A Model of Money and Credit, with Application to the Credit Card Debt Puzzle*

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Abstract

Many individuals simultaneously have significant credit card debt and money in the bank. The so-called credit card debt puzzle is, given high interest rates on credit cards and low interest rates on bank accounts, why not pay down this debt? Economists have gone to some lengths to explain this. As an alternative, we present a natural extension of the standard model in monetary economics to incorporate consumer debt, which we think is interesting in its own right, and which shows that the coexistence of debt and money in the bank is no puzzle.

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1 Introduction

A large number of households simultaneously have significant credit card debt and a significant amount of money in checking and savings accounts. Although there are many ways to measure this, a simple summary statistic is that 27% of U.S. households in 2001 had credit card debt in excess of $500, and over $500 in checking, savings and brokerage accounts. This is the so-called credit card debt puzzle: given 14% interest on credit card debt and 1 or 2% on bank accounts, why not pay down the debt? "Such behavior is *puzzling*, apparently inconsistent with no-arbitrage and thus inconsistent with any conventional model." (Gross and Souleles 2001, emphasis added).

Economists have gone to elaborate lengths to explain this phenomenon. For example, some people assume that consumers cannot control themselves (Laibson et al. 2000); others assume they cannot control their spouses (Bertaut and Haliassos 2002; Haliassos and Reiter 2003); still others hypothesize, counter to the facts, that all such households are on the verge of bankruptcy (Lehnert and Maki, 2001). We show that one does not have to resort to such extremes. This is not to say that these ideas have no merit, but simply that standard theory is not inconsistent with the observation that households simultaneously have substantial debt and money in the bank. By standard theory we mean modern monetary economics. These models are designed to study liquidity. They predict that agents may hold assets with low rates of return if they are liquid – i.e. if they have use as a medium of exchange.[1]

Our hypothesis accounts for the credit card debt puzzle in the following way. Households need money – more generally, liquid assets – for situations where credit cannot be used. The obvious and standard examples include taxis, cigarettes, and so on, although increasingly credit cards can be used for some of these, but we want to emphasize that there are also some big-ticket examples. For instance, usually rent or mortgage payments cannot be made by credit card. Thus, even if a household is revolving credit card debt, it needs to have money in the bank in order to meet these obligations. According to the Consumer Expenditure Survey, the median household that holds both debt and liquidity revolves $3,800 of credit card debt, has about $3,000 in the bank, and spends $1,993 per month on goods purchased with liquid assets

[1] Some version of this idea is in all of search-based monetary theory going back to Kiyotaki and Wright (1989).

(see Telyukova 2005). Moreover, according to the U.S. Statistical Abstract, 77% of consumer transactions in 2001 were done in liquid assets.

We develop a micro-founded model of monetary exchange to formalize these ideas. While we build upon recent work, and in particular Lagos and Wright (2005), we need to extend existing models to incorporate consumer credit, since the typical model in this literature does not have anything along these lines. This seems like a natural and interesting extension of modern monetary theory in its own right, and also allows us to argue that coexistence of consumer debt and money in the bank is not a puzzle. Of course, whether our approach is able to account quantitatively for salient aspects of the data remains to be seen, and is the subject of ongoing research (Telyukova 2005).[2]

2 The Basic Model

We build on Lagos and Wright (2005), hereafter LW. That model gives agents periodic access to a centralized market, in addition to the decentralized markets where, due to various frictions, money is essential for trade as in the typical search-based model. Having some centralized markets is interesting for its own sake, and also makes the analysis more tractable than in much of the literature on the microfoundations of money.[3] We will extend this framework along several dimensions.

We now describe the basic physical environment. In this section, we consider a special case; later on, we will generalize the model. Time is discrete and there is a [0,1] continuum of infinitely-lived agents. There is one general consumption good at each date that is nonstorable and perfectly divisible. Agents can produce the good in each period using labor as an input. There is also money in this economy, an object that is storable and perfectly divisible; it is

[2] To be concrete, there are several facets to the credit card debt puzzle. For example, in addition to having debt and liquidity in their portfolios at the same time, we observe households persistently revolving debt, something we do not address. In Telyukova (2005), the current model is extended to account for key empirical features of the credit card debt puzzle and to assess the validity of the theory quantitatively.

[3] See Molico (1997), Green and Zhou (1998), Camera and Corbae (1999), Zhou (1999) or Zhu (2003). A framework related to LW is described in Shi (1997). Earlier models, like Shi (1995) or Trejos and Wright (1995), were also tractable, but only because money was assumed to be indivisible.

intrinsically worthless but potentially has use as a medium of exchange. The money supply is fixed for now at M, but see below.

Although we frame the discussion as though agents literally use money to transact, it is now well known how to recast the model with agents depositing money into bank accounts and paying for goods using checks or debit cards. This is discussed in detail in He, Huang and Wright (2005). This is relevant for our purposes because what we have in mind is not necessarily cash, per se, but liquid assets generally. So when we say "agents carry money" in what follows, one should interpret this liberally as "agents hold liquid assets" or "have money in the bank".

In LW each period is divided into two subperiods. In one, say the morning, there is a centralized (frictionless, Walrasian) market. In the other, say the evening, there is a decentralized market where agents meet anonymously according to a random bilateral-matching process, which makes a medium of exchange essential. After each evening's meeting of the decentralized market, the next morning agents can consume, produce, and adjust their money holdings in the centralized market. Under the assumption of quasi-linear utility, it turns out that all agents will take the same amount of money out of the centralized and into the next decentralized market, which is a big simplification.

There is no role for credit in LW. Credit is not possible in the decentralized market, and not necessary in the centralized market. It is not possible in the decentralized market because of the assumption that agents are anonymous, which is needed to make money essential, and it is not necessary in the centralized market because of the assumption that all agents can produce, which is needed to make the distribution of money degenerate. Our idea is to introduce an intermediate subperiod, say afternoon, where at random some agents want to consume but cannot produce and vice-versa. There is a centralized market in this subperiod, where agents may use either credit or cash. This allows us to introduce consumer credit while maintaining both a role for a medium of exchange and the simplicity of LW.[4]

All agents want to consume in subperiod $j = 1$, and $u_1(x_1)$ is their common utility function. Only a random, and not necessarily the same, subset want to consume in $j = 2, 3$, and conditional

[4]We will generalize this below, but having three subperiods is sufficient to make the basic point. Berentsen, Camera and Waller (2005a) also have a third subperiod, but it is a second round of decentralized exchange, and hence there is no possibility of credit (but see also Berentsen, Camera and Waller 2005b).

on this, $u_j(x_j)$ is their utility function. All agents are able to produce in subperiod $j = 1$, and the disutility of working h_1 hours is linear, $c_1(h_1) = h_1$. A random subset are able to produce in subperiods $j = 2, 3$, and conditional on this, the disutility of working is some general convex function $c_j(h_j)$.When they can produce, all agents can transform labor one-for-one into output, $x_j = h_j$.[5] Let x_j^* denote efficient level of production – i.e. the solution to $u_j'(x_j^*) = c_j'(x_j^*)$. Let β_j be the discount factor between j and the next subperiod.

