Methods versus Substance: Measuring the Effects of Technology Shocks

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One Question and Three Methodologies

- Long-standing question in business cycle research: what fraction of the variation in hours worked is due to technology shocks?
- Empirical approaches: (i) Calibration of a DSGE model; (ii) Estimation of a DSGE model; (iii) VAR analysis.

Controversies

- How much theoretical structure should one impose on the theoretical model that is used for quantitative analysis.
- What is the role of formal econometrics – or
- Do DSGE model provide probability distributions that are realistic enough, such that one can derive meaningful statistical measures of uncertainty for quantitative model implications?

What We Do

- Study how the choice of methodology affects our answer of the question "What fraction..."
- We "fix" the data set and a basic theoretical framework (DSGE model).
- We (i) calibrate the DSGE model; (ii) estimate the DSGE model; (iii) estimate a structural VAR
- thereby varying (a) the identification scheme of key parameters and (b) the extent to which we impose theory on the data.

Households

- There is a continuum of households solving the following problem

$$\max \quad \boldsymbol{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - B \frac{H_t^{1+1/\nu}}{1+1/\nu} \right) \right] \tag{1}$$

$$\text{s.t.} \quad C_t + P_t X_t = W_t H_t + R_t P_t K_t \tag{2}$$

$$K_{t+1} = (1-\delta) K_t + X_t \tag{3}$$

- P_t is the price of the unit of the investment good (using the consumption good as numeraire).

Firms

- Firms rent capital and labor services from Households and produce consumption and investment goods.
- Technology:

$$C_t + \frac{X_t}{V_t} = A_t K_t^{\alpha} H_t^{1-\alpha}$$

- Profits:

$$\Pi_t = C_t + P_t X_t - W_t H_t - R_t P_t K_t$$

- For the firms to be willing to produce both consumption and investment goods it has to be the case that $P_t = 1/V_t$.

NIPA and Exogenous Processes

- NIPA: investment is measured as $I_t = X_t P_t$. Hence,

$$Y_t = C_t + I_t = A_t K_t^{\alpha} H_t^{1-\alpha}.$$

- We will specify an autoregressive law of motion for the (exogenous) technology processes:

$$\begin{aligned}
&(\ln A_t - \ln A_0 - \gamma_a t) \\
&\quad = \rho_{a,1}(\ln A_{t-1} - \ln A_0 - \gamma_a(t-1)) \\
&\qquad +\rho_{a,2}(\ln A_{t-2} - \ln A_0 - \gamma_a(t-2)) + \sigma_a \epsilon_{a,t} \\
&(\ln V_t - \ln V_0 - \gamma_v t) \\
&\quad = \rho_{v,1}(\ln V_{t-1} - \ln V_0 - \gamma_v(t-1)) \\
&\qquad +\rho_{v,2}(\ln V_{t-2} - \ln V_0 - \gamma_v(t-2)) + \sigma_v \epsilon_{v,t}.
\end{aligned}$$

Remark

- The specification of the model is incomplete at this point.
- We only specified technology shocks, but have not taken a stand on what generates the remaining variation in the data.
- If we would belief that technology shocks are the only shocks driving business cycle fluctuations, our question would be answered by now!

Answering the Question: Three Approaches

- A Calibration
- Bayesian estimation of the DSGE model
- A structural VAR, loosely based on the model
- We use data from 1955:Q1 to 2004:Q4

Calibration

- We will generate a quantitative answer with a model that is "incomplete."
- Rationale: making auxiliary assumptions about other shocks might distort the inference.
- Use data to identify plausible values for the parameters of the DSGE model.
- Simulate the DSGE model under these parameters and compute the ratio of the (expected) sample variance of simulated data to the actual sample variance of observed data.

How Can we Identify the Parameters?

- $1-\alpha$: average data on the labor share $W_t H_t / Y_t$.
- β: average data on real interest rates, also affects investment output ratio.
- V_t: According to our model, the investment-specific technology shock corresponds to the (reciprocal) relative price of investment goods P_t.
- P_t: take from the data, we follow Cummins and Violante (2002).

Capital and Neutral Technology

- Observations on the neutral technology:

$$A_t = Y_t / K_t^{\alpha} H_t^{1-\alpha}$$

- This requires observations on K_t (and H_t).
- K_t: Capital stock is constructed according to:

$$K_{t+1} = (1-\delta)K_t + X_t$$

- δ: average of the time-varying annual depreciation rates for total capital provided by Cummins and Violante (2002).
- $X_t = I_t / P_t$: use data on investment measured in terms of the consumption good, convert into physical units.

