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Abstract

Habit formation has been proposed as a possible solution for explaining the equity premium puzzle. This paper extends the class of models that support the habits explanation in order to account for heterogeneity in earnings, wealth, habits and consumption. I find that habit formation increases the equity premium. However, contrary to the earlier results in the literature, the habit hypothesis does not imply a price for risk much higher than the one implied by models with intertemporally separable preferences. The main reasons for this are general equilibrium ones. First, with just two assets available, households can smooth out consumption fluctuations very well. Therefore, the higher utility losses of uncertainty imposed by habits will not command a high price of risk because households manage to avoid this risk. Second, the composition of the set of agents pricing the assets is sensitive to changes in the model. In an economy with habits, pricing agents turn out to be households facing very small consumption fluctuations. In addition I characterize three important properties of the model economy that relate to portfolio choice: willingness to hold risky assets (1) increases with wealth, (2) decreases with labor earnings and (3) decreases with habit stock

Keywords: Equity Premium; Habit Formation; Incomplete Markets

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1 Introduction

The equity premium puzzle, as stated by Mehra and Prescott (1985), uncovers the inability of the standard macroeconomic models to generate, for given parameter constraints, a differential return between risky and risk free assets as large as the one found in data. In other words, quantitative macroeconomic models produce a compensation for risk that is too small compared to its empirical counterpart.

One strand of the literature on asset pricing has proposed habit formation as an explanation for the equity premium. Habit formation increases the utility losses from consumption fluctuations and therefore increases the compensation for risk required to hold risky assets. Habit formation is an interesting preference hypothesis because it disentangles preferences for consumption in different states of the world and preferences for consumption in different points in time. This feature allows to increase the curvature over the risk dimension while keeping fixed the curvature over the time dimension and therefore not generating counterfactual risk free rates. Constantinides (1990), Abel (1990), Heaton (1995), Campbell and Cochrane (1999) and Boldrin, Christiano, and Fisher (1997) show how the extra parameters introduced by the habit formation hypothesis can be used to match the data on equity premium.

In contrast to this partial equilibrium approach, there have been some attempts to show that habit formation in representative agent general equilibrium models does not bring big equity premia. Jermann (1998) shows in an equilibrium model without labor-leisure choice that the path of consumption generated in equilibrium once we add habit formation turns out to be too smooth. The same result is shown by Lettau and Uhlig (2000) once labor-leisure choice is included. This is the basic critique to the standard result of habit formation as a solution of the equity premium puzzle: if habit formation makes households dislike consumption fluctuations to a higher degree, where do the observed consumption fluctuations come from? In other words, households facing higher disutility of consumption fluctuations should change their decisions in order to face a smoother consumption profile, one that would be inconsistent with the observed consumption series. Jermann (1998) and Boldrin, Christiano, and Fisher (2001) propose a similar solution: do not let agents smooth their consumption fluctuations. Jermann (1998) introduces high adjustment costs for capital whereas Boldrin, Christiano, and Fisher (2001) considers a two-sector model with limited resource flexibility. In this way, households get prevented from smoothing consumption and habit formation will make them require a high premium for holding risky assets.

The original formulation of the puzzle, and the papers aforementioned, relied on the representative agent hypothesis, which implies that the consumption fluctuations faced by agents are equal to the fluctuations of aggregate consumption. However, consumption fluctuations at the household level can be much higher than its aggregate counterpart. In this direction, another branch of the asset pricing literature has attempted to allow for agents to differ in their earnings, wealth holdings and consequently in the consumption fluctuations they face. A higher variability of individual consumption may allow the equity premium to increase. Heaton and Lucas (1996) and Krusell and Smith (1997) show that this type of models can deliver a price of risk similar to the one found in data. However, this is the case only if transaction costs or tight borrowing constraints are included. Without them, an asset structure of just risk free bonds and shares suffices to smooth out consumption fluctuations well enough so that the associated fluctuations in marginal utility do not command a high return on risky assets. Again, the answer seems to be that unless we do not prevent agents from smoothing out the fluctuations in their income equilibrium models cannot account for the observed equity premium.

One open question, therefore, is whether the interaction between habit formation and heterogeneity can close the bridge. This paper includes the habit formation hypothesis in a standard general equilibrium model with heterogenous agents and incomplete markets. In the model economy, agents differ in habit stocks as well as earnings and wealth holdings. The source of these differences is the absence of markets to insure against idiosyncratic shocks to labor income. This exercise extends the class of models used so far to show that habit formation can be a good explanation for the equity premium by allowing for heterogeneity in earnings and wealth and for consumption decisions of households. Instead of looking at the predictions for the equity premium once certain statistics on consumption data are given, it does so for given properties of earnings data. Consumption fluctuations are therefore derived as the result of optimal decisions of households.

The paper will look at results for the market price of risk instead of the equity premium. As it is shown by Krusell and Smith (1997) (following Hansen and Jagannathan (1991)) the equity premium can be decomposed into two terms: the market price of risk (also known as Sharpe ratio) and the amount of risk. The amount of risk in the economy is given by the standard deviation of the returns on shares. In order to have a rich structure on the consumer side, the production side is kept simple, which means that a single parameter (the volatility of the aggregate shock) drives two important moments of the model: the amount of risk and the volatility of aggregate series.[1] I will calibrate this parameter to the volatility

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[1] In a related paper, Storesletten, Telmer, and Yaron (2001) consider an stochastic depreciation rate for capital to break this link.

of the aggregate series (basically aggregate consumption) and will leave the amount of risk of the economy free, which turns out to be nowhere around its empirical counterpart. This is why I do emphasize the results on the market price of risk and do not look at the equity premium itself.

The main result is that habit formation hypothesis cannot reproduce the empirical consumption fluctuations needed to deliver high market prices for risk. Compared to the model without habits, what I find is that habit formation increases the Sharpe ratio by as much as 45%. However, the value found is still below its empirical estimates. The main reasons for this result are general equilibrium ones. The first reason is that, as it happens with a representative agent framework, households prevent their consumption profile from fluctuating much. Instead of sitting and demanding a big compensation to hold risky assets they make sure they do not face such big consumption fluctuations. Precautionary savings are increased and income fluctuations are smoothed out. The result is that the degree of consumption smoothness achieved is high enough to prevent the habit formation preferences from generating large fluctuations in marginal utilities. To be precise, the average over the model population of next period's expected consumption fluctuations falls by more than 50% when adding habits to the standard model. The second reason for this result is related to the composition of the set of agents pricing the assets. In this type of economy not everybody solves their portfolio choice problem with interior solution. Typically, poorly self-insured agents, seeking for a hedge against risk, borrow as much as they can in risky assets to invest in bonds. On the contrary, well insured agents go as short as they can with bonds and invest in risky assets to obtain higher expected payoffs. Between these two limit cases, a fraction of the population in the model have an interior solution to the portfolio choice problem. These agents form what it will be called the set of pricing agents. As one increases the disutility of risk (introduction of habit formation), the composition of the set of pricing agents changes. It will be better insured agents who will have interior solution. Therefore, not only average consumption fluctuations fall but also the composition of the set of agents that matter for pricing risk changes towards agents that face relatively lower consumption fluctuations. The trade of risk among households turns out to be a quantitative important factor.

The result is robust to increases in consumption fluctuations. If we look at a habits economy recalibrated so that aggregate consumptions fluctuations are the same as in a non-habit economy the Sharpe ratio increase to almost three times its size without habit formation but still staying below empirical estimates. Therefore, it is not only the reduction in aggregate consumption fluctuations that matters but also the trade of risk between different types of households.

As pointed out by Mankiw (1986) and Constantinides and Duffie (1996), the interaction between idiosyncratic uncertainty and aggregate uncertainty is crucial for the model to generate big equity premia. The heterogeneous agents model adds value to the representative agent framework if the addition of idiosyncratic risk affects the price of the risky asset. For this to happen, the source of uncertainty for the risky asset (the aggregate shock) has to be somehow related to the idiosyncratic uncertainty. In particular, it is required that the variance of the process for individual earnings is higher in downturns than in peaks. In this way, equity turns out to be an asset that pays well when idiosyncratic uncertainty is low (in peaks earnings are less volatile) and pays bad when idiosyncratic uncertainty is high (in downturns earnings are more volatile).[2] The model in this paper captures this feature by calibrating an employment process that generates higher and longer unemployment rates in downturns.[3]

The paper also makes a novel contribution in the technical side. To the author's knowledge this is the first attempt to solve for an equilibrium where the state space of the household problem contains a distribution of agents and two endogenous individual state variables. The decisions of households depend on their amount of wealth, history of past consumption, idiosyncratic shock and the distribution of the economy's households over these variables, together with the aggregate shock.

An interesting feature of the model is that it allows us to look at the portfolio choice of the agents. The results are as follow: willingness to hold risky assets (1) increases with wealth, (2) decreases with labor earnings and (3) decreases with habit stocks. The first result is consistent with empirical studies. In the model, wealthier households face lower uncertainty since labor earnings represent a lower fraction of their income and they are further away from their borrowing constraint. The second result may be surprising. However, it should not be so. Once controlling for wealth, the role of the current earnings shock is solely to predict future earnings. Agents with high earnings also expect higher earnings in the future if, and only if, they remain employed. Since the source of earnings uncertainty related to the aggregate shock is the probability of being unemployed, they have more to lose by a downturn. The earnings variability conditional on aggregate shock is higher the higher the earnings level. Finally, the third result is also as expected. Households with higher habit stocks dislike fluctuations in consumption in a higher degree than households with lower

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[2] With a convex marginal utility this implies that households will require a higher compensation to face aggregate risk than in the representative agent economy. See Krusell and Smith (1997).

[3] The interaction between earnings and aggregate shock turns out to be an empirical question. Is the variance of earnings negatively related to the cycle? Looking at PSID data, Storesletten, Telmer, and Yaron (2001) answer in the affirmative.

habit stocks. Therefore, they are less willing to hold risky assets.

