Information, Liquidity, Asset Prices and Monetary Policy*

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Abstract

We study economies with multiple assets that are valued both for their return and their liquidity. Liquidity is modeled by having some trade occur in decentralized markets, with frictions, where certain assets are more likely to be accepted in trade. This is due to an information problem: while all agents recognize some assets, like currency, they are less sure about and hence less inclined to accept others. Recognizability is endogenized by letting agents invest in information, potentially generating multiple equilibria with different liquidity properties. We discuss implications for asset pricing and monetary policy. We show in particular that what looks like a cash-in-advance constraint is not invariant to policy. We also discuss some tentative implications for understanding recent financial market events, such as the prediction that small changes in the amount of, or in the cost of, information concerning asset quality can generate large negative responses in liquidity, output and welfare.

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Aringosa walked back to his black briefcase, opened it, and removed one of the bearer bonds. He handed it to the pilot. "What's this?" the pilot demanded. "A ten-thousand-euro bearer bond drawn on the Vatican Bank." The pilot looked dubious. "It's the same as cash." "Only cash is cash," the pilot said, handing the bond back. Dan Brown, *The Da Vinci Code.*

1 Introduction

We study economies in which assets are valued both for their rate of return and for their use as a medium of exchange, i.e. for their liquidity.[1] In general, assets are not perfect substitutes in our model: currency, for example, may be valued despite having an inferior rate of return because it may be more liquid, or more widely accepted as a means of payment. The reason that alternative assets are less liquid is based on information frictions: we assume that these assets may be high- or low-quality, and that a seller must invest in information in order to recognize the difference. We specify the environment so that agents who do not become informed will not accept assets that they cannot recognize. This is technically convenient because it allows us to use standard bargaining theory to determine the terms of trade, since agents only exchange objects that they recognize. Hence, information frictions can drive liquidity differentials across assets, without overly complicating the analysis of price determination.

Although we think the model is interesting even when liquidity differentials are exogenous, our main contribution is to endogenize agents' decisions to become informed, and thus their ability to transact in alternative assets. Thus, we can study how assets become more or less liquid in response to changes in the environment. For instance, inflation decreases demand

[1] It is by now well understood that it is both natural and efficient for some objects to emerge as media of exchange – which means they are highly liquid – when spot trade is difficult due to a standard double coincidence problem, and credit is imperfect due to limited commitment and incomplete record keeping. See Kocherlakota [?], Wallace [?], Corbae et al. [?], Araujo [?], and Aliprantis et al. [?], [?] for formal discussions in the context of monetary theory (as well as some other monetary papers cited below). In terms of asset pricing more generally, these insights from monetary economics have recently been put to good use by several people, including Duffie et al. [?], [?], Lagos [?], [?], Lagos and Rocheteau [?], [?], Lagos et al. [?], Geromichalos et al. [?], Nosal and Rocheteau [?], Rocheteau [?], Weill [?], [?] and Vayanos and Weill [?]. We are following in this new and we think productive program of using models with frictions, like those used in monetary theory to price currency, to think about asset pricing and liquidity more generally.

for real money balances and increases demand for alternative assets. This raises the market value of these alternatives and the benefit to being able to trade in the assets, which raises the incentive to acquire information. An increase in inflation can increase liquidity. We also highlight an important complementarity: if more sellers become informed, assets are more liquid and hence more valuable, increasing the incentive to invest in information and hence potentially generating multiple equilibria. One interpretation of these results is that the share of transactions apparently subject to a cash-in-advance constraint is endogenous, it is not necessarily uniquely determined, and it is not invariant to policy.

As a related example, one that is consistent with much experience, suppose we interpret easily recognizable assets as a local currency, such as pesos in Latin America, and the alternative as US dollars, which traditionally constitute a better store of value. When peso inflation is not too high, locals tend to use pesos as a means of payment; dollars do not circulate widely, nor are they universally recognized. If the peso inflation rate increases, transacting in local currency becomes more costly, and the economy begins to dollarize. Notice, however, that if the peso inflation rate later subsides, the dollar does not fall into disuse, because once the locals learn to recognize it for transactions they do not quickly forget. This imparts a natural hysteresis effect into dollarization, which has often been discussed, but never formalized in this way.[2]

The framework can in principle be used to analyze any vector of assets, including local and foreign currency, stocks, bonds, and mortgage-backed securities. As our leading example, however, we consider the case of two assets: money, and real claims like the claims to "trees" bearing "fruit" as dividends as in standard Lucas [?] asset-pricing theory. In the model, all agents have perfect information about currency: they recognize it and know what it is worth.

[2] See Uribe [?] for a discussion of the issues and literature. He also provides a interesting model, but simply assumes as a reduced form that the cost of accepting foreign currency is decreasing in the fraction of agents that accept it; we instead have an increase in the benefit of accepting it, and the effect works through an explicit information-based model.

But some agents may not be able to recognize the value of the alternative asset, and, as noted above, sellers who do not recognize an asset will not accept it (they won't give a buyer anything for it). The reason is that they are concerned a claim may be a counterfeit or worthless lemon – a "lemon tree," as it were.[3] In any case, this recognizability problem can give rise to a demand for both currency and real assets.

Although the way we model acceptability and recognizability is new, aspects of the economic ideas and policy implications go back to Tobin's [**?**] portfolio theory of the demand for money and Wallace's [**?**] overlapping generations models. As Wallace put it:

> Of course, in general, fiat money issue is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on the magnitude of the fiat money-financed deficit ... [T]he real rate-of-return distribution faced by individuals in equilibrium is less favorable the greater the fiat money-financed deficit. Many economists seem to ignore this aspect of inflation because of their unfounded attachment to Irving Fisher's theory of nominal interest rates. (According to this theory, (most?) real rates of return do not depend on the magnitude of anticipated inflation.) The attachment to Fisher's theory of nominal interest rates accounts for why economists seem to have a hard time describing the distortions created by anticipated inflation. The models under consideration here imply that the higher the fiat money-financed deficit, the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income. This seems to be what most citizens perceive to be the cost of anticipated inflation.

[3] More generally, the value of a claim can be random and the seller may not know the distribution, even if the buyer does; the possibility the claim may be totally worthless is simply a special case where its value can be 0. Also, we are aware that in general agents may accept assets even if they do not recognize them; we specify the details below so that it is optimal for sellers who do not recognize assets to reject them outright.

These words ring true, but many questions arise. How can the Fisher equation not hold? Why do different assets bear different returns in the first place? In the models Wallace mentions, it is *not* differences in liquidity – notice he talks about “saving” and defines returns in terms of the rate “at which present income can be converted into future income” but there is no mention of a transactions or medium of exchange role. This is where modern monetary theory can provide insight, with explicit descriptions of trading processes and liquidity. Early search-based models such as Kiyotaki and Wright [**?**] determine endogenously the acceptability of different objects in exchange, but are too crude to address the issues studied in this paper. Hence we use a multiple-asset version of the more recent model in Lagos and Wright [**?**]. While others have considered multiple assets in this environment, our focus is on differential liquidity and how this can be determined endogenously using information frictions.[4]

The rest of the paper is organized as follows. Section 2 lays out basic assumptions. Section 3 describes equilibrium when the probability that a random agent recognizes assets quality is taken as given. This is an important preliminary step in endogenizing information acquisition, and hence acceptability, since one needs to know what happens for any given information structure in order decide whether to invest. Section 4 describes equilibrium when agents choose to acquire information, thus endogenizing liquidity. Section 5 discusses implications for asset pricing and monetary policy. We also offer a tentative discussion of implications for recent financial market events, such as the prediction that small changes in the amount of, or in the cost of, information concerning asset quality can generate large negative responses in liquidity, output and welfare. Section 6 concludes.[5]

[4] Lagos and Rocheteau [**?**] and Geromichalos et al. [**?**] study two-asset versions of the model, but assets are equally liquid and bear the same return. Differential liquidity is considered by Lagos [**?**], [**?**], who argues it is theoretically and quantitatively important, but his differential is exogenous. To be clear, our goal is to make liquidity endogenous by incorporating recognizability and information acquisition. Also see Rocheteau [**?**].

