

Information and Liquidity*

Benjamin Lester
University of Western Ontario

Andrew Postlewaite
University of Pennsylvania

Randall Wright
University of Pennsylvania

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Abstract

We study how recognizability affects assets' acceptability, or liquidity. Some assets, like US currency, are readily accepted because sellers can easily recognize their value, unlike stock certificates, bonds or foreign currency, say. This idea is common in monetary economics, but previous models deliver equilibria where less-recognizable assets are always accepted with positive probability, never probability 0. This is inconvenient when prices are determined through bargaining, which is difficult with private information. We construct models where agents reject outright assets that they cannot recognize, at least for some parameters. Thus, information frictions generate liquidity differences without overly complicating the analysis.

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1 Introduction

An old idea in economics, going back at least to Menger [18], is that an object that serves as a medium of exchange, or perhaps a liquid asset more generally, either tends to have or ought to have certain desirable characteristics. These characteristics typically include portability, storability, divisibility and recognizability. We focus here on recognizability, and ask how this affects an asset's liquidity, or acceptability. For example, the fact that US dollars are readily accepted in many places would seem to have something to do with the idea that sellers recognize US dollars and are roughly familiar with their value, while they are less likely to recognize or know the value of alternative assets, such as foreign currency, bonds, stock certificates, and so on. Hence, these alternatives are less likely to be accepted – they are less liquid. This is a candidate explanation as to why such assets are not typically used as a means of payment, despite often having better rates of return than currency.

The goal of this paper is to study the relationship between acceptability and recognizability in games with information frictions. We are not the first to consider this. In the literature that uses search-based models to build a micro-foundation for monetary theory, papers that emphasize information include Williamson and Wright [29], Trejos [25], [26], Kim [24], and Berentsen and Rocheteau [5].¹ Those models can, for some parameters, deliver equilibria where certain objects are accepted in exchange with probability less than 1, and hence are less than perfectly liquid, because they are not easily recognized. This, in principle, can generate a medium-of-exchange role for a more recognizable object, like currency, even if it has an inferior rate of return. For other parameters, even when the quality of an asset

¹Other information-based models in the search literature include Li [17], Cuadras-Morató [7], Green and Weber [10], Velde, Weber and Wright [28], Ennis [8], Nosal and Wallace [20], and Hu [12]. The role of information has also been discussed in monetary economics by Brunner and Meltzer [6], Alchian [2], Freeman [9], and Bannerjee and Maskin [3], among others.

cannot be recognized, it is accepted with probability 1. It is never the case in those models that hard-to-recognize objects are outright rejected – they are always accepted with strictly positive probability.

This last result is inconvenient for the following reason. The early search-based literature assumes that both goods and assets are indivisible, so every trade entails a one-for-one swap. This allows one to focus on acceptability without having to determine the terms of trade. Although this makes the analysis easier, it is obviously a drawback when one wants to apply these theories to empirical or policy issues. When one focuses on frictions other than information problems, including the double-coincidence problem in bilateral exchange resulting from specialization, it is relatively straightforward to generalize the models to have divisible goods or assets, and use standard bilateral bargaining theory to determine prices. In models that focus on information frictions, this is *not* straightforward, because bargaining under private information is complicated, at best. This technical hurdle has slowed progress in monetary economics based on information frictions.²

For this reason, it would be nice to have models in which agents reject outright objects that they do not recognize. Then information problems could drive liquidity differences, but the analysis would be straightforward because agents only bargain over assets that they recognize (i.e. under full information); when an asset is not recognized it is simply not on the bargaining table. In this paper we show how to construct environments that can deliver this outcome. This constitutes a step toward developing a relatively tractable framework where information frictions can

²Models with indivisible goods and money include Kiyotaki and Wright [13], [14]. Extensions to divisible goods or money include Shi [22], [23], Trejos and Wright [27], Molico [19], Green and Zhou [11], and Lagos and Wright [15]. Some of the papers mentioned above, including Berentsen and Rocheteau [5], and some of those cited in footnote 1, do make progress with bargaining under private information, but it is fair to say it is a complicated problem. See Rocheteau [21] for recent work on the problem.

generate the coexistence of money and other assets. In a companion paper (Lester et al. [16]) we study models where it is assumed that assets are not accepted when they are not recognized. Although this approach is not unprecedented (see e.g. Freeman [9]), the goal here is to develop better micro-foundations, in the sense of providing an environment in which the result emerges endogenously.

Before we begin the formalities, to show why the results are not trivial, we review the logic in the models mentioned above. Suppose there is an object that can be either high value or low value. For simplicity, assume that the low quality object is completely worthless – a pure *lemon* in the sense of Akerlof [1], or perhaps a *counterfeit*. Buyers choose to acquire either high or low quality versions *ex ante*, before the market opens. High quality costs more, although it is not prohibitively costly in the sense that it would be produced in the absence of private information. After agents choose quality, a market opens where buyers and sellers meet bilaterally. Information frictions are captured as follows. With some probability a seller is able to recognize the quality of a buyer’s object, in which case we say he is informed; with complementary probability, he cannot distinguish quality, in which case we say he is uninformed. Also, for now suppose all objects are indivisible.

Consider a candidate equilibrium in which sellers do not accept objects that they do not recognize. Since sellers clearly do not accept lemons when they do recognize them, buyers with lemons cannot trade. So, given that it is not prohibitively costly, all buyers bring high quality to the market. Given this, sellers should accept objects even if they do not recognize them since, by assumption, there are no lemons in the market. Hence, the candidate equilibrium does not exist. Instead, there typically exist equilibria in which sellers accept objects they do not recognize with positive probability, sometimes probability one, and buyers may or may not bring some

low quality to market. Thus, hard-to-recognize assets may be less than perfectly acceptable, but they are never rejected outright. We illustrate this more formally below. Then we change the setup to overturn the results. The key change is that we allow buyers to decide which asset to bring to market *after* knowing whether the seller will be informed.³

With this modification, there now exist regions of the parameter space in which sellers never accept objects that they do not recognize. We prove this both when sellers' information is exogenous, and when they choose whether to become informed, since this choice is a critical element of some applications, including our other paper. We show that our results are robust to changes in the timing of the seller's decision to invest in verifying the quality of the asset. We then explicitly introduce a money-like asset, that is perfectly recognizable, and show it can be valued despite having an inferior return. Much of the analysis assumes indivisible goods and assets, but at the end, we make these divisible, so that the terms of trade can be determined endogenously. Although this complicates the analysis somewhat, the main results go through – it is still possible to generate robust equilibria where sellers do not accept objects that they do not recognize, at least approximately, in a sense to be made precise.

