Centro de Altísimos Estudios Ríos Pérez

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CAERP

Documento de Trabajo #22

Working Paper #22

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This version: July 7, 2004

Abstract

We study the effects of nominal debt on the optimal sequential choice of monetary policy. When the stock of debt is nominal, the incentive to generate unanticipated inflation increases the cost of the outstanding debt even if no unanticipated inflation episodes occur in equilibrium. Without full commitment, the optimal sequential policy is to deplete the outstanding stock of debt progressively until these extra costs disappear. Nominal debt is therefore a burden on monetary policy, not only because it must be serviced, but also because it creates a time inconsistency problem that distorts interest rates. The introduction of alternative forms of taxation may lessen this burden, if there is enough commtiment to fiscal policy. If there is full commitment to an optimal fiscal policy, then the resulting monetary policy is the Friedman rule of zero nominal interest rates.

JEL Classification Numbers: E40, E50, E58, and E60

*We would like to thank Jośe-Victor Ríos-Rull, Jaume Ventura, Juan Pablo Nicolini and Isabel Correia for their comments, as well as the participants in seminars and conferences where this work has been presented. Corresponding author: Ramon Marimon; Universitat Pompeu Fabra; Ramon Trias Fargas, 25-27; 08005 Barcelona (Spain); <ramon.marimon@upf.edu>.

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1 Introduction

Fiscal discipline has often been seen as a precondition to sustain price stability. Such is, for example, the rationale behind the Growth and Stability Pact in Europe. More precisely, it is understood that an economy with a large stock of nominally denominated government debt can benefit from inflation surprises that reduce the need for distortionary taxation in the future. This means that optimal monetary policy under full commitment (the Ramsey policy) can be time inconsistent. In other words, if a government with the ability to honor its commitments were to re-optimize at a later date, it may choose to deviate from its original plan. In this context, a constraint on the level of debt may reduce the impact of such time-inconsistency distortions.

In this paper we study the effects of nominal debt on the optimal sequential choice of monetary policy We analyze a very stylized monetary model to show how optimal monetary policy differs depending on whether there is indexed or nominal debt and on the degree of commitment of monetary authorities. The structure of the optimal taxation problems that we solve is the following: first, we assume that the government has to finance a given constant flow of expenditures with revenues levied using only seigniorage. To solve these optimal taxation problems, the government chooses the paths of seigniorage that maximize the household's utility subject to the implementability and budget constraints. Unexpected inflations are costly, because we assume that the consumption good must be purchased with cash carried over from the previous period, as in Svensson (1985). This timing of the cash-in-advance constraint implies that, if the government decided to surprise the household with an unexpected increase in inflation in any given period, the household's consumption would be smaller than planned because its predetermined cash balances would be insufficient to purchase the intended amount of consumption. When considering whether or not to carry out such a surprise inflation, the government compares the reduction in the household's current utility that results from this lower level of consumption with the increase in the household's future utility that results from the reduction in future seigniorage.

In Section 2 we describing the model economy in detail. Then, in Section 3 we characterize the optimal policy that obtains when the outstanding stock of government debt is indexed, which is our benchmark case. Our results build on those of Nicolini (1998), who shows that, when the utility function is logarithmic in consumption and linear in leisure, and the stock of government debt is indexed, the optimal monetary policy is to abstain from inflation surprises. This result follows from applying optimal taxation principles, and it means that the solution to the Ramsey problem is time consistent in this model economy. Furthermore, this solution is also stationary, and there is a unique interest rate that balances the government budget.

Next, we study the optimal monetary policy that obtains when the outstanding stock of government debt is nominal. In Section 4, we assume that there is full commitment to monetary policy. We show that interest rates are kept constant from the second period onwards, but that the initial interest rate is higher, since it is optimal to cancel part of the inherited stock of nominal debt. In this case, after the initial period, the interest rate is lower than the one that obtains in the equilibrium with indexed debt, since the government cannot reduce the inherited stock of debt by increasing the initial price. We characterize the rational expectations Ramsey equilibrium in which there are no surprise inflations even in period zero This equilibrium has the property that initial real liabilities are the same as in the case of indexed debt. Since there is no "free lunch" surprise inflation, the equilibrium that obtains with nominal debt and full commitment is less efficient than the time consistent equilibrium with indexed debt.

Next, in Section 5, we study the optimal policy that obtains in the absence of commitment, and we present the main result of this article. In this case, we restrict our attention to the Markov perfect equilibrium. We call this equilibrium recursive as in Cole and Kehoe (1996), and in Obstfeld (1997). Two interesting features of the optimal policy that obtains in this recursive equilibrium are that the optimal inflation tax is non-stationary, and that it converges to the inflation tax that obtains when there is no government debt. This last result arises because, in the recursive equilibrium, it is optimal for the government to deplete the stock of nominal government debt until it is asymptotically zero. An implication of this result is that the optimal nominal interest is initially higher than the one that obtains when the stock of debt is indexed while, in the limit, it is lower. This decreasing path for the nominal interest rate is another indication that nominal debt is indeed a burden for monetary policy. Not only the stock of debt has to be serviced, but it also distorts interest rates. In fact, it is because the optimal policy endogenizes these distortions that it monetizes the entire stock of debt asymptotically in order to eliminate them.

Next, in Section 6, we carry out a numerical example and we describe our findings by meanse of a numerical comparison of the equilibria that obtain in the various regimes.

In Section 7 we ask whether these results are robust to the introduction of additional taxes. This is important since, in advanced economies, seignorage is a minor source of tax revenues, and we want to know if our results still hold when government outlays are financed with other taxes. Specifically, we study the case of consumption taxes. First, we impose the natural assumption that taxes are chosen before the monetary policy decisions are made. We find that the same equilibria result when there are both seignorage and consumption taxes than when there is only seignorage, provided that the optimal monetary policy distortions can be supported with strictly postive nominal interest rates. However, when there is enough fiscal commitment, the fiscal authority can constrain the monetary authority to follow the Friedman rule, of zero nominal rates, from the outset. In this case, since negative interest rates cannot be sustained in equilibrium, the monetary authority has no incentive to monetize the debt and, as a result, it implements the optimal equilibrium that obtains with indexed debt (see Marimon, Nicolini and Teles, 2003).

The relationship between fiscal and monetary policy has been addressed in the unpleasant monetarist arithmetic literature of Sargent and Wallace (1981), and in the fiscal theory of the price level of Sims (1994) and Woodford (1996). In these approaches, however, policies are taken to be exogenous. This is not the case in our analysis, nor in the related work of Chari and Kehoe (1999), Rankin (2002), and Obstfeld (1997). These last two papers are the closest to ours. Both, however, assume that debt is real, and they focus only on monetary policy. They aim at characterizing the Markov perfect equilibrium when the source of the time inconsistency of monetary policy is related to the depletion of the real value of money balances. This source of time inconsistency is ambiguous: while in Lucas and Stokey (1983) the government would want to completely deplete the outstanding money balances, in Svensson (1985)'s set up, as was shown in Nicolini (1998), under certain elasticity conditions, the government problem would be time consistent. This ambiguity led Obstfeld (1997) to consider an ad-hoc cost of a surprise inflation. Our analysis differs from Obstfeld's both because we consider nominal debt, and because, in our model economy, the cost of unanticipated inflation arises from the timing of the cash-in-advance constraint, rather than being imposed ad-hoc. In a similar framework, Rankin (2002) shows that the size of the initial debt matters for the direction of the time inconsistency problem, but he does not provide a full characterization of the resulting dynamic equilibrium. He shows that, for general preferences, there can be a value of debt where the elasticity is unitary and, therefore, that there exists a steady state with positive debt.

