

# On the Multiplicity of Monetary Equilibria: Green-Zhou Meets Lagos-Wright\*

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## Abstract

Green and Zhou relax the assumption, made in early search-based models of monetary exchange, of indivisible money. Their paper and various extensions make much technical progress, and derive some interesting substantive results. In particular, they show there is an indeterminacy of steady-state monetary equilibria. We reconsider this result in the framework of Lagos and Wright, which is more tractable. We show that a similar multiplicity arises, and is much easier to derive and understand. We also compare the results to those in related nonmonetary models, and discuss how they depend on details, including the number of agents and the timing.

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# 1 Introduction

Green and Zhou (1998) make a contribution to monetary economics by showing how to relax the assumption of indivisible money in search-based models along the lines of Kiyotaki and Wright (1993), Shi (1995), and Trejos and Wright (1995).<sup>1</sup> In their paper and its various extensions, including Zhou (1999, 2003), Green and Zhou (2002), Kamiya and Sato (2004), Kamiya, Morishita and Shimizu (2005), and Kamiya and Shimizu (2006, 2007*a*, 2007*b*), significant advances are made along two fronts. First, these papers make progress on deriving analytic results in the Green-Zhou framework, which is a technical contribution because search-based models with divisible money are not especially tractable, mainly because one has to keep track of a nondegenerate distribution of money holdings as an endogenous state variable. Second, this work highlights some interesting substantive economic results, including an indeterminacy of steady-state, single-price, monetary equilibria – they show that there is a continuum of such equilibria.

In this note, we reconsider the substantive economic results in Green and Zhou in the framework of Lagos and Wright (2005), which is much more tractable than earlier search-based monetary models because, in the baseline version, the equilibrium distribution of money across agents of a given type is degenerate, at least as long as we have quasi-linear utility, or make some related alternative assumption, as discussed in Rocheteau et al. (2008). Here we use price posting rather than bargaining, and assume certain goods are indivisible, consistent with at least some versions of the Green-Zhou model, but otherwise we stay close to the Lagos-Wright framework. Since it is easy to analyze, one does not need all the machinery developed for the Green-Zhou model to derive similar results. In particular, the same multiplicity of steady-state, single-price, monetary equilibria arises, and the results are much simpler to demonstrate and to understand.

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<sup>1</sup>It is obviously desirable to allow divisible money because, among other reasons, it is otherwise hard to do much serious quantitative or policy analysis. Divisible money complicates the analysis considerably, however, since it means one has to track the distribution of cash balances across agents. Other approaches, different from Green and Zhou, to allowing divisible money in these types of models include Shi (1997) or Lagos and Wright (2005), who make assumptions on the environment to render the distribution of money degenerate, and Molico (2006) or Dressler (2008), who attack the problem with a nondegenerate distribution numerically.

To preview the intuition, note that if sellers think all buyers will carry exactly  $m$  dollars, over some range of parameters, their best response is to charge  $p = m$ ; and if buyers think all sellers will charge exactly  $p$ , over some range of parameters, their best response is to carry  $m = p$ . Hence, as in many coordination games, multiple values for  $p = m$  may be consistent with equilibrium. This logic is simplistic, however, for several reasons. First, it might make it appear the indeterminacy is nominal when in fact it can be real. That is, given the nominal money supply there is an interval of different price levels, and hence different real balances, consistent with equilibrium, and these can have different welfare implications. Also, one has to work out the exact bounds on the interval, since if  $m = p$  is too low or too high then sellers or buyers will not participate in the market. Still, the above intuition is basically valid, and one can think of the multiplicity here as coming from coordination.

Although it may seem obvious ex post, we think our results are useful because they illustrate certain aspects of what drives the Green-Zhou multiplicity. Additionally, our results provide insights into a class of nonmonetary price-posting models emanating from the work of Diamond (1970), as we discuss below. We also discuss how to break the multiplicity by changing the timing. Suppose that, rather than having sellers post  $p$  and buyers choose  $m$  simultaneously we let one side move first, and suppose also that, rather than assuming a continuum we have a finite number of agents. In this case we show that there is a unique equilibrium. Interestingly, this trick does not work in the basic Green-Zhou framework. Hence, we do not claim our model captures all of the features of that framework, only that it can capture some interesting features in a very tractable way.