2 Basic Assumptions

There is a continuum of buyers indexed by $b \in \mathcal{B} = [0, 1]$ and a continuum of sellers indexed by $s \in \mathcal{S} = [0, 1]$ (it is straightforward to allow different measures of buyers and sellers). Each seller s can produce a single good at cost k_g that yields utility y_g

³There are many interpretations of this timing assumption that seem plausible. One interpretation is that buyers realize a taste shock determining which good they want to consume from some set, and it is known that the producers of certain goods are informed while producers of other goods are not. Then the difference in assumptions amounts to saying whether the buyer knows which good he wants before or after committing to the quality of the asset he brings to the market.

when consumed by any buyer b , and utility 0 when consumed by s . Equivalently, s can be endowed with the good and we can interpret k_g as his utility from consuming it (i.e., k_g can be an opportunity cost rather than a production cost). Each buyer b is endowed with something we call a high quality asset that yields utility y_h to any agent when consumed. In addition, b can also produce or otherwise acquire a low quality asset at cost k_l that yields utility y_l to all agents. For now, both goods and assets are indivisible, and we ignore lotteries, so any trade must entail a one-for-one swap, but this is relaxed in Section 5.3.

Let l denote the strategy for b of acquiring a low quality asset and l' the strategy of not doing so, where in general we use a “prime” to denote negation. Let p_l denote the probability that b plays strategy l . We assume s cannot differentiate between high and low quality assets unless he acquires the necessary information, or perhaps the necessary technology.⁴ Sellers are heterogeneous with respect to the cost of acquiring this information: the information cost for $s \in \mathcal{S}$ is k_i^s , distributed according to CDF $F(k_i^s)$, where without loss of generality we order agents so that $k_i^s \leq k_i^{s'}$ for all $s' \geq s$. Denote by i a seller’s strategy of acquiring information, i' the strategy of remaining uninformed, and p_i^s the probability with which s plays strategy i . Let λ be the fraction of sellers who choose strategy i and become informed.

Buyers choose whether or not to acquire a low quality asset and, simultaneously, sellers choose whether to become informed. Then nature randomly matches buyers and sellers bilaterally. Letting the total number of matches be $\alpha \in (0, 1]$, α is also the probability any b or any s is matched (it is easy to generalize to probabilities

⁴We are agnostic about what this information or verification technology is in the model, but there are many examples in the real world. As simple examples, consider a seller paying for a service that allows him to verify personal checks, or buying a machine that allows him to accept debit/credit cards. One might also think about a seller who is not able to readily distinguish between two batches of mortgage-backed securities unless he hires an expert to analyze them. Historically, when things were simpler, sellers still had to invest in scales and touchstones to verify the weight and purity of metal coins.

α_b and α_s by assuming different measures of buyer and sellers). If unmatched, a seller neither produces nor consumes, and a buyer consumes his assets. If matched there are potential gains from trade, although this may be hindered by informational frictions. Within a match, b can observe whether s is informed, and s can distinguish between high and low quality assets if and only if he has acquired the requisite information or technology for verifying quality.⁵

Assume $y_h > 0 > y_l - k_l$ and $y_g > k_l$. The first assumption guarantees that b would never produce a low quality asset for personal consumption, although he might produce it for use in exchange. The second assumption guarantees he would produce a low quality asset if he knew that he could use it to acquire the good for sure. We also assume $0 \leq y_l < k_g < y_h < y_g$. This implies that, when b and s are matched, b would exchange either asset for the good, while if informed s would accept a high but not a low quality asset. This is the sense in which the low quality asset is a lemon. If uninformed, s must choose whether to accept something he cannot recognize. We denote by a the strategy of an uninformed seller of accepting assets he cannot recognize, a' the strategy of rejecting them, and p_a the probability of playing a .

3 Model 1

In our first specification, b must choose which asset to bring to the market *before* he knows the type of seller he might meet – informed or uninformed. Also, b can only bring *one* asset to the market. Hence, if b chooses to bring low quality, he consumes the high quality asset himself before the market opens.

⁵In Section 5.1 we introduce a cost for s of actually checking quality, in addition to the ex ante cost of acquiring the ability to do so.

3.1 Exogenous Information

We begin by characterizing behavior for a given fraction of informed sellers λ . The payoffs to b from strategies l and l' , conditional on p_a , are:

$$\begin{aligned}\pi_l &= y_h - k_l + (1 - \alpha)y_l + \alpha \{ \lambda y_l + (1 - \lambda) [p_a y_g + (1 - p_a)y_l] \} \\ \pi_{l'} &= (1 - \alpha)y_h + \alpha \{ \lambda y_g + (1 - \lambda) [p_a y_g + (1 - p_a)y_h] \}.\end{aligned}$$

In the first equation, if b produces low quality he first incurs cost k_l and consumes the high quality asset. Then, with probability $1 - \alpha$, he is unmatched and consumes the low quality asset, while with probability α he is matched. If matched, when s is informed or is uninformed and plays a' , b consumes the low quality asset, while b gets the good if s is uninformed and plays a .

Letting

$$\tilde{p}_a = \frac{\alpha \lambda (y_g - y_h) + k_l - y_l}{\alpha (1 - \lambda) (y_h - y_l)}, \quad (1)$$

the buyer's best response is:

$$p_l^*(p_a) = \begin{cases} 1 & \text{if } p_a > \tilde{p}_a \\ [0, 1] & \text{if } p_a = \tilde{p}_a \\ 0 & \text{if } p_a < \tilde{p}_a \end{cases}$$

Similarly, the payoffs to an uninformed s are:

$$\begin{aligned}\pi_a &= -k_g + p_l y_l + (1 - p_l) y_h \\ \pi_{a'} &= 0.\end{aligned}$$

Hence his best response is:

$$p_a^*(p_l) = \begin{cases} 1 & \text{if } p_l < \tilde{p}_l \\ [0, 1] & \text{if } p_l = \tilde{p}_l \\ 0 & \text{if } p_l > \tilde{p}_l \end{cases}$$

where

$$\tilde{p}_l = \frac{y_h - k_g}{y_h - y_l}. \quad (2)$$

We now have the following results conditional on λ (all proofs are in the Appendix).

Proposition 1. Let $\tilde{\lambda} = \frac{\alpha(y_h - y_l) - (k_l - y_l)}{\alpha(y_g - y_l)}$.

- If $\lambda > \tilde{\lambda}$, then the unique equilibrium is $p_a^* = 1$ and $p_l^* = 0$.
- If $\lambda = \tilde{\lambda}$, then $p_a^* = 1$ and p_l^* can take on any value in the interval $[0, \tilde{p}_l]$.
- If $\lambda < \tilde{\lambda}$, then the unique equilibrium is $p_a^* = \tilde{p}_a$ and $p_l^* = \tilde{p}_l$.

Notice that if either λ is large or α is small, all uninformed sellers accept assets that they cannot recognize and no buyers produce lemons.⁶ Also, even when $p_a^* = \tilde{p}_a < 1$, it is bounded away from zero as long as $\alpha\lambda > 0$. The intuition is clear: if sellers never accept assets they do not recognize then buyers never produce lemons, but then uninformed sellers can accept assets with impunity. We do not generally get the result that s rejects outright assets he does not recognize. It is the case that $p_a^* = 0$ if $k_l = y_l$ and $\alpha\lambda = 0$, but then the market completely shuts down, which is not interesting.