[5] In terms of some additional literature, the idea in monetary economics goes back at least to Menger [**?**] that recognizability is a crucial property of media of exchange. Alchian [**?**], Brunner and Meltzer [**?**], Freeman [**?**], and Banerjee and Maskin [**?**] discuss the connection between money and information using varying degrees of formal modeling. Our approach is closer to search-based monetary theory, where informational frictions have been

2 The Basic Model

As in Lagos and Wright [?], hereafter LW, in each period in discrete time a $[0,1]$ continuum of infinitely-lived agents participate in two distinct markets: a centralized market (CM) that is basically Walrasian; and a decentralized market (DM) where buyers and sellers meet bilaterally. This is a useful abstraction for our purposes because we can impose relevant frictions in the DM, but keep the analysis tractable due to the CM. Meetings in the DM are characterized by a double coincidence problem, which rules out barter, and anonymity, which rules out credit, and so some tangible medium of exchange is essential for trade (it is understood that we only need *some*, not *all*, DM meetings where barter and credit are difficult). At each date in the CM there is a consumption good x that agents can produce using labor h according to $x = h$, and utility is quasi-linear, $U(x) - h$. In the DM there is another good q that agents value according to $u(q)$ and can be produced at disutility cost $c(q)$. We define x^* and q^* by $U'(x^*) = 1$ and $u'(q^*) = c'(q^*)$, and assume $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $U'(0) = u'(0) = \infty$.

There are two assets (the generalization to n assets is straightforward). The first is interpreted as fiat money, and the second as a real asset like the claims to "trees" in the standard Lucas [?] model, yielding a dividend δ in terms of "fruit" in units of x in the next CM. We introduce informational frictions as follows. Generally, an agent in the DM might give you either a high- or low-quality asset, where the latter is completely worthless – a pure lemon or a counterfeit. For example, consider a stock certificate in a profitable company. A counterfeit is a

incorporated by Williamson and Wright [?], Trejos [?], [?], Li [?], Cuadras-Moreto [?], Kim [?], Velde et al. [?], Berentsen and Rocheteau [?], Ennis [?], Faig and Jerez [?], Nosal and Wallace [?], Cavalcanti and Nosal [?], Hu [?], Rocheteau [?], and Kim and Lee [?]. An alternative approach to liquidity going back to Glosten and Milgrom [?] and Kyle [?] also considers bilateral transactions between asymmetrically informed agents; while similar in spirit, those models are technically different, and have nothing to say about the substantive issues addressed here, including the role of money and monetary policy. Other work that has previously considered making the number of transactions that require cash endogenous include Shreft and Lacker [?], Ireland [?], Freeman and Huffman [?], Freeman and Kydland [?], and recent work by Dong [?] that includes additional references.

fake copy of the certificate that does not actually entitle the holder to anything. Alternatively, a lemon might be a legitimate certificate of stock in a different company that has zero profit. These two cases can be thought of as "bad claims to good trees" and "good claims to bad trees" in our economy. For our purposes, what is important is that buyers can produce worthless assets whenever they like at no cost.[6]

In the frictionless CM, all agents (or perhaps the market) can distinguish between high- and low-quality assets. But in the anonymous bilateral DM, only informed sellers – those who have acquired the requisite information – can make this distinction. One story, stepping outside the formal model just a little, is that in the CM there are third parties available to certify quality, but they are not around in the DM. For ease of exposition we assume fiat money is universally recognized.[7] We emphasize that currency may be valued in equilibrium, while bad claims cannot be valued, despite the superficial similarity that both are "intrinsically worthless" in the formal sense that they pay 0 dividends. As discussed in a related context by Cavalcanti and Wallace [**?**], [**?**], as long as individuals can produce their own worthless claims at no cost, there is no equilibrium where they value those produced by others. The key difference with currency of course is that you cannot produce it yourself.

Since buyers can produce worthless claims whenever they want at 0 cost, sellers who cannot verify the quality of a claim will never accept it. A seller without a technology for verifying checks, e.g., will not accept one if he knows you can always sign someone else's name. This is

[6] Thus, in the epigram, Aringosa offers the pilot a ten-thousand-euro bearer note drawn on the Vatican Bank, which the pilot took to be worthless. The idea is that Aringosa could have written himself such a note, for any amount, say by adding zeros to the demonination, at negligible cost, and it could be worthless either because the bank has no obligation to honor it, or because the bank in question may not even exist.

[7] We can include a recognizability problem with currency without changing the basic results, but having universally recognized money seems a good benchmark. Currency and coins have always been designed with recognizability in mind when they were stamped with the likeness of the monarch, when weight and fineness were standardized, etc. Also, as Nobu Kiyotaki pointed out to us, in the olden days most people could recognize coins, although many could not even read, let alone understand a piece of paper claiming some payoff in a future contingency. Literacy may have improved since then, but people still have trouble evaluating the worth of complicated financial instruments, including mortgage-backed securities.

different from some of the papers mentioned above, including Williamson and Wright [?], where agents make ex ante choices to bring good or bad assets to the market. In those models, sellers always accept assets with positive probability, and sometimes probability 1, even if uninformed. The logic is as follows: since low-quality assets are worthless, informed sellers never accept them; if we conjecture that uninformed sellers never accept them, then buyers with low quality cannot trade; hence no one brings low quality to the market. But then uninformed sellers have no reason to reject. The difference here is that buyers can always produce worthless assets on the spot.[8]

Our environment, if somewhat special, is attractive precisely because sellers who do not recognize an asset reject it outright. Why is this important? Given bilateral meetings in the DM, as is standard and natural, we assume the terms of trade are determined through bargaining (although there are other options, as discussed below). In general, bargaining with information frictions is complicated at best. In our set up, however, because a seller never accepts something he does not recognize, buyers can only pay with assets he does recognize. Consequently, all bargaining occurs under full information. In this way informational frictions can be instrumental in determining acceptability and liquidity, but we avoid the usual problems with bargaining under asymmetric information. Although alternative assumptions are also worth study, we think the big gain in tractability makes our specification reasonable.

To focus on steady state equilibria, assume there is a fixed supply of "trees" denoted A, while the supply of money M grows according to $\hat{M} = \gamma M$ (for any variable z, $\hat{z}$ denotes its value next period). Although M changes over time, real balances will be constant in steady state, just like the stock of real assets A. Changes in the money supply, $\hat{M} - M = (\gamma - 1)M$, are accomplished using lump sum transfers, or taxes if $\gamma < 1$ (it is actually equivalent to assume

[8] In a companion paper, Lester et al. [?], we analyze a class of related games to show which details are crucial to get sellers to reject unrecognized assets, including the assumption that buyers can costlessly produce worthless claims at any time.

that the government uses new money to buy goods in the CM given the quasi-linear preference specification). In what follows, we assume $\gamma > \beta$, where β is the discount factor, although we can consider the limiting case of the Friedman rule, where $\gamma \to \beta$. Let ϕ be the CM price of money and ψ the CM price of the real asset, both in terms of x. In the CM all prices are taken parametrically; as mentioned above, in the DM agents bargain bilaterally.

It is well known (Lagos and Rocheteau [?]; Geromichalos et al. [?]) that someone with a units of a real asset in the CM may not want to bring it all to the DM, since the bargaining solution generally depends on a buyer's available wealth. Thus, he may want to hold a large amount of a, because it is a good store of value, but not bring it all to the bargaining table (there is no similar effect on m since it has no dividend; the *only* reason to acquire m is to use it for payments). In principle, this detail could be avoided if the buyer made take-it-or-leave it offers in the DM, since then the terms of trade do not deteriorate when he has more wealth in a meeting This does not work for our purposes, however, since we want sellers to make ex ante investments in information, which they would never do if they get none of the ex post gains from trade. Hence, we use a more general bargaining solution, and allow agents to potentially acquire some a that they do not bring to the bargaining table: they choose a portfolio in the CM comprised of m units of cash and a_2 real assets that they take to the DM, plus a_1 real assets they do not take to the DM.