Finally, an additional contribution of this paper is the full characterization and the computation of the optimal policy in a recursive equilibrium with a state variable. In this respect, our work is closely related to the recent work of Krusell, Martín and Ríos-Rull (2003) who characterize the recursive equilibria that obtain in an optimal labor taxation problem.

2 The model economy

In our model economy there is a representative household and a government. The government issues currency, M^g_{t+1}, and nominal debt, B^g_{t+1}, to finance an exogenous and constant level of public consumption, g. Initially, we abstract from all other sources of public revenues. In each period $t \geq 0$ the government budget constraint is the following:

$$M^g_t + B^g_t(1+i_t) + p_t g \leq M^g_{t+1} + B^g_{t+1} \tag{1}$$

where i_t is the nominal interest rate paid on debt issued by the government at time $t-1$, and p_t is the price of one unit of the date t composite good. The initial stock of currency, M^g_0, and initial debt liabilities, $B^g_0(1+i_0)$, are given. A government policy is, therefore, a specification of $\{M^g_{t+1}, B^g_{t+1}, g\}$ for $t \geq 0$.

We assume that the household's preferences over consumption and labor can be represented by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t] \tag{2}$$

where $c_t > 0$ denotes consumption at time t, n_t denotes labor at time t, and $0 < \beta < 1$ is the time discount factor. We assume that the utility of consumption satisfies the standard assumptions of being strictly increasing and strictly concave. For reasons that will become clear below, in most of this article we assume that the utility is logarithmic in consumption, i.e., $u(c) = \log(c)$.

We assume that consumption in period t must be purchased using currency carried over from period $t-1$ as in Svensson (1985). This timing of the cash-in-advance constraint implies that the representative household takes both M_0 and $B_0(1+i_0)$ as given when solving its maximization problem, and it is crucial to obtain the results that we report here. The specific form of the cash-in-advance constraint faced by the representative household is:

$$p_t c_t \leq M_t \tag{3}$$

for every $t \geq 0$.

To simplify the productive side of this economy, we assume that labor can be transformed into either the private consumption good or the public consumption good on a one-to-one basis. Consequently, the competitive equilibrium real wage is $w_t^* = 1$, and the economy's resource constraint is:

$$c_t + g \leq n_t \tag{4}$$

both for every $t \geq 0$.

Each period the representative household also faces the following budget constraint:

$$M_{t+1} + B_{t+1} \leq M_t - p_t c_t + B_t(1+i_t) + p_t n_t \tag{5}$$

where M_{t+1} and B_{t+1} denote, respectively, the currency and the stock of nominal government debt that the household carries over from period t to period $t+1$.

Finally, we assume that the representative household faces a no-Ponzi games condition:

$$\lim_{T \longrightarrow \infty} \beta^T B_{T+1} = 0 \tag{6}$$

2.1 A competitive equilibrium

Definition 1 *A competitive equilibrium for an economy with nominal debt is a government policy, $\{M_{t+1}^g,\ B_{t+1}^g, g,\}_{t=0}^{\infty}$, an allocation $\{M_{t+1}, B_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, and a price vector, $\{p_t, i_{t+1}\}_{t=0}^{\infty}$, such that:*

(i) given M_0^g *and* $B_0^g(1+i_0)$*, the government policy and the price vector satisfy the government budget constraint described in expression (1);*

(ii) when households take M_0*,* $B_0(1+i_0)$ *and the price vector as given, the allocation maximizes the problem described in expression (2), subject to the cash-in-advance constraint described in expression (3), the household budget constraint described in expression (5), and the no-Ponzi games condition described in expression (6); and*

(iii) the price vector is such that all markets clear, that is: $M_t^g = M_t$*,* $B_t^g = B_t$*, and* g *and* $\{c_t, n_t\}_{t=0}^{\infty}$ *satisfy the economy's resource constraint described in expression (4), for every* $t \geq 0$*.*

When the stock of debt is indexed, outstanding government liabilities are fixed in real terms. Let $b_{t+1} = B_{t+1}^g / p_t$ be the real value of the end-of-period stock of debt. *A competitive equilibrium for an economy with indexed debt* is defined as a government policy, $\{M_{t+1}^g, b_{t+1}^g, g, \}_{t=0}^{\infty}$, an allocation $\{M_{t+1}, b_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, and a price vector, $\{p_t, i_{t+1}\}_{t=0}^{\infty}$, such that the conditions *(i)*, *(ii)* and *(iii)* of Definition 1 are satisfied when nominal liabilities are replaced by real liabilities. Specifically, in this case we assume that the initial real debt liabilities are $b_0\beta^{-1}$

Given our assumptions on the utility of consumption u, it is straightforward to show that the competitive equilibrium allocation of this economy satisfies both the economy's resource constraint (4) and the household's budget constraint (5) with equality, and that the first order conditions of the Lagrangian of the household's problem are both necessary and sufficient to characterize the solution to the household's problem. Furthermore, it is also straightforward to show that, when $i_{t+1} > 0$, the cash-in-advance constraint (3) is binding.

The competitive equilibrium allocation of an economy with nominal debt can be completely characterized by the following conditions that must hold for every $t \geq 0$:

$$\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1}, \tag{7}$$

$$1 + i_{t+1} = \beta^{-1}\frac{p_{t+1}}{p_t}, \tag{8}$$

$$c_t = \frac{M_t}{p_t}, \tag{9}$$

the government budget constraint (1), the resource constraint (4), and the no-Ponzi games condition (6).

These conditions imply that the intertemporal government budget constraint described in expression (10) is also satisfied in equilibrium.

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{\beta}{\alpha} - n_t \right) = \frac{B_0(1+i_0)}{p_0}. \tag{10}$$

The competitive equilibrium allocation of an economy with indexed debt is also completely characterized by (7), (8) and (9), but, in this case, expression (8) must be satisfied also in period zero (i.e., for $t = -1$). In other words, when debt is indexed and p_{t-1} is given, the government policy must be such that i_t adjusts to p_t in order to satisfy Fisher's equation even in period zero. The present value budget government constraint is the same as (10) in its revenue side, while its liabilities are given in real terms, independently of the initial price. That is, $\sum_{t=0}^{\infty} \beta^t \left(\frac{\beta}{\alpha} - n_t \right) = b_0 \beta^{-1}$.