Generally, an individual's state variable is (m_{tj}, b_{tj}), denoting money and debt in period $t = 1, 2, ...$, subperiod $j = 1, 2, 3$, but we drop the t subscript when there is no risk of confusion. Let $W_j(m_j, b_j)$ be the value function in subperiod j. The value of money in the centralized markets at $j = 1, 2$ is ϕ_j; that is, $p_j = 1/\phi_j$ is the nominal price of the consumption good in subperiod j. There is no ϕ_3 since there is no centralized market at $j = 3$, although there will be an implicit price defined by bilateral trades in the decentralized market. Similarly, the real interest rate in the centralized markets at $j = 1, 2$ is r_j, but there is no r_3. Our covention for notation is as follows: if you bring debt b_j into subperiod $j = 1, 2$ you owe $(1 + r_j)b_j$.

The plan now is to consider each subperiod in turn. After this, we put the markets together and describe equilibrium.

2.1 Subperiod 1

In the morning, there is a standard centralized market. Given the state (m_1, b_1), agents solve[6]

$$\begin{aligned} W_1(m_1, b_1) &= \max_{x_1, h_1, m_2, b_2} \{u_1(x_1) - h_1 + \beta_1 W_2(m_2, b_2)\} \\ \text{s.t. } x_1 &= h_1 + \phi_1(m_1 - m_2) - (1 + r_1)b_1 + b_2. \end{aligned}$$

where x_1 is consumption, h_1 is labor, (m_2, b_2) gives the money and debt taken into subperiod 2, ϕ_1 is the value of money and r_1 is the interest due in subperiod 1 (of course this interest does not have to be paid now, and can be rolled over into b_2). Substituting h_1 from the budget

[5] The real wage w is constant, and normalized to 1, because implicitly we have firms with a linear technology; nothing of substance changes if we introduce more general firms and determine w endogenously (Aruoba and Wright 2003).

[6] To rule out Ponzi schemes, we implicitly impose a credit limit $b_j \leq B$, but it will not bind in equilibrium. Similarly, we implicitly have nonnegativity constraints, but assume they will not bind. See LW for conditions on fundamentals to guarantee that this is valid.

constraint into the objective function, we have

$$W_1(m_1, b_1) = \max_{x_1, m_2, b_2} \left\{ u_1(x_1) - [x_1 + \phi_1(m_2 - m_1) + (1 + r_1)b_1 - b_2] + W_2(m_2, b_2) \right\}.$$

The first-order conditions are

$$1 = u_1'(x_1) \tag{1}$$

$$\phi_1 = \beta_1 W_{2m}(m_2, b_2) \tag{2}$$

$$-1 = \beta_1 W_{2b}(m_2, b_2). \tag{3}$$

The first condition implies $x_1 = x_1^*$ for all agents. The other two determine (m_2, b_2), independent of x_1 and (m_1, b_1), a generalization of one of the basic results in LW. As long as W_2 is strictly concave, there is a unique solution. It is simple to check that the conditions to guarantee strict concavity in m in LW also apply here, so we can use this to conclude $m_2 = M$ for all agents. However, W_2 is actually linear in b_2, which means we cannot pin down b_2 for any individual.

This is no surprise with a perfectly competitive credit market and quasi-linear utility: given the equilibrium interest rates (see below), agents are indifferent to working a little more today and less tomorrow, so for any one of them we can raise h_1 and lower b_2. We cannot do this in the aggregate, of course, since average labor input $\bar{h}_1$ must equal total output x_1^*. We resolve this payoff-irrelevant indeterminacy by focusing on equilibria where we impose that when two agents have the same set of solutions to a maximization problem, they choose the same one. Nothing of substance hinges on this; other equilibria are payoff equivalent and observationally equivalent at the aggregate level. It simply means that we have $b_2 = \bar{b}_2$ for all agents.

Aggregating budget equations across agents,

$$\bar{x}_1 = \bar{h}_1 + \phi_1(\bar{m}_1 - \bar{m}_2) - (1 + r_1)\bar{b}_1 + \bar{b}_2.$$

We have $\bar{h}_1 = \bar{x}_1 = x_1^*$ and $\bar{m}_1 = \bar{m}_2 = M$, clearly, and $\bar{b}_1 = 0$ because average debt must be 0. Hence in equilibrium $b_2 = \bar{b}_2 = 0$ for all agents. This simply says that in equilibrium agents settle all past debts in subperiod 1; they are happy to do so, given quasi-linear utility. Hence we have

$$m_2 = M \text{ and } b_2 = 0 \text{ for all agents.}$$

To close the analysis of subperiod 1, we have the envelope conditions

$$W_{1m}(m_1, b_1) = \phi_1 \quad (4)$$
$$W_{1b}(m_1, b_1) = -(1 + r_1). \quad (5)$$

So W_1 is linear in (m_1, b_1).

2.2 Subperiod 2

In the afternoon, some agents want to consume but cannot produce, and vice-versa. To ease the presentation, assume these events are i.i.d., and each period a measure $\pi \leq 1/2$ of agents want to consume but cannot produce, and the same measure π can produce but do not want to consume (to reduce the notation, we assume that no agent does both, but this is easy to relax). Feasibility requires $x_2^C = h_2^P$, where x_2^C is the consumption of those who want to consume and h_2^P the production of those able to produce (it is also easy to relax the assumption that there is the same measure of producers and consumers). The expected value of entering the subperiod 2 centralized market is

$$W_2(m_2, b_2) = \pi W_2^C(m_2, b_2) + \pi W_2^P(m_2, b_2) + (1 - 2\pi) W_2^N(m_2, b_2),$$

where W_2^C, W_2^P and W_2^N are the value functions for a consumer, a producer and a nontrader.