Let $V(m, a_1, a_2)$ be the value function of an agent in the DM with portfolio (m, a_1, a_2), and $W(y)$ the value function in the CM with $y = \phi m + (\delta + \psi)(a_1 + a_2)$, since in a frictionless market only the total value of a portfolio matters. The CM problem is

$$\begin{aligned} W(y) &= \max_{x,h,\hat{m},\hat{a}_1,\hat{a}_2} \{U(x) - h + \beta V(\hat{m}, \hat{a}_1, \hat{a}_2)\} \\ \text{s.t. } x &= h + y - \phi\hat{m} - \psi(\hat{a}_1 + \hat{a}_2) + T, \end{aligned}$$

where $T = (\gamma - 1)M$ is the transfer. In principle there may be constraints on $h \in [0, \bar{h}]$, but we assume they are not binding (see LW for conditions to make this valid). Then, substituting for h, we take first order conditions for the other variables:

$$x \quad : \quad U'(x) = 1 \tag{1}$$

$$\hat{m} \quad : \quad \phi \geq \beta V_1(\hat{m}, \hat{a}_1, \hat{a}_2), \ = \text{ if } \hat{m} > 0 \tag{2}$$

$$\hat{a}_1 \quad : \quad \psi \geq \beta V_2(\hat{m}, \hat{a}_1, \hat{a}_2), \ = \text{ if } \hat{a_1} > 0 \tag{3}$$

$$\hat{a}_2 \quad : \quad \psi \geq \beta V_3(\hat{m}, \hat{a}_1, \hat{a}_2), \ = \text{ if } \hat{a_2} > 0. \tag{4}$$

Notice that x and $(\hat{m}, \hat{a}_1, \hat{a}_2)$ do not depend on y, and that $W'(y) = 1$. These results are standard, and (2)-(4) look like the portfolio conditions in the generalization of LW by Lagos and Rocheteau [**?**] and Geromichalos et al. [**?**]. What happens next, however, now depends on the information structure in the DM.

3 Equilibrium with Exogenous Information

In order to determine who invests, we need to analyze the DM for any given information structure. First, we specify the double coincidence problem in a standard way as follows: every agent in the DM is assumed to have an equal probability λ of a bilateral meeting where he wants to buy from his partner and a meeting where he wants to sell to his partner.[9] The important new element is that there are two types of meetings where you want to buy: with probability ρ, you are in a *type 2 meeting*, in which the seller accepts either m or a_2 because he is informed about asset quality; and with probability $1 - \rho$, you are in a *type 1 meeting*, in which the seller is uninformed and hence accepts only m. We take the fraction of informed agents $\rho \in (0, 1)$

[9] There are no barter meetings, where you both want to sell to and buy from the same person, but it would easy to let this happen with a nonzero probability without changing any substantive results. Also, we can get very similar results in a version with two types, where one type can only be buyers in the DM and the other can only be sellers in the DM (see Rocheteau and Wright [**?**]). This may seem more natural, for some issues, and allows one to consider extensions like a generalized matching technology, but since it complicates the notation without changing the basic point we present only the representative agent version here.

as given for now, making the model similar to a random-matching version of a cash-in-advance specification, but ρ is endogenized below.

We determine the DM terms of trade using the generalized Nash bargaining solution.[10] Consider a type j meeting between a buyer with portfolio (m, a_1, a_2) and a seller with $(\tilde{m}, \tilde{a}_1, \tilde{a}_2)$. The former pays p_j to the latter for q_j units of the good, determined by

$$\max \ \left[u(q_j) + W(y - p_j) - W(y)\right]^{\theta} \left[-c(q_j) + W(\tilde{y} + p_j) - W(\tilde{y})\right]^{1-\theta} \tag{5}$$

subject to the constraint $p_j \leq y_j$, where y and $\tilde{y}$ are total wealth of the buyer and seller respectively, and y_j describes the wealth the buyer can use in a type j meeting: $y_1 = \phi m$ and $y_2 = \phi m + (\psi + \delta)a_2$. There are two cases, since either $p_j \leq y_j$ is binding or it is not. In each case we can take the first order conditions from (5) and solve (see LW for details) to get:

Lemma 1. *The solution to (5) is*

$$q_j = \min\left\{z^{-1}(y_j), q^*\right\} \ \text{ and } p_j = \min\left\{y_j, y^*\right\},$$

where the function z is defined by

$$z(q) \equiv \frac{\theta u'(q)c(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)},$$

$y^* = z(q^*)$, *and* $u'(q^*) = c'(q^*)$.

The DM value function satisfies

$$V(m, a_1, a_2) = \lambda_1\left[u(q_1) + W(y - p_1)\right] + \lambda_2\left[u(q_2) + W(y - p_2)\right] + (1-\lambda)W(y) + k, \tag{6}$$

where $\lambda_1 = \lambda(1-\rho)$, $\lambda_2 = \lambda\rho$, and k is a constant unimportant for what follows. There are three relevant events described in (6): an agent is a buyer in a type 1 meeting; an agent is a buyer in

[10] There are alternative approaches for determining the terms of trade in the DM. With symmetric information, LW use generalized Nash bargaining; Aruoba et a. [?] consider several alternative bargaining solutions; Rocheteau and Wright [?] analyze price taking and price posting; Galenianos and Kircher [?] and Dutu et al. [?] use auctions in versions with some multilateral meetings. With private information, Ennis [?] and Faig and Jerez [?] use posting, and Guerrieri [?] uses price taking. We like bargaining because it is natural and easy in this setting. Again, it is easy because sellers only accept and hence only bargain over assets they recognize.

a type 2 meeting; and an agent is not a buyer. In the third case, he may be a seller or may not have the opportunity to trade at all, and this effects his continuation value. But since Lemma 1 implies the terms of trade do not depend on the seller's state and $W(y)$ is linear, conveniently, we need not know what happens when an agent is a seller in order to determine his portfolio.

Hence, in order to determine (m, a_1, a_2), we first differentiate V, where we get the derivatives of q_j with respect to these arguments from Lemma 2. This yields

$$V_1(m, a_1, a_2) = \phi\left[\lambda_1 \ell(q_1)\mathbf{1}\{y_1 < y^*\} + \lambda_2 \ell(q_2)\mathbf{1}\{y_2 < y^*\} + 1\right] \quad (7)$$

$$V_2(m, a_1, a_2) = \psi + \delta \quad (8)$$

$$V_3(m, a_1, a_2) = (\psi + \delta)\left[\lambda_2 \ell(q_2)\mathbf{1}\{y_2 < y^*\} + 1\right] \quad (9)$$

where $\mathbf{1}\{z\}$ is an indicator function equaling 1 if and only if z is true, and $\ell(q) \equiv \frac{u'(q)}{z'(q)} - 1$. Notice for future reference that $\ell(q)$ is the *liquidity premium* – the value of an additional unit of wealth available in a type j meeting, over and above its return if it were simply carried to the next CM. We assume $\ell'(q) < 0$, which is true under known conditions.[11] Combining (7)-(9) and (2)-(4), we arrive at the conditions determining portfolio demand:

$$m \quad : \quad \phi \geq \beta\hat{\phi}\left[\lambda_1 \ell(\hat{q}_1)\mathbf{1}\{\hat{y}_1 < y^*\} + \lambda_2 \ell(\hat{q}_2)\mathbf{1}\{\hat{y}_2 < y^*\} + 1\right], \ = \ \text{if } \hat{m} > 0 \quad (10)$$

$$a_1 \quad : \quad \psi \geq \beta(\hat{\psi} + \delta), \ = \ \text{if } \hat{a}_1 > 0 \quad (11)$$

$$a_2 \quad : \quad \psi \geq \beta(\hat{\psi} + \delta)\left[\lambda_2 \ell(\hat{q}_2)\mathbf{1}\{\hat{y}_2 < y^*\} + 1\right], \ = \ \text{if } \hat{a}_2 > 0. \quad (12)$$

One can define an equilibrium for a fixed value of ρ in terms of time paths for asset holdings (m, a_1, a_2), asset prices (ϕ, ψ), the DM terms of trade (p_j, q_j), $j = 1, 2$, and the CM allocation (x, h), for every agent, satisfying the utility maximization conditions derived above, the bargaining solution, and market clearing. A steady state equilibrium implies all real variables, including

[11] One can show that $\ell' < 0$ if θ is close to 1, or if c is linear and u displays decreasing absolute risk aversion. The point of $\ell' < 0$ is that it guarantees a unique solution to the CM problem at least for a_2 and m (it is possible that agents are indifferent about a_1, but this is not relevant for anything interesting). Alternatively, the method in Wright [**?**] can be used with some more work to establish generic uniqueness even if ℓ is not monotone.