In the remaider of this article, in order to make meaningful comparisons, we compare the indexed debt economy with initial liabilities $b_0\beta^{-1}$ with nominal debt economies whose initial real liabilities $\frac{B_0(1+i_0)}{p_0}$ are equal to $b_0\beta^{-1}$ *in equilibrium*.

3 Optimal policy with indexed debt

In this section we study the optimal policy when the stock of government debt is indexed (I). This is the benchmark against which we compare the optimal policy that obtains when the stock of government debt is nominal —that is, not indexed— which is the main focus of this article.

In the model economy with indexed government debt, the definition of an optimal monetary equilibrium is the following

Definition 2 *For a given level of government expenditures, g, and initial values of currency, M_0, and real government debt, b_0, an optimal monetary equilibrium with indexed debt (I) is a government policy, a price vector, and an implied allocation, such that: (i) the household utility is maximized, (ii) the government policy, the allocation, and the price vector are a competitive equilibrium with indexed debt.*

We follow the standard implementability approach of letting the government choose the allocation directly. That is, we use expressions (7), (8), and (9), the equilibrium conditions $M_t^g = M_t$ and $B_t^g = B_t$, and $b_{t+1} = B_{t+1}/p_t$ to replace the nominal variables from the government budget constraint (1).

Consequently, the household's utility is maximized when the government chooses the policy that implements the c_t and b_{t+1} that maximize expression (2), subject to the following implementability condition:

$$u'(c_{t+1}) c_{t+1} \frac{\beta}{\alpha} + b_{t+1} = c_t + b_t \beta^{-1} + g, \; t \geq 0 \tag{11}$$

and the no Ponzi games condition (6).

This problem is recursive only when $u(c) = \ln(c)$. In this case, the price elasticity is unitary and, as Nicolini (1998) has shown, the optimal monetary policy is time-consistent. This problem can be written recursively as follows:

$$V(b) = \max_{c,b'} \{\log(c) - \alpha(c+g) + \beta V(b')\} \tag{12}$$

subject to:

$$b' = c + \beta^{-1} b - \gamma, \tag{13}$$

where $\gamma \equiv \frac{\beta}{\alpha} - g$.

The first order condition for c is:

$$\frac{1}{c} - \alpha = -\beta V'(b') \tag{14}$$

The interpretation of this equation is that the marginal gain of increasing consumption is equated to the marginal cost of increasing future debt. Using the envelope theorem, we obtain that

$$V'(b) = V'(b'), \tag{15}$$

and, substituting expression (14) into this expression, we obtain that

$$\frac{1}{c} - \alpha = \frac{1}{c'} - \alpha \tag{16}$$

which implies that the optimal level of consumption, c_I^*, is constant and equal to:

$$c_I^* = \gamma - \left(\beta^{-1} - 1\right) b_0. \tag{17}$$

Notice that expression (15) implies that the real value of the government debt is stationary and, consequently, that $b_I^* = b_0$.

Finally, the stationary value of the nominal interest rate is:

$$1 + i_I^* = \left[\alpha\gamma - \alpha\left(\beta^{-1} - 1\right) b_0\right]^{-1}. \tag{18}$$

and the evolution of prices and currency are given recursively by: $p_{I,0}^* = M_0/c_I^*$ and $M_{I,t+1}^* = \beta p_{I,t}^*/\alpha$, and $p_{I,t}^* = M_{I,t}^*/c_I^*$.[1]

[1]Notice that this last equality has been obtained from expressions (7), (8) and (9).

4 Optimal policy with nominal debt and full commitment

In this section we study the optimal monetary policy that obtains when the outstanding stock of government is nominal —i.e., not indexed— and the government can fully commit to the Ramsey monetary policy (R) after a given initial period that we denote by $t = 0$. When the stock of debt is not indexed, the fact that consumption must be purchased with currency carried over from the previous period, implies that seignorage at $t = 0$ is a tax that can be levied without affecting the commitment to future interest rates. This may create an incentive for a Ramsey government to increase the initial seignorage tax at $t = 0$, and to use its proceeds to reduce the outstanding stock of debt.

Rational forward-looking households are aware of this incentive and, therefore, they would have anticipated it when making their previous period decisions. In terms of the initial level of consumption, this implies that the resulting optimal c_0 must have been correctly anticipated by the households or, equivalently, that the ex-ante and the ex-post real interest rates must coincide in a rational expectations equilibrium (see Chari and Kehoe, 1999).

To clarify this further, consider the government budget constraint in period zero:

$$p_0 g + M_0^g + B_0^g(1 + i_0) = M_1^g + B_1^g \tag{19}$$

Using the optimality conditions (7), (8), and (9) and the definition of b_1, expression (19) can be rewritten as:

$$g + c_0 + \frac{B_0^g(1 + i_0)c_0}{M_0^g} = \frac{\beta}{\alpha} + b_1 \tag{20}$$

When debt is nominal, a fully committed Ramsey government would choose the value of $c_0 = c^*_{R,0}$ that maximizes the representative household utility subject to (20).[2] This optimal choice would result in $p^*_{R,0} = M_0/c^*_{R,0}$ and the initial real liabilities would be $B_0^g(1+i_0)/p^*_{R,0} = b_0(1+r^*_{R,0})$, where $r^*_{R,0}$ is the ex-post real interest rate that is consistent with $c_0 = c^*_{R,0}$.

Let $\overline{p}_0$ and $\overline{c}_0$ be such that they satisfy:

$$\frac{B_0^g(1 + i_0)}{\overline{p}_0} = \frac{B_0^g(1 + i_0)}{M_0^g}\overline{c}_0 = b_0\beta^{-1}. \tag{21}$$

That is, $\overline{p}_0$ is the price that satisfies Fisher's equation (8) in period zero, and $\overline{c}_0$ is the optimal consumption plan for $t = 0$ that is consistent with an initial real interest rate $r_0 = \beta^{-1}$.

[2]Naturally, the government is subject to additional implementability constraints, one for each $t > 0$, that we describe below.

Now we impose the additional consistency condition that $p_0 = \overline{p}_0$ in equilibrium. In other words, we impose the condition that the decisions taken in the past must satisfy an ex-post, rational expectations, consistency condition in period zero. This requires that there must be a fixed point between expectations and realizations —or, equivalently, between the ex-ante and the ex-post real interest rates— implicitly defined by $c_0 = \overline{c}_0$.

Notice that imposing this additional consistency restriction in period zero prevents the "free lunches" that would result from surprise inflations, and allows us to compare the optimal policies that obtain in model economies with indexed debt and those in model economies with nominal debt. We can compare equilibria that result under different regimes having the same initial real government liabilities.[3]

Definition 3 *A **full commitment** Ramsey equilibrium with nominal debt (R) is a competitive equilibrium with nominal debt, and a value for the expected consumption at time $t = 0$, $\overline{c}_0$, that satisfy the following conditions:*

(i) given $\overline{c}_0$, the equilibrium allocation solves the following problem:

$$Max \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - \alpha(c_t + g)\right] \tag{22}$$

subject to the implementability constraints

$$\gamma + b_1 - c_0 - c_0 b_0 \frac{\beta^{-1}}{\overline{c}_0} = 0 \tag{23}$$

$$\gamma + b_{t+1} - c_t - \beta^{-1} b_t = 0, \text{ for } t \geq 1 \tag{24}$$

(ii) $c_0 = \overline{c}_0$.