## 2 The Model

There is a  $[0, 1]$  continuum of agents who live forever in discrete time with discount factor  $\beta \in (0, 1)$ . Following Lagos-Wright, each period is divided into two subperiods. In the second subperiod all agents consume a good  $X$  and supply labor  $H$  in a centralized market, CM, with competitive Walrasian pricing. In the first subperiod they trade specialized goods  $x$  and labor  $h$  in a decentralized market, DM, with random bilateral matching. In a random match between agents  $i$  and  $j$ , the probability  $i$  wants to

consume a specialized good  $j$  can produce but not vice-versa, a so-called single-coincidence meeting, is  $\sigma \in (0, \frac{1}{2}]$ , and the probability  $i$  wants to consume a good  $j$  can produce and vice-versa, a so-called double-coincidence meeting, is  $\delta = 0$  (it is a simple generalization to allow  $\delta > 0$  without changing the main results). When  $i$  wants to consume what  $j$  produces, we call the former a buyer and the latter a seller in that meeting.<sup>2</sup>

We assume with no loss in generality that  $H$  produces  $X$  and  $h$  produces  $x$  one-for-one (it is a simple generalization to allow general concave technologies). Trade in the DM requires money  $m$  because of well-known reasons: by assumption, consumption goods are nonstorable, which rules out barter; and agents are anonymous, which rules out credit. See Kocherlakota (1997), Wallace (2001), Corbae et al. (2003), Araujo (2004), and Aliprantis et al. (2006) for formal discussions of these ideas, especially the role of anonymity. This specification of the DM corresponds to what one sees in almost all recent work on the foundations of monetary theory, including the papers mentioned in the introduction, although the earlier work restricted individual money holdings to  $m \in \{0, 1\}$  while here we allow, as in most modern models,  $m \in [0, \infty)$ .

We make two additional assumptions about the DM, consistent with at least some versions of the Green-Zhou framework, but different from Lagos-Wright: we assume the decentralized market good  $x$  is indivisible; and we assume the terms of trade are determined by price posting, as opposed to bargaining. Thus, each agent sets a  $p$  such that in any meeting where a buyer likes his specialized good, he commits to producing a unit of it as long as the buyer hands over  $p$  dollars. One interpretation, discussed in Curtis and Wright (2004) in a different context, is that agents stock vending machines with a unit of  $x$  and program it to accept  $p$  dollars, then agents sample each other's machines.<sup>3</sup> Given these assumptions, if we shut down the CM our model collapses to Green-Zhou. But as in Lagos-Wright, the CM can simplify the analysis dramatically, mainly because it allows agents to rebalance their money

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<sup>2</sup>Later we consider a version where some agents are always buyers in the DM and others are always sellers in the DM.

<sup>3</sup>This story may or may not be helpful in explaining how sellers can commit to  $p$  and not succumb to bargaining. Actually, to fit the equations in the baseline model more accurately, one should assume agents do not stock the vending machines in advance but program them with the ability to produce the good on demand; one can amend the model so they do stock the machines in advance, however, and it would not change the basic results much (see footnote 6 below).

holdings after each round of DM trade.

Let  $u$  and  $-c$  be the utilities of consuming and producing an indivisible specialized good  $x$ , conditional on it being one the buyer consumes and the seller produces, in the DM. Let  $U(X) - H$  be the utility of consuming  $X$  and working  $H$  in the CM, which are perfectly divisible, where  $U(X)$  satisfies standard assumptions. In general, an equilibrium is described by, among other things, a distribution of prices  $F(p)$  and a distribution of money holdings  $G(m)$  across agents in the DM.<sup>4</sup> Let  $V(m, p)$  be the expected payoff of an agent entering the DM with  $m$  dollars and a posted price of  $p$ , and  $W(m)$  the expected payoff of an agent entering the CM with  $m$ . Let  $\phi$  be the CM price of  $m$  in terms of  $X$ . Then

$$\begin{aligned} W(m) &= \max_{X, H, m_+, p_+} \{U(X) - H + \beta V_+(m_+, p_+)\} \\ \text{st } X &= H + \phi(m + T - m_+), \end{aligned}$$

where in general a subscript  $+$  indicates next period, so  $m_+$  is the money with which the agent exits the CM and enters the next DM. Also, as is standard,  $T$  is a lump sum monetary transfer (or tax if negative) consistent with increases (or decreases) in the aggregate money supply described by  $M_+ = (1 + \pi)M$ , with  $\pi > \beta - 1$ . In steady state  $\phi/\phi_+ = 1 + \pi$  gives the inflation rate.<sup>5</sup>