For a nontrader,

$$\begin{aligned} W_2^N(m_2, b_2) &= \max_{m_3, b_3} \beta_2 W_3(m_3, b_3) \\ \text{s.t. } 0 &= \phi_2(m_2 - m_3) - (1 + r_2) b_2 + b_3, \end{aligned}$$

where r_2 interest due on debt brought into the subperiod (which again can be rolled over to the next subperiod). Note that although a nontrader neither consumes not produces, he can adjust his portfolio.[7] One can rewrite

$$W_2^N(m_2, b_2) = \max_{m_3} \beta_2 W_3 \left[m_3, \phi_2(m_3 - m_2) + (1 + r_2) b_2 \right],$$

[7] Hence, it might appear that calling these agents nontraders is inaccurate, but we will see that in fact they do not trade in equilibrium.

which implies the solution (m_3^N, b_3^N) satisfies

$$W_{3m}(m_3^N, b_3^N) = -\phi_2 W_{3b}(m_3^N, b_3^N) \tag{6}$$

plus the budget equation. The envelope conditions are

$$\begin{aligned} W_{2m}^N(m_2, b_2) &= \beta_2 W_{3m}(m_3^N, b_3^N) && (7)\\ W_{2b}^N(m_2, b_2) &= \beta_2(1+r_2)W_{3b}(m_3^N, b_3^N). && (8) \end{aligned}$$

For a consumer,

$$\begin{aligned} W_2^C(m_2, b_2) &= \max_{x_2, m_3, b_3} \{u_2(x_2) + \beta_2 W_3(m_3, b_3)\} \\ \text{s.t. } x_2 &= \phi_2(m_2 - m_3) - (1+r_2)b_2 + b_3. \end{aligned}$$

One can rewrite

$$W_2^C(m, b) = \max_{m_3, b_3} \{u_2 [\phi_2(m_2 - m_3) - (1+r_2)b_2 + b_3] + \beta_2 W_3(m_3, b_3)\},$$

which implies the solution (x_2^C, m_3^C, b_3^C) satisfies

$$\begin{aligned} \phi_2 u_2'(x_2^C) &= \beta_2 W_{3m}(m_3^C, b_3^C) && (9)\\ -u_2'(x_2^C) &= \beta_2 W_{3b}(m_3^C, b_3^C) && (10) \end{aligned}$$

plus the budget equation. Using these, we write the envelope conditions as

$$\begin{aligned} W_{2m}^C(m_2, b_2) &= \phi_2 u_2'(x_2^C) = \beta_2 W_{3m}(m_3^C, b_3^C) && (11)\\ W_{2b}^C(m_2, b_2) &= -(1+r_2)u_2'(x_2^C) = (1+r_2)\beta_2 W_{3b}(m_3^C, b_3^C). && (12) \end{aligned}$$

For a producer,

$$\begin{aligned} W_2^P(m_2, b_2) &= \max_{h_2, m_3, b_3} \{-c_2(h_2) + \beta_2 W_3(m_3, b_3)\} \\ \text{s.t. } 0 &= h_2 + \phi_2(m_2 - m_3) - (1+r_2)b_2 + b_3. \end{aligned}$$

We rewrite

$$W_2^P(m_2, b_2) = \max_{m_3, b_3} \{-c_2 [\phi_2(m_3 - m_2) - b_3 + (1+r_2)b] + \beta_2 W_3(m_3, b_3)\}$$

and the solution (h_2^P, m_3^P, b_3^P) satisfies

$$\phi_2 c_2'(h_2^P) = \beta_2 W_{3m}(m_3^P, b_3^P) \tag{13}$$

$$-c_2'(h_2^P) = \beta_2 W_{3b}(m_3^P, b_3^P) \tag{14}$$

plus the budget equation. The envelope conditions are

$$\begin{aligned} W_{2m}^P(m_2, b_2) &= \phi_2 c_2'(h_2^P) = \beta_2 W_{3m}(m_3^P, b_3^P) \\ W_{2b}^P(m_2, b_2) &= -(1+r_2)c_2'(h_2^P) = (1+r_2)\beta_2 W_{3b}(m_3^P, b_3^P). \end{aligned}$$

We cannot conclude here that (m_3, b_3) is independent of (m_2, b_2), as we could for subperiod 1, where we had (m_2, b_2) independent of (m_1, b_1), since we are not necessarily assuming quasilinearity here. Thus, if x_2^C depends on (m_2, b_2) then so will (m_3^C, b_3^C), in general, unless u_2 is linear, and if h_2^P depends on (m_2, b_2) then so will (m_3^P, b_3^P), in general, unless c_2 is linear. This actually does not make a big difference in equilibrium, since we already established that $(m_2, b_2) = (M, 0)$ for all agents. But even if, e.g., all consumers choose the same (x_2, m_2, b_2), so far we have nothing to say about the comparison of (m_3, b_3) across consumers, producers and nontraders.

In any case, we can combine envelope conditions to write

$$W_{2m}(m_2, b_2) = \beta_2 \pi [W_{3m}(m_3^C, b_3^C) + W_{3m}(m_3^P, b_3^P) + \tfrac{1-2\pi}{\pi} W_{3m}(m_3^N, b_3^N)] \tag{15}$$

$$W_{2b}(m_2, b_2) = \beta_2 (1+r_2) \pi [W_{3b}(m_3^C, b_3^C) + W_{3b}(m_3^P, b_3^P) + \tfrac{1-2\pi}{\pi} W_{3b}(m_3^N, b_3^N)] \tag{16}$$

Again, note that the continuation value $W_3(m_3, b_3)$ does not depend on whether one was a consumer, producer or nontrader in subperiod 2, except inasmuch as this affects the state (m_3, b_3).

2.3 Subperiod 3

In the evening, agents enter the decentralized market where trade occurs via anonymous bilateral meetings. Because of anonymity, you cannot use credit: I will not take your promise for payment tomorrow because I understand that you can renege without fear of punishment (Kocherlakota 1998; Wallace 2001). I will, however, take cash. I will also take a check in the version of the model laid out in He, Huang and Wright (2005), because there a check is a claim on your bank and not you personally (think of travellers' checks). Similarly, I would take a debit card, which is tantamount to cash. So you can pay with money, or with money in the bank, but you cannot get credit.[8]

Before presenting the value functions, consider a single-coincidence meeting, where one agent wants to consume and the other can produce. Call the former agent the *buyer* and the latter the *seller*. They bargain over the amount of consumption for the buyer x_3 and labor by the seller h_3, and also a dollar payment d from to the former to the latter. Since feasibility implies $x_3 = h_3$ we denote their common value by q. If (m_3, b_3) is the state of a buyer and $(\tilde{m}_3, \tilde{b}_3)$ the state of a seller, the outcome satisfies the generalized Nash bargaining solution,

$$(q, d) \in \arg\max \ S(m_3, b_3)^{\theta} \tilde{S}(\tilde{m}_3, \tilde{b}_3)^{1-\theta} \text{ s.t. } d \leq m_3, \tag{17}$$

where the constraint says the buyer cannot transfer more cash than he has, θ is the bargaining power of the buyer, and

$$\begin{aligned} S(m_3, b_3) &= u_3(q) + \beta_3 W_{1,+1}(m_3 - d, b_3) - \beta_3 W_{1,+1}(m_3, b_3) \\ \tilde{S}(\tilde{m}_3, \tilde{b}_3) &= -c_3(q) + \beta_3 W_{1,+1}(\tilde{m}_3 + d, \tilde{b}_3) - \beta_3 W_{1,+1}(\tilde{m}_3, \tilde{b}_3) \end{aligned}$$

are the surpluses. A +1 in the subscript denotes next period. Using (4) and (5), the surpluses simplify to

$$\begin{aligned} S(m_3, b_3) &= u_3(q) - \beta_3 \phi_{1,+1} d \\ \tilde{S}(\tilde{m}_3, \tilde{b}_3) &= -c_3(q) + \beta_3 \phi_{1,+1} d \end{aligned}$$

Now the following result is a straightforward generalization of LW; see the Appendix for the the proof.