This full commitment Ramsey equilibrium is characterized by the following conditions:

$$\frac{1}{c^*_{R,0}} - \alpha = \left[\frac{1}{c^*_{R,1}} - \alpha\right]\left[1 + b_0 \frac{\beta^{-1}}{c^*_{R,0}}\right] \tag{25}$$

$$c_{t+1} = c^*_{R,1}, \text{ for } t \geq 1. \tag{26}$$

Notice that, as long as $b_0 > 0$, $c^*_{R,0}$ is smaller than $c^*_{R,1}$, and that

$$c^*_{R,0} = c^*_{R,1} - \beta^{-1} b_0 (1 - \alpha c^*_{R,1}). \tag{27}$$

[3]Notice that, when the utility function is logarithmic in consumption, there is a one-to-one mapping between initial conditions (B_0, M_0) and b_0. Specifically, $b_0 \beta^{-1} = B_0/(\alpha M_0)$. This follows from the equalities $(1 + i_0)/p_0 = (1 + i_0)c_0/M_0 = 1/(\alpha M_0)$.

Moreover, from expressions (23) and (24), we obtain that:

$$c_{R,1}^{*}=\gamma\left[1+\alpha\left(\beta^{-1}-1\right)b_{0}\right]^{-1} \tag{28}$$

Finally, rewriting expression (25), we obtain that the path of the nominal interest rate is

$$i_{R,1}^{*}=\frac{i_{0}}{1+\frac{(1+i_{0})B_{0}}{M_{0}}} \tag{29}$$

It is useful to compare the full commitment Ramsey equilibrium allocation with the equilibrium allocation that obtains when debt is indexed, since indexed debt can be viewed as an extreme form of commitment. The full commitment Ramsey equilibrium with nominal debt is characterized by an ex-post nominal interest rate that is higher than the one that obtains in the optimal equilibrium with indexed debt in period $t=0$, and that is lower afterwards.

The reason for this is that the government wants to take advantage of the lump-sum character of monetizing part of the outstanding stock of nominal debt, since there is no time zero indexation that the government must internalize. In other words, in our economy with unitary price elasticity —$u(c)=\ln(c)$— nominal debt adjusts less than one-to-one to any price change. It follows that to cancel part of the stock of nominal debt in the initial period is part of the optimal tax policy. This is because in the full commitment Ramsey equilibrium expectations are given and the initial price increase does not question the government's commitment to the new optimal monetary policy.

When the stock of debt is indexed, the first order condition (16) requires that the marginal values of consumption are equated, even in period zero. In contrast, when the stock of debt is optimal and there is full commitment, the first order condition (25) shows that the marginal value of consumption in period zero is discounted, since a marginal reduction in c_0, results in a lower b_1.

Moreover, notice that, for any given b_0, the indexed debt solution, c_I^*, is a feasible solution for a fully committed Ramsey planner.[4] However, when the stock of debt is nominal, the fully committed Ramsey planner has an additional taxation instrument, namely to monetize part of the outstanding stock of debt, and c_I^* is not a best reply to the expectations $\bar{c}_0=c_I^*$. As it often happens with Nash equilibria, the fact that the government has an additional taxation instrument does not imply that the Ramsey equilibrium that obtains when the stock debt is nominal (i.e., $\bar{c}_0=c_{R,0}^*$) is more efficient than the equilibrium that obtains when the stock of debt is indexed. In fact, as the following proposition states, the converse is true.

Proposition 4 *Assume that $u(c)=\log(c)$. Then, for any given $b_0>0$, the optimal policy that obtains in an equilibrium with indexed debt is more efficient than the optimal policy that obtains in a full commitment Ramsey equilibrium with nominal debt.*

[4]In this case the expected initial consumption would be $\bar{c}_0=c_I^*$.

Proof. It is enough to show that the household's value obtained in the full commitment Ramsey equilibrium is lower than the value obtained in the equilibrium with indexed debt. That is,

$$
\begin{aligned}
& \left\{\log(c^*_{R,0}) - \alpha(c^*_{R,0} + g) + \beta(1-\beta)^{-1}\left[\log(c^*_{R,1}) - \alpha(c^*_{R,1} + g)\right]\right\} \\
< \quad & \left\{(1-\beta)^{-1}\left[\log(c^*_I) - \alpha(c^*_I + g)\right]\right\}
\end{aligned}
$$

Since preferences are linear in labor and strictly concave in consumption, the above inequality follows from Jensen's inequality as long as that $c^*_I = (1-\beta)c^*_{R,0} + \beta c^*_{R,1}$. But this equality follows immediately from the definitions of $c^*_{R,0}, c^*_{R,1}$, and c^*_I, i.e.,

$$
\begin{aligned}
& (1-\beta)c^*_{R,0} + \beta c^*_{R,1} \\
= \quad & (1-\beta)\left[c^*_{R,1} - b_0\beta^{-1}\left(1 - \alpha c^*_{R,1}\right)\right] + \beta c^*_{R,1} \\
= \quad & \left(\gamma - \left(\beta^{-1} - 1\right) b_0\right) \\
= \quad & c^*_I
\end{aligned}
$$

■

5 Optimal policy with nominal debt and no commitment

When the government cannot commit to its monetary policy, the incentive to monetize part of the debt discussed in the previous section arises every period and, consequently, the price level each period becomes a function $p_t = p(b_t, M_t)$. Since the representative household has rational expectations, it takes the government policy function as given. Furthermore, its expected future prices, $\bar{p}_t$, are formed in period $t-1$, and they are the same function of the state of the economy at the beginning of period t, i.e. $\bar{p}_t = p(b_t, M_t)$.

Consequently, in this case, the nominal interest rate will satisfy the following version of the equilibrium Fisher's equation:

$$
1 + i_t = \frac{\overline{p}_t}{\beta p_{t-1}} = \frac{p(b_t, M_t)}{\beta p_{t-1}} \tag{30}
$$

If we substitute expression (30) into the implementability condition which, in general, can be written as

$$
\gamma + b_{t+1} = c_t + b_t(1 + i_t)\frac{p_{t-1}}{p_t} \tag{31}
$$

we obtain

$$
\gamma + b_{t+1} = c_t + b_t\beta^{-1}\frac{\overline{p}_t}{p_t} \tag{32}
$$

And, since from the cash-in-advance constraint we have that $c_t = M_t/p_t$ and, consequently, that $\overline{c}_t = M_t/\overline{p}_t$, expression (32) can be rewritten as

$$\gamma + b_{t+1} = c_t + b_t\beta^{-1}\frac{c_t}{\overline{c}_t} \tag{33}$$

Notice that, once again, the optimal policy problem can be written as a recursive dynamic program with a single state variable, b_t. Specifically, the government has to find a policy, $c = C(b)$, that solves the following problem:

$$V(b) = \text{Max}\{\log(c) - \alpha(c+g) + \beta V(b')\} \tag{34}$$

s.t.

$$b' \leq c + b\beta^{-1}\frac{c}{\overline{C}(b)} - \gamma \tag{35}$$

and $C(b) = \overline{C}(b)$.