(q_1, q_2), are constant over time, which implies ϕm and ψa_2 are constant, and $\phi/\hat{\phi} = \hat{M}/M = \gamma$; a monetary steady state has $\phi > 0$, $\hat{m} > 0$, $q_1 > 0$, and (10) holds with equality. Such a model would be similar to Lagos and Rocheteau [?] and Geromichalos et al. [?], except that they have $\rho = 1$ (all assets are perfectly recognizable/acceptable). Although the main focus here is on making ρ endogenous, it is also interesting to ask how the model behaves for a fixed $\rho \neq 1$.

To this end, notice from Lemma 1 that q_j is an increasing function of y_j and $q_1 \leq q_2 \leq q^*$. It is easy to show $q_j \leq \bar{q}$, where $\bar{q}$ maximizes the buyer's surplus $u(q) - p = u(q) - z(q)$, and $\bar{q} \leq q^*$ with strict inequality unless $\theta = 1$. Notice that $\ell(\bar{q}) = 0$. We next claim that $a_2 > 0$ for all $\lambda_2 > 0$. The proof is in Appendix A; the intuition is that, because it is costly to carry cash, agents do not bring enough to buy $\bar{q}$ and hence also bring at least some a_2 to the DM.

Lemma 2. $a_2 > 0$ *for all* $\lambda_2 > 0$.

Given $a_2 > 0$ and $m > 0$, in any monetary equilibrium, it remains to determine whether $a_1 = 0$ or $a_1 > 0$. To answer this, let $\tilde{q} < \bar{q}$ be defined by $\ell(\tilde{q}) = (\gamma - \beta)/\beta\lambda_1$, and let

$$\bar{A} = [z(\bar{q}) - z(\tilde{q})](1 - \beta)/\delta > 0.$$

The next result, the proof of which is in Appendix B, demonstrates that $A \leq \bar{A}$ (the real asset is relatively scarce) implies $a_1 = 0$, and $A > \bar{A}$ (the real asset is plentiful) implies $a_1 > 0$.

Proposition 1. *(i) If* $A \leq \bar{A}$ *there exists a unique steady state monetary equilibrium, and in this equilibrium,* (q_1, q_2) *solves*

$$A\delta = [z(q_2) - z(q_1)]\{1 - \beta[\lambda_2 \ell(q_2) + 1]\} \quad (13)$$

$$\gamma = \beta[\lambda_1 \ell(q_1) + \lambda_2 \ell(q_2) + 1], \quad (14)$$

prices are $\phi = z(q_1)/M$ *and* $\psi = [z(q_2) - z(q_1)]/A - \delta$*, and* $(m, a_1, a_2) = (M, 0, A)$*.*

(ii) If $A > \bar{A}$ *there exists a unique steady state equilibrium, and in this equilibrium,* $(q_1, q_2) = (\tilde{q}, \bar{q})$*, prices are* $\phi = z(\tilde{q})/M$ *and* $\psi = \beta\delta/(1 - \beta)$*, and* $(m, a_1, a_2) = (M, A - \bar{A}, \bar{A})$*.*

4 Equilibrium with Endogenous Information

The results in Proposition 1 are key to endogenizing ρ. Suppose that agent $j \in [0, 1]$ has the ex ante choice of whether or not to acquire at cost $\kappa(j)$ the information, or perhaps the technology, that allows him to recognize asset quality. We label agents so that $\kappa(j)$ is increasing and we assume that $\kappa(j)$ is differentiable. An agent accepts a in the DM if and only if he pays this cost, since this is the only way to distinguish genuine from worthless claims, and any buyer who meets an uninformed seller accepting a clearly offers a worthless claim.[12] The fraction of agents that incurs the cost to become informed about assets therefore determines the fraction ρ that accept these assets in the DM.

One can imagine several interpretations of $\kappa(j)$. It is typically thought to be costly to learn how to use a new medium of exchange, for a variety of reasons, as has been documented in episodes of dollarization (Uribe [?]; Guidotti and Rodriguez [?]; Dornbusch et al. [?]). Stepping outside the formal model, a financial institution that wants to accept a pool of asset-backed securities, say, must set up a department with analysts that can ascertain their value. Other costs may be technological, as in the case of debit or credit cards, where sellers must buy a machine to verify buyers' creditworthiness or transfer funds from one institution to another. Agents choose to make investments allowing them to accept certain types of assets, taking as given what others are doing. Naturally, one might think that coordination will be central in determining equilibrium, as in the literature on payment networks, even though their models are quite different (see Hunt [?] and Rochet and Tirole [?] for surveys).

Conditional on a fraction $\rho \in [0, 1]$ of other agents becoming informed, the benefit to a given agent of becoming informed is that he can now accept real assets in the DM. This benefit can

[12] As is standard, buyers are assumed to know whether the seller is informed in a given meeting. Also, there is no marginal cost to an informed seller of verifying asset quality, once he has paid to acquire the requisite information, and there is no cost to a buyer of coming up with worthless assets. In Lester et al. [?], we relax some of these assumptions in related albeit simpler environments.

be written

$$\Pi(\rho) \equiv \beta\lambda \left\{ z[q_2(\rho)] - c[q_2(\rho)] \right\} - \beta\lambda \left\{ z[q_1(\rho)] - c[q_1(\rho)] \right\}. \tag{15}$$

Since $z(q) - c(q)$ is the seller's surplus from trade, $\Pi(\rho)$ is the expected discounted surplus a seller gets in a type 2 meeting rather than a type 1 meeting. Proposition 1 implies that $q_1(\rho)$ and $q_2(\rho)$ are well-defined objects in (15), as that result fully characterizes the (unique) equilibrium for any given ρ. The best response condition is for agent j to acquire information if $\Pi(\rho) \geq \kappa(j)$. If some agents invest in information and others do not, in equilibrium, the marginal agent is indifferent, $\Pi(\rho) = \kappa(\rho)$ (if a fraction ρ invest the marginal agent is $j = \rho$, given we order agents by their costs).

To be slightly more formal, a steady-state equilibrium is now defined by a portfolio (m, a_1, a_2), asset prices (ϕ, ψ), DM terms of trade (q_j, p_j), a CM allocation (x, h), and a fraction of informed agents ρ such that: (i) given ρ, the other variables satisfy the equilibrium conditions in the previous section; and (ii) given the other variables, which generate $\Pi(\rho)$ according to (15), we have $\rho = 0$ if $\Pi(0) < \kappa(0)$, $\rho = 1$ if $\Pi(1) > \kappa(1)$, and $\Pi(\rho) = \kappa(\rho)$ if $\rho \in (0, 1)$. Define the cumulative distribution function $F : \mathbb{R} \to [0, 1]$ by letting $F(z)$ be the fraction of agents for whom $\kappa \leq z$. Then define the mapping $\mathcal{E} : [0, 1] \to [0, 1]$ as $\mathcal{E}(\rho) = F[\Pi(\rho)]$. The equilibrium value ρ^* is a fixed point of this mapping. Since Π is upper hemi-continuous, and F is continuous by assumption, standard fixed point theorems establish existence of ρ^*. And given ρ^*, the other endogenous variables are determined as discussed above.