Substituting  $H$  from the budget constraint and taking the FOC for  $m_+$ , we get  $\phi = \beta \partial V_+ / \partial m_+$  (see Lagos-Wright for a discussion of conditions to guarantee an interior solution and the differentiability and concavity of  $V$  with respect to  $m_+$ ). Thus, the solution to the problem of choosing  $m_+$  is independent of the  $m$  brought to the CM. This does not of course mean that all agents choose the same  $m_+$  since there could be multiple solutions to this problem. Indeed, in a related but different model – different because, first, they allow multilateral meetings, and second, they use auctions instead of price posting – Galenianos and Kircher (2007) and Dutu, Julien and King (2007) show there is a continuum of solutions to their analogous problem, and equilibrium implies a nondegenerate distribution  $G(m)$ . Here we look

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<sup>4</sup>More generally, one should specify the joint distribution of  $m$  and  $p$ , although it is natural to think they would be independent. We ignore this because we are interested in this paper in equilibria with degenerate distributions. But as discussed below, it might also be interesting to consider other types of equilibria.

<sup>5</sup>There is no equilibrium with  $\pi < \beta - 1$ . We can consider the limit as  $\pi \rightarrow \beta - 1$ , which is the Friedman rule. Defining the nominal interest rate  $i$  by the Fisher equation  $1 + i = (1 + \pi)/\beta$ , the Friedman rule means  $i = 0$ .

for equilibria with degenerate distributions. Also, notice that  $W$  is linear with  $\partial W/\partial m = \phi$ .

In the DM, if a buyer meets a seller with a random price  $p$ , he has to decide whether to make a purchase. Let  $R(m)$  be his reservation price, above which he will not buy, given his money balances  $m$ . It is obvious and simple to show that  $m = R$  in equilibrium: why take more money than you will ever spend, given it is costly when  $\pi > \beta - 1$ ? Then

$$V(m, p) = (1 - 2\sigma)W(m) + \sigma \left\{ \int_0^m [u + W(m - \tilde{p})] dF(\tilde{p}) + [1 - F(m)] W(m) \right\} \\ + \sigma \{ [1 - G(p)] [-c + W(m + p)] + [1 - \sigma + \sigma G(p)] W(m) \}.$$

The first term is the payoff to not meeting anyone, the second is the payoff in a meeting where the agent might want to buy, and the third the payoff in a meeting where he sells.

Using the linearity of  $W$  we have

$$V(m, p) = \sigma \int_0^m (u - \phi \tilde{p}) dF(\tilde{p}) + \sigma [1 - G(p)] (\phi p - c) + W(m).$$

Assuming, for the moment, for the sake of illustration, that  $F$  is differentiable, notice  $\partial V/\partial m = \sigma(u - \phi m)F'(m) + \phi$ . Updating this one period and inserting into the FOC  $\phi = \beta \partial V_+/\partial m_+$ , we have

$$\phi = \beta \phi_+ + \beta \sigma (u - \phi_+ m) F'(m).$$

The second term on the right is a *liquidity premium* on cash, which means agents may carry  $m > 0$  despite the fact that it has bad rate of return given  $\pi > \beta - 1$  (this assumes  $m$  is valued in trade; there is of course always a nonmonetary equilibrium where it is not valued and no DM trade occurs).

As in most of the literature on the Green-Zhou model, we focus on single-price equilibria, where  $F$  is degenerate. We consider only equilibria where  $G$  is also degenerate, which makes sense in the Lagos-Wright framework. Now it should be obvious that if  $F$  were degenerate at  $p^*$ , all agents should bring either  $m = 0$  or  $m = p^*$  to the DM: given it is costly to carry money, when  $\pi > \beta - 1$ , there is no incentive to bring  $m > p^*$  or  $m \in (0, p^*)$ . It should also be obvious that if everyone brings  $m^*$ , posting  $p < m^*$  would not be a best response, and posting  $p > m^*$  results in no trade. Therefore, we look for

equilibria with  $p^* = m^*$ . What needs to be established is: when do agents prefer to post  $p = m^*$ , as opposed to not selling at all; and when do they prefer bringing  $m = p^*$ , as opposed to bringing  $m = 0$  and not buying at all?