Definition 5 *A recursive monetary equilibrium for this economy (M) is a value function $V(b)$, policy functions $\{C^*(b), b^*(b)\}$, and a function $\overline{\mathrm{C}}(b)$ such that*

(i) Given function $\overline{\mathrm{C}}(b)$, the value function and the policy functions solve the problem described in expressions (34) and (35), and

(ii) $C^*(b) = \overline{\mathrm{C}}(b)$

To characterize the recursive monetary equilibrium, notice that the first order conditions of the problem described in expressions (34) and (35) are, first,

$$\frac{1}{c} - \alpha = -\beta V'(b')\left[1 + b\beta^{-1}\frac{1}{\overline{C}(b)}\right] \tag{36}$$

This condition equates the marginal gain of one additional unit of consumption to its marginal cost associated with higher debt needed to finance this consumption, plus the additional debt that results from the lower current period price level.

Second, using the envelope theorem, we obtain that

$$V'(b) = V'(b')\left[\frac{c}{\overline{C}(b)} - \frac{c}{\overline{C}(b)}\frac{b\overline{C}'(b)}{\overline{C}(b)}\right]. \tag{37}$$

Given that, in equilibrium $c = \overline{C}(b)$, expression (37) can be rewritten as:

$$V'(b) = V'(b')\left[1 - \epsilon_c(b)\right] \tag{38}$$

where $\epsilon_c(b)$ is the elasticity of function $C(b)$. That is, the marginal increase of b_t has value $V'(b_t)$, but the corresponding increase of b_{t+1} has two components, the direct effect of increasing the stock of debt —as in the indexed debt case— and the indirect effect that arises from the fact that higher values of debt are associated with higher interest rates, $\epsilon_c(b) \leq 0$. This higher costs arise because a larger stock of nominal debt increases the incentive to monetize the debt and, along a rational expectations equilibrium path, these additional distortions are anticipated.

Using expression (36), expression (38) becomes

$$\frac{\frac{1}{c} - \alpha}{\left[1 + b\beta^{-1}\frac{1}{c}\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + b'\beta^{-1}\frac{1}{c'}\right]}\left[1 - \epsilon_c(b')\right] \tag{39}$$

or, equivalently,

$$\frac{\frac{1}{c} - \alpha}{\left[1 + \frac{(1+i)B}{M}\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + \frac{(1+i')B'}{M'}\right]}\left[1 - \epsilon_c(b')\right] \tag{40}$$

In contrast with expression (16), that characterizes the optimal policy that obtains with indexed debt, and where the marginal values of consumption are simply equated, expression (40) shows that in a recursive monetary equilibrium with nominal debt, the marginal values of consumption must be discounted, since a higher consumption means a lower price level and, therefore, a higher stock of debt in the future.

Recall that, this discounting already showed up when the fully committed Ramsey planner evaluated the marginal value of consumption in period zero according to expression (25). Now, the uncommitted recursive planner reoptimizes every period and, therefore, the marginal values of consumption must be discounted every period, as long as there is outstanding debt.

Condition (40) also shows that the discounted marginal values of consumption are distorted further by the incentive to increase the current price that arises when the end-of-period stock of debt, b', is positive: $[1 - \epsilon_c(b_{t+1})]$.

In the previous section we have shown that the optimal policy that obtains when the stock of debt is indexed is more efficient than the optimal policy followed by a fully committed Ramsey planner. That the latter is more efficient than the optimal policy that obtains in a non-commitment recursive monetary equilibrium, follows from the standard argument of comparing the commitment and non-commitment policies achieved with the same tax instruments and rational expectations consistency conditions. Namely, the Ramsey planner can choose the recursive equilibrium allocation —satisfying the required consistency condition— but the recursive equilibrium allocation is dominated by the Ramsey equilibrium allocation.

6 Numerical solutions

To carry out our numerical example, we use the following values for the model economy parameters: $\alpha = 0.45$, $\beta = 0.98$, $b_0 = 0.17865$ and $g = 0.00822$. Notice that our period corresponds to a year and that we choose a very high level of nominal debt in relation to government expenditures ($b_0 \simeq 22g$). As we will see, results for lower values of the initial stock of debt can be obtained from our calculations. The results that we obtain for the time paths of the stocks of debt, nominal interest rates, and consumption in the three cases analyzed in the previous sections are reported in Figures 1, 2, and 3, respectively.

Our calculations confirm that the optimal monetary policy with indexed debt is indeed stationary, while this is not the case when debt is nominal (see Figures 1, 2 and 3). When the debt is indexed, its stock is time-invariant, while when the debt is nominal, it is optimal to reduce the initial stock. Under full commitment this reduction is only carried out in the first period, while under no commitment, the stock of debt is depleted progressively until it is completely cancelled (see Figure 1).

We also find that the long-run interest rate that obtains when debt is indexed is higher than those that obtain when debt is nominal. In this latter case, the long-run interest rate under full commitment is higher than the one that obtains when there is no commitment (see Figure 2).

Finally, when we compare the welfare levels in the three different regimes, we find that the value of the optimal consumption path is highest in the economy with indexed debt and no taxes. In particular, the value of the optimal consumption is 0.012% smaller when there is nominal debt and full commitment, and 0.133% smaller when there is nominal debt and no commitment.

The algorithm used to compute the solution is described in the Appendix and the code that implements the algorithm can be obtained from the first author upon request.

7 Additional taxes

In most advanced economies, seignorage is a minor source of revenue, and government liabilities are financed mostly with consumption and income taxes. This leads us to generalize our model economy to include consumption taxes, τ.[5] In this economy, a fiscal policy is a sequence $\{\tau_t\}_{t=0}^{\infty}$.

Since the consequences of fiscal policy take longer to be realized than those of monetary policy, we assume that tax rates are announced at the beginning of the period, before the monetary policy decisions are made. That is, given the current state of the economy (b_t, M_t, t), the fiscal authority chooses $\tau_t = \tau(b_t, M_t, t)$, and then the monetary

[5]As it will become clear from our analysis, the introduction of other taxes does not change the nature of the main results reported in this section.

authority chooses $p_t = p(b_t, M_t, \tau_t, t)$.

In this section we first study the case of an arbitrary fiscal policy that allows the monetary policy to adapt to it optimally choosing a path of nonnegative interest rates. Next, we study the case in which the fiscal authority can fully commit to an optimal policy, and we show that is part of such policy to finance all the outstanding government liabilities with the consumption tax, an to constrain the monetary authority to implement a zero nominal interest rate.