3.2 Endogenous Information

We now endogenize λ .⁷ The payoffs to seller s from strategies i and i' are:

$$\begin{aligned}\pi_i^s &= -k_i^s + \alpha(1 - p_l)(y_h - k_g) \\ \pi_{i'}^s &= \alpha \{p_a [-k_g + p_l y_l + (1 - p_l)(y_h)]\}.\end{aligned}$$

The payoff to s from information, conditional on λ other sellers being informed, is derived as follows. First, if $\lambda \in [0, \tilde{\lambda})$, given Proposition 1, we have

$$\pi_i^s - \pi_{i'}^s = \frac{\alpha(k_g - y_l)(y_h - k_g)}{y_h - y_l} - k_i^s.$$

⁶In words, if there are a sufficient number of informed sellers, buyers have no incentive to produce low quality assets and thus uninformed sellers need not worry about lemons; similarly if the probability of a match is low buyers have no incentive to produce lemons, since they are only worth producing if they can be traded, and again uninformed sellers can safely accept assets.

⁷This is in the spirit of the extension by Kim [24] of Williamson and Wright [29], and is interesting because as we said above, endogenous information is a key element of the applications in our companion paper.

In this case s invests if and only if $k_i^s \leq \tilde{k}_i \equiv \frac{\alpha(k_g - y_l)(y_h - k_g)}{y_h - y_l}$ (assuming he invests when indifferent). Second, if $\lambda \in (\tilde{\lambda}, 1]$, then $p_l = 0$, so $\pi_i^s - \pi_i^s = -k_i^s$ and no seller with $k_i^s > 0$ acquires information. Finally, for $\lambda = \tilde{\lambda}$, there is a continuum of equilibria in p_a and p_l strategies, and across these equilibria the net benefit of information varies over the interval $[0, \tilde{k}_i]$.

Insert Figure 1 About Here.

Consider the mapping Λ that for any given λ gives the proportion of sellers that have a (weak) net benefit of becoming informed:

$$\Lambda(\lambda) = \begin{cases} F(\tilde{k}_i) & : \lambda \in [0, \tilde{\lambda}) \\ [F(0), F(\tilde{k}_i)] & : \lambda = \tilde{\lambda} \\ F(0) & : \lambda \in (\tilde{\lambda}, 1] \end{cases} \quad (3)$$

We plot this mapping in Figure 1. Equilibrium is now a fixed point of this correspondence, and is characterized in the following proposition.

Proposition 2. *There exists an equilibrium, and it is unique.*

- If $\tilde{\lambda} > F(\tilde{k}_i)$ then $\lambda^* = F(\tilde{k}_i)$, $p_l^* = \tilde{p}_l$, and $p_a^* = \frac{\alpha F(\tilde{k}_i)(y_g - y_h) + k_l - y_l}{\alpha[1 - F(\tilde{k}_i)](y_h - y_l)}$.
- If $F(0) \leq \tilde{\lambda} \leq F(\tilde{k}_i)$, then $\lambda^* = \tilde{\lambda}$, $p_l^* = \frac{F^{-1}(\tilde{\lambda})}{\alpha(k_g - y_l)}$, and $p_a^* = 1$.
- If $\tilde{\lambda} < F(0)$ then $\lambda^* = F(0)$ and $p_l^* = 0$ and $p_a^* = 1$.

If $F(\tilde{k}_i)$ is small, so that a large fraction of sellers face high costs of becoming informed, then few sellers invest in information. In this case, some buyers produce low quality assets, and they are accepted with positive probability. But when $F(\tilde{k}_i)$ is large, many sellers become informed, no low quality assets are produced, and sellers always accept. As in the case with λ given exogenously, for reasonable parameters, equilibria tend to have a strictly positive probability that uninformed sellers accept

assets.⁸ This fraction cannot be too small for the reason discussed above: if $p_a^* \approx 0$, then no buyers choose l , but then uninformed sellers should accept assets with probability 1. This fraction cannot, in general, be too large either: for sellers to have an incentive to acquire information, p_l must be sufficiently high, which requires λ not too large.

We think the model above, while highly stylized, gives some insights into issues concerning recognizability and liquidity; it certainly allows one to make the point that a lemons problem in the asset market can hinder trade. One would obviously like to extend this type of model in many directions, including endogenous price formation. Much of the related literature allows buyers and sellers to bargain over the terms of trade. The problem is that bargaining under private information is complicated. It would therefore be desirable to have sellers who do not recognize an asset to simply refuse to accept it. However, we have seen that $p_a^* = 0$ is not likely in this model. In the next section, we show that a simple adjustment in the timing can overturn this result.

4 Model 2

Suppose b can choose which asset to bring to the market *after* knowing s 's type. One interpretation is that he learns which type of good he wants to consume, and knows whether sellers of that good are informed. Although variations are possible, to keep the model close to the previous section, assume here that b first decides whether to acquire a low quality asset and then learns if the seller is informed. Also, as before, b can only bring one asset to the market. If he acquires the low quality asset and s is informed, he brings the high and consumes the low quality asset. If he acquires

⁸To be specific, a necessary condition for $p_a^* = 0$ is $F(\tilde{k}_i) = 0$, or the cost of information for all sellers is prohibitively large. Even if $F(\tilde{k}_i) = 0$, we have $p_a^* > 0$ if $k_l > y_l$.

the low quality asset and s is uninformed, he consumes the high and brings the low quality asset, since one would never acquire low quality and then not try to use it, given $k_l > y_l$. If he does not acquire the low quality asset, of course, he can only bring high quality even if he learns s is uninformed.

4.1 Exogenous Information

The payoffs to b from the strategies l and l' are now:

$$\begin{aligned}\pi_l &= -k_l + (1 - \alpha)(y_h + y_l) + \alpha \{ \lambda(y_g + y_l) + (1 - \lambda)[y_h + p_a y_g + (1 - p_a)y_l] \} \\ \pi_{l'} &= (1 - \alpha)y_h + \alpha \{ \lambda y_g + (1 - \lambda)[p_a y_g + (1 - p_a)y_h] \}.\end{aligned}$$

In the first equation, a buyer who acquires a lemon incurs cost k_l . With probability $1 - \alpha$ he is unmatched and consumes both assets, while with probability α he is matched. If the seller is informed, b must trade the high quality asset. If the seller is uninformed, b consumes the high quality asset and tries to trade the lemon. If s accepts, b gets the good; otherwise b consumes the low quality asset.

Let

$$\hat{p}_a = \frac{(k_l - y_l)}{\alpha(1 - \lambda)(y_h - y_l)}. \quad (4)$$

Then b 's best response is as in Model 1, except we replace \tilde{p}_a with \hat{p}_a . Notice that $\hat{p}_a < \tilde{p}_a$, since given the option of deciding whether to try to trade a lemon after knowing the seller's type, b has a greater incentive to acquire it in the first place. The payoffs to an uninformed seller and the best response are identical to that in Model 1. Then we have the following equilibrium characterization.⁹

Proposition 3. *Let $\hat{\lambda} = \frac{\alpha(y_h - y_l) - (k_l - y_l)}{\alpha(y_h - y_l)}$.*

- *If $\lambda > \hat{\lambda}$, then the unique equilibrium is $p_a^* = 1$ and $p_l^* = 0$.*

⁹The proof is omitted, as it is nearly identical to the proof of Proposition 1.