The rest of the paper continues as follows. Section 2 develops and details the model economy. Then, section 3 shows how I calibrate it to US data. Section 4 finds the optimality conditions and explains their implications. Then section 5 simulates a representative agent version of the model. Section 6 states the benchmark heterogeneous economy and discusses the results compared to the representative agent model and to a world with tight borrowing constraints. Section 7 analyzes the interaction between habit formation and heterogeneous agents. Finally, section 8 concludes. The computational method, explained in section 4, is presented in more detail in the appendix A.

2 The model economy

The basic structure of the economies in this paper is the standard growth model with aggregate uncertainty and heterogeneous agents with incomplete markets. The market incompleteness is the lack of insurance for the idiosyncratic shocks to labor earnings.

2.1 Preferences

Preferences are stated generally to include habit formation. Non-habits preferences can be represented as a special parametric case. Households derive utility from both present and past consumption. Present consumption will be denoted by $c \in \mathcal{R}_+$ and past consumption will accumulate in a stock of habits denoted by $h \in \mathcal{R}_+$. The habit stock evolves according to the law of motion $h_{t+1} = \psi(c_t, h_t)$ with partial derivatives $\psi_c \in (0, 1]$ and $\psi_h \in [0, 1)$. Per period utility will be denoted by $u(c_t, h_t)$. Standard conditions on u_c apply. Habit formation hypothesis requires $u_h < 0$ and $u_{ch} > 0$. Agents are infinitely-lived and their total utility will be the infinite discounted sum of period utilities: $\sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t\right)$. Preferences are identical over households.

2.2 Technology

Output Y is produced using aggregate capital K and aggregate labor L. Output can be either consumed or invested to form productive capital. Capital is the only productive asset and depreciates at an exogenous rate $\delta \in [0, 1]$. Aggregate labor is the sum of the economy's efficiency units of labor. We write the constant returns to scale production function as $F(z, K, L)$, where $z \in Z \equiv \{z_g, z_b\}$ is an exogenous stochastic technology level. It follows a Markov process represented by the transition probability matrix $\Gamma_z(z, z') \equiv \Pr(z_{t+1} = z' | z_t = z)$.

Households are subject to idiosyncratic shocks to labor earnings. I decompose the idiosyncratic risk into two parts. First, there is the employment shock. Employment possibilities $e \in E = \{0, 1\}$ come stochastically and depend on the aggregate technology level z. At every period of time, households may $(e = 1)$ or may not $(e = 0)$ be given an employment opportunity. Since agents do not value leisure the employment opportunity will be taken. Conditional on z and z' the process is iid and Markov with transition matrix $\Gamma_e(z, z', e, e') \equiv \Pr(e_{t+1} = e' | e_t = e, z_t = z, z_{t+1} = z')$. Second, when given an employment opportunity, agents also get an endowment of efficiency units of labor. Efficiency units of labor, represented by $\xi \in \Xi \equiv \{\xi_1, \xi_2, ..., \xi_{n_e}\}$, follow an iid process with Markov transition matrix $\Gamma_\xi(\xi, \xi') \equiv \Pr(\xi_{t+1} = \xi' | \xi_t = \xi)$. Notice that this process is independent of the aggregate shock z. When not given an employment opportunity households are assumed to operate a home technology that provides them with d units of consumption.

Since there is a continuum of households, a law of large number applies and the share of employed and unemployed people is only function of the aggregate shock. Total population is normalized to one and the unconditional expectation of efficiency units of labor $E[\xi]$ is also normalized to one. Unemployment rates of the economy in good and bad times are called u_g and u_b. Therefore, the amount of efficiency units L in the economy is a function $L(z)$ of the aggregate shock, with $L(z_g) = 1 - u_g$ and $L(z_b) = 1 - u_b$.

2.3 Market arrangements

Agents and firms trade two different assets: one-period risk free claims to consumption units $b \in B \equiv [\underline{b}, \infty)$ and shares for firm's ownership $s \in S \equiv [\underline{s}, \infty)$. One unit of the risk free bond b entitles a known payment of R^b units next period. One unit of shares s entitles a stochastic payment of R^s units next period, which will be a function of next period's aggregate shock. Notice that both assets are restricted by a lower bound. If the absolute value of this lower bounds are lower than or equal to the lower bound on total wealth $\underline{\omega}$, it means that holdings on one asset can not be used as collateral for debt on the other asset. The lower bound on total wealth can be imposed as an exogenous borrowing constraint or can arise endogenously as the maximum borrowing that allows the agents to repay their debts in all states of the world with non-negative consumption (a concept know, since Aiyagari (1994), as natural borrowing limit). There are no state contingent markets for the idiosyncratic shocks. Therefore, neither employment nor efficiency units shocks can be insured against. The absence of state contingent markets plus the limitations in borrowing are the ingredients that allows the model to depart from the representative agent framework.[4] Firms rent capital and labor in competitive factor markets, paying the marginal product of capital R^s to the former and the marginal product of labor w per efficiency unit to the latter). Being a closed economy, households' aggregate savings will form the productive capital available in the economy.

2.4 Timing

The model period starts with households carrying certain amount of wealth ω, certain amount of habits h and knowing their idiosyncratic shocks e and ξ and the aggregate state formed by the distribution μ and the shock z. Households decide consumption c, bonds b and shares s. Firms decide the capital and labor demands. Then, the goods, bonds and shares markets clear. This sets productive capital K' and gross return on bonds R^b as functions of today's state z and μ. Then, nature provides the new shocks z', e' and ξ'. Labor market clears. Production $F(z', K', L')$ takes place. Factor inputs are rewarded their marginal products $w(\mu, z, z')$ and $R^s(\mu, z, z')$. Finally, the new individual wealth ω', individual habit stock h' and distribution of households over their individual state space μ' are determined.

Notice that the beginning of the model period does not coincide with the new realizations of the exogenous shocks and that therefore prime and non-prime variables (the latter referred to variables after the realization of the shocks) coexist within the same time period. I chose this timing schedule for simplicity reasons. If the model period coincided with the timing of the shocks, then households would see their state space increased. Instead of total wealth ω they would carry b and s from period to period. Since ultimately households decisions depend on the amount of wealth and its compositions turns out to be irrelevant, this choice simplifies the problem.[5]

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[4] See Huggett (1993) and Aiyagari (1994) for details. Quadrini and Ríos-Rull (1997) contains a review on this topic.

[5] This time schedule was first proposed by Krusell and Smith (1996). Notice that in presence of transaction costs in the asset markets the composition of wealth would matter and this timing would not bring any benefit. Notice also that if one considers wealth ω as the relevant state variable the role of idiosyncratic shocks e and ξ is solely to predict next period's wealth.

2.5 The firm's problem

Households own the capital. Therefore, firms choose capital and labor solving the following static maximization problem:

$$\max_{K,L} \left\{ F\left(z,K,L\right) + \left(1-\delta\right)K - R^{s}K - wL \right\}$$

R^{s} and w are the gross return on shares and the wage rate, and will be set equal to the marginal product of factors according to the FOC of the problem:

$$\begin{aligned} w &= F_{L}\left(z,K,L\right) && (1) \\ R^{s} &= F_{K}\left(z,K,L\right) + \left(1-\delta\right) && (2) \end{aligned}$$

2.6 The household's problem

We formulate the problem recursively. Each individual state is given by the vector j formed by the agents's wealth ω, stock of habits h, employment opportunity e and efficiency units endowment ξ, plus the distribution of agents μ over this vector and the aggregate shock z.[6,7] We define household wealth $\omega \in \Omega = [\underline{\omega}, \infty)$ as the sum of bonds, shares, the income generated by them and the labor earnings. μ is a probability measure over a $\sigma - algebra$ generated by the set $J \equiv \Omega \times R_{+} \times E \times \Xi$. The transition function for the measure μ is given by $\mu' = Q\left(\mu, z, z'\right)$.[8]

Agents maximize the discounted sum of expected utilities by choosing consumption c, risk free bonds b and shares s subject to the feasibility constraints, the budget constraint, the law of motion for habits, the transition matrices for the exogenous shocks and the transition function for the aggregate state. The gross return on bonds depends on today's aggregate state (so it is known at the time of taking decisions) and the gross return on shares depends on today's aggregate state and also on next period's realization of the aggregate shock. The problem can be written as:

$$v\left(j,z,\mu\right) = \max_{c,b,s} \left\{ u(c,h) + \beta E_{e',\xi',z'|e,\xi,z}\left[v\left(j',z',\mu'\right)\right] \right\} \qquad (3)$$

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[6] Notice that when $e=0$ the efficiency units endowment will be equal to zero.

[7] See section 2.4 for a clear definition of the model timing.

[8] Since the process for the employment shock depends on z and z' so will the transition function for the distribution of agents over the individual state vector j.

subject to

$$c = \omega - b - s \tag{4}$$

$$h' = \psi(c, h) \tag{5}$$

$$\omega' = bR^b(\mu, z) + sR^s(\mu, z, z') + \nu' \tag{6}$$

$$\nu' = \Upsilon_{e'=1} w(\mu, z, z')\, \xi' + \Upsilon_{e'=0} d \tag{7}$$

$$(c, \omega, b, s) \geq (0, \underline{\omega}, \underline{b}, \underline{s}) \tag{8}$$

where ν' stands for labor earnings and Υ_l is an indicator function that takes value 1 when the statement l is true and 0 otherwise. The expression $E_{l'|l}[var]$ is the operator for the mathematical expectation of var with respect the distribution of l' conditional on l. The laws of motion for e', ξ' and z' are implicit in the expectation operator. We are looking for the policy functions $c = g^c(j, z, \mu)$, $b = g^b(j, z, \mu)$ and $s = g^s(j, z, \mu)$.