Lemma 1. *$\forall (m_3, b_3)$ and $(\tilde{m}_3, \tilde{b}_3)$, the solution to the bargaining problem is*

$$q = \begin{cases} g^{-1}(\beta_3 m_3 \phi_{1,+1}) & \text{if } m_3 < m_3^* \\ q^* & \text{if } m_3 \geq m_3^* \end{cases} \quad \text{and } d = \begin{cases} m_3 & \text{if } m_3 < m_3^* \\ m_3^* & \text{if } m_3 \geq m_3^* \end{cases} \tag{18}$$

[8] Since the model is highly stylized, it is not clear exactly what a credit card might be; whatever it is, it cannot be used in this market. Although it is perhaps good to be agnostic about this, it is also good to have a story. One story is that agents can produce fake credit cards. Note that fake debit cards are not possible, because these involve instant settlement using money that is already on deposit at the bank. The same is true for travellers' checks, or personal checks that are verified using a check card. The point is that although we think the model captures nicely the distinction between cash and credit, one has to work a little to use it to discuss credit cards and other means of payment.

where q^* *solves* $u_3'(q^*) = c_3'(q^*)$*, the function* $g(\cdot)$ *is given by*

$$g(q) = \frac{\theta u_3'(q) c_3(q) + (1-\theta) u_3(q) c_3'(q)}{\theta u_3'(q) + (1-\theta) c_3'(q)}, \tag{19}$$

and $m_3^* = g(q^*)/\beta_3 \phi_{1,+1}$.

Clearly, the bargaining solution (q, d) depends on the buyer's money holdings m_3, but on no other element of (m_3, b_3) or $(\tilde{m}_3, \tilde{b}_3)$; hence we write $q = q(m_3)$ and $d = d(m_3)$ from now on. Of course, q and d at t also depend on ϕ_1 at $t+1$, but this is left implicit in the notation. We argue in the Appendix that, as in LW, $m_3 < m_3^*$ in any equilibrium. So from Lemma 1, buyers always spend all their money m_3 and receive $q = g^{-1}(\beta_3 m_3 \phi_{1,+1})$. Notice $g' > 0$; thus, if the buyer brings an additional dollar to this market, the terms of trade change according to $\partial q/\partial m_3 = \beta_3 \phi_{1,+1}/g'(q) > 0$ and $\partial d/\partial m_3 = 1$.

Define

$$z(q) = \frac{u_3'(q)}{g'(q)}. \tag{20}$$

As is usual in this type of model, it is useful to make the assumption that $z'(q) < 0$ (see e.g. Rocheteau and Wright 2005). This assumption is not completely standard, as it involves third derivatives of utility.[9] We will not dwell on this here, except to say that conditions on preferences to guarantee that the assumption holds can be found in LW, and also to note that it always holds (for any preferences) when $\theta \approx 1$, because at $\theta = 1$, $g(q) = c_3(q)$ and $z(q) = u_3'(q)/c_3'(q)$.

This completes the analysis of bargaining in a single-coincidence meeting. Let σ denote the probability of such a meeting – i.e. the probability that two agents meet, one wants to consume the other's good but cannot produce, and the other can produce but does not want to consume.[10]

[9] Since g depends on u_3' and c_3', g' depends on u_3'' and c_3'', and the monotonicity of u'/g' depends on third derivatives.

[10] In general, in the standard version of the model with many specialized commodities, one interprets a single-coincidence meeting as one where one agent wants what the other can produce, but not vice-versa. Since there is only one good in this paper, we interpret it instead as a meeting where one agent wants to consume but cannot produce, while the other can produce but does not want to consume. In the standard model, it is easy to allow some double-coincidence meetings, where both agents want what the other can produce. Here we can similarly allow some agents to both consume and produce in subperiod 3 – in which case they do not even need a meeting – but it adds little of interest.

Given the above results, the value function $W_3(m_3, b_3)$ satisfies

$$\begin{aligned} W_3(m_3, b_3) &= \sigma\left\{u_3[q(m_3)t] + \beta_3 W_1[m_3 - d(m_3), b_3]\right\} \\ &\quad + \sigma\mathbb{E}\left\{-c_3[q(\tilde{m}_3)] + \beta_3 W_1[m_3 + d(\tilde{m}_3), b_3]\right\} \\ &\quad + (1-2\sigma)\beta_3 W_1[m_3, b_3], \end{aligned} \tag{21}$$

where $\mathbb{E}$ is the expectation of $\tilde{m}_3$ (the money holdings of other agents, which may be nondegenerate even though all agents carry the same amount of money out of subperiod 1, because they may leave subperiod 2 with different amounts, depending on whether they are consumers, producers or nontraders).

Differentiating (**??**), using the linearity of W_1 derived in (4) and (5), and $q'(m_3) = \beta_3\phi_{1,+1}/g'(q)$, we have

$$W_{3m}(m_3, b_3) = \beta_3\phi_{1,+1}\left\{\sigma z\left[q(m_3)\right] + 1 - \sigma\right\} \tag{22}$$

$$W_{3b}(m_3, b_3) = -\beta_3(1 + r_{1,+1}), \tag{23}$$

where $z(q)$ is given by (20). Here (22) gives the marginal value of money in the decentralized market as a weighted average of the values of using it in the decentralized market and of carrying it forward to the next subperiod. According to (23), the marginal value of debt is simply the value to rolling it over into subperiod 1 at $t+1$, since credit is not adjusted in the decentralized market.