Existence is therefore routine. In terms of the types of equilibria that might exist, there is always one with $\rho^* = 0$, in which no one invests and no one brings a to the DM (although this may be ruled out by assuming that some agents are exogenously informed). If the buyer has all the bargaining power, $\theta = 1$, then $\rho^* = 0$ is the only equilibrium, since no seller will invest if he gets no gain from trade. Also, if costs were prohibitive – say, $\kappa(0) > u(q^*) - c(q^*)$ – then

$\rho^* = 0$ is obviously again the unique equilibrium. Whenever $\rho^* = 0$ the DM looks like a pure cash-in-advance market. Alternatively, if $\kappa(j)$ is low enough for all j, the natural outcome is that everyone invests, so $\rho^* = 1$, real assets work perfectly as a medium of exchange, and there is no role for money as long as A is sufficiently large.[13]

Perhaps most interesting are interior equilibria, $\rho^* \in (0, 1)$, in which case real assets are accepted in some but not all DM meetings. Obviously, such an equilibrium exists when the cost of becoming informed is sufficiently low for some agents and sufficiently high for others. To make this more precise, consider the situation where all agents carry $m = 0$ and just enough a to purchase $\bar{q}$. Clearly $\Pi(1) \leq \bar{\Pi} = \beta\lambda[z(\bar{q}) - c(\bar{q})]$. Also, consider

$$\begin{aligned} \lim_{\rho\to 0} q_1(\rho) &\equiv \hat{q}_1 = \ell^{-1}(i/\lambda) \\ \lim_{\rho\to 0} q_2(\rho) &\equiv \hat{q}_2 = z^{-1}\left[(1+r)A\delta/r + z(\hat{q}_1)\right] > \hat{q}_1, \end{aligned}$$

and let

$$\underline{\Pi} = \lim_{\rho\to 0} \Pi_1(\rho) = \beta\lambda\left[z(\hat{q}_2) - c(\hat{q}_2)\right] - \beta\lambda\left[z(\hat{q}_1) - c(\hat{q}_1)\right] > 0.$$

This provides us with conditions to guarantee that some but not all agents invest in information.

Proposition 2. *Equilibrium with endogenous ρ^* always exists. If $\kappa(0) < \underline{\Pi}$ and $\kappa(1) > \bar{\Pi}$ then there exists an equilibrium with $\rho^* \in (0, 1)$.*

INSERT FIGURES **??** - **??** ABOUT HERE

Clearly, there may be multiple equilibria, as illustrated by the numerical examples in Figures **??**-**??**. This multiplicity is not unexpected given the network nature of the game, as discussed in the payment literature mentioned above, or more generally given the role of coordination in

[13] Even if $\rho^* = 1$, when $A < \bar{A}$ and real assets are scarce, m is still essential, as in Lagos and Rocheteau [**?**] and Geromichalos et al. [**?**]. One difference in Lagos and Rocheteau [**?**] is that there, real assets are reproducable capital in a standard neoclassical production function, as opposed to the assets here and in Geromichalos et al. [**?**], which are in fixed supply. This changes some details but not the basic point. See Wallace [**?**] for related results in the overlapping generations model.

determining equilibrium (see e.g., Cooper and John [?]). However, our model has the advantage that this result arises from an explicit equilibrium asset market effect. When ρ is bigger it is easier to spend a in the DM, or in other words the asset is more liquid. This increases the demand for a in the CM, bidding up the price ψ. This makes agents more willing to pay the cost of information so that they are able to transact in the a in the DM, and this rationalized the bigger ρ. In terms of economic intuition, this asset market effect is at the heart of the potential multiplicity.[14]

5 Discussion

Now that we have results about the nature of equilibrium we can discuss some implications of the theory. Before proceeding, it is useful to introduce additional notation and reformulate certain conditions slightly differently. Imagine an asset that costs 1 unit of x in the current CM and pays $1 + r$ units in the next CM, but cannot be traded in the DM (say, because it is not a tangible asset, but merely a book entry). In equilibrium its real return satisfies $1 + r = 1/\beta$. Now imagine an asset that costs 1 dollar in the current CM and pays $1 + i$ dollars in the next CM, and similarly cannot be traded in the DM. Its return, the nominal interest rate, satisfies $1 + i = \phi/\hat{\phi}\beta$. Arbitrage implies $1 + i = (1 + r)\phi/\hat{\phi}$, which is a version of the Fisher equation. But we emphasize that this condition applies in general only across assets that are *illiquid*, in the sense that they cannot be traded in the DM. As we will soon see, things are different when liquidity comes into play.

Given $1 + i = \phi/\hat{\phi}\beta$, we can equivalently discuss monetary policy in terms of either the

[14] This intuition for multiplicity is based on $\Pi(\rho)$ being increasing. We cannot prove this, in general, since the asset market effect in the CM and the nonlinearity induced by bargaining in the DM are somewhat complicated, but $\Pi(\rho)$ is increasing in every example we studied. In any case, we are not claiming that one always gets multiple equilibria, only that it is possible. It would still be nice to find some assumptions to guarantee that $\Pi(\rho)$ is increasing, however, because we could then dispense with any continuity assumptions on the distribution of κ and prove a fixed point exists using Tarski's Theorem.

nominal rate i or inflation $\phi/\hat{\phi} = \gamma$; we use the former. Thus, rewrite (13)-(14) as

$$(1+r)A\delta = [z(q_2) - z(q_1)][r - \lambda_2 \ell(q_2)] \quad (16)$$

$$i = \lambda_1 \ell(q_1) + \lambda_2 \ell(q_2). \quad (17)$$

Let $q_2 = \alpha(q_1)$ and $q_1 = \mu(q_2)$ denote the implicit functions characterized by (16) and (17). It is routine to show that $\mu(\cdot)$ is decreasing and $\alpha(\cdot)$ is increasing, and they intersect for some $q_1 \in [0, \bar{q}]$. For $A \leq \bar{A}$, the intersection of α and μ determines the equilibrium $(q_1, q_2) \in [0, \bar{q}]^2$. For $A > \bar{A}$, the intersection of α and μ occurs at $q_2 > \bar{q}$, but since $q_2 > \bar{q}$ is not possible the equilibrium is $(q_1, q_2) = (\tilde{q}, \bar{q})$. The unique monetary steady state is therefore characterized by the intersection of $q_1 = \mu(q_2)$ and $q_2 = \bar{\alpha}(q_1) = \min\{\alpha(q_1), \bar{q}\}$, as shown for the cases $A < \bar{A}$ and $A > \bar{A}$ in Figures **??** and **??**.

INSERT FIGURES **??** - **??** ABOUT HERE

This makes it easy to analyze changes in parameters for a given information structure ρ by shifting curves. First observe that as long as $A < \bar{A}$, we have $q_2 < \bar{q}$, and a bears a liquidity premium $\ell(q_2) > 0$. In this case, $\psi > \beta\delta/(1-\beta) = \delta/r$, and the price of asset a exceeds the present value of its dividend stream, because this price reflects not only fundamentals (dividends) but also the value of the asset as a means of payment in the DM. To be precise, in steady state (12) at equality yields

$$\psi = \frac{\beta\delta\left[1 + \lambda_2 \ell(q_2)\right]}{1 - \beta\left[1 + \lambda_2 \ell(q_2)\right]} = \frac{\delta}{r}\left[1 + \frac{(1+r)\lambda_2 \ell(q_2)}{r - \lambda_2 \ell(q_2)}\right],$$

which exceeds the fundamental price whenever $\lambda_2 > 0$ or $q_2 < \bar{q}$. In this case there is a sense in which the Fisher equation does not hold for asset a (see below), and hence its and return in general will not be independent of monetary policy.