2.7 Equilibrium

An equilibrium for this economy is a set of functions $\left\{v, g^c, g^b, g^s, R^b, R^s, w\right\}$ and a law of motion Q such that:

1. Factor prices satisfy the firms' optimality conditions:

$$\begin{aligned} w(\mu, z, z') &= F_L(z', K', L(z')) \\ R^s(\mu, z, z') &= F_K(z', K', L(z')) + (1-\delta) \end{aligned}$$

2. Given pricing functions $\left\{R^b, R^s, w\right\}$, a law of motion Q and the exogenous transition matrices $\{\Gamma_z, \Gamma_e, \Gamma_\xi\}$, functions $\left\{v, g^c, g^b, g^s\right\}$ solve the household problem

3. Labor market clears

$$L = L(z) = 1 - u_z$$

4. Shares market clears

$$\int g^s(j, z, \mu)\, d\mu = K'$$

5. Bonds market clears

$$\int g^b(j, z, \mu)\, d\mu = 0$$

6. Goods market clears

$$F(z, K, L) + (1-\delta) K = \int g^c(j, z, \mu)\, d\mu + \int g^b(j, z, \mu)\, d\mu + \int g^s(j, z, \mu)\, d\mu$$

7. The law of motion $Q(\mu, z, z')$ for the measure μ is generated by the optimal decisions $\{g^c, g^b, g^s\}$, the law of motion for habits ψ and the transition matrices $\{\Gamma_e, \Gamma_\xi\}$

Notice that conditions four and five imply

$$K' = \int g^b(j, z, \mu)\, d\mu + \int g^s(j, z, \mu)\, d\mu \tag{9}$$

which makes explicit the dependence of R^s and w on z and μ.

3 Calibration

The model economies are targeted to reproduce some long-run features of the US economy. To start with, we need to specify functional forms for our production function, instant utility function and law of motion for habits. Consistently with the lack of trend in US data for the factor shares, production function is assumed to be Cobb-Douglas,

$$F(z, K, L) = zK^{1-\theta}L^{\theta}$$

Utility function is assumed to be of the standard CES class. Habit formation is modelled as in Abel (1990), Fuhrer (2000) and in Diaz, Pijoan-Mas, and Rios-Rull (2003). They characterize it by using the following utility function:

$$u(c, h) = \frac{\left(ch^{-\gamma}\right)^{1-\sigma}}{1-\sigma} \quad \text{with} \quad \gamma \in (0, 1)$$

and the following law of motion for habits:

$$\psi(c, h) = (1-\lambda)\, h + \lambda c \quad \text{with} \quad \lambda \in (0, 1]$$

Notice that the non-habits case has a representation under this formulation by setting γ equal to zero.[9] The choice of parameter values is discussed in section 7.

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[9] There is an alternative way of modelling habit formation in which habit stock enters utility function as a survival consumption level

$$u(c, h) = \frac{\left(c - \gamma h\right)^{1-\sigma}}{1-\sigma}$$

In representative agent frameworks this is the only way to have the Arrow-Pratt coefficient of risk aversion depending on the cycle and therefore to have a negative correlation between equity premium and the cycle. In a heterogeneous agents framework this is not necessary since the negative correlation between the equity premium and the cycle can be obtained with power utility as it is shown in section 7. As Diaz, Pijoan-Mas, and Rios-Rull (2003) discuss, the choice of the *survival consumption* formulation presents serious problems when a model is calibrated to individual data. Notice that in the *survival consumption* formulation utility is not defined when consumption falls below the habit level. Consumption at the individual level can and does fluctuate much more than aggregate consumption and therefore it is not possible to keep it above the habit level.

The model period is imposed to be a quarter.[10] Macroeconomic ratios are set to reproduce data from NIPA. We need to define measurements from the US economy consistent with our model economies. Following Cooley and Prescott (1995) I define consumption as the sum of its non-durable and services components. I expand the capital concept to include the stock of durable and the stock of inventories. Government consumption, investment and capital are ignored. Consequently, the measurement for investment will consist of private fixed investment, expenditure in durable goods and net exports. The series for output is the sum of the consumption and investment series just described. All series are deflated by the corresponding implicit price deflator and stated in per capita terms. For the period 1946 to 2001 these measurements deliver a capital output ratio of 12.56 and an investment output ratio of 0.35. These ratios will determine the values of the discount factor β and the capital stock depreciation δ. The labor share θ is set equal to 0.60 as in Cooley and Prescott (1995). The intertemporal elasticity of substitution is set to 0.5.[11]

The lower bounds on total wealth and each type of asset are set as follow. In a model without aggregate uncertainty there is a natural lower bound $\underline{\omega}$ on total wealth ω given by the present value of the worst possible sequence of idiosyncratic shocks (see Aiyagari (1994) for details).[12] With aggregate uncertainty the interest rate to discount the stream of future earnings is not constant. I use an arbitrary upper bound for this interest rate. The idea is that if the borrowing limit computed with this interest rate is close enough to the idea of natural borrowing limit very few, if any, households will be there and therefore we can think of the model as having no borrowing constraints in total wealth.[13] The lower bounds $\underline{b}$ and $\underline{s}$ are set such that investment in one asset cannot be used as collateral for borrowing by means of the other asset. This implies that $|\underline{\omega}| \geq |\underline{b}|$ and $|\underline{\omega}| \geq |\underline{s}|$. I set both of them to equality so that the borrowing limit on total wealth can be reached using any of them.

The employment shock is characterized by four two by two transition matrices which gives 8 independent parameters. This means we need 8 equations. First, I set average duration

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[10] This comes to a higher computer cost than an annual model period because it demands higher discount factors and therefore convergence procedures take longer. However, the explicit consideration of an employment shock cannot be accommodated to a period length of a year.

[11] As it will be shown later, this implies changing the value of σ when we introduce habit formation preferences.

[12] This natural borrowing limit implies that households can repay the interest services of their loans with positive consumption with probability one. Since marginal utility when consumption approaches zero tends to infinity, in equilibrium all agents will hold assets above this point.

[13] The lower bound $\underline{\omega}$ will be determined by the stream of the lowest possible labor income endowment, ie, home production, discounted by the highest possible gross return, which is determined by the marginal product of capital measured at the lower limit on aggregate capital and the highest realization of the aggregate shock.

of unemployment spells in good and bad times equal to 1.5 and 2.5 quarters respectively. Second, to avoid aggregate labor being a state variable four extra conditions are imposed: (1) employment in good times must be the same regardless of last period being in good or bad times and (2) employment in bad times must be the same regardless of last period being in good or bad times:[14]

$$\begin{aligned}(1-u_g)\,\Gamma_e(z_g,z_g,1,1)+u_g\Gamma_e(z_g,z_g,0,1) &= e_g\\ (1-u_b)\,\Gamma_e(z_b,z_g,1,1)+u_b\Gamma_e(z_b,z_g,0,1) &= e_g\end{aligned}$$

$$\begin{aligned}(1-u_g)\,\Gamma_e(z_g,z_b,1,1)+u_g\Gamma_e(z_g,z_b,0,1) &= e_b\\ (1-u_b)\,\Gamma_e(z_b,z_b,1,1)+u_b\Gamma_e(z_b,z_b,0,1) &= e_b\end{aligned}$$

Third, probability for the unemployed finding a job when moving from good to bad times is set to zero and probability for the employed to enter unemployment when moving from bad to good times is also set to zero:[15]

$$\begin{aligned}\Gamma_e(z_g,z_b,0,1) &= 0\\ \Gamma_e(z_b,z_g,1,0) &= 0\end{aligned}$$

The only thing remaining is to set the unemployment levels for good and bad times. I target the average and standard deviation of the BLS unemployment rate for the period 1948-2001, which are 5.63% and 1.61% respectively. This gives values for the unemployment rates in good and bad times: $u_g = 0.0417$ and $u_b = 0.0719$.[16]

Regarding the aggregate shock I set the average duration of good and bad times equal to 8 quarters each. The levels of the shock will be model dependent and set to have the fluctuations of aggregate output or aggregate consumption as in data.

Regarding the efficiency units shock I establish three points and try to replicate some cross-section and time series statistics of US data. Table 1 shows these statistics (column 1) together with the ones produced by the chosen parameterization of the efficiency units shock (column 2). Table 2 presents the parameters for the efficiency units shock that generate the statistics in column 2 of table 1.[17]

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[14] Aggregate labor becomes therefore just a function of the aggregate shock.

[15] Notice that these 2 extra conditions satisfy:

$$\begin{aligned}\Gamma_e(z_g,z_b,0,1) &< \Gamma_e(z_g,z_g,0,1)\\ \Gamma_e(z_b,z_g,1,0) &< \Gamma_e(z_b,z_b,1,0)\end{aligned}$$

Table 1: **Statistics of US data and simulated process**

	data	model
share of earnings of top 20%	60.2%	61.7%
share of earnings of bott 40%	3.8%	4.5%
gini index of earnings	0.61	0.55
persistence top 20%	68%	46%

Note: Cross section statistics in the first column are from SCF98. Persistence is the probability that those people on the top 20% in PSID 1989 are still there in PSID 1994

Table 2: **Stochastic process for efficiency units.**

	ξ_1	ξ_2	ξ_3
	30.0	8.0	1.0
	$\Gamma_{\xi}(\xi_1,.)$	$\Gamma_{\xi}(\xi_2,.)$	$\Gamma_{\xi}(\xi_3,.)$
$\Gamma_{\xi}(.,\xi_1)$	0.9850	0.0025	0.0050
$\Gamma_{\xi}(.,\xi_2)$	0.0100	0.9850	0.0100
$\Gamma_{\xi}(.,\xi_3)$	0.0050	0.0125	0.9850

Finally, the home production parameter b is set equal to 5% of the average quarterly earnings.[18]

4 Solution of the model

Computation of this class of models is very demanding. In order to predict next period's prices agents need to know the distribution of households μ over shocks, asset holdings and habit stocks and its law of motion $Q(\mu, z, z')$. Therefore, the state space contains an object (the probability measure μ) of infinite dimensionality that cannot be stored by a computer. To get around this problem, I follow the *partial information* approach used by Krusell and Smith (1997) which is itself an extension of the algorithm previously used by Castañeda,

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[16] Under the constraint that u_g and u_g are equidistant from the average.

[17] Cross section statistics are from SCF98. Persistence is the probability that those people in the top 20% in PSID 1989 are still there in PSID 1994. Notice that this corresponds to 20 model periods. See Budría, Díaz-Gimenez, Quadrini, and Ríos-Rull (2002) for details.

[18] The endowment when unemployed is important because it determines how much debt agents can hold while still being able to pay for interest with non-negative consumption. It needs to be set small enough so that unemployment is not a desirable state.