2.4 Equilibrium

Definition 1. *An* equilibrium *is a set of value functions* $\{W_1, W_2, W_3\}$*, decision rules* $\{x_1, h_1, m_2, b_2, x_2^C, h_2^P, m_3^i, b_3^i, q, d\}$*,* $i \in \{C, P, N\}$*, and a set of bounded paths for prices* $\{r_1, r_2, \phi_1, \phi_2\}$*, such that:*

1. *Agents solve their maximization problem at each subperiod, and the relevant decision rules satisfy first-order conditions (1)-(3) together with (15) and (16) in subperiod 1; conditions (6), (9) and (10), (13) and (14), with (22) and (23) in subperiod 2; while q and d satisfy the bargaining solution (18).*

2. Markets clear at each subperiod, and $\forall t$*:*

$$\begin{aligned}
\int x_1 dF_1(m_1, b_1) &= \int h_1 dF_1(m_1, b_1) \\
\int x_2 dF_2(m_2, b_2) &= \int h_2 dF_2(m_2, b_2) \\
\int m_2 dF_1(m_1, b_1) &= \int m_3 dF_2(m_2, b_2) = M \\
\int b_2 dF_1(m_1, b_1) &= \int b_3 dF_2(m_2, b_2) = 0,
\end{aligned}$$

where F_j *is the distribution of the individual state in subperiod* j*.*

We now put everything together to solve for an equilibrium that satisfies this definition. We establish a theorem that describes several conditions that any steady-state equilibrium must satisfy. However, recall that when we say any equilibrium, we mean any equilibrium with the following property: we impose that in subperiod 1, if two agents have multiple solutions for b_2, they both choose the same solution. As we discussed above, due to linearity in utility, there are other equilibria, but they are payoff-equivalent for individuals and observationally equivalent at the aggregate level. In particular, in these other equilibria, b_2 and h_1 may be different for individuals, although not at the aggregate level, but in any case hours in the other subperiods h_2 and h_3, as well as money holdings m_j and consumption x_j in every subperiod, will be identical for every individual to what is described below.

Theorem 1. *In any steady state monetary equilibrium:*

1. *In subperiod 1, all agents choose* $x_1 = x_1^*$*,* $m_2 = M$ *and* $b_2 = 0$*.*
2. *In subperiod 2,*

   consumers choose $x_2 = x_2^*$*,* $m_3 = M$ *and* $b_3 = x_2^*$*;*

   producers choose $h_2 = x_2^*$*,* $m_3 = M$ *and* $b_3 = -x_2^*$*;*

   nontraders choose $m_3 = M$ *and* $b_3 = 0$*.*
3. *In subperiod 3, in every trade* $d = M$ *and* q *solves the usual LW condition*

$$1 + \frac{\rho}{\sigma} = \frac{u_3'(q)}{g'(q)}. \tag{24}$$

where ρ *is the rate of time preference defined by* $\frac{1}{1+\rho} = \beta_1\beta_2\beta_3$*.*

4. Prices are given by:

$$\begin{aligned}
r_1 &= \frac{u_2'(x_2^*) - \beta_2\beta_3}{\beta_2\beta_3},\ r_2 = \frac{\rho - r_1}{1+r_1}, \\
\phi_1 &= \frac{g(q)}{\beta_3 M},\ \textit{and}\ \phi_2 = \frac{\phi_1\left[\sigma z(q) + 1 - \sigma\right]}{1+r_1}.
\end{aligned}$$

Proof: To begin, insert the envelope condition for W_{3b} from (23) into the first order conditions for consumers and producers with respect to b_3, (10) and (14), to get

$$u_2'(x_2^C) = \beta_2\beta_3(1+r_{1,+1}) \tag{25}$$

$$c_2'(h_2^P) = \beta_2\beta_3(1+r_{1,+1}). \tag{26}$$

Hence, $u_2'(x_2^C) = c_2'(h_2^P)$, and $x_2^C = h_2^P = x_2^*$. Similarly, insert the envelope condition for W_{3m} from (22) in to the first order conditions for consumers and producers with respect to m_3, (9) and (13), to get

$$\phi_2 u_2'(x_2^C) = \beta_2\beta_3\phi_{1,+1}\left\{\sigma z\left[q(m_3^C)\right] + 1 - \sigma\right\} \tag{27}$$

$$\phi_2 c_2'(h_2^P) = \beta_2\beta_3\phi_{1,+1}\left\{\sigma z\left[q(m_3^P)\right] + 1 - \sigma\right\} \tag{28}$$

Given $z(q)$ is decreasing and $q(m)$ is increasing for all $m < m_3^*$, and since $x_2^C = h_2^P = x_2^*$, we conclude that $m_3^C = m_3^P$.

Similarly, inserting the envelope conditions (23) and (22) into the first order condition for a nontrader,

$$\phi_{1,+1}\left\{\sigma z\left[q(m_3^N)\right] + 1 - \sigma\right\} = \phi_2(1+r_{1,+1}) \tag{29}$$

Exactly the same condition results from combining (25) and (27) for a consumer, or (26) and (28) for a producer. Hence, we conclude $m_3^N = m_3^C = m_3^P = M$, and everyone carries the same amount of money into subperiod 3. From the budget equations, this means debt is given by

$$\begin{aligned}
b_3^C &= x_2^* + (1+r_2)b_2 \\
b_3^P &= -x_2^* + (1+r_2)b_2 \\
b_3^N &= (1+r_2)b_2.
\end{aligned}$$

This completes the description of subperiod 2. Moving back to subperiod 1, clearly (1) implies $x_1 = x_1^*$. Inserting the envelope conditions for W_2 and W_3 into the first order conditions

(2) and (3) for m_1 and b_1, we have

$$\begin{aligned}\phi_1 &= \beta_1\beta_2\beta_3\phi_{1,+1}\sigma z[q(M)] + 1 - \sigma \quad (30)\\ 1 &= \beta_1\beta_2\beta_3(1+r_2)(1+r_{1,+1}), \quad (31)\end{aligned}$$

where we have used in the first case that W_{3m} depends on m_3 but not b_3, and everyone has the same $m_3 = M$. By construction, in the candidate equilibrium (31) holds, and this implies that any choice of b_1 is consistent with optimization; hence we can set $b_1 = 0$. On the other hand, (30) implies

$$(1+\rho)\frac{\phi_1}{\phi_{1,+1}} = \sigma z[q(M)] + 1 - \sigma. \quad (32)$$

In steady state this implies (24).

The only things left to determine are the prices. Now r_2 is defined in terms of r_1 by (31), and we get r_1 from (25) with $x_2 = x_2^*$. Given q, Lemma 1 tells us $\phi_1 = g(q)/\beta_3 M$, and (29) gives

$$\phi_2 = \frac{\phi_1[\sigma z(q) + 1 - \sigma]}{(1+r_1)}.$$

This completes the proof. ■

3 Discussion

Remark 1: In any – not only steady state – monetary equilibrium, essentially everything in the previous Theorem is true, except (32) does not reduce to (24). However, we can insert the bargaining solution $q = g^{-1}(\beta_3 m_3 \phi_{1,+1})$ from Lemma 1, which holds at every date in any equilibrium, to get

$$(1+\rho)\frac{g(q)/\beta_3 M}{g(q_{+1})/\beta_3 M} = \sigma z(q_{+1}) + 1 - \sigma. \quad (33)$$

A monetary equilibrium is now any (bounded, positive) solution $\{q_t\}$ to this difference equation, as is standard. Now there exist many solutions to (33), but this does not affect the other aspects of equilibrium, including e.g. $x_j^C = h_j^P = x_j^*$ for $j = 1, 2$ .This is an example of the dichotomy in Aruoba and Wright (2003) – in the basic LW model, one can solve for the real allocations in the centralized and decentralized markets independently. Here, the real allocations in the centralized markets of subperiods 1 and 2 are not affected by the allocation in the decentralized market of the third subperiod.