So far...

- We have used data and model implications to identify

$$\alpha, \beta, \delta, A_t, V_t$$

- and we can easily estimate the AR coefficients for the technology processes.

Labor Supply Elasticity

- Link labor supply elasticity to steady state relationship. Suppose preferences are of the form

  $$\ln C_t + \ln(1 - H_t)$$

  Then Frisch elasticity is given by $(1 - H^*)/H^*$. If households work $1/3$ of their time then Frisch elasticity is 2.
- Most variation in hours occurs on the extensive margin. Use indivisible labor model, which leads to $\nu = \infty$ (we'll take $\nu = 100$).
- Look at prime age white males who have no choice but to work 40.xx hours a week: $\nu = 0.2$.

Model Simulation

- We can surround the calibrated parameters by probability distributions.
- Draw parameter vector and simulated 200 observations from model.
- Compute the ratio of the variance of hours based on actual and model generated data.

Sample Variance Ratios for Hours: Model / Data

	$\nu = 0.2$		$\nu = 2$		$\nu = 100$	
Shock	Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
			Stochastic Trend			
A	.002	[.001, .003]	0.05	[0.02, 0.07]	0.14	[0.07, 0.21]
V	.010	[.002, .017]	0.23	[0.06, 0.42]	0.83	[0.25, 1.45]
			Deterministic Trend			
A	.003	[.001, .005]	0.09	[0.04, 0.13]	0.31	[0.14, 0.45]
V	.007	[.003, .010]	0.22	[0.09, 0.34]	0.98	[0.34, 1.54]

DSGE Model Estimation

- The DSGE model will be estimated based on the investment price series, labor productivity, and hours worked.
- So far: three observables and two shocks. To overcome degeneracy of likelihood, we introduce a preference shock:

$$\ln(B_t/B) = \rho_b \ln(B_{t-1}/B) + \sigma_b \epsilon_{b,t}.$$

- Kalman Filter produces (latent) investment, capital, and neutral technology process.

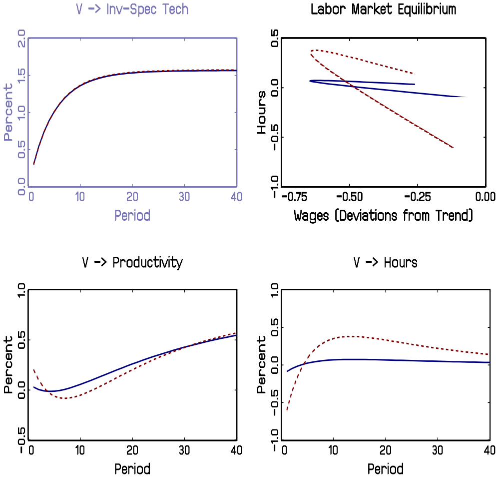
Priors

- Provide a convenient method of incorporating additional information in the estimation.
- The calibration used information about the labor share, real interest rates, and capital depreciation to determine α, β, and δ.
- We can use the same information to specify priors for these parameters.
- We use fairly uninformative priors for the remaining parameters.

Labor Supply Elasticity – Using P_t Data

- How is the labor supply elasticity identified in the estimation?
- Likelihood tries to match sample and model-implied autocovariances. What's the economics?
- Our model has hard-wired the restriction that $-\ln P_t$ is exogenous and can be interpreted as investment-specific technology shock.
- This restriction essentially identifies the dynamic response of the autoregressive system to an innovation $\epsilon_{v,t}$.
- Resulting shifts in the labor market equilibrium can identify labor supply elasticity.