Díaz-Giménez, and Ríos-Rull (1998) and Krusell and Smith (1998).[19] There are some novel issues in this implementation since consideration of habit formation increases the state space both at the aggregate and individual level. The approach is based on assuming that by only using part of the information contained in μ agents can predict tomorrow's aggregate state (and hence prices) almost as well as by using the whole distribution. Krusell and Smith (1998) show that typically the first moments of μ are enough. One finding of this paper is that the marginal distribution of agents over habits (or its first moment) does not bring any additional information in predicting tomorrow's state once we are considering the marginal distribution of assets (or its first moment).[20]

Technically, the idea is to replace the equilibrium equation (9) by

$$K' = f^{K}\left(z, K, H\right) \tag{10}$$

and introduce a new equation to predict aggregate habits

$$H' = f^{H}\left(z, K, H\right) \tag{11}$$

We also need to approximate $R^{b}\left(z, \mu\right)$, which is a direct function of the distribution of agents, by:

$$R^{b} = f^{R^{b}}\left(z, K, H\right) \tag{12}$$

Under this approximation, the state space of the household problem is reduced. Instead of μ, consumers only need K and H to predict prices. Then, the first order conditions of the model will be:

$$\begin{aligned} u_c\left(c,h\right) + \psi_c\left(c,h\right)\beta E_{e',\xi',z'|e,\xi,z}\left[v_h\left(j',z',K',H'\right)\right] = \\ \beta E_{e',\xi',z'|e,\xi,z}\left[v_\omega\left(j',z',K',H'\right)R^{s}\left(z',K',R^{b}\right)\right] \end{aligned} \tag{13}$$

$$\begin{aligned} u_c\left(c,h\right) + \psi_c\left(c,h\right)\beta E_{e',\xi',z'|e,\xi,z}\left[v_h\left(j',z',K',H'\right)\right] = \\ \beta E_{e',\xi',z'|e,\xi,z}\left[v_\omega\left(j',z',K',H'\right)R^{b}\right] \end{aligned} \tag{14}$$

where $\psi_c\left(c,h\right) = \lambda$ under the habit formulation used throughout this paper.

These equations tell us that the utility loss of giving up one unit of consumption today (direct utility loss plus the discounted expected value of tomorrow's effect in the habit stock) must equal the mathematical expectation over different states of the discounted value

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[19] See a detailed explanation of this type of algorithms in Ríos-Rull (1998). Young (2002) contains a discussion on the method.

[20] See the computational appendix.

of tomorrow's extra wealth units obtained investing on each type of asset. Notice that these equations imply:

$$E_{e',\xi',z'|e,\xi,z}\left[v_{\omega}\left(j',z',K',H'\right)\left(R^{s}\left(z',K',R^{b}\right)-R^{b}\right)\right]=0$$

and applying the law of iterated expectations:

$$E_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v_{\omega}\left(j',z',K',H'\right)\right]\left(R^{s}\left(z',K',R^{b}\right)-R^{b}\right)\right]=0 \tag{15}$$

which is what we will call the pricing equation. It tells us that the mathematical expectation of the difference between the returns of each asset weighted by the expected marginal value of wealth in each state, must equal zero.[21] This is the condition that non constrained optimizing agents will satisfy. Obviously, some agents will be in a corner solution by setting $b=\underline{b}$ or $s=\underline{s}$ and will not be able to satisfy equation (15).

The envelope conditions give us the value of one extra unit of wealth and one extra unit of habits:

$$\begin{aligned} v_{\omega}\left(j,z,K,H\right)= &\quad u_{c}\left(c,h\right)+\psi_{c}\left(c,h\right)\beta E_{e',\xi',z'|e,\xi,z}\left[v_{h}\left(j',z',K',H'\right)\right] && (16)\\ v_{h}\left(j,z,K,H\right)= &\quad u_{h}\left(c,h\right)+\psi_{h}\left(c,h\right)\beta E_{e',\xi',z'|e,\xi,z}\left[v_{h}\left(j',z',K',H'\right)\right] && (17)\end{aligned}$$

where $\psi_{h}\left(c,h\right)=1-\lambda$.

For given forecasting laws 10, 11 and 12, solving the household problem will amount to solve the first order conditions 13 and 14 together with constraints 4, 5, 6 and 8. To do so I approximate the derivatives of the value function v_{ω} and v_{h} piece-wise linearly and use the envelope conditions 16 and 17 to update them. Then, I will also have to iterate in the forecasting laws space to find the forecasting laws that are consistent with the equilibrium. A detailed explanation of the procedure can be found in the computational appendix (appendix A).

4.1 Equity premium

The equity premium is the difference between the expected return on the risky asset and the return in the risk free asset. We can write it formally in the context of our model as

$$E_{z'|z}\left[R^{s}\left(z',K',R^{b}\right)\right]-R^{b}$$

From the pricing equation 15, and using the definition of covariance, we can write

$$E_{z'|z}\left[R^{s\prime}-R^{b}\right]=-\frac{cov_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v_{\omega}'\right],R^{s\prime}-R^{b}\right]}{E_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v_{\omega}'\right]\right]}$$

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[21] Or more clearly, the value of investing in each assets has to be the same.

where *cov* is the covariance operator. To simplify we use v'_ω as an abbreviation for $v_\omega\left(j', z', K', H'\right)$ and $R^{s\prime}$ as an abbreviation for $R^s\left(z', K', R^b\right)$. Further expanding we get:

$$\begin{aligned}&E_{z'|z}\left[R^{s\prime}\right]-R^b=\\&-SD_{z'|z}\left[R^{s\prime}\right]\ CV_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_\omega\right]\right]\ Corr_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_\omega\right],R^{s\prime}\right]\end{aligned}$$

The first term, the conditional standard deviation of the return of the risky asset, is generally known as the amount of risk. The product of the second and third terms form what is generally called Sharpe ratio and it measures the price of risk. The second term is the coefficient of variation on aggregate shock of the expectation of the marginal value of wealth. It measures the utility cost of aggregate fluctuations. The third term is the correlation on the aggregate shock between the expectation of the marginal value of wealth and the returns on the risky asset. The correlation term gives the sign of the equity premium. If high returns of the risky asset are associated with low marginal values of wealth, the risky asset entitles a positive risk premium. On the contrary, if the risky asset pays more when the marginal value of wealth is high it will then be seen as an insurance mechanism and it will entitle a negative premium. Recall that the aggregate shock has only two possible realizations. Conditional on e, ξ and z, the expectation over e' and ξ' of wealth ω' has only two possible realizations depending on z'. When $z'=z_g$ (conversely $z'=z_b$) the return on the risky asset is high (low) and the expectation of the marginal wealth is low (high).[22] Therefore, the correlation term will be exactly -1. We then rewrite:

$$E_{z'|z}\left[R^{s\prime}\right]-R^b=SD_{z'|z}\left[R^{s\prime}\right]\ CV_{z'|e,\xi,z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_\omega\right]\right] \tag{18}$$

5 Representative agent economy

I start by looking at the results for a general equilibrium production economy where the dynamics of aggregate variables are given by the dynamics of a single representative household. This economy will be used as a benchmark to understand the insights introduced by the explicit consideration of heterogeneity. The representative agents economy is a particular case of our general setting. Employment opportunities come with probability one. The representative agent receives an endowment of efficiency units equal to $1-u_g$ when $z=z_g$ and $1-u_b$ when $z=z_b$. In this way aggregate output fluctuations are the same as in the heterogeneous agent economy. It can be thought of as an heterogeneous agents economies where there are contingent markets for both the efficiency units shock and the employment

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[22] Here, it is crucial the assumption that expected labor earnings are higher in good times than in bad times.

shock. The amplitude of aggregate shock fluctuations is set such that the rate of growth of consumption equals that in data. I call this economy RA.

I simulate this economy and present some statistics in the second column of table 3. Together with the statistics for this economy, first column of table 3 presents the same statistics for US data. Lettau and Uhlig (1997), using US quarterly data from 1948 to 1996, estimate a value of 0.27 for the Sharpe ratio.[23] As it can be seen, the price of risk obtained in our simulated economy is a very small 0.010 which is more than one order of magnitude below the corresponding estimate.

Table 3: **Statistics of simulated economies. Representative Agent.**

	data	RA
k/y	12.56	12.56
$SD\left(g_C\right)$	0.52	0.52
$Sharpe$	0.27	0.010
$\int CV\left[Ec'\right]d\mu$	–	0.52%
$\int CV\left[Ec'\right]/CV\left[E\nu'\right]d\mu$	–	0.11

Note: First column refers to US quarterly data from 1946 to 2001 except $SD(g_C)$ that refers to 1948-1996. $SD(g_C)$ is the unconditional standard deviation of the rate of growth of aggregate consumption in quarterly terms. The last two rows report the average (over all households) of the conditional coefficient of variation of the expected consumption c' and of the ratio of the conditional coefficient of variation of the expected consumption c' and the conditional coefficient of variation of the expected labor earnings ν'. The coefficient of variation operator refers to the distribution of z' conditional on z. The expectation operator on consumption and earnings refers to the joint distribution of e' and ξ' conditional on e, ξ, z and z'.

This result illustrates the essence of the equity premium puzzle. For reasonable parameter values, the price of risk that households require to hold risky assets is nowhere around the observed value in data. This is so even in this parameterization where I am being purposely generous with the economy volatility, since the model is allowed to show big fluctuations of macroeconomic variables to make sure that the production economy generates fluctuations in aggregate consumption similar to those in data.

6 Heterogeneous agents economy

In this section I look at the market price of risk once we allow for heterogeneity in earnings and wealth and therefore in consumption. The analysis with habit formation is deferred to

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[23] For the return on shares they use the SP500 index and for the risk free rate the Treasury bills.

the next section. I will show that market incompleteness matters in quantitative terms. The main reason is that with different types of agents we do not have to look at the dynamics of average consumption in the economy but at the dynamics of consumption for those agents pricing the risk.