Remark 2: Suppose $M_{+1} = (1+\pi)M$, that is, money supply is changing over time at constant rate π. Then we cannot have a steady state as defined above, but it is natural to look for an equilibrium where all real variables are constant, including q and real balances ϕM. Inserting this into (32), we have

$$(1+\rho)(1+\pi) = \sigma z[q(M)] + 1 - \sigma.$$

Indeed, if we use the Fisher equation for the nominal interest rate, the left hand side is simply $1+i$, and so we have the standard result

$$1 + \frac{i}{\sigma} = z(q).$$

Hence, q is decreasing in i (and, therefore, π). But again, this does not affect ${}^{C}_{j} = h^P_j = x^*_j$ for $j = 1, 2$ due to the dichotomy.

Remark 3: It should be clear that there is no puzzle that some agents, and in particular consumers in subperiod $j = 2$, carry debt and cash simultaneously. Thus, we can address the co-existence, within any period, of liquid assets and consumer debt in agent's portfolios. However, it is also clear, that in this model, agents will not roll over debt from any period t to $t+1$, as they pay it off at the beginning of the period. This is the result of quasilinearity in first-subperiod preferences. Moreover, we have also concluded that agents will only hold as much cash as they expect to spend as a buyer in the third subperiod. So, we are able to address one of the aspects of the credit card debt puzzle in the data, but not the other two salient ones. As mentioned in the beginning, an extension and generalization of the model studied here can deliver all of these aspects qualitatively, and is the subject, along with quantitative evaluation, of ongoing research.

4 Model Extensions

4.1 Simultaneously Operating Markets

The timing of the subperiods and the structure of the associated markets as we described them above need not be defined in that order for the model's implications to hold. In particular, the subperiods' markets do not have to be sequential, and instead can all be open simultaneously at each subperiod. In this section, we describe one such general setup.

Figure 1: Market Structure of the Simultaneous-Market Model

Figure 1 demonstrates the market structure that we have in mind. We generalize by letting each period be subdivided into n subperiods. At the beginning of period t, during subperiod 1, we retain the usual frictionless centralized market from LW. Again, here agents are assumed to have preferences that are nonlinear in consumption, but linear in labor.

In each of the subsequent subperiods $s \in \{2, ..., n\}$, two markets are open simultaneously. One is a Walrasian market in the spirit of the afternoon market presented before: we retain the friction that at least some of the consumers cannot produce. Here, the ability to produce is determined in each subperiod by a stochastic shock on productivity, denoted by ϵ_s, where the subscript s stands for the subperiod, and the t subscript is omitted. The shocks ϵ_s are i.i.d.[11]

Simultaneously, the decentralized market is open as well. This is the standard LW decentralized market, where agents meet randomly and trade bilaterally, solving a Nash bargaining problem. The meetings are, as always, anonymous, so credit cannot be used in this market.

The agents progress through the markets as follows: everyone enters the centralized market with the associated value function W_1 at subperiod 1. At the end of the subperiod, an of any of the subsequent subperiods in the centralized market, with some probability δ the household moves to the decentralized market; with probability $1 - \delta$, it stays in the centralized market. In contrast, any agent who finds himself in the decentralized market at any point must, by assumption, move back to the centralized market in the following subperiod. At the end of subperiod n, regardless of which market they are in, all agents move to the centralized market, where preferences are again quasilinear in subperiod 1 of the next period.

Thus, in the first subperiod, everyone solves the same problem, with the usual variables

[11] Distribution?

taken as states:

$$\begin{aligned} W_1(m_1, b_1) &= \max_{x_1,h_1,m_2,b_2} \{U_1(x_1) - h_1 + \beta_1(1-\delta)EW_2(\epsilon_2, m_2, b_2) + \beta_1\delta V_2(m_2, b_2)\} \\ s.t. \;\; x_1 &= h_1 + \phi_1(m_1 - m_2) + b_2 - (1+r_1)b_1 \end{aligned}$$

In subperiod $s \in \{2, ..., n-1\}$, agents in the centralized market solve the following problem, where the current realization of the shock ϵ_s is also taken as a state:

$$\begin{aligned} W_s(\epsilon_s, m_s, b_s) = \max_{x_s,h_s,m_{s+1},b_{s+1}} \{U_s(x_s, h_s) &+ \beta_s(1-\delta)EW_{s+1}(\epsilon_{s+1}, m_{s+1}, b_{s+1}) \\ &+ \beta_s\delta V_{s+1}(m_{s+1}, b_{s+1})\} \\ s.t. \;\; x_s = \epsilon_s h_s + \phi_s(m_s - m_{s+1}) &+ b_{s+1} - (1+r_s)b_s \end{aligned}$$

Meanwhile, agents who are in the decentralized market in this subperiod solve a Nash bargaining problem, where for simplicity we assume take-it-or-leave-it offers by buyers. Then, d_s must still satisfy $d_s = m_s$ as before, while q_s will solve

$$c(q_s) = \beta_s EW_{s+1}(\epsilon_{s+1}, m_s + d_s, (1+r_s)b_s) - \beta_s EW_{s+1}(\epsilon_{s+1}, m_s, (1+r_s)b_s). \tag{34}$$

The value function in this market, given that buyer's surplus will be 0, is

$$\begin{aligned} V_s(m_s, b_s) &= \sigma[u(q_s) + \beta_s EW_{s+1}(\epsilon_{s+1}, m_s - d_s, (1+r_s)b_s)] \\ &+ (1-\sigma)\beta_s EW_{s+1}(\epsilon_{s+1}, m_s, (1+r_s)b_s) \end{aligned}$$

To finish describing the problem, at the end of subperiod n, everyone must return to the tandard LW centralized market again, where preferences are quasilinear. Thus, in subperiod n, an agent in the centralized market solves

$$\begin{aligned} W_n(\epsilon_n, m_n, b_n) &= \max_{x_n,h_n,m_{1,+1},b_{1,+1}} \{U_n(x_n, h_n) + \beta_n EW_{1,+1}(\epsilon_{1,+1}, m_{1,+1}, b_{1,+1})\} \\ s.t. \;\; x_n &= \epsilon_n h_n + \phi_n(m_n - m_{1,+1}) + b_{n+1} - (1+r_n)b_n, \end{aligned}$$

where the $+1$ subscript denotes, as before, period $t+1$. The problem for the decentralized market during subperiod n is the same as for any subperiod s.