At this point we invoke market clearing and set  $m = M$  at every date. Given a path for  $M$  induced by an exogenous value for  $\pi$ , a steady-state monetary equilibrium is a path for  $\phi$  such that real balances  $\phi M$  are constant. A sale at  $p_+ = M_+$  in the next DM covers the cost of production iff  $c \leq W_+(M_+) - W_+(0) = M_+ \phi_+$ . Also, conditional on having  $m_+ = M_+$ , in the DM, agents prefer buying at  $p_+ = M_+$  to not trading iff  $u \geq W_+(M_+) - W_+(0) = \phi_+ M_+$ . Hence, we need  $\phi_+ M_+ \in [c, u]$  for trade to be profitable for sellers and buyers ex post. We actually need something more for buyers: we need to check that  $m_+ = M_+ > 0$  makes them better off than  $m_+ = 0$  ex ante, when they choose  $m_+$  in the CM. So we must check  $\beta V_+(M_+, p_+) - \phi M_+ \geq \beta V_+(0, p_+)$ , which reduces to  $\sigma u \geq M_+ [\phi/\beta - (1 - \sigma)\phi_+]$ . Letting the real interest rate  $r$  be given by  $1 + r = 1/\beta$  and the nominal rate  $i$  by  $i = (1 + \pi)(1 + r) - 1$ , this reduces to  $M_+ \phi_+ \leq \sigma u / (i + \sigma)$ , which provides an upper bound on  $\phi_+ M_+$  less than the one for ex post trade as long as  $i > 0$ .

What we have shown is that any value of real balances in the interval  $\left[ c, \frac{\sigma}{i + \sigma} u \right]$  is consistent with all the equilibrium conditions. As a special case, with  $\pi = 0$  and  $M$  constant, the interval in question becomes  $\left[ c, \frac{\sigma}{r + \sigma} u \right]$ ; as another, at the Friedman rule  $\pi = \beta - 1$  it becomes  $[c, u]$ . In general, notice that higher  $\pi$  or  $i$  reduces the size of the interval, and for big enough  $i$  the interval and hence the monetary equilibrium collapses. This indeterminacy is essentially the same as that discussed in the Green-Zhou framework, even if our model is much simpler. The main reason our model is simpler is that in the Green-Zhou framework one has to worry about the endogenous distribution  $G(m)$ , even when  $F(p)$  is degenerate, while here  $G(m)$  is also degenerate.

**Proposition 1** *For  $i < \sigma(u - c)/c$  there exists a continuum of steady-state, single-price monetary equilibria where real balances  $\phi M$  can be anything in the interval  $\left[ c, \frac{\sigma}{i + \sigma} u \right]$ ; for  $i > \sigma(u - c)/c$  there is no such equilibrium.*

### 3 Discussion

In this section we do several things. For one, we present an alternative version of the above model, along the lines of Rocheteau and Wright (2005), to develop some further intuition and connect to other literatures. For another, we show how the model can be reformulated in terms of real balances, which is useful for several reasons. We also discuss some of the critical assumptions by demonstrating how a modification of the setup yields a unique equilibrium. And we address a point raised by an associate editor concerning the nature of our results.

To start with the last point, one might think that Green-Zhou indeterminacy has something to do with a discontinuity in the value of money, because in any single-price equilibrium, with  $p = p^*$ , having  $m = kp^*$  for any integer  $k$  is a lot better than having  $m = kp^* - \varepsilon$  even if  $\varepsilon$  is small. We think this is a correct interpretation of the baseline Green-Zhou economy, where in equilibrium all revenue accumulates in multiples of  $p^*$ , so that after any history of trade starting with  $kp^* - \varepsilon$  an agent cannot buy more than he could if he started with  $(k - 1)p^*$ , and so the extra  $p^* - \varepsilon$  is worth nothing. Thus, the value function is a step function with steps at integer multiples of  $p^*$ . This is far less obvious in our model since agents can top off or spend down money balances in the CM, where the value function  $W$  is not only continuous but linear in  $m$ .