- If $\lambda = \hat{\lambda}$, then $p_a^* = 1$ and p_l^* can take on any value in the interval $[0, \tilde{p}_l]$.
- If $\lambda < \hat{\lambda}$, then the unique equilibrium is $p_a^* = \hat{p}_a$ and $p_l^* = \tilde{p}_l$.

Notice something here that is *not* true in Model 1. Let $k_l \rightarrow y_l$. Then $\hat{\lambda} \rightarrow 1$, and we are almost certainly in the equilibrium with $p_a^* = \hat{p}_a$ and $p_l^* = \tilde{p}_l$. Moreover, in this equilibrium, as $k_l \rightarrow y_l$, we have $\hat{p}_a \rightarrow 0$. In words, as the net cost of acquiring a low quality asset gets small, the lemons problem becomes severe, and uninformed sellers accept assets with probability close to 0. In the limiting case $k_l = y_l$, sellers *never* accept assets they cannot recognize; when $k_l = y_l$, p_l^* is indeterminate, but $p_a^* = 0$ and uninformed sellers always reject assets outright. We think $k_l \approx y_l \approx 0$ is not an unreasonable case – it simply says that lemons, or perhaps counterfeits, are very cheap to produce and worth very little to consume. This delivers the result that uninformed sellers do not accept assets for fear of getting a lemon.

4.2 Endogenous Information

First, when $\lambda \in [0, \hat{\lambda})$, we have $\pi_i^s - \pi_{i'}^s = \alpha(y_h - k_g) - k_i^s$, and s invests if and only if $k_i^s \leq \hat{k}_i \equiv \alpha(y_h - k_g)$. Second, if $\lambda \in (\hat{\lambda}, 1]$, then $p_l = 0$ and so no seller with $k_i^s > 0$ becomes uninformed. Finally, for $\lambda = \hat{\lambda}$, there is a continuum of equilibria and across these equilibria the net benefit of information varies over $[0, \hat{k}]$. Following the analysis in the previous section, we have the next result.

Proposition 4. *There exists an equilibrium, and it is unique.*

- If $\hat{\lambda} > F(\hat{k}_i)$ then $\lambda^* = F(\hat{k}_i)$, $p_l^* = \tilde{p}_l$, and $p_a^* = \frac{k_l - y_l}{\alpha[1 - F(\hat{k}_i)](y_h - y_l)}$.
- If $F(0) \leq \hat{\lambda} \leq F(\hat{k}_i)$, then $\lambda^* = \hat{\lambda}$, $p_l^* = \frac{F^{-1}(\hat{\lambda})}{\alpha(y_h - y_l)}$, and $p_a^* = 1$.
- If $\hat{\lambda} < F(0)$ then $\lambda^* = F(0)$ and $p_l^* = 0$ and $p_a^* = 1$.

Again, by contrast with Model 1, in this model it is quite likely that equilibria may entail uninformed sellers refusing to accept assets with probability arbitrarily close to (and in the limit, equal to) 1. Suppose $\hat{\lambda} > F(\hat{k}_i)$, so that the cost of information is high for a significant number of sellers. Then $p_i^* = \tilde{p}_i$. Now suppose again that $k_l \rightarrow y_l$. First, this implies $\hat{\lambda} \rightarrow 1$, so we are in the first type of equilibrium. Second, it implies $p_a^* \rightarrow 0$, so uninformed sellers approximately reject assets outright. Buyers almost never succeed in trading low quality assets, since informed sellers always reject them and uninformed sellers reject them with probability close to 1, but they would try to trade them if uninformed sellers accepted.

5 Extensions

We extend Model 2 in several directions. First, we introduce costly asset quality verification – it costs something for an informed seller to check quality. Second, we introduce another asset meant to represent currency, as best we can in a static model, in the sense that it is universally recognized but has a relatively low return. Finally, we let goods and assets be divisible. To reduce notation, in this section, we set $\alpha = 1$.

5.1 Costly Verification

A central feature of the analysis here is the existence of two kinds of sellers, informed and uninformed. So far, sellers could become informed at some cost, at which point they can verify asset quality for free. We now generalize this so that sellers have to pay a cost k_v to verify quality even if they are informed (one can interpret this cost as ∞ if they are uninformed). This captures the idea that a seller may incur an initial cost to acquire, for instance, a machine to accept credit cards, after which he still must pay a fee to verify the quality of an asset in a particular transaction.

We assume as before $y_h > k_l$, and additionally $y_h > k_g + k_v$ and $k_g - y_l > k_v$.

The game proceeds as follows: first b chooses l or l' , and is then matched with an informed or uninformed s . If b produced a low quality asset, he then chooses whether to bring it or his high quality asset (again, he can bring only one). As before, if b played l and trades with an uninformed s , he always brings low quality. An uninformed seller then chooses whether to accept, $a_{i'}$, or not accept, a'_i , and $p_a^{i'}$ his probability of playing $a_{i'}$. An informed seller has the following decisions to make. First, he can either accept a_i or not accept a'_i , and p_a^i is the probability of playing a_i . Then, if he is willing to accept, he can either verify v or not verify v' quality, and p_v is the probability of playing v . If s verifies, he then accepts high and rejects low quality. If b meets an informed s , he has to make a non-trivial decision concerning which asset to bring – if he brings low quality he can pass it and consume y_h if s does not verify, but he cannot trade if s does verify. We denote by f the decision of a buyer to bring low quality to a match with an informed seller, f' the negation, and p_f the probability of playing f .

As usual, we start with the case in which λ is given, so that equilibrium is characterized by (p_l, p_f) and $(p_a^{i'}, p_a^i, p_v)$, the buyer's and seller's strategies, respectively.

Proposition 5. *Let $\bar{\lambda} = \frac{y_h - k_l}{y_h - y_l}$, $\bar{p}_l = \frac{y_h - k_g}{y_h - y_l}$, $\underline{p}_l = \frac{k_v}{k_g - y_l}$, $\bar{p}_v = \frac{y_h - y_l}{y_g - y_l}$, $\underline{p}_v = \frac{y_h - k_l}{\lambda(y_g - y_l)}$, $\bar{p}_f = \underline{p}_l / \bar{p}_l$, $\bar{p}_a = \frac{k_l - y_l}{(1 - \lambda)(y_h - y_l)}$, and $\bar{k}_v = \frac{(y_h - k_g)(k_g - y_l)}{y_h - y_l}$. If $k_v < \bar{k}_v$, then:*

- if $\lambda > \bar{\lambda}$, then the unique equilibrium is $p_a^{i*} = p_a^{i'*} = 1$, $p_v^* = \underline{p}_v$, $p_l^* = \underline{p}_l$, and $p_f^* = 1$.
- if $\lambda = \bar{\lambda}$, then $p_a^{i*} = p_a^{i'*} = 1$, $p_v^* = \bar{p}_v$, p_l^* can take on any value in the interval $[\underline{p}_l, \bar{p}_l]$, and $p_f^* = \underline{p}_l / p_l$.
- if $\lambda < \bar{\lambda}$, then the unique equilibrium is $p_a^{i*} = 1$, $p_a^{i'*} = \bar{p}_a$, $p_v^* = \bar{p}_v$, $p_l^* = \bar{p}_l$,

and $p_f^* = \bar{p}_f$.