We now solve the problem of the agent in each subperiod and each market. We start at subperiod 1 of some period, and go backwards to $t-1$.

4.1.1 Subperiod 1

For the subperiod-1 household problem, we get the following first-order conditions:

$$U_1(x_1) = 1 \tag{35}$$

$$\beta_1(1-\delta)EW_{2m}(\epsilon_2, m_2, b_2) + \beta_1\delta V_{2m}(m_2, b_2) = \phi_1 \tag{36}$$

$$\beta_1(1-\delta)EW_{2b}(\epsilon_2, m_2, b_2) + \beta_1\delta V_{2b}(m_2, b_2) = -1, \tag{37}$$

while the envelope conditions are

$$W_{1m}(\epsilon_1, m_1, b_1) = \phi_1 \tag{38}$$

$$W_{1b}(\epsilon_1, m_1, b_1) = -(1+r_1) \tag{39}$$

So we have, again, linearity of the first-subperiod value function in m_1, b_1, and a degenerate distribution of the assets resulting from quasilinearity of preferences, as can be seen from the relevant first-order conditions.

4.1.2 Subperiod n

Solving backwards now, we turn to the last subperiod. The first-order conditions of the centralized market are

$$\epsilon_n U_{nx}(x_n, h_n) + U_{nh}(x_n, h_n) = 0 \tag{40}$$

$$\phi_n U_{nx}(x_n, h_n) = \beta_n W_{1,+1,m}(m_{1,+1}, b_{1,+1}) \tag{41}$$

$$U_{nx}(x_n, h_n) = -\beta_n W_{1,+1,b}(m_{1,+1}, b_{1,+1}) \tag{42}$$

and using the envelope conditions (**??**) and (38), we get

$$\phi_n U_{nx}(x_n, h_n) = \beta_n \phi_{1,+1} \tag{43}$$

$$U_{nx}(x_n, h_n) = \beta_n(1+r_{1,+1}). \tag{44}$$

The envelope conditions for subperiod n are, inserting (**??**) and (43) directly,

$$W_{nm}(\epsilon_n, m_n, b_n) = \phi_n U_{nx}(x_n, h_n) = \beta_n \phi_{1,+1} \tag{45}$$

$$W_{nb}(\epsilon_n, m_n, b_n) = -(1+r_n)U_{nx}(x_n, h_n) = -\beta_n(1+r_n)(1+r_{1,+1}) \tag{46}$$

where the last equalities come from using (**??**) and (40). Observe that the subperiod-n centralized-market value functions are again linear in the current portfolio variables.

In the decentralized market in subperiod n, because of linearity of $W_1(m_1, b_1)$ in m_1, we have, from (34)

$$c(q_n) = \beta_n \phi_{1,+1} m_n \tag{47}$$

so that

$$q_n = c^{-1}(\beta_n \phi_{1,+1} m_n). \tag{48}$$

The envelope conditions from the period-n decentralized market are

$$V_{nm}(m_n, b_n) = \beta_n \phi_{1,+1}[\sigma \frac{u'(q_n)}{c'(q_n)} + 1 - \sigma] \tag{49}$$

$$V_{nb}(m_n, b_n) = -\beta_n (1 + r_n)(1 + r_{1,+1}) \tag{50}$$

4.1.3 Subperiod $n-1$

We go back another subperiod and solve explicitly, before generalizing to s. Unlike in subperiod n, in any subperiod prior to it, agents can go from the centralized market to either the centralized or the decentralized market. The first-order conditions for money and debt become, after plugging in the envelope conditions above,

$$\phi_{n-1} U_{n-1,x}(x_{n-1}, h_{n-1}) = \beta_{n-1} \beta_n \phi_{1,+1}(\delta[\sigma \frac{u'(q_n)}{c'(q_n)} + 1 - \sigma] + 1 - \delta) \tag{51}$$

$$U_{n-1,x}(x_{n-1}, h_{n-1}) = \beta_{n-1} \beta_n (1 + r_n)(1 + r_{1,+1}) \tag{52}$$

The envelope conditions of this subperiod are

$$W_{n-1,m}(\epsilon_{n-1}, m_{n-1}, b_{n-1}) = \beta_{n-1} \beta_n \phi_{1,+1}(\delta[\sigma \frac{u'(q_n)}{c'(q_n)} + 1 - \sigma] + 1 - \delta) \tag{53}$$

$$W_{n-1,b}(\epsilon_{n-1}, m_{n-1}, b_{n-1}) = -\beta_{n-1} \beta_n (1 + r_{n-1})(1 + r_n)(1 + r_{1,+1}) \tag{54}$$

Again, notice that the centralized-market value function W remains linear in the current holdings of money and debt, (m_{n-1}, b_{n-1}). In the decentralized market, the quantity exchanged is given by the difference of payoffs expressed in terms of $W_n(\epsilon_n, m_n, b_n)$, but from (**??**) we know that it is linear in m_n, so that

$$q_{n-1} = c_{n-1}^{-1}(\beta_{n-1} \beta_n \phi_{1,+1} m_{n-1}). \tag{55}$$

Envelope conditions for the decentralized market are thus

$$V_{n-1,m}(m_{n-1}, b_{n-1}) = \beta_{n-1}\beta_n\phi_{1,+1}[\sigma\frac{u'(q_{n-1})}{c'(q_{n-1})} + 1 - \sigma] \tag{56}$$

$$V_{n-1,b}(m_{n-1}, b_{n-1}) = -\beta_{n-1}\beta_n(1 + r_{n-1})(1 + r_n)(1 + r_{1,+1}) \tag{57}$$

Observations: from above, it clearly generalizes for all subperiods that $W_(m, b) = V_b(m, b)$ Moreover, they are both linear in b.

4.1.4 Subperiod $s \in \{2, ..., n-1\}$.

The above analysis extends back, with the following implications. In the centralized market, m_{s+1}, x_s, h_s are not functions of the past history of portfolio choices or shocks, that is, of (ϵ_s, m_s, b_s). This is given by linearity of the function $W_s(m_s, b_s)$ in m_s (from the relevant Envelope Condition, though a general formula is messy to construct), its obvious independence of ϵ_s, and its linearity in b_s in any general subperiod as follows:

$$W_{sb}(\epsilon_s, m_s, b_s) = \beta_s\beta_{s+1}...\beta_n(1 + r_s)(1 + r_{s+1})...(1 + r_n)(1 + r_{1,+1}) \tag{58}$$

The independence of consumption and labor from the state variables is given by the relevant first-order condition

$$\epsilon_s U_{sx}(x_s, h_s) = -U_{sh}(x_s, h_s), \tag{59}$$

where $U_{sx}(x_s, h_s)$ is independent of (ϵ_s, m_s, b_s) from the first-order condition in debt (equivalently, from the above envelope condition).

