i = 1, 2, 3, 4$.

C Computation

This Appendix describes the algorithm that we have used to compute the equilibrium allocations of the model economies. The outline of the algorithm is the following:

We use a standard non-linear equation solver (specifically a modification of Powell's hybrid method, implemented in subroutine DNSQ from the SLATEC package). We use this solver to search for a zero of a system of 26 equations in 26 unknowns. The equations include the steady-state equilibrium condition for the capital-labor ratio and the 25 equations that specify the steady-state values of our 25 additional targets. The unknowns include the guesses for the capital-labor ratio and for aggregate output, and the values of 24 of our free model economy parameters.[27]

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[27] We make a change of variables to ensure that the choices that the algorithm makes of the transition probabilities are always positive and that the sign restrictions are satisfied. See the discussion in Sections 4.2.10 and 4.3 and in Footnote 24.

For each specification of the model economy parameters, to compute the equilibrium we do the following

- *Step 1:* We compute the households' decision rules. We do this using a piecewise linear decision rule method. The decision rule grid is not equally spaced. Given the very large range of possible asset holdings needed to achieve the observed wealth concentration, we use a large number of grid points. Specifically we use 100 grid points per realization of the shock, which gives us a total of about 800 grid points. In every iteration and in every gridpoint, a non-linear equation has to be solved. Monotonicity of this equation and the assumed piecewise linearity of the decision rules simplify the computations. However, in our model economy the labor decision is endogenous and this adds one extra level of complexity. For every evaluation of the Euler equation another non-linear equation has to be solved —the contemporaneous first order condition. Occasionally, this first order condition does not have a solution. Only when this happens, the feasibility constraint on the upper bound of leisure is binding. In this case we set the work effort to zero. See Ríos-Rull (1998) for details.
- *Step 2:* We compute the Markov process associated with the decision rules of each household-type that satisfies the necessary conditions for existence of a unique stationary distribution, x^* (see Aiyagari (1994) or Huggett (1995)). We approximate this distribution with a piecewise linearization of its associated distribution function. Again, the grid for this approximation has many points — exactly 80,000— and they are particularly close to each other near the origin. Again, see Ríos-Rull (1998) for details.
- *Step 3:* We compute the model economy's distributional and aggregate statistics. This step effectively involves the computation of integrals with respect to the stationary distribution, x^*. We evaluate these integrals directly using our approximation to the distribution function for every statistic except for those that measure mobility and the intergenerational correlation of earnings. To compute these other statistics we use a large sample of households drawn from x^*. Again see Ríos-Rull (1998) for details.