Nonetheless, in our model the DM value function  $V$  is discontinuous in  $m$ , in any equilibrium with  $p^* < \sigma u / (i + \sigma)$ , since  $m = p^*$  allows one to make a DM purchase while  $m = p^* - \varepsilon$  does not, and the instantaneous utility  $u$  from a purchase strictly exceeds the continuation value of carrying  $m$  to the next CM given the opportunity cost of acquiring the money has already been paid. Therefore, as long as  $p^* < \sigma u / (i + \sigma)$ , buyers are discretely better off with  $m = p^*$  than they are with  $m = p^* - \varepsilon$ . This contributes to the strong coordination effect in our model: when all sellers are charging  $p^*$  it is clear that agents want to carry  $p^*$  to the DM here, even though  $p^* - \varepsilon$  is not worth nothing, as it was in Green-Zhou, because in our setup it can be used in the next CM. Because of this observation, we agree that a discontinuity in  $V$  is at the heart of the indeterminacy result in our model, too.

To consider now changes in the basic assumptions, suppose that as in Rocheteau and Wright (2005) agents come in two *permanently* different types, called buyers and sellers, distinguished as follows: in the DM buyers can consume good  $x$  but cannot supply  $h$ , while sellers can supply  $h$  but do not consume  $x$ . We can normalize the measure of buyers to 1 and let the measure of sellers be  $n$ . In all other respects the environment is the same. If we change the setup in this way, by having permanent buyers and sellers, in the basic Green-Zhou framework the market would shut down, because sellers would have no incentive to produce for  $m$  in one DM if they never get to be buyers in a future DM. This is not the case here, however, since sellers can always spend their money in the CM. It should also be clear that sellers in this version of the model never carry  $m$  into the DM, but simply set  $p$ , while buyers choose only  $m$  and have no  $p$  to set.

Let  $V^b(m)$  and  $V^s(p)$  be the now type-dependent DM payoffs, and similarly for  $W^b(m)$  and  $W^s(m)$ . Then the CM problems look like the previous model, except buyers do not set  $p_+$  and sellers choose  $m_+^s = 0$ , while in the DM

$$\begin{aligned} V^b(m) &= \sigma_b \int_0^m (u - \phi \tilde{p}) dF(\tilde{p}) + W^b(m) \\ V^s(p) &= \sigma_s [1 - G(p)] (\phi p - c) + W^s(0), \end{aligned}$$

where  $\sigma_j$  is the probability of a single-coincidence DM meeting for  $j$ , satisfying the identity  $\sigma_b = n\sigma_s$ . As in the previous model, while there could potentially be other equilibria, consider single-price, steady-state monetary equilibria. Again, if  $F$  is degenerate at  $p^*$  all buyers should bring  $m_+ = 0$  or  $m_+ = p^*$  to the DM, and if  $G$  is degenerate at  $m^*$  all sellers will post  $p = m^*$  if they want to trade at all. In fact, Proposition 1 goes through as stated except  $\sigma_b$  replaces  $\sigma$ .<sup>6</sup>

One reason this alternative model is interesting is that it looks like a monetary version of the literature on price posting in nonmonetary economies emanating from Diamond (1970). In Diamond, sellers post prices and buyers search, as in our model, but the results are very different: there is a unique

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<sup>6</sup>It appears sellers have only a static trade off, since they produce only after meeting a buyer; it is however easy to alternatively assume they produce in the CM, in which case we need  $\beta\sigma_s\phi M \geq c$  instead of  $\phi M \geq c$ , which is basically a renormalization. The difference between sellers and buyers is the former do not make an ex ante investment in cash.

equilibrium; in equilibrium all sellers post the same  $p^*$ ; and  $p^*$  extracts all the surplus from buyers. The analog here is  $\phi M = u$ , since this means sellers get the entire surplus. In our economy,  $\phi M = u$  is *not* an equilibrium, since buyers have to make a costly (as long as  $i > 0$ ) ex ante choice to bring  $m$  to the market. Also, there are many other single-price equilibria. The usual Diamond-style logic does not apply: in his model, if all sellers charge the same price  $p$  buyers would in principle be willing in a given meeting to pay  $p + \varepsilon$  for  $\varepsilon > 0$  rather than continue searching, so sellers will want to shade up their price, as long as in real terms it is less than  $u$ . Here buyers are *willing* in a meeting to pay  $p + \varepsilon$  but *not able* since they only have  $m = p$ .