7.1 The model economy with consumption taxes

When the government levies consumption taxes, the household problem becomes:

$$\max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \alpha n_t\right] \tag{41}$$

subject to:

$$p_t(1 + \tau_t)c_t \leq M_t \tag{42}$$

$$M_{t+1} + B_{t+1} \leq M_t - p_t(1 + \tau_t)c_t + B_t(1 + i_t) + p_t n_t \tag{43}$$

and to:

$$\lim_{T \longrightarrow \infty} \beta^T B_{T+1} = 0 \tag{44}$$

Now, expressions (7), (8) and (9), that characterize the households's optimal choice, become:

$$\frac{u'(c_{t+1})}{\alpha} = (1 + i_{t+1})(1 + \tau_{t+1}) \tag{45}$$

$$1 + i_{t+1} = \beta^{-1} \frac{p_{t+1}}{p_t} \tag{46}$$

and

$$c_t \leq \frac{M_t}{p_t(1 + \tau_t)} \tag{47}$$

These conditions must hold for every $t \geq 0$. Notice that expression (45) reflects the fact that the household makes its plans based on its expectations about both interest rates and taxes. The intertemporal condition (46) is exactly the same as expression (45)[6] and the cash-in-advance constraint (47) now includes consumption taxes.

The intertemporal government budget constraint in this economy is now:

$$p_t g + M_t^g + B_t^g(1+i_t) \leq p_t \tau_t c_t + M_{t+1}^g + B_{t+1}^g \tag{48}$$

and the feasibility condition (4) does not change.

7.2 Optimal monetary policy when the fiscal authority moves first

We now consider the general case in which: (i) the fiscal authority moves first and its policy rule is $\tau_t = \tau(b_t, M_t, t)$; (ii) the monetary authority moves last and its policy rule is $p_t = p(b_t, M_t, \tau_t, t)$, and (iii) the household makes its plans in period $t-1$ based on expectations $\overline{p}_t = p(b_t, M_t, \tau_t, t)$. Specifically we assume that for all $t \geq 0$, the following rational expectations condition is satisfied:

$$1 + i_t = \frac{\overline{p}_t}{\beta p_{t-1}} = \frac{p(b_t, M_t, \tau_t, t)}{\beta p_{t-1}} \tag{49}$$

planned consumption $\bar{c}_t$ satisfies expression (45), and the cash-in-advance constraint (47) is satisfied with equality for $i_t > 0$.

With this general formulation, when the stock of debt is indexed, $p_t = \overline{p}_t$ must be satisfied for all $t \geq 0$. When the stock of debt is nominal and the monetary authority is fully committed, $p_t = \overline{p}_t$ must be satisfied for all $t \geq 1$, while $p_0 = \overline{p}_0$ must be satisfied only in equilibrium. Finally, when the stock of debt is nominal and there is no commitment to monetary policy, $p_t = \overline{p}_t$, must be satisfied for all $t \geq 0$ only in equilibrium.

In the economy with consumption taxes the general implementability condition is

$$u'(c_{t+1})\, c_{t+1} \frac{\beta}{\alpha} + b_{t+1} = b_t \beta^{-1} \frac{c_t}{\bar{c}_t} + c_t + g \tag{50}$$

which, when the utility of consumption is logarithmic, simplifies to

$$\gamma + b_{t+1} - c_t - b_t \beta^{-1} \frac{c_t}{\bar{c}_t} = 0 \tag{51}$$

[6] Notice that when the government uses labor taxes, $\tau_t^n = \tau^n(b_t, M_t, t)$, expression (46) becomes $1 + i_{t+1} = \beta^{-1} \frac{p_{t+1}(1-\tau_{t+1}^n)}{p_t(1-\tau_t^n)}$ and, therefore, the equilibrium nominal interest rate will be affected by fiscal policy. As far as our results are concerned, this change makes a difference only when the stock of debt is indexed debt and the fiscal authority is not fully committed.

Notice that this expression is exactly the same as expression (33).

Now, the following additional restrictions must be satisfied: *(i)* when the stock of debt is indexed, $\frac{c_t}{\tilde{c}_t} = 1$ must be satisfied for all $t \geq 0$ along any path (i.e. both in and out of equilibrium); *(ii)* when the stock of debt is nominal and there is full commitment to monetary policy, $\frac{c_t}{\tilde{c}_t} = 1$ must be satisfied for all $t \geq 1$ along any path, while $\frac{c_0}{\tilde{c}_0} = 1$ is only a Ramsey equilibrium restriction; and *(iii)* when the stock of debt is nominal and there is no commitment to monetary policy, $\frac{c_t}{\tilde{c}_t} = 1$ for all $t \geq 0$ is only a recursive equilibrium restriction.

Consequently, the monetary authority faces the same problem with consumption taxes than the one faced when there was only seignorage, for any degree of monetary commitment. Therefore, the allocations that obtain for the various types of debt and monetary policy commitment technologies are exactly the same as those that obtained before. This result is established in the following subsections:

Consumption taxes and indexed debt. In this case policies are stationary and we obtain the stationary equilibrium allocation $c_I^* = \gamma - \left(\beta^{-1} - 1\right) b_0$ described in Section 3. Notice, however that now interest rates $\tilde{\imath}_t = i(b_t, \tau_t)$ are set as to satisfy:[7]

$$\frac{u'(c^*)}{\alpha} = [1 + i(b_t, \tau_t)](1 + \tau_t) \tag{52}$$

And when the stock of debt is indexed the time-invariant equilibrium nominal interest rate is

$$\tilde{\imath}_{I,t} = \tilde{\imath}_I(b_0, \tau_t) = \left[\alpha\left(\gamma - \left(\beta^{-1} - 1\right) b_0\right)(1 + \tau_t)\right]^{-1} - 1 \tag{53}$$

Where, as we have already mentioned, we assume that $\tilde{\imath}_I(b_0, \tau_t) \geq 0$. The equilibrium paths of prices and currency are recursively given recursively by: $\tilde{p}_t = \frac{\tilde{M}_t}{c^*(1+\tau_t)}$ and $\tilde{M}_{t+1} = \frac{\beta}{\alpha}\tilde{p}_t$.

Consumption taxes, nominal debt and full commitment to monetary policy. In this case we obtain the Ramsey equilibrium allocation $c_{R,0}^* = c_{R,1}^* - \beta^{-1} b_0 (1 - \alpha c_{R,1}^*)$ and $c_{R,1}^* = \gamma\left[1 + \alpha\left(\beta^{-1} - 1\right) b_0\right]^{-1}$ described in Section 4.

Now, the equilibrium interest rates are

$$\tilde{\imath}_{R,0} = \tilde{\imath}_R(b_0, \tau_0, 0) = \left[\alpha c_{R,0}^*(1 + \tau_0)\right]^{-1} - 1 \tag{54}$$

[7] Henceforth we use tildes to distinguish the variables of the model economy with consumption taxes from the corresponding variables of the model economy without consumption taxes.

and

$$\tilde{\imath}_{R,t} = \tilde{\imath}_R(b_t, \tau_t, t) = \left[\alpha c^*_{R,1}(1+\tau_t)\right]^{-1} - 1 \tag{55}$$

for all $t \geq 1$, and where we assume that with $\tilde{\imath}_R(b_t, \tau_t, t) \geq 0$ for all t.