Alternatively, if $k_v \geq \bar{k}_v$, then $p_l^* = \bar{p}_l$ and $p_f^* = 1$. If $k_v = \bar{k}_v$, then p_v^* can take on any value in the interval $[0, \bar{p}_v]$, while if $k_v > \bar{k}_v$ then $p_v^* = 0$. Finally, $(p_a^{i*}, p_a^{i'*}) \in [0, 1]^2$ satisfy

$$k_l - y_l = \lambda p_a^i [y_h - y_l - p_v^*(y_g - y_l)] + (1 - \lambda) p_a^{i'} (y_h - y_l). \quad (5)$$

First, note that if k_v is small then $p_a^{i*} = 1$ in all equilibrium; so long as it is not too costly to verify, an informed seller will always agree to trade. But he verifies randomly ($p_v^* < 1$); it cannot be an equilibrium for informed sellers to verify with probability 1 since then buyers never bring low quality. Alternatively, if k_v is big then $p_v = 0$, and s never verifies, which means there is no distinction between informed and uninformed sellers. This leads to an indeterminacy in p_a^{i*} and $p_a^{i'*}$; since sellers are all, essentially, uninformed, b only cares about the average probability of acceptance, which is a linear combination of p_a^{i*} and $p_a^{i'*}$. Thus, when $k_v > \bar{k}_v$, notice that $k_l - y_l = [\lambda p_a^i + (1 - \lambda) p_a^{i'}] (y_h - y_l)$. However, the key economic result for our purposes here is that $p_a^{i'*} \rightarrow 0$ as $k_l \rightarrow y_l$ in any equilibrium. If $k_v < \bar{k}_v$, as $k_l \rightarrow y_l$, we have $\bar{\lambda} \rightarrow 1$ and hence the relevant case is where $p_a^{i'*} = \bar{p}_a \rightarrow 0$. And if $k_v \geq \bar{k}_v$, then as $k_l \rightarrow y_l$ we again get $p_a^{i'*} \rightarrow 0$. Therefore, in all equilibria, uninformed sellers stop accepting assets as $k_l \rightarrow y_l$.

We now characterize equilibrium when sellers choose whether or not to become informed, given $k_v < \bar{k}_v$ (this is the interesting case, since $k_v > \bar{k}_v$ implies that $p_v = 0$ and no seller becomes informed). As before, endogenizing information does not change the key result that uninformed sellers stop accepting assets when $k_l \rightarrow y_l$.

Proposition 6. *Suppose $k_v < \bar{k}_v$ and let $\bar{k}_i = y_h - k_g - \frac{k_v(y_h - y_l)}{k_g - y_l}$. There exists an equilibrium, and it is unique.*

- If $\bar{\lambda} > F(\bar{k}_i)$ then $\lambda^* = F(\bar{k}_i)$, $p_a^{i*} = 1$, $p_a^{i'*} = \frac{k_l - y_l}{[1 - F(\bar{k}_i)](y_h - y_l)}$, $p_v^* = \bar{p}_v$, $p_l^* = \bar{p}_l$, and $p_f^* = \bar{p}_f$.
- If $F(0) \leq \bar{\lambda} \leq F(\bar{k}_i)$, then $\lambda^* = \bar{\lambda}$, $p_a^{i*} = p_a^{i'*} = 1$, $p_v^* = \bar{p}_v$, $p_l^* = \frac{F^{-1}(\bar{\lambda}) + p_l}{y_h - y_l}$, and $p_f^* = p_l^* / \underline{p}_l$.
- If $\bar{\lambda} < F(0)$ then $\lambda^* = F(0)$, $p_a^{i*} = p_a^{i'*} = 1$, $p_v^* = \frac{y_h - k_l}{F(0)(y_g - y_l)}$, $p_l^* = \underline{p}_l$, and $p_f^* = 1$.

5.2 Money

Consider Model 2, with exogenous λ , but now suppose in addition to potentially acquiring a low quality asset, b can also acquire at cost k_m a new asset called money. Money is universally recognized and yields utility y_m to all agents (in Lester et al. [16] y_m is determined endogenously as a continuation value). Let m be the buyer's strategy of acquiring money, m' the negation, and p_m the probability of playing m . In addition to the previous assumptions, we have $y_g > k_m > y_m > k_g$ and $y_h = y_m > y_l$. The first set of inequalities implies that b and s want to trade, and that b would not acquire money for consumption alone. In the second assumption, we set $y_h = y_m$ so that b has no reason to acquire money other than its liquidity value.¹⁰

On the one hand, if b plays strategy m' , the game proceeds as before. First, the buyer chooses whether to acquire the low quality asset (strategy l) or not (strategy l'). Then with probability λ (probability $1 - \lambda$) b is matched with an informed (uninformed) s . After observing the seller's type, b chooses an asset to bring to the match; again, he can only bring one. Then, if s is uninformed he chooses whether to accept. On the other hand, if b plays m , the following ensues. First, b chooses

¹⁰Had we set $y_m < y_h$, given the indivisibility of goods and assets a buyer would have incentive to acquire money to receive more favorable terms of trade.

whether or not to acquire low quality, and then is matched with s , whose type he observes. Then b chooses which asset to bring to the match. If s is informed, b and s are indifferent between trading high quality assets or money, given $y_h = y_m$, so we assume they use money. If s is uninformed and b has low quality, b decides whether to bring low quality or money (he clearly never brings high quality).

As in the analysis above, under the assumption $k_l > y_l$, b will never acquire a low quality asset and then not attempt to use it in trade with an uninformed s . Therefore, since $k_m > y_m$, a buyer will never choose to acquire money if he also acquires a low quality asset; if he meets an uninformed agent he will bring his low quality asset, and if he meets an informed agent he can use his high quality asset. In other words, b may acquire a low quality asset or money, but never both. As a result, he has two relevant strategies: p_m is the probability that he acquires money, and p_l is the probability that he acquires the low quality asset conditional on not acquiring money.

Since s always accepts money, his only decision is whether or not to accept an asset he cannot recognize. The payoff from accepting is

$$\pi_a = -k_g + p_l y_l + (1 - p_l) y_h,$$

while, of course, the payoff from rejecting is $\pi_{a'} = 0$. Conditional on strategy m' , it is straightforward to write b 's payoffs from strategies l and l' :

$$\begin{aligned}\pi_l &= -k_l + \lambda(y_g + y_l) + (1 - \lambda)[y_h + p_a y_g + (1 - p_a) y_l] \\ \pi_{l'} &= \lambda y_g + (1 - \lambda)[p_a y_g + (1 - p_a) y_h].\end{aligned}$$

Therefore, b will only acquire a low quality asset if p_a is above some threshold, given by

$$\check{p}_a = \frac{k_l - y_l}{(1 - \lambda)(y_h - y_l)}.$$

Finally, conditional on p_a and p_l , the payoffs to b from acquiring money or not are:

$$\begin{aligned}\pi_m &= -k_m + y_g + y_h \\ \pi_{m'} &= p_l \{-k_l + \lambda(y_l + y_g) + (1 - \lambda)[y_h + p_a y_g + (1 - p_a)y_l]\} \\ &\quad + (1 - p_l) \{\lambda y_g + (1 - \lambda)[p_a y_g + (1 - p_a)y_h]\}.\end{aligned}$$

Given these payoffs, we characterize equilibrium in the proposition below. The proof is again relegated to the appendix.