In the decentralized market, on the other hand, with TOL offers by the buyers, we have $d_s = m_s$ as long as we do not impose borrowing constraints, and $q_s = q(m_s)$, as follows:

$$\begin{aligned} q(m_s) &= c^{-1}(\beta_s\beta_{s+1}...\beta_n\phi_{1,+1}m_{n-2}) \Rightarrow \\ q'(m_s) &= \frac{\beta_s\beta_{s+1}...\beta_n\phi_{1,+1}}{c'(q_s)} \end{aligned} \tag{60}$$

The envelope conditions of the decentralized market will be

$$V_{sm}(m_s, b_s) = \beta_s\beta_{s+1}...\beta_n\phi_{1,+1}[\sigma\frac{u'(q_s)}{c'(q_s)} + 1 - \sigma] \tag{61}$$

$$V_{sb}(m_s, b_s) = -\beta_s\beta_{s+1}...\beta_n(1 + r_s)(1 + r_{s+1})...(1 + r_n)(1 + r_{1,+1}) \tag{62}$$

These imply that everyone will carry the same amount of money at all times, regardless of their previous portfolio and productivity history. The only time money holdings change will be when the agents enter the decentralized market, and buyers spend the money, while sellers accumulate it. In the subperiod after the decentralized market, sellers will use their excess money to pay down their debt (or save, if they don't have any), while buyers will borrow the amount of money necessary in case of an entrance into the decentralized market.

In all the periods of the centralized market, agents will maintain their money holdings at the same level, and will borrow to consume in all the subperiods of the centralized market. Once every period, they will use the "quasilinear" market to pay off their debt entirely. Notice that between the subperiods, agents can revolve debt.

Thus we again have captured the essential aspects of the credit card debt puzzle. Those agents who have a history of low-productivity shocks in the centralized market will borrow and revolve their debt, while maintaining a positive balance on their liquid account. There is again precautionary demand for liquidity among those who end up as a seller in the decentralized market, but this precautionary demand only lasts for one subperiod thereafter.

A Properties of the general Nash Bargaining Solution in the Decentralized Market

In this Appendix we do several things. First we derive the bargaining solution given in Lemma 1. The necessary and sufficient conditions for (17) are

$$\theta\left[\beta_3\phi_{1,+1}d - c_3(q)\right]u_3'(q) = (1-\theta)\left[u_3(q) - \beta_3\phi_{1,+1}d\right]c_3'(q) \tag{63}$$

$$\theta\left[\beta_3\phi_{1,+1}d - c_3(q)\right]\beta_3\phi_{1,+1} = (1-\theta)\left[u_3(q) - \beta_3\phi_{1,+1}d\right]\beta_3\phi_{1,+1} - \lambda\left[u_3(q) - \beta_3\phi_{1,+1}d\right]^{1-\theta}\left[\beta_3\phi_{1,+1}d - c_3(q)\right]^{\theta} \tag{64}$$

where λ is the Lagrange multiplier on $d \leq m_3$. There are two possible cases: If the constraint does not bind, then $\lambda = 0$, $q = q^*$ and $d = m^*$. If the constraint binds then q is given by (63) with $d = m_3$, as claimed.

We now argue that $m_3 < m_3^*$. First, as is standard, in any equilibrium $\phi_{1,+1} \leq (1+\rho)\phi_1$; this just says the nominal interest rate i is nonnegative. In fact, again as is standard, although we allow $i \to 0$, we only consider equilibria where $i > 0$, so that $\phi_{1,+1} < (1+\rho)\phi_1$. Now suppose $m_3 > m_3^*$ at some date for some agent. Since the bargaining solution tells us he never spends more than m_3^*, he could reduce m_3 by reducing h_1 at t, then increase h_1 at $t+1$ so that he need not change anything else. It is easy to check that this increases utility, so $m_3 > m_3^*$ cannot occur in any equilibrium.

Hence $m_3 \leq m_3^*$. To show the strict inequality, suppose $m_3 = m_3^*$ for sme agent. Again he can reduce h_1 at t and carry less money. If he is a buyer in subperiod 3, he gets a smaller q, but the continuation value is the same since by the bargaining solution he still spends all his money. If he does not buy then he can increase h_1 at $t+1$ so that he need not change anything else. It is easy to check that the net gain from carrying less money is positive, exactly as in LW.

B Simultaneous Markets: Subperiod $n-1$

This is temporary, for reference only.

In the decentralized market, the first-order conditions, aside from the one for consumption, are

$$
\begin{aligned}
\phi_{n-2}U_{n-2,x}(x_{n-2},h_{n-2}) &= \beta_{n-2}\beta_{n-1}\beta_n\phi_{1,+1}\{(1-\delta)\delta[\sigma\frac{u'(q_n)}{c'(q_n)}+1-\sigma]+(1-\delta)^2) \\
&+ \delta[\sigma\frac{u'(q_{n-1})}{c'(q_{n-1})}+1-\sigma]\} \\
U_{n-2,x}(x_{n-2},h_{n-2}) &= -\beta_{n-2}\beta_{n-1}\beta_n(1+r_{n-1})(1+r_n)(1+r_{1,+1})
\end{aligned}
$$

Again, envelope conditions for the centralized market mimic the RHS's of these equations:

$$
\begin{aligned}
W_{n-2,m}(\epsilon_{n-2},m_{n-2},b_{n-2}) &= \beta_{n-2}\beta_{n-1}\beta_n\phi_{1,+1}\{(1-\delta)\delta[\sigma\frac{u'(q_n)}{c'(q_n)}+1-\sigma]+(1-\delta)^2) \\
&+ \delta[\sigma\frac{u'(q_{n-1})}{c'(q_{n-1})}+1-\sigma]\} \\
W_{n-2,b}(\epsilon_{n-2},m_{n-2},b_{n-2}) &= -\beta_{n-2}\beta_{n-1}\beta_n(1+r_{n-2})(1+r_{n-1})(1+r_n)(1+r_{1,+1})
\end{aligned}
$$

In the decentralized market, we have, again, that $c(q_{n-2})$ is a function of the difference $EW_{n-1}(m_{n-1}+d_{n-1},b_{n-1})-EW_{n-1}(m_{n-1},b_{n-1})$, but by (**??**), $W_{n-1}(m_{n-1},b_{n-1})$ is linear in m_{n-1} (because $q_n=fn(m_n)\neq fn(m_{n-1})$ by (**??**)). Thus:

$$
q_{n-2}=c^{-1}(\beta_{n-2}\beta_{n-1}\beta_n\phi_{1,+1}m_{n-2})
$$

Envelope conditions for the decentralized market are built the same way as above. These argument extend back. We can now generalize to subperiod s.

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