First of all I define the benchmark economy without habits. I calibrate it as stated in section 3. The absence of habits implies $\gamma = 0$. An intertemporal elasticity of substitution equal to 0.5 requires σ to equal 2. The amplitude of the aggregate shock fluctuations is set such that the standard deviation of aggregate consumption growth, g_C, equals 0.52 as in data. I call this economy MV (multivariate). Together with MV I define an identical economy MVC with the same calibration targets but with the restriction that households cannot borrow. The results for economies MV and MVC are reported in the second and third columns of table 4.

Table 4: **Statistics of simulated economies. Benchmark Economy.**

	data	MV	MVC
k/y	12.56	12.56	12.56
$SD\left(g_C\right)$	0.52	0.52	0.52
$Sharpe$	0.27	0.011	0.190
$\int CV\left[Ec'\right] d\mu$	–	0.57%	0.62%
$\int CV\left[Ec'\right]/CV\left[E\nu'\right] d\mu$	–	0.15	0.20

Note: First column refers to US quarterly data from 1946 to 2001 except $SD(g_C)$ that refers to 1948-1996. $SD(g_C)$ is the unconditional standard deviation of the rate of growth of aggregate consumption in quarterly terms. The last two rows report the average (over all households) of the conditional coefficient of variation of the expected consumption c' and of the ratio of the conditional coefficient of variation of the expected consumption c' and the conditional coefficient of variation of the expected labor earnings ν'. The coefficient of variation operator refers to the distribution of z' conditional on z. The expectation operator on consumption and earnings refers to the joint distribution of e' and ξ' conditional on e, ξ, z and z'.

Regarding the price of risk, we see that the Sharpe ratio obtained in economy MV is 0.011. The value obtained in our simulated economy is one order of magnitude below its empirical estimate. At first sight the interaction of idiosyncratic risk and aggregate risk does not seem to be quantitatively important: the price of risk in this economy is just 10% higher than the value found in the representative agent economy. However, in spite of the small change there are interesting economic factors at work that make this quantitative result interesting on its own.

To start, it is not clear in qualitative terms whether the price of risk should be higher or lower than in the representative agent economy. On the one hand, Krusell and Smith (1997) show that, with convex marginal utility, if variance of consumption is higher in downturns than in peaks there will be an increase in the market price of risk. However, on the other hand households in this economy can engage in asset trading among each other to lower their exposure to risk and therefore to prevent the variance of earnings from translating into consumption. Precisely, by borrowing in shares and investing in bonds households can create portfolios whose return is negatively correlated with the aggregate shock. This trade of risk through the trade of assets is something that our representative agent of section 5 cannot do. Overall, the effect is that the explicit consideration of heterogeneity in an equilibrium economy rises the market price of risk albeit in a modest size.

When agents are not allowed to borrow, as it is the case in economy MVC, the Sharpe ratio is a much larger 0.190. Without borrowing the ability of households to smooth out fluctuations is seriously lowered and hence the higher earnings variance of the heterogeneous agent model drives the price of risk to almost 20 times its size in the representative agent economy. The comparison of the Sharpe ratios in economies MV and MVC gives us a measure of the quantitative importance of borrowing as a mechanism of insurance.

The market price of risk in the economy without borrowing gets quite close to the measured value of data although it still stays below it. This result is close to the one found, in a similar framework, by Krusell and Smith (1997), who report for an economy without borrowing a Sharpe ratio of 0.21. Krusell and Smith (1997) consider an heterogeneous agents model where the only source of idiosyncratic uncertainty is given by an employment shock. The main difference is that they set an employment shock much more volatile than the one used in this paper without considering the efficiency units shock. They set $1-u_g = 0.96$ and $1-u_b = 0.90$, a pair of values that deliver a standard deviation for the unemployment rate of 3.0%, a figure almost twice as big as the value computed from BLS data. A major drawback of their choice is that it generates countercyclical wages.[24] The employment shock, in an economy without borrowing, cannot be washed away and therefore it makes agents demand a higher premium for holding the risky asset.

Another way to see these results is by looking at the fluctuations faced by individual households. Row 4th of tables 3 and 4 displays the average over the whole population of the coefficient of variation (with respect to the conditional distribution of the aggregate shock) of the expected consumption next period (with respect to the conditional idiosyncratic shock

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[24] The marginal product of labor is decreasing in labor and increasing in the aggregate shock z. In peaks the increase in labor due to such a volatile employment process is so big that more than offsets the increase in z.

distribution). Technically,

$$\int CV_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[c'\right]\right] d\mu$$

which I abbreviate in the tables by $\int CV\left[Ec'\right] d\mu$. This statistic gives a measure of how much individual consumption fluctuates. For economy RA this statistic is 0.52%, for MV is 0.57% and for MVC is 0.62%. Notice that the three economies are calibrated to have the same volatility of aggregate consumption. In spite of this, the heterogeneous agents economies produce higher individual level consumption fluctuations than the representative agent economy. This is true even when we allow for borrowing in bonds and shares, and therefore when giving households an extra dimension to smooth out earnings fluctuations. A measure of the success of households in smoothing out earnings fluctuations can be given by the average over the whole population of the ratio between the coefficient of variation of expected consumption and the coefficient of variation of expected labor earnings:

$$\int \left(\frac{CV_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[c'\right]\right]}{CV_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[\nu'\right]\right]}\right) d\mu$$

which I abbreviate in the tables by $\int CV\left[Ec'\right]/CV\left[E\nu'\right] d\mu$. This statistic is reported in row 5th of tables 3 and 4. As expected the non-borrowing heterogeneous agents economy MVC displays the highest ratio. Households in the borrowing economy MV can smooth out earnings fluctuation more than those in the non-borrowing economy MVC. When opening up the possibility of trade the ratio falls from 0.20 to 0.15. As the result for the market price of risk suggests, in equilibrium households in economy MV do not smooth out earnings fluctuations as much as the representative agent does.

To understand how the trade of assets affects the price of risk is important to see the portfolio choices of different types of households. In figure 1 we have the policy functions for economy MV. Each panel, corresponding to each possible earnings state, displays the portfolio choice for each possible value of wealth ω. There are two distinct patterns. First, as wealth increases households put more shares in their portfolio. Second, for a given level of wealth, the higher the efficiency level the lower the amount of shares. This means that low-efficiency households are buying risk from high-efficiency ones in exchange of higher expected payoffs.

Regarding the first result, higher wealth means (1) having a lower proportion of labor earnings in next period's expected income (2) and being further away from the borrowing

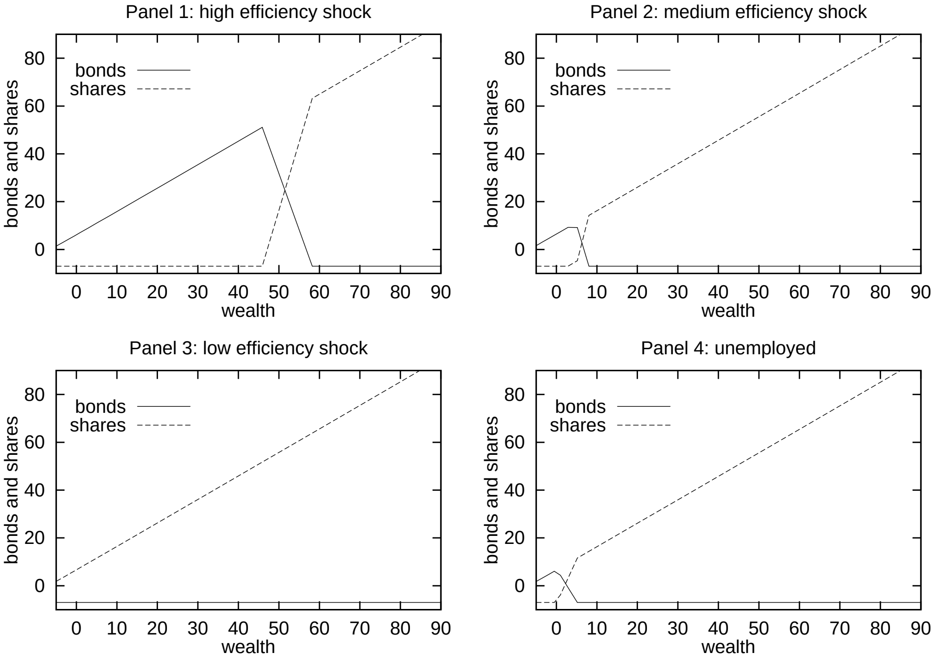

Figure 1: **Simulated policy functions. Economy** MV

constraints. Therefore, (1) the variability of expected income is lower and (2) it translates to a lesser degree into consumption variability. Wealth-poor agents go as short as possible in shares and invest in bonds. They sacrifice expected returns but in exchange get a portfolio that pays well when their marginal value of wealth is high (downturns) and pays bad when their marginal value of wealth is low (peaks).

The second result is at first surprising. Notice, however, that it should not be so. Once we control for wealth, the sole role of the efficiency units shock is to predict next period's efficiency units endowment.[25] The process for ξ is such that $E\left[\xi'/\xi, e'=1\right]$ is increasing in ξ. I.e., conditional on being employed next period, the expected amount of efficiency units next period is increasing in the amount of efficiency units in the current period. Therefore, the higher ξ, the larger the difference between being employed and unemployed and hence the larger the conditional variability of expected labor earnings. This result relies on unemployment risk being unrelated to the earnings position.[26] A final comment here is that unemployed households are the least willing to hold risk.

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[25] The amount of resources brought by the shock are already accounted for in the wealth holdings.

[26] The result does not need to hold when unemployment probability is related to the skill level. High efficiency workers may face a higher differential in earnings between employed and unemployed status but they might also have a lower probability of unemployment. Overall, conditional variance of earnings could be lower for high earners than for low earners

Empirical results by Bertaut and Starr-McCluer (2000) are not inconsistent with these findings. Using SCF data, Bertaut and Starr-McCluer (2000) estimate households' portfolio choices as function of wealth, education, income and other household observable variables. They find that (1) wealth always increases the share of risky assets in the portfolio and (2) income decreases the share of risky assets in the portfolio. The control for education means that we have to read the result on income as net of fixed heterogeneity, which make proxy the model's lack of correlation between earnings and aggregate uncertainty.