Proposition 7. *Let $\check{\lambda} = \frac{y_h - k_l}{y_h - y_l}$ and $\check{k}_m = (\check{\lambda} - \lambda)(y_g - y_h)$.*

- *If $\lambda < \check{\lambda}$, then $p_a^* = \check{p}_a$ and $p_l = \check{p}_l$. The value of p_m^* depends on k_m : if $k_m < \check{k}_m$ then $p_m^* = 1$, if $k_m > \check{k}_m$ then $p_m^* = 0$, and if $k_m = \check{k}_m$ then p_m^* can take any value in $[0, 1]$.*
- *If $\lambda = \check{\lambda}$, then $p_a^* = 1$, p_l^* can take any value in the interval $[0, \check{p}_l]$, and $p_m^* = 0$.*
- *If $\lambda > \check{\lambda}$, then $p_a^* = 1$, and $p_l^* = p_m^* = 0$.*

There are several interesting features of this model. First, money is only used if λ is small (the information problem is relatively severe). Thus, if $\lambda > \check{\lambda}$ then all sellers accept assets, which are rarely low quality, and there is no need for money. Alternatively, if $\lambda < \check{\lambda}$ then assets are accepted with probability $p_a^* < 1$ by uninformed sellers, and money can have a role as a medium of exchange despite its inferior return (as long as this return is not too bad, or k_m is not too high). Second, as $k_l \rightarrow y_l$, we have $\check{\lambda} \rightarrow 1$, so we must have $\lambda < \check{\lambda}$ and we must be in the first type of equilibrium. Then as long as k_m is not too large, money is used with probability one, and sellers who cannot recognize assets never accept them.¹¹

¹¹Though we do not include it here, it is easy to show as in the previous sections that these results go through with endogenous information.

5.3 Divisibility

The models analyzed above assume assets and goods are indivisible. This obviously simplifies the analysis because it makes the terms of trade trivial – either the asset is exchanged for the good or it is not. If the asset is divisible, any quantity of it might be traded for the good, and similarly if the good is divisible. In fact, even if goods and assets are indivisible, once we allow agents to use lotteries (as in Berentsen, Molico and Wright [4]) there are many possible trades. In all these cases we need to determine the terms of trade. The central point of the analysis above is that, under plausible conditions, sellers will not accept assets if they cannot distinguish good from worthless ones. We now show that something similar holds with divisible assets and goods; there are again plausible scenarios under which uninformed sellers do not accept assets.

Consider a variant of Model 2, where for simplicity λ is exogenous, but assets and goods are divisible. As before, b is endowed with one unit of the high quality asset, but now it gives utility y_h per unit to both b and s . To facilitate the presentation, we assume here that b is also endowed with one unit of the low quality asset, so we do not have to determine the probability p_l that he acquires it; equivalently, we can assume he can acquire it at cost $k_l = 0$, and consider equilibria where all buyers do acquire it. The low quality asset yields utility y_l per unit consumed. The cost of producing the good is k_g per unit, and the buyer receives utility y_g per unit consumed. Also, assume $y_l < k_g$.¹²

As above, b observes s 's type – informed or uninformed – and then chooses which asset to offer in trade (again he can only bring one type of asset to the market).

¹²The fact that utility is linear is really just a choice of units, but is also convenient because a simple reinterpretation covers the case where the goods and assets are actually indivisible and agents trade using lotteries.

The amount of the asset that the buyer brings to the transaction is observed by the seller. To pin down the terms of trade, we assume that in any match nature selects with equal probability either b or s to make a take-it-or-leave-it offer, where an offer consists of an amount d of the asset that is exchanged for an amount q of the good. If an offer is accepted, s produces and the deal is consummated; otherwise there is no trade.

We allow agents to offer menus of possible exchanges. As is usual in such problems, there is no benefit to b from offering a menu to s since the former knows everything there is to know about the latter, and hence can predict which element of the menu will be accepted. The situation is different for an uninformed s , since he does not know b 's type, which in this case refers to his asset being low or high quality. The seller can predict which element of a menu b will accept conditional on his type, however, so without loss of generality he can offer a menu with at most two possible trades. We will show now that, as $y_l \rightarrow 0$, the quantity of the good traded when s is uninformed becomes arbitrarily small, and hence it is a good approximation to say that he rejects assets outright.

Suppose nature chooses b to make an offer, and let (q, d) denote the terms of trade in equilibrium. The payoff of s is $\pi^s = dy - qk_g$, where y is either y_h or y_l depending on which asset b has. Given s is uninformed, b 's payoffs when he trades low and high quality are:

$$\begin{aligned}\pi_l^b &= y_h + (1 - d)y_l + qy_g \\ \pi_h^b &= y_l + y_h(1 - d) + qy_g.\end{aligned}$$

It is trivial to see $\pi_l^b > \pi_h^b$, and it is a dominant strategy for b to use low quality in the transaction. Hence, $\pi^s = dy_l - qk_g$. As $y_l \rightarrow 0$, it must be that $q \rightarrow 0$ if s is willing to trade at all.

Now suppose that s makes the offer. As discussed above, he offers a menu: (q_l, d_l) and (q_h, d_h) , where the first is (weakly) preferred by b with low quality and the second is (weakly) preferred by b with high quality. The payoff to s from this menu is $\pi^s = d_h y_h - q_h k_g$ or $\pi^s = d_l y_l - q_l k_g$ depending on b 's type. The payoff to b with low quality from the different menu choices are

$$\begin{aligned}\pi_{lh}^b &= y_h + y_l(1 - d_h) + q_h y_g \\ \pi_{ll}^b &= y_h + y_l(1 - d_l) + q_l y_g.\end{aligned}$$

Similarly, if b has high quality,

$$\begin{aligned}\pi_{hl}^b &= y_h(1 - d_l) + q_l y_g \\ \pi_{hh}^b &= y_h(1 - d_h) + q_h y_g.\end{aligned}$$

The incentive compatibility constraints are $\pi_{ll}^b \geq \pi_{lh}^b$ and $\pi_{hh}^b \geq \pi_{hl}^b$.

Under the condition $y_h > y_l$, it is trivial to observe that whichever transaction b accepts, he does strictly better by bringing low rather than high quality. Thus, there is no incentive compatible mechanism that separates buyers with different quality assets, and hence s knows that b will bring low quality. Then s chooses (d, q) to solve the problem

$$\max dy_l - k_g q \text{ s.t. } qy_g \geq dy_l.$$

It is clear that as $y_l \rightarrow 0$, $q \rightarrow 0$. This completes the argument. Note that for $y_l > 0$ it is not the case that assets are literally rejected: when y_l is very small, uninformed sellers do accept assets, but only in exchange for a q that is very small. So it is a good approximation to say that uninformed sellers reject outright assets in this case. It would be a simple extension to show that for this reason b may want to acquire money, for its liquidity, due to its superior recognizability, even if it is dominated in rate of return.