$x =$	i	A	δ	λ	ρ
$\frac{\partial q_1}{\partial x}$	$-$	$-$	$-$	$+$	$-$
$\frac{\partial q_2}{\partial x}$	$-$	$+$	$+$	$+$	?
$\frac{\partial \phi}{\partial x}$	$-$	$-$	$-$	$+$	$-$
$\frac{\partial \psi}{\partial x}$	$+$	$+$	$+$	$+$	$+$

Table 1: Effects of parameters when $A < \bar{A}$

The effects of parameter changes on asset prices and allocations when $A < \bar{A}$ are shown in Table 1 (see Appendix D for details). To start with one obviously policy relevant change, consider an increase in i. This shifts the μ curve southwest and leaves α unchanged, reducing q_1 and q_2. Intuitively, as i increases agents try to economize on m, reducing its CM price ϕ and DM value $q_1 = z^{-1}(\phi M)$. Given this, agents want to hold more a, raising its CM price ψ but on net lowering $q_2 = z^{-1}(\phi M + \psi A)$. Notice the observed accounting return on a between meetings of the CM, $1 + \delta/\psi$, decreases with i. Hence the Fisher equation apparently does not hold for a, since the observed return on a is not independent of inflation or the nominal rate. This would not happen if a were never traded in the DM; only when a bears a liquidity premium does its observed return depend on i. So the Fisher equation holds in some circumstances, or for some assets, but not others – not for those that bear a liquidity premium.

One can similarly analyze other parameter changes for a given ρ. Thus an increase in δ or A shifts the α curve northwest but leaves μ unchanged, leading to a fall in q_1 and rise in q_2, with resulting changes in ϕ and ψ. Intuitively, as dividends increase agents substitute into a and out of m, which affects both their DM and CM values. Similarly, an increase in λ shifts μ right and α left, but the net effects on q_1 and q_2 are positive. What is more relevant for thinking about endogenizing information is the effect of changes in ρ. As Table 1 shows, an increase in ρ reduces ϕ and increases ψ. Intuitively, in their portfolio problem, at the margin, agents want to move out of m and into a when the latter becomes more liquid. Since money is now less valuable, the amount you get in the DM when you use only cash q_1 falls. The effect on q_2 is ambiguous, since this is what you get when you pay with both cash and real assets, when the former becomes less and the latter more valuable.[15]

Before discussing information and liquidity in more detail, we want to briefly mention one more application that seems interesting even with ρ fixed. This is to ask how a change in i affects not only those agents or markets that use cash directly, but also those that do not. In Appendix C we sketch a simple extension with two distinct decentralized markets, call them markets B and C, where a fraction b and $1-b$ of the agents go between meetings of the CM. In market B, sellers always accept *both* a and m, and so everyone brings $m=0$ to this market, while in market C a fraction ρ of transactions require *cash* exactly as in the baseline model. We show that $\partial q_2/\partial i < 0$ in market B. Intuitively, as i increases, agents going to market C shift out of m and into a, driving up ψ. Thus, inflation lowers the return on assets and hence the equilibrium utility for agents participating in market B, even though they never use cash.

Moving to the model with endogenous ρ, the first thing we want to emphasize is that small changes in parameters, including policy or the cost of information, can have potentially large effects on equilibrium. Consider a change in i. Since q_1 and q_2 are affected by i for any given ρ, $\Pi(\rho)$ shifts with i, and this affects the set of equilibria. We illustrate one possibility in Figure **??**, where an increase in i introduces an additional interior equilibrium with a relatively large fraction of sellers acquiring information. Similarly, consider a change in the cost of information, illustrated in Figure **??** by an upward shift in $\kappa(\cdot)$. Again, a small change in the environment

[15] All of the comparative static results reported in the text are for the case $A < \bar{A}$. When $A > \bar{A}$, we have $q_2 = \bar{q}$ and $q_1 = \tilde{q}$ where $\tilde{q}$ solves $\ell(\tilde{q}) = i/\lambda(1-\rho)$. In this case, $\partial \tilde{q}/\partial i < 0$, $\partial \tilde{q}/\partial \lambda > 0$, and $\partial \tilde{q}/\partial \rho < 0$, while neither A nor δ affect q_1, and none of these variables affects q_2. Also, when $A > \bar{A}$, ϕ is decreasing in i and ρ and increasing in λ, while as we already have remarked $\psi = \beta\delta/(1-\beta)$ is pinned down by fundamentals. Also, notice that $\tilde{q} < \bar{q}$ if $i > 0$, but $\tilde{q} \to \bar{q}$ as $i \to 0$. In fact, as $i \to 0$, $\bar{A} \to 0$, which means equilibrium entails $a_1 > 0$. This says that at the Friedman rule $i = 0$ we have $q_1 = q_2 = \bar{q}$ and all assets bear the same return $1 + r = 1/\beta$. Although the focus here is not on welfare, for completeness we mention that $i = 0$ is the optimal policy, but it only gives the first best $q = q^*$ when buyers have all the bargaining power $\theta = 1$. Of course, if $\theta = 1$ then no seller will invest in information, but this is actually efficient when $i = 0$.

can change the set of equilibria a lot: in this case, when costs shift upward, the equilibrium with a high value of ρ^* disappears.

INSERT FIGURES ??-?? ABOUT HERE

All of this suggests that there is a problem with the typical reduced-form model that assumes a certain fraction of goods are exogenously subject to cash-in-advance constraints. When this fraction is endogenized, it becomes clear that in general it is neither uniquely determined nor invariant to policy. It is true that models with some cash goods and some credit goods allow for adjustment on the intensive margin – higher inflation typically makes agents consume lower quantities of the former and higher quantities of the latter – but there is no adjustment on the extensive margin determining *which* trades use cash (with a few exceptions, including some of the papers in footnote 5). Moreover, the kind of theory we are proposing seems relevant for discussing a variety of questions where cash-in-advance constraints are clearly inappropriate, in the sense that they assume the answer, such as asking about why and when economies use one currency or another.

In several episodes in Latin America, inflation induced the adoption of an alternative medium of exchange with a better rate of return – namely, the US dollar (see Guidotti and Rodriguez [?] for a discussion of countries such as Bolivia, Mexico, Peru, and Uruguay). Similar episodes of currency substitution have been observed in Eastern Europe and the Middle East (see Feige [?]). It is easy to see the following in our model: When peso inflation is not too high, locals tend to use pesos as a means of payment; dollars do not circulate widely, and they are not universally recognized. If the peso inflation rate increases, transacting in local currency becomes more costly, and the economy begins to dollarize as more agents learn to use dollars. Notice, however, that if the peso inflation rate later subsides, the dollar does not fall into disuse, because once the locals learn to recognize and use it, they do not quickly forget. This imparts a natural hysteresis effect into dollarization, as has been often discussed in the literature, but not formalized in this way (see Uribe [?]).

We also think it is interesting to analyze recent financial market events in the context of this model.[16] Even the relatively simple version with ρ exogenous seems useful for this purpose. Suppose we start in steady state where many people – a large fraction ρ_1 – are able to identify high- versus low-quality assets. Then exchange is relatively smooth, so output and welfare are high. Now suppose that there is a shock and information gets worse – we change to $\rho_0 < \rho_1$. This can occur for many reasons: perhaps some new complicated assets are developed that are harder to evaluate; perhaps there are real developments in the economy, such as in the mortgage market, that make existing assets harder to evaluate. This fall in ρ, an aggregate liquidity shock, implies that the market value ψ of assets drops, the value of alternatives like currency ϕ increases, transactions q_1 and q_2 are affected, and generally output and welfare can fall.