To sum up, I find that the interaction between idiosyncratic risk and aggregate risk is quantitatively important. First, an heterogeneous agents economy where households are not allowed to borrow displays a price of risk almost 20 times higher than a representative agent economy. Second, once we allow them to borrow, households do a good job in insuring themselves against income fluctuations. The ratio of consumption volatility over earnings volatility falls and so does the market price of risk. And third, even with borrowing, an heterogeneous agents economy displays a price of risk 10% higher than its representative agent counterpart. The reason for this is that, even if aggregate consumption fluctuations are the same, individual consumption fluctuations are not: households in the heterogeneous agents economies, even when allowed to trade risk, cannot flatten their consumption profiles as much as the representative agent does.

7 Habit formation

Habit formation preferences have been proposed as an explanation for the equity premium. A common feature of the literature is to calibrate a process for consumption from data. Then, given the process for consumption, the first order conditions of the model generate the price of risk consistent with it. This paper takes a different approach. It does not calibrate the process for consumption but the process for earnings. Then, forward-looking consumers decide weather to let earnings fluctuations translate into consumption fluctuations or to use the assets available in order to try to smooth them out. When moving from a setting without habits to one with habits, not only the marginal utility of consumption changes but also the consumption fluctuations do. In addition, there is also a change in the structure of pricing agents and therefore of the agents whose consumption fluctuations are relevant for asset pricing.

In this section I introduce habit formation into the general model. I need to choose values for the parameters γ and λ. However, it is not clear which values to pick.[27] There are

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[27] The optimal way of doing this would be to set a minimum distance estimation method with the Sharpe

some empirical papers estimating habit parameter values. However, results are very different among them, depending on the data and model specification.[28] The strategy followed in this paper is to allow for a 'strong' habit process and interpret the results as an upper bound. Firstly, I set $\gamma = 0.75$. γ determines the weight of habits on the utility function. The value chosen implies that households care much more about relative consumption than about consumption level.[29] Secondly, I set $\lambda = 0.25$. This generates a highly persistent habits process. A value of 1 would mean that only previous period consumption matters whereas a value smaller than 1 means that the whole history of past consumption enters the habit stock. Small values of λ imply that consumption in the distant past still has a lot of weight on current period's habit stock (or in other words, that current period's consumption hardly modifies next period's habit stock). A persistent habit process is used by Constantinides (1990) and Heaton (1995) to obtain sizeable equity premia. In addition, Diaz, Pijoan-Mas, and Rios-Rull (2003) shows in a similar model without aggregate risk that the more persistent the process, the higher the effect of habits in the consumption/savings decision. Finally, I set $\sigma = 5$ in order to keep the intertemporal elasticity of substitution equal to 0.5.[30]

I define a new model economy MVH similar to MV with the changes of these three preference parameters just described and the discount factor β readjusted to generate an amount of savings equal to economy MV and equal to data. Column 2 of table 5 shows some statistics. The price of risk increases about 45% from 0.011 to 0.016. However, the value obtained is still more than one order of magnitude below its empirical counterpart. Notice, however, that economy MVH produces a standard deviation for aggregate consumption growth g_C of

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ratio, consumption fluctuations and other moments as targets. The problem with this method is that it is computationally very expensive. Solving the model for a given calibration takes already quite a lot of time. Besides, one must be very careful about the parameter values given and the initial conditions used. A mechanical algorithm trying different parameter values to match certain moments of data is far beyond the set of feasible exercises to perform with this model.

[28] Some empirical studies on habit formation are Fuhrer (2000), Dynan (2000) and Heaton (1995). See section Diaz, Pijoan-Mas, and Rios-Rull (2003) for a survey.

[29] Notice that utility function can be rewritten as

$$u(c,h) = \frac{\left(c^{1-\gamma}\left(\frac{c}{h}\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma}$$

which shows the role of γ. Consumers care for the level of present consumption and for present consumption relative to past consumption (or habits). γ gives the weight of the latter.

[30] as shown in Diaz, Pijoan-Mas, and Rios-Rull (2003), intertemporal elasticity of substitution becomes

$$\frac{1}{\gamma + (1-\gamma)\,\sigma}$$

Economies without habits ($\gamma = 0$) keep the property of intertemporal elasticity of substitution being the inverse of risk aversion. However, when habits are introduced, the link between preferences over different states and over different time periods is broken.

0.26, half its counterparts in the non-habits economy and data. Households more sensitive to consumption fluctuations, not only demand a higher compensation for holding risky assets but also make sure they face smaller consumption fluctuations. They change their consumption/savings decisions. Habit formation preferences increase the volatility in marginal utilities for a given consumption process. However, when consumption decisions are endogenous this increased volatility in marginal utilities will not fully translate into the market price of risk because households will decrease the consumption fluctuations they face. This is a force keeping the price of risk from increasing. Notice that our measure of volatility for individual consumption also falls, from 0.57% in economy MV to 0.26% in economy MVH.

Table 5: **Statistics of simulated economies. Habits Economies.**

	data	MVH	MVH'
k/y	12.56	12.56	12.56
$SD\left(g_C\right)$	0.52	0.26	0.52
$Sharpe$	0.27	0.016	0.031
$\int CV\left[Ec'\right] d\mu$	–	0.26%	0.52%
$\int CV\left[Ec'\right]/CV\left[E\nu'\right] d\mu$	–	0.077	0.073

Note: First column refers to US quarterly data from 1946 to 2001 except $SD(g_C)$ that refers to 1948-1996. $SD(g_C)$ is the unconditional standard deviation of the rate of growth of aggregate consumption in quarterly terms. The last two rows report the average (over all households) of the conditional coefficient of variation of the expected consumption c' and of the ratio of the conditional coefficient of variation of the expected consumption c' and the conditional coefficient of variation of the expected labor earnings ν'. The coefficient of variation operator refers to the distribution of z' conditional on z. The expectation operator on consumption and earnings refers to the joint distribution of e' and ξ' conditional on e, ξ, z and z'.

To have a habits economy consistent with consumption fluctuations in data I reset the amplitude of the aggregate shock fluctuations such that the equilibrium aggregate consumption fluctuations generated by the model match those measured in data. I call this new economy MVH'. Results are in the third column of table 5. This new habits economy produces a much larger Sharpe ratio, 0.029, more than 2.5 times higher than the one obtained in the non-habits economy MV and almost twice as big as the one in the MVH economy. However, it is still one order of magnitude below empirical estimates. Even if aggregate consumption fluctuates as much as in a non-habits economy, individual consumption does not. Our measure of fluctuations in individual consumption for economy MVH' is 0.52%, lower than in the non-habits case.

The incomplete markets settings give us insight in another important equilibrium effect. Asset prices do not depend on the average of individual consumption fluctuations but on the consumption fluctuations of those individuals that solve the portfolio choice problem with an interior solution. The addition of habit formation changes households decisions and the way they trade assets. This implies that, in equilibrium, the composition of the set of agents that are not constrained in their portfolio choice problem changes. If agents faced higher disutility of given consumption fluctuations and were forced to keep the same portfolio composition they would ask for a higher compensation for holding risky assets. But they are not forced to keep the same portfolio structure. Indeed, what happens is that households that were holding bonds and asset will switch to specialization in bonds, and households that were specialized in shares will start to introduce some bonds in their portfolios to lower their exposure to risk. The change in the composition in the set of pricing agents is a force preventing the price of risk from increasing too much. Throughout the paper I refer to this mechanism as composition effect.

To see the importance of the composition effect some statistics are presented in table 6. I define I as the set of all households in the economy, I_b as the set of households whose choice of bonds equals $\underline{b}$, I_s as the set of households whose choice of shares equals $\underline{s}$ and I_p as the set of pricing agents, i.e., the set of agents with interior solution to their portfolio choice problem.

Table 6: **Statistics of simulated economies. Composition effect.**

	MV	MVH	MVH'
$\int_I d\mu$	1.00	1.00	1.00
$\int_{I_p} d\mu$	0.38	0.54	0.61
$\int_{I_b} d\mu$	0.41	0.30	0.24
$\int_{I_s} d\mu$	0.21	0.16	0.15
$\int_I CV\left[Ev'_\omega\right] d\mu$	0.89%	1.36%	2.91%
$\int_{I_p} CV\left[Ev'_\omega\right] d\mu$	0.90%	1.28%	2.89%
$\int_{I_b} CV\left[Ev'_\omega\right] d\mu$	0.66%	1.08%	2.16%
$\int_{I_s} CV\left[Ev'_\omega\right] d\mu$	1.33%	2.15%	4.28%

Note: The first four rows report the number of agents in each set. Set I stands for the whole population, set I_p for those agents unconstrained in their portfolio choice problem, set I_b for those agents constrained in bonds and set I_s for those agents constrained in shares. The last four rows report the average (over the corresponding set of agents) of the conditional coefficient of variation of the expected marginal value of wealth v'_ω. The coefficient of variation operator refers to the distribution of z' conditional on z. The expectation operator refers to the joint distribution of e' and ξ' conditional on e, ξ, z and z'.

The first four rows of table 6 show the proportion of households in each category for economies MV, MVH and MVH'. As it can be seen, the structure of constrained and unconstrained people in each economies varies substantially. In the economy without habit formation only 38% of households solve their portfolio choice problem with an interior solution. Adding habits increases this proportion. In economy MVH the proportion of households in the set of pricing agents rises to 54%. The higher volatility habits economy MVH' sees this proportion further increased to 61%. Clearly, individual decisions change when we modify the model, and more precisely, the set of pricing agents of these economies differs from each other. The next four rows gives us some information on who are these constrained and unconstrained people. I present, for each set of agents, the average coefficient of variation on the expected marginal value of wealth. In all of the three economies we can see that the agents that go as short as possible in shares are those who face higher volatility in their marginal utilities. These agents build portfolios that pay well in downturns and bad in peaks. Reversing the returns of shares implies giving up expected value in exchange of insurance against earnings volatility. The important result to highlight is that, when adding habits, the volatility of marginal utilities increases less for the set of pricing agents than for the overall economy. Precisely, economy MVH displays an average volatility of the marginal value of wealth of 1.36% whereas MV shows a value of 0.89%, a figure that is 53% higher. In contrast, the average volatility for the set of pricing agents differs in a smaller 42% between the two economies. Economy MVH', also sees the average volatility of marginal utilities for the set of pricing agents increase less than the average for the overall economy.