6 Conclusion

It is an old idea that recognizability contributes to liquidity, and this is one reason currency may be useful. We have captured this idea with tractable games that incorporate information frictions. In particular, we have identified environments in which sellers reject outright objects that they do not recognize, or when this is a good approximation. In such environments, information problems can drive liquidity differences. In the companion paper Lester et al. [16], we use models in which this is the case to study monetary policy and related applications. These models point the way to a number of problems for future research.

Appendix

Proof of Proposition 1. Equilibrium is characterized by the intersection of $p_l^*(p_a)$ and $p_a^*(p_l)$. Since $k_g > y_l$ by assumption, we know that $\tilde{p}_l \in (0, 1)$. Likewise, assuming $\alpha > 0$ and $\lambda < 1$, since $k_l > y_l$ we have that $\tilde{p}_a > 0$. Given the definitions of $p_l^*(p_a)$ and $p_a^*(p_l)$, it is clear that if $\tilde{p}_a < 1$ then the unique equilibrium is $p_a^* = \tilde{p}_a$ and $p_l^* = \tilde{p}_l$. If $\tilde{p}_a = 1$, then $p_l^*(p_a)$ and $p_a^*(p_l)$ coincide at $p_a^* = 1$ for all values $p_l^* \in [0, \tilde{p}_l]$. If $\tilde{p}_a > 1$, the unique intersection is at $p_a^* = 1$ and $p_l^* = 0$. Noting that $\tilde{p}_a = 1$ if and only if $\lambda = \tilde{\lambda}$, the remainder of the proof follows immediately. ■

Proof of Proposition 2. Existence is immediate, since Λ is non-empty valued, convex valued, and upper hemi-continuous. Uniqueness is assured by the monotonicity of Λ . The three different cases are then easily illustrated in figure ???. The solid line corresponds to the case in which $\tilde{\lambda} > F(\tilde{k}_i)$, so that the fixed point occurs at $\lambda^* = F(\tilde{k}_i)$. The dashed line corresponds to $F(0) \leq \tilde{\lambda} \leq F(\tilde{k}_i)$, so that $\lambda^* = \tilde{\lambda}$. In this equilibrium, we pin down p_l^* by noting that the marginal seller must be indifferent between strategies i and i' , which is only true if $k_i^s = F^{-1}(\tilde{\lambda}) = p_l \alpha (k_g - y_l)$. Finally, the dotted line corresponds to the case of $F(0) > \tilde{\lambda}$, so that $\lambda^* = F(0)$. ■

Proof of Proposition 5. We first derive the payoffs and best responses at each decision node. The payoffs to an informed seller from strategies v and v' are

$$\begin{aligned}\pi_v &= -k_v + (1 - p_f p_l)(y_h - k_g) \\ \pi_{v'} &= -k_g + p_f p_l y_l + (1 - p_f p_l) y_h.\end{aligned}$$

The best response is $p_v = 1$ if $p_f p_l > \underline{p}_l$, $p_v = 0$ if $p_f p_l < \underline{p}_l$, and $p_v \in [0, 1]$ if

$p_f p_l = \underline{p}_l$. The payoffs to a buyer from strategies f and f' are:

$$\begin{aligned}\pi_f &= -k_l + y_h + p_v y_l + (1 - p_v) y_g \\ \pi_{f'} &= -k_l + y_l + y_g.\end{aligned}$$

The best response is $p_f = 1$ if $p_v > \bar{p}_v$, $p_f = 0$ if $p_v < \bar{p}_v$, and $p_f \in [0, 1]$ if $p_v = \bar{p}_v$.

The payoffs to an informed seller from strategies a_i and a'_i are:

$$\begin{aligned}\pi_a^i &= -p_v k_v + p_l p_f (1 - p_v) (y_l - k_g) + (1 - p_l p_f) (y_h - k_v) \\ \pi_{a'}^i &= 0.\end{aligned}$$

Let

$$\bar{p}_l = \frac{y_h - k_g - p_v k_v}{y_h - y_l - p_v (k_g - y_l)}.$$

The best response is $p_a^i = 1$ if $p_f p_l < \bar{p}_l$, $p_a^i = 0$ if $p_f p_l > \bar{p}_l$, and $p_a^i \in [0, 1]$ if $p_f p_l = \bar{p}_l$. Alternatively, the payoffs to an uninformed seller from strategies $a_{i'}$ and $a'_{i'}$ are

$$\begin{aligned}\pi_a^{i'} &= -k_g + p_l + (1 - p_l) y_h \\ \pi_{a'}^{i'} &= 0.\end{aligned}$$

The best response is $p_a^{i'} = 1$ if $p_l > \bar{p}_l$, $p_a^{i'} = 0$ if $p_l < \bar{p}_l$, and $p_a^{i'} \in [0, 1]$ if $p_l = \bar{p}_l$.

Finally, the payoffs from strategies l and l' are:

$$\begin{aligned}\pi_l &= -k_l + (1 - \lambda) \left[p_a^{i'} (y_g + y_h) + (1 - p_a^{i'}) (y_h + y_l) \right] \\ &\quad + \lambda \left\{ p_a^i [p_f [y_h + p_v y_l + (1 - p_v) y_g] + (1 - p_f) (y_l + y_g)] + (1 - p_a^i) (y_h + y_l) \right\} \\ \pi_{l'} &= (1 - \lambda) \left[p_a^{i'} y_g (1 - p_a^{i'}) y_h \right] + \lambda [p_a^i y_g + (1 - p_a^i) y_h].\end{aligned}$$

Note that $k_v < \bar{k}_v \Leftrightarrow \underline{p}_l > \bar{p}_l \geq \bar{\bar{p}}_l$, with $\bar{p}_l > \bar{\bar{p}}_l$ if $p_v > 0$. Alternatively, $k_v \geq \bar{k}_v \Leftrightarrow \underline{p}_l < \bar{p}_l \leq \bar{\bar{p}}_l$, with $\bar{p}_l < \bar{\bar{p}}_l$ if $p_v > 0$. Finally, $k_v = \bar{k}_v \Leftrightarrow \underline{p}_l = \bar{p}_l = \bar{\bar{p}}_l$.

We consider each of these three cases.

First, suppose $k_v < \bar{k}_v$. We claim that in any equilibrium $p_f p_l = \underline{p}_l$. If $p_f p_l < \underline{p}_l < \bar{p}_l \leq \bar{p}_l$, then $p_a^i = p_a^{i'} = 1$ and $p_v = 0$. But then the best response of the buyer is $p_l = p_f = 1$, a contradiction. Alternatively, if $p_f p_l > \underline{p}_l$, then in any equilibrium it must be that $p_a^i > 0$ and $p_v = 1$. But if $p_v = 1$, then $p_f = 0$, a contradiction. So it must be that $p_f p_l = \underline{p}_l$, and thus $p_l \in [\underline{p}_l, 1]$.

Suppose that $p_l = \underline{p}_l < \bar{p}_l \leq \bar{p}_l$, so that $p_f = 1$ and $p_a^i = p_a^{i'} = 1$ are best responses. Then, since $p_l \in (0, 1)$, p_v must be such that $\pi_l = \pi_{l'}$. Solving yields $p_v = \underline{p}_v$. It remains to check whether $p_f = 1$ is a best response. This is true when $\pi_f \geq \pi_{f'}$, or equivalently when $\lambda \geq \bar{\lambda}$.