Finally, the intertemporal condition between interest rates in period zero —that corresponds to expression (29)— is

$$\frac{(1+\tilde{\imath}_{R,0})(1+\tau_0) - 1}{1 + (1+\tilde{\imath}_{R,0})(1+\tau_0)\frac{B_0}{M_0}} = (1+\tilde{\imath}_{R,1})(1+\tau_1) - 1 \tag{56}$$

Consumption taxes, nominal debt and no commitment to monetary policy. In this case policies are also stationary and we obtain the recursive equilibrium allocation described in Section 5. Specifically, the intertemporal condition (40) now takes the form:

$$\frac{\frac{1}{c} - \alpha}{\left[1 + (1+\tilde{\imath}_M)(1+\tau)\frac{B}{M}\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + (1+\tilde{\imath}'_M)(1+\tau^{c\prime})\frac{B'}{M'}\right]}\left[1 - \epsilon_c(b')\right] \tag{57}$$

It follows that in the economy with nominal debt and no commitment to monetary policy, the path of depletion of the stock of debt in real terms coincides with the one characterized in Section 5 and computed in Section 6 even though the consumption tax revenues would allow for a faster debt depletion rate.

7.3 Optimal fiscal policy with commitment

In the three regimes discussed in the previous section exactly how the equilibrium allocations are supported is indetermined since the household only cares about the effective nominal rate of return, $(1+i)(1+\tau)$. For instance, it is always possible to set taxes in a way that the resulting monetary policy follows the Friedman rule of zero nominal interest rates, even though in our economy there is no efficiency gain from following such a rule.[8] More precisely, as long as monetary responses to realized fiscal policies result in non-negative interest rates, fiscal policy is not effective in this economy since the monetary authority can adapt to any fiscal policy in order to support the allocation that obtained the economy without consumption taxes. Moreover, such adaptation is the optimal policy in every case. But this may not be the only scenario in which monetary authorities operate.

[8]This may not be true in a more general model economy. For instance, this is not true if we introduce a distinction between cash and credit goods. In this case, the Friedman rule would eliminate the distortion between cash and credit goods created by the cash-in-advandce constraint. This notwithstanding, the distortions introduced by the presence of a positive stock of nominal debt would still be there, just as in the economy with only cash goods.

To see this, suppose that the stock of debt is nominal and that there is full commitment to monetary policy. Let the fiscal authority set $\tau(b_t, M_t, t) = \tau^*(b_0)$, where $\tau^*(b_0)$ corresponds to the tax rate that fully finances the government liabilities in the allocation that obtains with indexed debt. That is,

$$(1+\tau^*(b_0)) = \frac{u'(\gamma - \left(\beta^{-1}-1\right) b_0)}{\alpha} = \frac{u'(c_I^*)}{\alpha} \tag{58}$$

If the monetary authority tries to monetize part of the existing stock of nominal debt and to use the resulting revenues to increase future consumption —say, maintaining a constant c_1— then, it must be the case that $c_0 < c_I^* < c_1$. But such allocation requires that $(1+\tilde{\imath}_{R,0})(1+\tau^*(b_0)) > (1+\tau^*(b_0)) > (1+\tilde{\imath}_{R,1})(1+\tau^*(b_0))$ which implies $\tilde{\imath}_{R,0} > 0 > \tilde{\imath}_{R,1}$. However, negative interest rates can not be an equilibrium in this economy since then the household would like to borrow unboundedly. Therefore, given that it is not possible to raise future consumption with negative taxes, there is no gain in partially monetizing the stock of nominal debt in period zero. It follows that, if the fiscal authority wants to maximize expression (2), it will set $\tau(b_t, M_t, t) = \tau^*(b_0)$. The same argument applies when there is no commitment to monetary policy.[9] The following proposition summarizes this result:

Proposition 6 *Assume that fiscal authorities maximize the welfare of the representative household and can fully commit to their policies. Then the equilibrium allocation is the optimal equilibrium allocation that obtains when the stock of debt is indexed, regardless of the nature of the debt and independently of the degree of commitment of the monetary authority.*

8 Concluding comments

This paper discusses the different ways in which nominal and indexed debt affect the sequential choice of optimal monetary and debt policies. To this purpose, we study a general equilibrium monetary model where the costs of an unanticipated inflation arise from a cash-in-advance constraint with the timing as in Svensson (1985), and where government expenditures are exogeneous. In our environment, as in Nicolini (1998), when the utility function is logarithmic in consumption and linear in leisure and debt is indexed, there is no time-inconsistency problem. In this case, the optimal monetary policy is to maintain the initial level of indexed debt, independently of the level of commitment of a Ramsey government.

In contrast, for the same specification of preferences, when the initial stock of government debt is nominally denominated, a time inconsistency problem arises. In this case, the government is tempted to inflate away its nominal debt liabilities. When the government cannot commit to its planned policies, the optimal sequential policy consists

[9] Marimon, Nicolini, and Teles (2003) make a similar argument.

in progressively depleting the outstanding stock of debt, so that it converges asymptotically to zero. Optimal nominal interest rates in this case are also decreasing, and they converge to zero, as long as there is no need to use seignorage to finance government expenditures different from debt servicing. Hence, the optimal monetary policy in this economy coincides in the long term with the one that obtains in an economy which has no outstanding debt, and from which these time-inconsistency distortions are obviously absent.

Such equilibrium path is not chosen when the initial stock of government debt is nominally denominated and the government can fully commit to its planned policies. In this case, it is optimal to increase the inflation tax in the first period, and to keep a lower and constant inflation tax for the rest of the future.

We show that in the rational expectations equilibria of our economies there are no surprise inflations and that, for a given initial level of outstanding debt, the most efficient equilibrium is the one that obtains when debt is indexed, the equilibrium with nominal debt and full commitment comes second, and the equilibrium with nominal debt and no commitment is the least efficient. This result highlights the sense in which nominal debt is indeed a burden on optimal monetary policy.

It should be noted that the source of the inefficiencies and of the monetary policy distortions discussed in this paper is not the desire to run a soft budgetary policy that increases the debt liabilities of the government. Every policy discussed in this article is an optimal policy, subject to the appropriate institutional and commitment constraints, and it is implemented by a benevolent and far-sighted government who does not face either uncertainty or the need for public investment, and who would, therefore, prefer to reduce debt liabilities. The source of the inefficiencies is the distortion created by the lack of commitment that results from the mere exsitence of an outstanding stock of nominal debt. Therefore, our results highlight the need to implement policy and institutional arrangements that either guarantee high commitment levels, or that reduce the allowed levels of nominal debt. This notwithstanding, our results also show that a constraint on deficits may be ineffective to reduce the distortions created by nominal debt since they are independent of the size of the deficits.