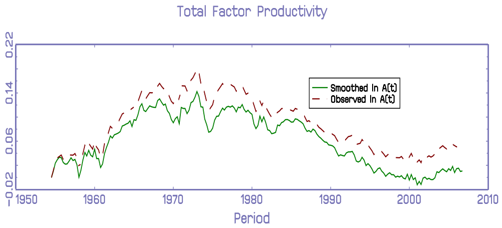
Labor Market Equilibrium: $\nu = 0.2$ versus $\nu = 2$

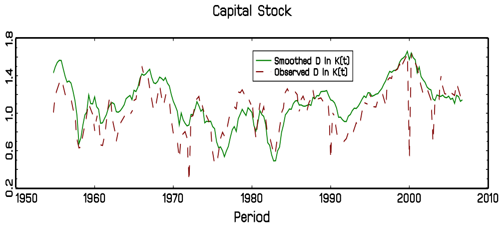


Posterior Estimates

	Deterministic Trend		Stochastic Trend	
	Mean	90% Cred. Intv	Mean	90% Cred Intv
		Parameter Values		
ν	0.670	[0.296, 1.038]	0.302	[0.050, 0.533]
		Variance Ratios for Hours: Model / Data		
A	0.030	[0.010, 0.060]	0.010	[0.000, 0.020]
V	0.060	[0.010, 0.100]	0.010	[0.000, 0.030]
A, V	0.100	[0.020, 0.180]	0.020	[0.000, 0.050]
$\ln p(Y)$	2264.74		2267.60	

Latent Capital Growth and Total Factor Productivity





Change of Observables

- Estimate model with data on: investment X_t, labor productivity, hours.
- Intuition: tight prior on δ and α implies that we are constructing A_t from observables.
- Labor supply elasticity is probably identified along response to neutral technology.
- Deterministic trend: $\hat{\nu}$ rises from 0.67 to 0.78.
- Stochastic trend: $\hat{\nu}$ rises from 0.30 to 0.60

VAR Analysis

- (Structural) VAR:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + \Phi_\epsilon \epsilon_t$$

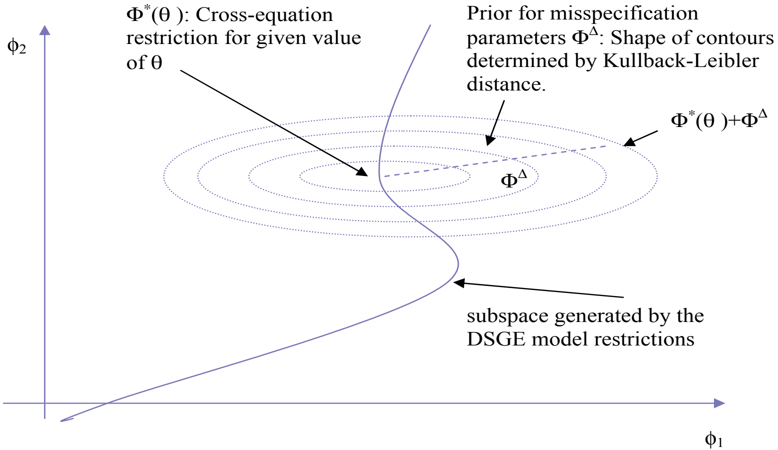
- Let y_t be composed of the *growth rates* of the investment goods price and labor productivity, and the *log level* of hours worked .
- Our interpretation: the vector ϵ_t is composed of the two technology shock innovations as well as the innovation to a third shock.
- One can think of the third shock as preference shock, but we don't have to take a stand. The innovations are normalized to have unit variance.

VAR and DSGE Model

- We can link the DSGE model and the VAR by assuming that we estimate a VAR based on infinitely many observations generated from the DSGE model, conditional on structural parameters θ:

$$\Phi^*(\theta) = [\Gamma_{XX}(\theta)]^{-1}\Gamma_{XY}(\theta)$$

- A concern when estimating a DSGE model is that we are imposing invalid cross-coefficient restrictions on the data.
- VARs are in general less restrictive and try to let the data speak.
- To relax the cross-coefficient restrictions, we can use a prior distribution that has a lot of mass *near* the restrictions but does not dogmatically impose them. (Del Negro and Schorfheide, 2004).
- We use a hyperparameter λ to scale the prior variance: larger λ, the more tightly the prior contours are concentrated.