Now suppose $p_l \in (\underline{p}_l, \bar{p}_l)$, $p_f = \underline{p}_l/p_l$, and $p_a^i = p_a^{i'} = 1$. Since $p_f \in (0, 1)$, it must be that $p_v = \bar{p}_v$. Again, we know from the results above that p_v , p_f , p_a^i and $p_a^{i'}$ are best responses, and it remains to determine when p_l is optimal. Given these strategies, $\pi_l = \pi_{l'}$ if and only if $\lambda = \bar{\lambda}$.

Next, if $p_l = \bar{p}_l$, then we must have $p_f = \underline{p}_l/\bar{p}_l$ and $p_a^i = 1$. Again, $p_f \in (0, 1)$ requires that $p_v = \bar{p}_v$. We know that p_v , p_a^i , and p_f are best responses, and that any $p_a^{i'} \in [0, 1]$ is a best response. It remains to determine the value of $p_a^{i'}$ such that $\pi_l = \pi_{l'}$. Solving yields $p_a^{i'} = \bar{p}_a$. Note that $\bar{p}_a \geq 0$ for all $k_l \geq y_l$ but that $p_a \leq 1$ requires $\lambda \leq \bar{\lambda}$.

Finally, suppose $p_l \in (\bar{p}_l, 1]$, with $p_f = \underline{p}_l/p_l$, $p_a^{i'} = 0$, $p_a^i = 1$, and $p_v = \bar{p}_v$. We know from the results above that p_f , $p_a^{i'}$, p_a^i , and p_v are best responses, so what remains to check is whether p_l is a best response. This requires $\pi_l \geq \pi_{l'}$, which requires $-k_l + y_l \geq 0$, which is not true under our parametric assumptions. Therefore, an equilibria with $p_l > \bar{p}_l$ does not exist. This completes the characterization of equilibria when $\underline{p}_l < \bar{p}_l$.

Suppose instead that $k_v \geq \bar{k}_v$, so that $\underline{p}_l \geq \bar{p}_l \geq \bar{p}_l$. We first establish that it

must be that $p_l = \bar{p}_l$. If $p_l < \bar{p}_l$ then $p_a^{i'} = 1$. Also, $p_f p_l \leq p_l < \bar{p}_l \leq \underline{p}_l$ implies that $p_v = 0$. This implies that $p_f = 1$ and $\bar{p}_l = \bar{\bar{p}}_l$, so that $p_a^i = 1$. However, if $p_a^{i'} = p_a^i = 1$ and $p_v = 0$, then any $p_l < 1$ cannot be consistent with equilibrium behavior. Similarly, if $p_l > \bar{p}_l$, then $p_a^{i'} = 0$, so it must be that $p_a^i > 0$, $p_v < 1$, and $p_f > 0$. For $p_a^i > 0$ to be optimal, it must be that $p_f p_l \leq \bar{\bar{p}}_l$, which implies that $p_f < 1$, so that we must have $p_v = \bar{p}_v$. Again, plugging this in reveals $\pi_l < \pi_{l'}$, a contradiction.

Therefore, it must be that $p_l = \bar{p}_l$. More specifically, we must have that $p_f = 1$, because if $p_f < 1$ then $p_v = 0$, which would imply $p_f = 1$ is a best response. Therefore, we have $p_l = \bar{p}_l$ and $p_f = 1$. If $\bar{p}_l = \underline{p}_l = \bar{\bar{p}}_l$, then any values for $(p_a^{i'}, p_a^i, p_v) \in [0, 1]^3$ constitute a best response for the seller, and the only conditions that must be satisfied are that $p_v \leq \bar{p}_v$ and $\pi_l = \pi_{l'}$. Alternatively if $\underline{p}_l > \bar{p}_l$, then $p_f p_l = \bar{p}_l < \underline{p}_l$, so $p_v = 0$ and $\bar{p}_l = \bar{\bar{p}}_l$. Therefore, any values $(p_a^{i'}, p_a^i)$ that satisfy $\pi_l = \pi_{l'}$ will be consistent with equilibrium behavior for b and s . ■

Proof of Proposition 7. First, note that there cannot be an equilibrium with $p_a > \check{p}_a$. If there were, the buyer would choose strategy $p_l = 1$ and this implies $\pi_a < \pi_{a'}$. A strategy $p_a > \check{p}_a > 0$ cannot be consistent with $\pi_a < \pi_{a'}$. Therefore, it must be that $p_a \leq \check{p}_a$. There are three relevant cases: \check{p}_a can be strictly less than, equal to, or strictly greater than 1. We consider each case below.

First suppose that $\check{p}_a < 1$, or equivalently that $\lambda < (y_h - k_l)/(y_h - y_l) \equiv \check{\lambda}$. If $p_a < \check{p}_a < 1$, then the buyer's optimal strategy must be $p_l = 0$, which implies $\pi_a = y_h > 0 = \pi_{a'}$, a contradiction. So it must be that $p_a = \check{p}_a$. Also, since $\check{p}_a \in (0, 1)$, p_l must be such that $\pi_a = \pi_{a'}$. This implies that $p_l = (y_h - k_g)/(y_h - y_l) \equiv \tilde{p}_l$. Given

these values of p_a and p_l , we have that

$$\begin{aligned}\pi_m &= -k_m + y_g + y_h \\ \pi_{m'} &= y_h + (y_g - y_h) \left[\lambda + \frac{k_l - y_l}{y_h - y_l} \right].\end{aligned}$$

It follows that $\pi_m \geq \pi_{m'}$ if and only if $k_m \leq \check{k}_m$.

Now suppose that $\check{p}_a = 1$, or equivalently that $\lambda = \check{\lambda}$. Again, if $p_a < \check{p}_a = 1$, then the buyer's optimal strategy must be $p_l = 0$, which implies $\pi_a = y_h > 0 = \pi_{a'}$, a contradiction. So it must be that $p_a = \check{p}_a = 1$, and thus $\pi_a \geq \pi_{a'}$, which is true if and only if $p_l \leq \tilde{p}_l$. Given these values of p_a and p_l , we have that

$$\begin{aligned}\pi_m &= -k_m + y_g + y_h \\ \pi_{m'} &= y_g.\end{aligned}$$

Since $y_h = y_m$ and $y_m - k_m < 0$, it follows that $\pi_{m'} > \pi_m$ and $p_m = 0$.

Last, suppose that $\check{p}_a > 1$, or equivalently that $\lambda > \check{\lambda}$. Then for any $p_a \in [0, 1]$, $p_a < \check{p}_a$, so that $p_l = 0$. Naturally, then, it must be that $p_a = 1$. Again, given these values of p_a and p_l , we have that

$$\begin{aligned}\pi_m &= -k_m + y_g + y_h \\ \pi_{m'} &= y_g,\end{aligned}$$

and so it must be again that $p_m = 0$. ■

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Figure 1: $\Lambda : \lambda \mapsto \lambda$