The introduction of additional forms of taxation further clarifies the interplay between the various forms of debt and commitment possibilities. Under the natural assumption that monetary choices are made after the tax rates have been decided, we show that the equilibrium allocations that obtain when we introduce consumption taxes are the same as that obtain when there is only seignorage, provided that the revenues levied through seignorage are enough to allow for an optimal monetary policy with non-negative interest rates. However, as in Marimon, Nicolini and Teles (2003), if there is full commitment to an optimal fiscal policy, the fiscal authorities, anticipating monetary policy distortions, choose to fully finance government liabilities, and the resulting monetary policy is the Friedman rule of zero nominal interest rates. Moreover, this policy results in the efficient equilibrium that obtains in the economy with indexed debt.

In summary, we show that fiscal discipline may be needed to achieve efficiency and price stability, even when monetary authorities pursue optimal policies. However, our analysis shows that fiscal discipline applies to the level of the debt and not to the level of the deficit or, alternatively, to the issuing of indexed debt.

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Appendix: Computation

In order to compute the recursive monetary equilibrium defined in Section 5, we must solve the following dynamic program:

$$V(b) = \max_{c,b'}\{\log(c) - \alpha(c+g) + \beta V(b')\} \tag{59}$$

s.t.

$$b' \leq c + b\beta^{-1}\frac{c}{\overline{C}(b)} + g - \frac{\beta}{\alpha} \tag{60}$$

for a given $\overline{C}(b)$.

However, computational considerations lead us to solve the following transformed problem:

$$V(x) = \max_{c,x'}\{\log(c) - \alpha(c+g) + \beta V(x')\} \tag{61}$$

s.t.

$$\beta x'\hat{C}(x') \leq c(1+x) + g - \frac{\beta}{\alpha} \tag{62}$$

for a given $\hat{C}(b)$ and where $x = b/\beta\hat{C}(x)$.

In order to solve this problem we use the following algorithm:

- Step 1: Define a discrete grid on x
- Step 2: Define a decreasing discrete function $\hat{C}_0(x)$
- Step 3: Iterate on the Bellman operator described in equations (35) and (62) until we find the converged $V^*(x)$, $x'^*(x)$,$c^*(x)$
- Step 4: If $c^*(x) = \hat{C}_0(x)$, we are done. Else, let $\hat{C}_0(b) = c^*(b)$ and go to Step 3.

Finally, to recover the policy functions of the original problem we undo the transformation as follows: From $x = b/\beta\hat{C}(x)$, we obtain that $\hat{b}(x) = \beta x\hat{C}(x)$, which can be computed directly from the solution to the transformed problem described above. Next we invert $\hat{b}(x)$ and we obtain $x = \hat{b}^{-1}(b)$. Finally, we use this expression to obtain $C(b) = \hat{C}[\hat{b}^{-1}(b)]$ and $b'(b) = \hat{b}\{x'[\hat{b}^{-1}(b)]\}$.

Figure 1: The optimal stocks of indexed debt and of nominal debt with full commitment and with no commitment

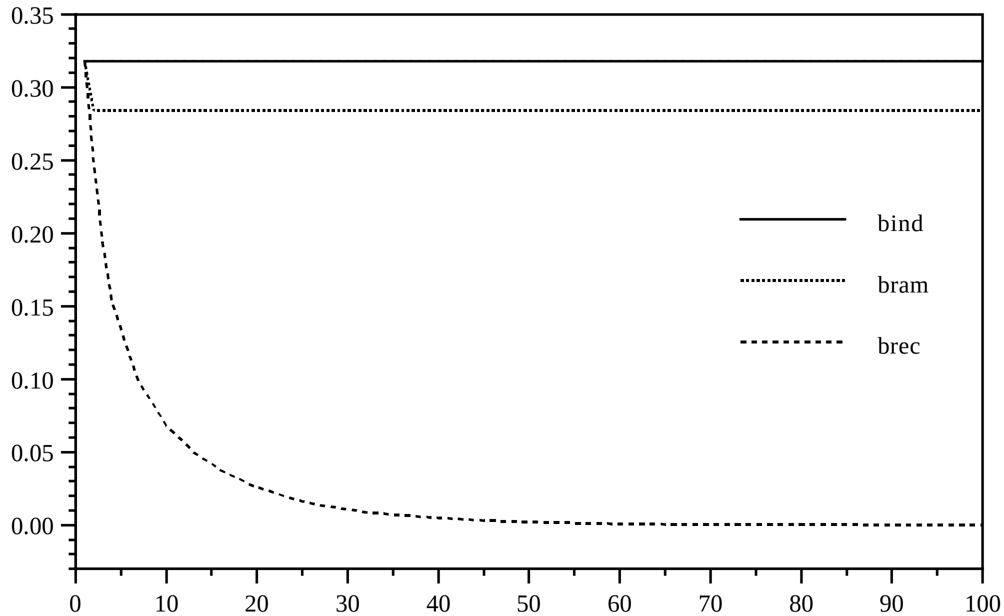

Figure 2: The optimal paths of nominal interest rates with indexed debt and with nominal debt with full commitment and with no commitment

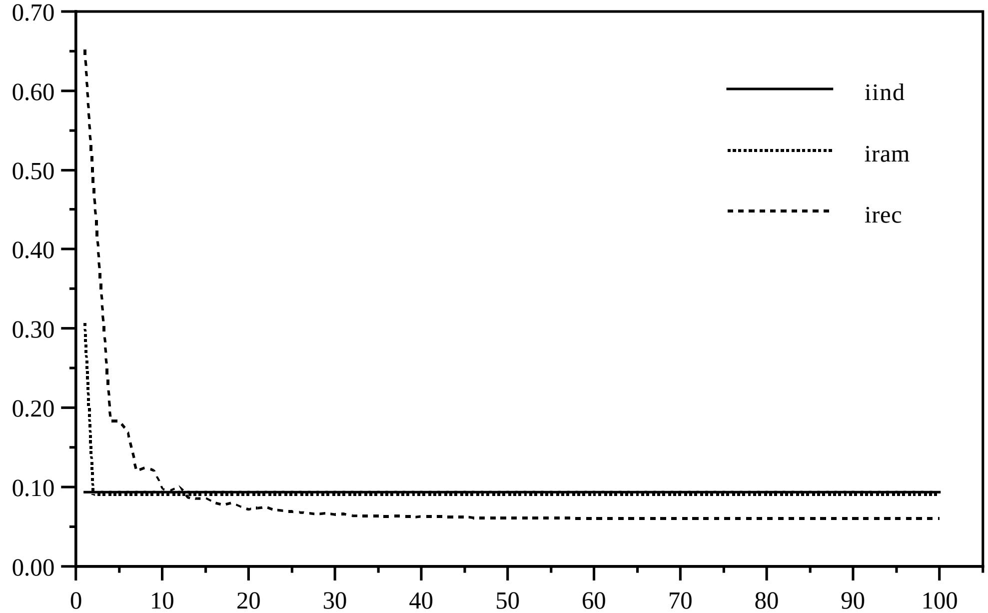

Figure 3: The optimal paths of consumption with indexed debt and with nominal debt with full commitment and with no commitment