$\Phi^*(\theta)$: Cross-equation restriction for given value of θ
Prior for misspecification parameters Φ^{Δ}: Shape of contours determined by Kullback-Leibler distance.
$\Phi^*(\theta)+\Phi^{\Delta}$
Φ^{Δ}
subspace generated by the DSGE model restrictions
ϕ_2
ϕ_1

Identification

- To answer our substantive questions we need to identify the technology shocks.
- In the DSGE model we can calculate:

$$\left(\frac{\partial y_t}{\partial \epsilon_t'}\right)_{DSGE} = A(\theta),$$

  say. Then use QR decomposition of $A(\theta)$ to decompose $A(\theta)$ into a lower triangular matrix and an orthonormal matrix $\Omega^*(\theta)$.
- For the VAR analysis we can now use:

$$\Phi_\epsilon = \Sigma_{tr}\Omega^*(\theta)$$

- Hence, along the restriction function the VAR impulse responses to structural shocks will closely resemble the DSGE model impulse responses, at least in the short run.

The DSGE-VAR

- We now have the following hierarchical model:
  - Likelihood function: $p(Y|\Phi, \Sigma)$
  - Prior for DSGE model parameters: $p(\theta)$
  - Prior for VAR parameters: $p(\Phi, \Sigma, \Omega|\theta, \lambda)$
- Joint distribution (conditional on λ):

$$p(Y|\Phi, \Sigma)p(\Phi, \Sigma|\theta, \lambda)p(\Omega|\theta)p(\theta)$$

- Use MCMC methods described in Del Negro and Schorfheide (2004) to generate draws from the joint posterior distribution.

Choosing the Hyperparameter λ

- We can study the fit of the DSGE model and determine by how much the cross-coefficient restrictions need to be relaxed by examining the marginal likelihood function of the hyperparameter λ:

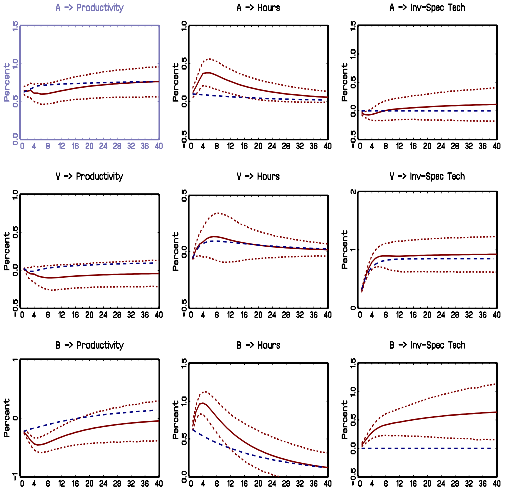
$$p(Y|\lambda) = \int p(Y|\Phi, \Sigma_u,)p(\Phi, \Sigma, \Omega, \theta|\lambda)d(\theta, \Phi, \Sigma, \Omega). \qquad (4)$$

- The marginal likelihood penalizes the in-sample-fit of the estimated VAR by a measure of complexity. The larger λ, the more restricted the prior, the smaller the model complexity, and the smaller the penalty.

DSGE-VAR Estimates

	DSGE-VAR($\lambda = \infty$)		DSGE-VAR($\lambda = 1$)	
	Mean	90% Cred. Intv	Mean	90% Cred Intv
Parameter Values				
ν	0.229	[0.056, 0.395]	0.484	[0.151, 0.815]
Variance Ratios for Hours: Model / Data				
A	0.004	[0.000, 0.009]	0.128	[0.004, 0.249]
V	0.010	[0.000, 0.021]	0.030	[0.001, 0.070]
A, V	0.014	[0.000, 0.029]	0.158	[0.001, 0.298]
$\ln p(Y)$	2278.14		2322.83	

DSGE-VAR($\lambda = 1$) versus DSGE-VAR($\lambda = \infty$)



Conclusions

- *Calibration of DSGE Model:* careful choice of observations to identify the model parameters. Except: labor supply elasticity is tricky! Quantitative statements rely heavily on theory. *Values: 0.01 to 1.29.*
- *Estimation of DSGE Model:* "complete" the model with additional shocks. Identify the labor supply elasticity from the relative response of hours and productivity to an investment-specific technology shock. While identification of many estimated DSGE models mysterious, our analysis was fairly transparent. *Values: 0.02 to 0.1*
- *(DSGE)-VAR analysis*: suggests that our theoretical model imposes invalid cross-equation restrictions. In turn we relaxed these restrictions while maintaining identification of the technology shocks. The response of hours was not constrained by a single Frisch elasticity. *Values: 0.16*

Finally...

- Methodologies do not really change the answers, but the identification used for the aggregate labor supply elasticity does.
- Econometrics helps documenting model fit and can provide measures of parameter uncertainty, but is no substitute for thinking carefully about identification.