Therefore, the set of agents with non-corner solutions in the portfolio problem changes. It will be better-insured agents who will be pricing risk in economy MVH. This means that the agents pricing the assets in the economy with habits face lower consumption fluctuations than the agents pricing the assets in the economy without habits.

The portfolio choices of individuals, as a function of wealth, are similar to those in the non-habits economies. The amount of shares increases with wealth and decreases with efficiency units. Figure 2 shows these policies for a fixed value of the habit stock. It remains to be explored the additional state variable, the habit stock. Figure 3 shows the policy functions, for a given wealth level, as a function of habit. We observe that the amount of shares decreases with the habit stock. The picture is similar for different levels of wealth (not shown). This is not surprising. Since the habit stock increases the volatility of marginal utilities, for the same level of wealth agents that have enjoyed a history of higher consumption are less willing to take risk than agents that have enjoyed a history of lower consumption. In other words, agents that are coming from poverty are less reluctant to takes risk because if things go wrong they do not lose as much as agents that have already got used to a certain status.

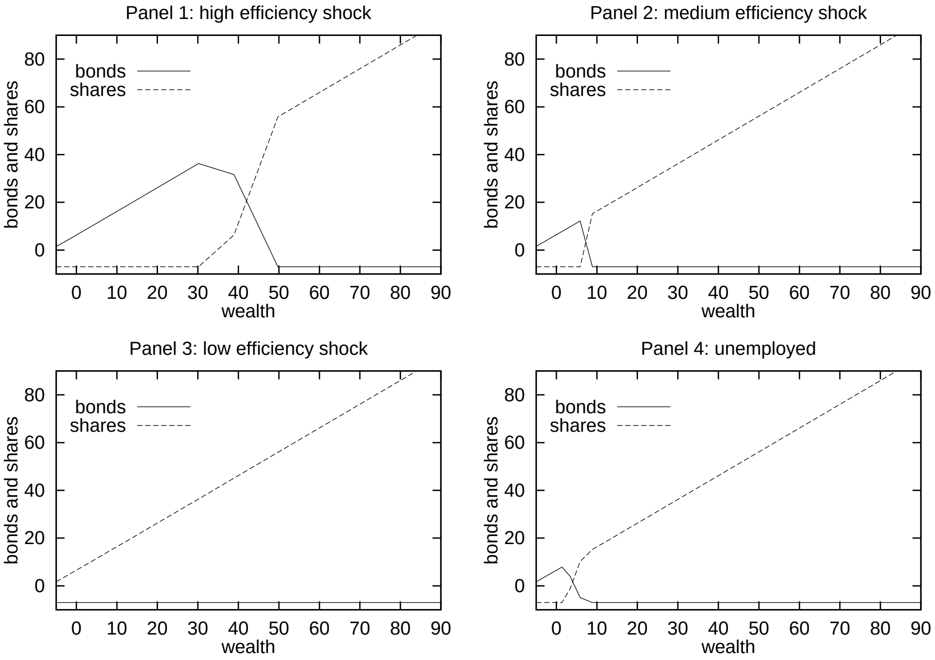

Figure 2: **Simulated policy functions. Economy** MVH

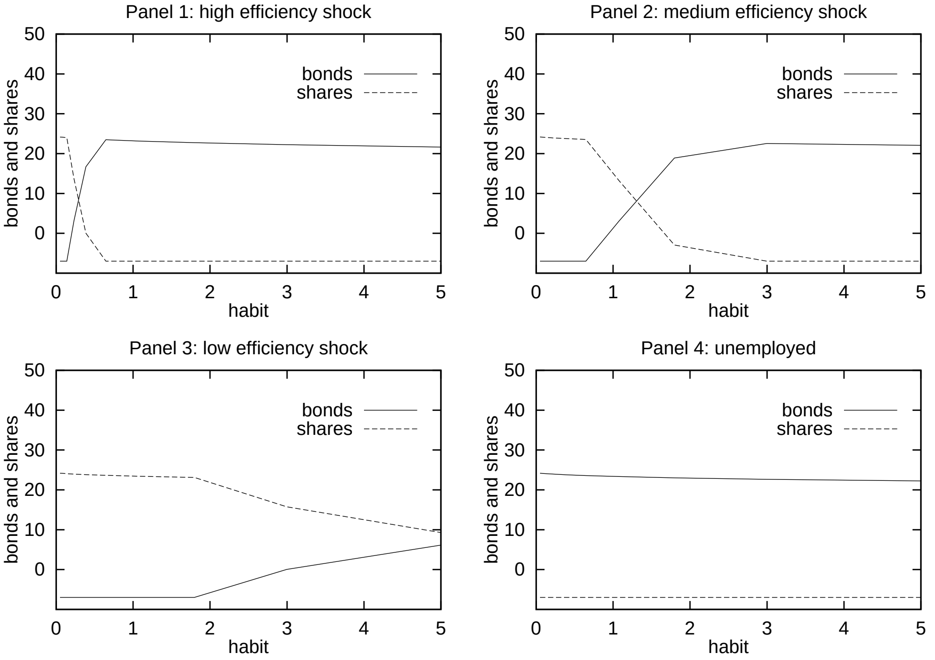

Figure 3: **Simulated policy functions. Economy** MVH

A final quantitative result to highlight is the correlation between the price of risk and the cycle. The simulated economy MVH produces a correlation between the Sharpe ratio and the rate of growth of output equal to -0.99 whereas the correlation between the hp-detrended series for output and the hp-detrended Sharpe ratio is -0.37. In terms of choosing a functional form for the habit formation preferences, Campbell and Cochrane (1999) argue that the survival consumption approach is consistent with the fact that the price of risk is countercyclical whereas the power utility is not. This is true in a representative agent framework. However, as the numbers presented show, in a heterogeneous agents setting the power utility can also generate this result.

To sum up, the hypothesis of habit formation delivers a market price of risk higher than the hypothesis of time separable preferences. The reason is that habit-forming agents are less reluctant to take risk because they want a smoother pattern of variations of consumptions. As pointed out by Lettau and Uhlig (2000) and Jermann (1998) habit formation preferences in a production economy imply that consumption fluctuations are reduced. If we calibrate a habits economy to display the same fluctuations of aggregate consumption as in data, the price of risk is still lower than the one estimated in data. There are two reasons for this. First, what matters for the price of risk is the volatility of individual consumption. Even if we recalibrate our model economies to display a volatility of aggregate consumption as in data, individual level volatility is still lower than in an economy without habit formation. And second, the price of risk does not depend on everybody's consumption profiles but on the consumption profiles of those agents that are not constrained in their portfolio choice problem. This set of agents is not invariant to preference hypothesis. When considering habit formation preferences the composition of this set of agents changes and it will be better insured agents those pricing the risk. In few words, the explicit consideration of general equilibrium factors holds the equity premium from increasing.

8 Conclusions

This paper shows how the habit formation hypothesis, which has been proposed as a possible explanation for the equity premium, is perhaps not such a good candidate. In a model where agents differ in earnings, wealth, habit stocks and therefore consumption decisions, the market price of risk stays below its empirical estimates.

There are two general equilibrium features that prevent the Sharpe ratio from growing too much. Firstly, precautionary savings. With habit formation households trying to avoid higher utility losses from consumption fluctuations will use asset markets to avoid them.

Secondly, what I call composition effect. When the disutility of risk increases (as it does with habit formation) the pool of agents pricing this risk changes. Pricing agents will be households that, relative to the overall population, face smaller consumption fluctuations. This prevents the increase in disutility of risk to fully translate into an increase in the equilibrium price of risk.

The literature on habits and asset pricing has tended to take consumption fluctuations as given without worrying about how forward-looking agents generated them. The argument is that, for given statistics of consumption data, increasing the disutility from consumption fluctuations should increase the premium attached to the risky asset. However, in a general equilibrium model with heterogeneous agents anticipation of higher disutility from fluctuations makes households save in advance in order to smooth their consumption profiles. The result is that the consumption fluctuations generated by the model with habits are smaller than the ones generated by a model without habits. Another way to see this result is that, for our calibrated process on earnings, the habit formation hypothesis is inconsistent with the empirical consumption fluctuations.

In short, what we observe is that households have changed their behavior. By taking the consumption process as given and then changing the preferences of households, one forces the agents in the model to bring all the adjustment on prices (equity premium) without giving a chance to change quantities (consumption fluctuations). However, once we introduce habits households do not just sit and ask for a high compensation to hold shares in their portfolios. The higher utility losses due to habit formation are mitigated by smoothing out consumption fluctuations to a higher degree and by the change of who is pricing the risk. Overall, the price of risk is higher but not as much as it would be if we kept the consumption fluctuations constant.

One interesting feature of this setup is the portfolio decision of agents according to their state. Higher wealth implies higher willingness to hold risky assets, higher earnings imply lower willingness to hold risky assets and higher habit stocks imply lower willingness to hold risky assets. These are clear testable implications for the heterogeneous agents model. The failure of the model in generating large equity premia should not disregard these portfolio choice implications. What the model says is that it is difficult to see the equity premium as a risk premium given the earnings shocks that households face, even if one considers habit formation. Recent work by Ellen McGrattan and Edward Prescott claims that it might well be that the equity premium is not a risk premium at all once intangible assets, foreign assets and different taxation issues are taken into account.[31]

Admittedly, the structure of earnings uncertainty is quite simple. It is assumed that efficiency units of labor are unrelated to aggregate risk. However, the results of the paper suggest that we would require a very high covariance between aggregate risk and idiosyncratic uncertainty to move the quantitative predictions of equilibrium model economies to the corresponding Sharpe ratio measured in data.