The above discussion concerns what happens when there is an exogenous decline in the liquidity of assets. Since we have a model where liquidity is endogenous, we can also ask what happens when there is an increase in parameters affecting the cost or benefit of information, including the distribution of κ across agents, or policy. Since there may be multiple equilibria, different outcomes are possible, but the natural prediction of the effect of an increase in κ is that fewer agents acquire information, this reduces liquidity, and then we can see what happens from the previous paragraph. However, it is important to note that there are significant feedback effects: as fewer agents are willing to bear the cost of becoming or staying informed, in equilibrium, the value of assets falls, making more agents unwilling to bear the cost, and so on. This kind of feedback can be seen using Table 1. Consider an increase in i. On impact this

[16] This is only meant to be suggestive, since we clearly do not think the model captures all the interesting aspects of the crisis. For instance, our baseline model assumes that buyers can produce worthless assets whenever they like, which may not be a good literal representation of the current situation if one thinks this is characterized by a fixed quantity of assets of unknown quality. But we still think it is interestsing to use insights from the theory to discuss the issues.

causes ψ to rise and ϕ to fall, even for a given ρ. Then as ρ increases in response to higher i, ψ rises and ϕ falls further.[17]

So the theory highlights multiplier effects that may generate relatively large responses to relatively small shocks to the cost of information, even if the equilibrium set does not change qualitatively. More ominously, it is easy to see how a catastrophe (mathematical and economic) can result from a small increase in the cost of information, as an equilibrium with a relatively high value of ρ might vanish even if the best response curve shifts only a little, as in Figures **??** and **??**. This can force the economy to discontinuously move to a new equilibrium with lower ρ and, again, some of the unpleasant implications discussed above. Although there is much to be done to understand recent financial events, we think that some version of an information-based theory with an emphasis on liquidity may be quite relevant, and that models like the one analyzed here may have something to contribute.

6 Conclusion

We analyzed a model where assets can differ in terms of liquidity, based on recognizability. Given the proportion of informed sellers, there is a unique equilibrium, in which there may or may not be a liquidity premium on certain assets depending on parameters. We then endogenized information and liquidity. Although the theory applies to any combination of assets, because recognizability has long been thought relevant for monetary economics, we discussed in detail the role of money. We showed how monetary policy affects asset prices and equilibrium allocations (even those individuals or markets that never use cash). Different from much of monetary economics, we endogenize the set of assets that are accepted in transactions through investment in information. This highlights complementaries that can generate multiple equilibria, and shows

[17] Table 1 is useful in this analysis because it shows the sign of the initial effect of i on asset prices is the same as the effect that ensues after ρ also changes.

how liquidity depends on policy. It is not appropriate to take cash-in-advance constraints as invariant. We also think that the theory has many practical implications, including results on dollarization and hysteresis.

Potentially the general approach also has something to contribute to understanding financial crises like the current situation, although more work remains on information frictions in models where exchange is modeled explicitly. In our set up, sellers who do not recognize an asset simply refuse to accept it. This is convenient, as it allows us to use simple bargaining theory, but it is also extreme. It would be interesting, if more difficult, to study specifications where agents sometimes trade for assets whose quality is unknown. We do not expect the economics to be necessarily very different, however. In our model, agents who do have imperfect information about assets refuse to accept them, but in principle, the outcome should be similar if, say, they accepted them at a discount. This version would still generate a role for more recognizable assets, including currency, and could still be used to discuss the impact of changes in the amount of, or the cost of, information regarding asset quality. For technical reasons, we studied an extreme version, but we think it provides some steps in the right direction.

Appendix

A. Proof of Lemma 2: Suppose $a_2 = 0$. Then $q_1 = q_2 \equiv q_0$. Given $\gamma > \beta$, we know $q_0 < \bar{q} \leq q^*$ by standard results (when $a_2 = 0$ there are no claims traded in the DM and the model is equivalent to the baseline LW model). Since $q_0 < q^*$, from (10) at equality we have

$$(\lambda_1 + \lambda_2)\ell(q_0) + 1 = \phi/\beta\hat{\phi} = \gamma/\beta > 1,$$

which implies $\ell(q_0) > 0$. Since $a_2 = 0$, market clearing implies $a_1 = A > 0$, and (11) holds at equality. Thus, $\psi = \beta(\hat{\psi} + \delta)$. Then (12) implies $\lambda_2 \ell(q_0) \leq 0$, a contradiction. ■

B. Proof of Proposition 1: Suppose $A \leq \bar{A}$. We first show that there exists a unique pair (q_1, q_2) that satisfy (13) and (14). We then show these conditions are equivalent to the necessary and sufficient conditions for equilibrium.

By the implicit function theorem:

$$\mu'(q_1) = -\frac{\beta\lambda_1\ell'(q_1)}{\lambda_2\ell'(q_2)} < 0$$

$$\alpha'(q_1) = \frac{-z'(q_1)\Big[1 - \beta[\lambda_2\ell(q_2) + 1]\Big]}{\beta\lambda_2 e'(q_2)(z(q_2) - z(q_1)) - z'(q_1)\Big[1 - \beta[\lambda_2\ell(q_2) + 1]\Big]} > 0$$

Let $\breve{q}$ satisfy $\ell(\breve{q}) = \frac{\gamma-\beta}{\beta\lambda_1} + \frac{\lambda_2}{\lambda_1}$, with $\breve{q} < \tilde{q} \leq \bar{q}$. Since $\ell'(q) < 0$ and $\lim_{q\to\infty} \ell(q) = -1$, it is easy to see that $\lim_{q_1\to\breve{q}^+} \mu(q_1) = \infty$. Moreover, we claim $\lim_{q_1\to\breve{q}^+} \alpha(q_1) < \infty$. Suppose not. That is, suppose $\lim_{q_1\to\breve{q}^+} \alpha(q_1) = \infty$. Then using (13) we have

$$A\delta = \lim_{q_1\to\breve{q}^+} [z\big(\alpha(q_1)\big) - z(q_1)][1 - \beta + \beta\lambda_2].$$

This implies $A\delta \geq [z(\bar{q}) - z(\breve{q})][1 - \beta + \beta\lambda_2]$, which implies $\frac{\delta}{1-\beta} > \frac{z(\bar{q})-z(\breve{q})}{A}$, a contradiction. Therefore, $\lim_{q_1\to\breve{q}^+} \mu(q_1) > \lim_{q_1\to\breve{q}^+} \alpha(q_1)$.

Now consider (14) with $q_1 = \bar{q}$, so that $\frac{\gamma}{\beta} = \lambda_2\ell(q_2) + 1$. This implies $\ell(q_2) \leq 0$, so that $\mu(\bar{q}) \leq \bar{q}$. Now consider (13). If $q_2 = \bar{q}$ then $A\delta = [z(\bar{q}) - z(q_1)](1 - \beta)$. Since $\frac{A\delta}{1-\beta} > 0$, $\alpha^{-1}(\bar{q}) < \bar{q}$. Since $\alpha' > 0$, $\alpha(\bar{q}) > \bar{q} \geq \mu(\bar{q})$. Since μ and α are continuous, $\mu' < 0$ and $\alpha' > 0$, $\mu(q') > \alpha(q')$ for some $q' < \bar{q}$, and $\alpha(\bar{q}) \geq \mu(\bar{q})$, we conclude that there exists a unique pair (q_1, q_2) with $q_1 > 0$ and $q_2 \leq \bar{q}$ that satisfy (13) and (14).

It is left to show that (13) and (14) are equivalent to the necessary and sufficient conditions for an equilibrium with $m > 0$, $a_1 = 0$, and $a_2 > 0$. Since $m > 0$, (10) holds with equality in equilibrium. Since $\gamma = \phi/\phi'$, clearly (14) and (10) are equivalent. Since $a_2 > 0$, (12) must also hold with equality. We know that $a_1 = 0 \Rightarrow a_2 = A$. Also, $z(q_1) = \phi M$ and $z(q_2) = \phi M + (\psi + \delta)A$ implies the asset pricing equation

$$\psi = \frac{z(q_2) - z(q_1)}{A} - \delta. \tag{18}$$

Substituting this into (13) yields (12).

Now suppose $A > \bar{A}$. We claim that there does not exist a pair (q_1, q_2) with $q_2 < \bar{q}$ that satisfy (13) and (14). To see this, let $\tilde{\tilde{q}}$ be the value of q_1 such that $\alpha(q_1) = \bar{q}$. It is easy to show that $A > \bar{A} \Rightarrow \tilde{\tilde{q}} < \tilde{q}$, which implies that $\mu(\tilde{\tilde{q}}) > \bar{q}$, so there does not exist a $q_1 < \tilde{\tilde{q}}$ satisfying $\mu(q_1) = \alpha(q_1)$. Therefore, $q_2 = \bar{q}$. From (10), $q_1 = \tilde{q}$ and the corresponding prices follow immediately. ■

C. Results for the Cashless Market: Since market C is is identical to the DM in that model, the first order conditions are (10)-(12). To derive the first order conditions for agents going to the other market, for any variable z associated with market C write the analog for market B as z^B. Then

$$V^B(m^B, a_1^B, a_2^B) = (1 - \lambda^B)W^B(y^B) + \lambda^B \left[u(q_2^B) + W^B\left(y^B - p_2^B\right)\right],$$

since all market B meetings are type 2 meetings. Differentiation yields the analogs of (10)-(12). Let us focus on the case where q_2, $q_2^B < \bar{q}$. One can show a_2, $a_2^B > 0$, and $m^B = 0$. Also, the bargaining solutions are $z(q_1) = \phi m$, $z(q_2) = \phi m + (\psi + \delta)a_2$, and $z(q_2^B) = (\psi + \delta)a_2^B$, and market clearing implies $A = (1 - b)a_2 + ba_2^B$. Given all this, routine manipulation allows us to describe (q_1, q_2, q_2^B) by

$$\begin{aligned}
i &= (1 - \rho)\lambda\ell(q_1) + \rho\lambda\ell(q_2) \\
(1 + r)A\delta &= \Big\{(1 - b)[z(q_2) - z(q_1)] + bz(q_2^B)\Big\}[r - \rho\lambda\ell(q_2)] \\
(1 + r)A\delta &= \Big\{(1 - b)[z(q_2) - z(q_1)] + bz(q_2^B)\Big\}[r - \lambda^B\ell(q_2^B)].
\end{aligned}$$

In fact, since $q_2 = h(q_2^B) \equiv \ell^{-1}\left[\frac{\lambda^B}{\rho\lambda}\ell(q_2^B)\right]$, these reduce to two equations in (q_1, q_2^B):

$$\begin{aligned} i &= (1-\rho)\lambda\ell(q_1) + \lambda^B\ell(q_2^B) \\ (1+r)A\delta &= \left\{(1-b)[z\left[h(q_2^B)\right] - z(q_1)] + bz(q_2^B)\right\}[r - \lambda^B\ell(q_2^B)]. \end{aligned}$$

We have

$$\begin{aligned} \frac{\partial q_1^B}{\partial i} &= \frac{(1-b)\Big\{z'[h(q_2^B)]h'(q_2^B)[r-\lambda^B\ell(q_2^B)]\Big\} - \Big\{(1-b)[z\big(h(q_2^B)\big) - z(q_1)] + bz(q_2^B)\Big\}\lambda^B\ell'(q_2^B)}{\Psi} \\ \frac{\partial q_2^B}{\partial i} &= \frac{(1-b)z'(q_1)[r-\lambda^B\ell(q_2^B)]}{\Psi} \end{aligned}$$

and so both take the sign of

$$\begin{aligned} \Psi &= (1-\rho)\lambda_1\ell'(q_1)\Big\{(1-b)\Big[z'[h(q_2^B)]h'(q_2^B)[r-\lambda^B\ell(q_2^B)]\Big] \\ &- \Big[(1-b)[z\big(h(q_2^B)\big) - z(q_1)] + bz(q_2^B)\Big]\lambda^B\ell'(q_2^B)\Big\} + (1-b)z'(q_1)[r-\lambda^B\ell(q_2^B)]\lambda^B\ell'(q_2^B) < 0. \end{aligned}$$

D. Results in Table 1: Let Δ denote the determinant of the following matrix:

$$\begin{bmatrix} \lambda_1\ell'(q_1) & \lambda_2\ell'(q_2) \\ [\lambda_2\ell(q_2) - r]\,z'(q_1) & [r-\lambda_2\ell(q_2)]z'(q_2) - [z(q_2) - z(q_1)]\lambda_2\ell'(q_2) \end{bmatrix}$$

From (16), an equilibrium with $q_2 \geq q_1$ requires $r - \lambda_2\ell(q_2) \geq 0$, so $\Delta < 0$. Then we have

$$\begin{aligned} \frac{\partial q_1}{\partial i} &= \frac{[r-\rho\lambda\ell(q_2)]z'(q_2) - [z(q_2)-z(q_1)]\rho\lambda\ell'(q_2)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial i} &= \frac{[r-\rho\lambda\ell(q_2)]z'(q_1)}{\Delta} < 0 \\ \frac{\partial q_1}{\partial \delta} &= \frac{-(1+r)A\rho\lambda\ell'(q_2)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial \delta} &= \frac{(1+r)A(1-\rho)\lambda\ell'(q_1)}{\Delta} > 0 \\ \frac{\partial q_1}{\partial A} &= \frac{-(1+r)\delta\rho\lambda\ell'(q_2)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial A} &= \frac{(1+r)\delta(1-\rho)\lambda\ell'(q_1)}{\Delta} > 0 \\ \frac{\partial q_1}{\partial \rho} &= \frac{\lambda[\ell(q_1)-\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_2) - [z(q_2)-z(q_1)]\rho\lambda\ell'(q_2)\lambda\ell(q_1)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial \rho} &= \frac{\lambda[\ell(q_1)-\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_1) + [z(q_2)-z(q_1)](1-\rho)\lambda\ell'(q_1)\lambda\ell(q_2)}{\Delta} \\ \frac{\partial q_1}{\partial \lambda} &= \frac{(1-\rho)\ell(q_1)[z(q_2)-z(q_1)]\rho\lambda\ell'(q_2) - [(1-\rho)\ell(q_1)+\rho\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_2)}{\Delta} > 0 \\ \frac{\partial q_2}{\partial \lambda} &= \frac{(1-\rho)\ell(q_2)[z(q_2)-z(q_1)]\rho\lambda\ell'(q_1) - [(1-\rho)\ell(q_1)+\rho\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_1)}{\Delta} > 0 \end{aligned}$$

Given $z(q_1) = \phi M$ and $z(q_2) - z(q_1) = (\psi + \delta)A$, we have

$$
\begin{aligned}
\frac{\partial \psi}{\partial i} &= \frac{[z(q_2) - z(q_1)]\rho\lambda\ell'(q_2)z'(q_1)}{A\Delta} > 0 \\
\frac{\partial \phi}{\partial i} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial i} < 0 \\
\frac{\partial \psi}{\partial \delta} &= \frac{[z(q_2) - z(q_1)](1-\rho)\lambda\ell'(q_1)\rho\lambda\ell'(q_2) + [1 + \rho\lambda\ell(q_2)][(1-\rho)\lambda\ell'(q_1)z'(q_2) + \rho\lambda\ell'(q_2)z'(q_1)]}{\Delta} \\
\frac{\partial \phi}{\partial \delta} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \delta} < 0 \\
\frac{\partial \psi}{\partial A} &= \frac{[z(q_2) - z(q_1)](1-\rho)\lambda\ell'(q_1)\rho\lambda\ell'(q_2)}{A\Delta} < 0 \\
\frac{\partial \phi}{\partial A} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial A} < 0 \\
\frac{\partial \psi}{\partial \rho} &= \frac{[z(q_2) - z(q_1)][(1-\rho)\lambda\ell'(q_1)\lambda\ell(q_2)z'(q_2) + \rho\lambda\ell'(q_2)\lambda\ell(q_1)z'(q_1)]}{\Delta} > 0 \\
\frac{\partial \phi}{\partial \rho} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \rho} < 0 \\
\frac{\partial \psi}{\partial \lambda} &= \frac{(1-\rho)[z(q_2) - z(q_1)][\ell(q_2)\ell'(q_1)z'(q_2) - \ell(q_1)\ell'(q_2)z'(q_1)]}{\Delta} \\
\frac{\partial \phi}{\partial \lambda} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \lambda} > 0
\end{aligned}
$$