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[31] See McGrattan and Prescott (2000a) and McGrattan and Prescott (2000b)

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Appendix

A Computational Procedures

This appendix explains the computer algorithm used to solve the model. The algorithm is based on the *partial information* approach used by Krusell and Smith (1997). They were already extending previous work by Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and Krusell and Smith (1998). Ríos-Rull (1998) explains it in good detail. Basically, as stated in section 4, solving the household problem implies, maximizing equation (3) subject to the constrains (4), (5), (6) and (8) and to the forecasting rules (10), (11) and (12). The problem is that the forecasting rules f^K, f^H and f^{R^a} are not known. We start explaining how to solve the household problem for given forecasting rules and then we discuss how to find the equilibrium ones.

A.1 Solving the household's problem

For the household problem the state space is given by the vector $j = \{\omega, h, e, \xi\}$ plus z, K and H. We collapse e, ξ and z into one variable ϵ that can take $n_\epsilon = n_z (n_\xi + 1) = 8$ different values. We are therefore left with the two endogenous individual state variables ω and h, the exogenous stochastic shock ϵ and the exogenous (at the household level) aggregate variables K and H. Define labor earnings in terms of the newly defined exogenous stochastic process ϵ as $\nu(\epsilon, K, H)$. Households have to solve the following system formed by the FOC, the constraints and the forecasting rules:

$$\begin{aligned}
0 &= u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega', h', \epsilon', K', H')\right] - \beta E_{\epsilon'|\epsilon}\left[v_\omega(\omega', h', \epsilon', K', H')\, R^s\left(\epsilon', K', R^b\right)\right] \\
0 &= u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega', h', \epsilon', K', H')\right] - \beta E_{\epsilon'|\epsilon}\left[v_\omega(\omega', h', \epsilon', K', H')\, R^b\right] \\
c &= \omega - b - s \\
h' &= \lambda c + (1-\lambda) h \\
\omega' &= bR^b + sR^s\left(\epsilon', K', R^b\right) + \nu(\epsilon', K', H') \\
K' &= f^K(\epsilon, K, H) \\
H' &= f^H(\epsilon, K, H) \\
R^b &= f^{R^b}(\epsilon, K, H) \\
&(\omega, b, s, c) \geq (\underline{\omega}, \underline{b}, \underline{s}, 0)
\end{aligned} \tag{19}$$

which for a given pair $\{v^0_\omega, v^0_h\}$ delivers the policy functions $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$. Then, substituting both of them into the right hand side of the EC

$$\begin{aligned}
v^1_\omega(\omega, h, \epsilon, K, H) &= u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v^0_h(\omega', h', \epsilon', K', H')\right] \\
v^1_h(\omega, h, \epsilon, K, H) &= u_h(c,h) + (1-\lambda)\beta E_{\epsilon'|\epsilon}\left[v^0_h(\omega', h', \epsilon', K', H')\right]
\end{aligned}$$

we get a new pair $\{v^1_\omega, v^1_h\}$. These two systems define a mapping T from the cartesian product of the space where v_ω and v_h belong into itself. Solving the household problem amounts to finding a fixed point of this mapping, i.e., a pair such that $\{v_\omega^*, v_h^*\} = T\{v_\omega^*, v_h^*\}$. The space where v_ω and v_h is unknown. We need to specify a class of functions that the computer can understand to approximate for this space. For every value of ϵ, we approximate $\{v_\omega, v_h\}$ piece-wise linearly in a four-dimensional grid.[32] Given an initial guess $\{v_\omega^0, v_h^0\}$, we solve the system (19) to get the policy functions $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$. Then, using the envelope conditions, we obtain a new pair $\{v_\omega^1, v_h^1\}$. If the new pair $\{v_\omega^1, v_h^1\}$ is close to $\{v_\omega^0, v_h^0\}$ we have find an approximation to the fixed point of the mapping T and we take $\left\{g^{0,c}, g^{0,b}, g^{0,s}\right\}$ as the solution of the model. If not, we update $\{v_\omega^0, v_h^0\} = \{v_\omega^1, v_h^1\}$ and start again. Notice that there is no contraction theorem for this mapping, which means that there is no guarantee to succeed by using this successive approximations approach. For the iterations to make good progress, it turns out to be very important to select proper initial conditions $\{v_\omega^0, v_h^0\}$.

A.2 Finding the equilibrium forecasting rules

The nature of the stationary stochastic equilibrium implies finding a distribution μ. The *partial information* approach is based on finding a vector of forecasting functions $f \equiv \left\{f^K, f^H, f^{R^a}\right\} \in \mathcal{F} \equiv \mathcal{F}^K \times \mathcal{F}^H \times \mathcal{F}^{R^a}$ consistent with rational expectations. I.e., given that agents forecast K, H and R^b with certain f, the simulated economy should display this same behavior. Or in other words, the simulated series for K, H and R^b should be *well* predicted by f. The idea is to start with an initial f^0, solve the household's problem defined in section A.1, simulate the economy for a long series of periods and estimate a new f^1 within the same parametric class $\mathcal{F}$. Krusell and Smith (1997) show that one needs to make one correction to this procedure. Precisely, the market for bonds does not clear in every period and state. In order to achieve the bond market clearing in every period and state, we define the following problem:

$$v\left(\omega, h, \epsilon, K, H, R^b\right) = \max_{c,a,s}\left\{u(c,h) + \beta E_{\epsilon'|\epsilon}\left[v\left(\omega', h', \epsilon', K', H'\right)\right]\right\} \tag{20}$$

subject to

$$\begin{aligned}
c &= \omega - a - s \\
h' &= \lambda c + (1-\lambda) h \\
\omega' &= bR^b + sR^s\left(\epsilon', K', R^b\right) + \nu\left(\epsilon', K', H'\right) \\
K' &= f^K\left(\epsilon, K, H\right) \\
H' &= f^H\left(\epsilon, K, H\right) \\
R^b &= f^{R^b}\left(\epsilon, K, H\right) \\
(\omega, b, s, c) &\geq (\underline{\omega}, \underline{b}, \underline{s}, 0)
\end{aligned}$$

This problem differs from the original one in the fact that R^b is an state variable for today, although tomorrow's R^b is perceived to follow the forecasting rule f^{R^b}. I.e., tomorrow's value

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[32] In the K and H dimension there is not much curvature, so we use much fewer points than in the ω and h dimensions. We typically use 6 points for the aggregate variables, 35 for wealth ω and 10 for habit stock h. For these two variables the grid is much thicker at its beginning than at its end since it is for small values that there is more curvature.

function is given by problem 3. In this manner one can find an R^b that exactly clears the bond market. Solution of this problem delivers $g^c\left(\omega, h, \epsilon, K, H, R^b\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^b\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^b\right)$. At this stage we can state the algorithm as follows

1. Take an initial f^0
2. Solve the household' problem given by 19
3. Simulate the economy.
    (a) Set an initial distribution of agents over ω, h and ϵ.
    (b) Iterate to find the R^b that clears the bond market. Give an initial $R^{b,0}$ and solve the problem 20 to find $g^c\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$. If there is an excess of lending in the bond market try $R^{b,1} < R^{b,0}$, if there is an excess of borrowing try $R^{b,1} > R^{b,0}$. Go on until finding an $R^{b,*}$ the clears the market. [33,34]
    (c) Get next period distribution over ω, h and ϵ by using $g^c\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$ and drawing a new value for the shock ϵ.
    (d) Come back to step (b). Do it for a large number of periods.
4. Use the simulated series for K, H and $R^{b,*}$ to estimate f^1.
5. Compare f^1 and f^0. If they are similar we are done, if not start again by setting $f^0 = f^1$ and going back to point 2.

There is just one last issue to be clarified. Which is the proper class $\mathcal{F}$ where to define our forecasting rules? In a problem without habit formation Krusell and Smith (1997) show that a linear function on the first moment of the wealth distribution is enough. We set the following rules

$$\begin{aligned}
\log K' &= \left\{ \begin{array}{ll} cf_{kg0} + cf_{kgk}\log K + cf_{kgh}\log H & \text{if } z = z_g \\ cf_{kb0} + cf_{kbk}\log K + cf_{kbh}\log H & \text{if } z = z_b \end{array} \right\} \\
\log H' &= \left\{ \begin{array}{ll} cf_{hg0} + cf_{hgk}\log K + cf_{hgh}\log H & \text{if } z = z_g \\ cf_{hb0} + cf_{hbk}\log K + cf_{hbh}\log H & \text{if } z = z_b \end{array} \right\} \\
\log R^b &= \left\{ \begin{array}{cc} \begin{array}{c} cf_{Rg0} + cf_{Rgk}\log K + cf_{Rgh}\log H + cf_{Rgkk}\left(\log K\right)^2 \\ +cf_{Rghh}\left(\log H\right)^2 + cf_{Rgkh}\log K \log H \end{array} & \text{if } z = z_g \\ \begin{array}{c} cf_{Rb0} + cf_{Rbk}\log K + cf_{Rbh}\log H + cf_{Rbkk}\left(\log K\right)^2 \\ +cf_{Rbhh}\left(\log H\right)^2 + cf_{Rbkh}\log K \log H \end{array} & \text{if } z = z_b \end{array} \right\}
\end{aligned}$$

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[33] Or until $R^{b,1} \simeq R^{b,0}$

[34] An alternative (and computationally cheaper) approach would be to solve the problem generally for R^b and then interpolate different values $R^{b,0}$, $R^{b,1}$, ... until market clears. The problem with this is its inexactitude. We would need an extremely fine grid on R^b to make the results along different periods of the simulation consistent among them.

Our findings are that we do not need so much information. Aggregate habits do not improve the forecasting. This actually means that aggregate habits turns out not to be an state variable of the system. Forecasting rules end up being:

$$\begin{aligned}\log K' &= \left\{\begin{array}{ll} cf_{kg0} + cf_{kgk}\log K & \text{if } z = z_g \\ cf_{kb0} + cf_{kbk}\log K & \text{if } z = z_b \end{array}\right\} \\ \log R^b &= \left\{\begin{array}{ll} cf_{Rg0} + cf_{Rgk}\log K + cf_{Rgkk}(\log K)^2 & \text{if } z = z_g \\ cf_{Rb0} + cf_{Rbk}\log K + cf_{Rbkk}(\log K)^2 & \text{if } z = z_b \end{array}\right\}\end{aligned}$$