Also, as pointed out before, we want to emphasize that there is a real and not just nominal indeterminacy here. This may not be immediately apparent when prices are quoted in nominal terms, since all equilibria have  $p = M$ , and differ only in  $\phi$ . To illustrate the results more clearly, and to aid in additional discussion, we now assume that prices are quoted in terms of real balances. In the CM, sellers post a quantity  $z$  of real balances required to buy their good. Note that the aggregate quantity of real balances, which is equal to  $\phi M$ , is not fixed; it is determined endogenously by agents' choices in the CM. Reinterpreting  $p$  from now on as a real price, we have

$$\begin{aligned} V^b(z) &= \sigma_b \int_0^z (u - \tilde{p}) dF(\tilde{p}) + W^b(z) \\ V^s(p) &= \sigma_s [1 - G(p)] (p - c) + W^s(0). \end{aligned}$$

Consider equilibria where both  $G$  and  $F$  are degenerate. Again, it should be obvious that if  $F$  is degenerate at  $p^*$ , all agents should bring either  $z = 0$  or  $z = p^*$  to the DM; and if everyone brings  $z^*$ , all sellers who want to participate should post  $p = z$ . What needs to be established is: when do the sellers prefer posting  $p = z^*$  as opposed to not selling at all; and when do the buyers prefer bringing  $z = p^*$  as opposed to bringing 0? The relevant condition for sellers is  $p \geq c$ . The relevant condition for buyers, after some algebra, is  $p \leq \sigma_b u / (i + \sigma_b)$ , exactly as before given we now interpret  $p$  as a real price. Any  $p \in \left[ c, \frac{\sigma_b u}{i + \sigma_b} \right]$  is an equilibrium. It is obvious that these different equilibria have different real

implications: the higher is  $p$  the greater is sellers' real surplus in any trade and hence greater is their equilibrium payoff; and vice-versa for buyers.

Finally, we discuss what happens when we alter two assumptions: (1) we let the sets of buyers and sellers be finite; and (2) we change the timing. Consider then the following game between sellers and buyers in the CM. Let sellers move first and choose  $p_+$ . Then buyers observe the price and choose real balances  $z_+$  and other CM variables. The payoffs for buyers and sellers are  $U(X) - H + \beta V_+^b(z_+)$  and  $U(X) - H + \beta V_+^s(p_+)$ . Assume first that there is a single buyer and a single seller. Then, clearly, there is a unique subgame perfect equilibrium and  $p^* = z^* = \sigma_b u / (i + \sigma_b)$ . We have eliminated the coordination issue in the baseline model, with a continuum of agents. In that model, if all sellers start at  $p^*$ , and one deviates to  $p' \neq p^*$ , the rest stay at  $p^*$  because all buyers continue to choose  $z = p'$ .

Thus, with one seller, there is a unique equilibrium given the timing specified above. Also, closer in spirit to the Diamond result, the seller here gets the maximum surplus, except that this takes into account buyers' ex ante costly choice of  $m$  - i.e. his price is  $p^* = \sigma_b u / (i + \sigma_b)$  and not  $u$ . Suppose now there is a finite number  $N$  of sellers. Suppose  $N - 1$  of the sellers post  $p' < p^*$  and one seller posts  $p' + \varepsilon$ . Buyers clearly choose either  $z = p'$  or  $z = p' + \varepsilon$ , and prefer the latter if

$$\varepsilon(1 + \pi) \leq \beta \left[ \sigma_b \frac{u - p'}{N} + \left(1 - \frac{\sigma_b}{N}\right) \varepsilon \right]$$

The cost of carrying the additional amount  $\varepsilon$  of real balances is  $\varepsilon(1 + \pi)$ , since they depreciate at the rate of inflation. The benefit is the increase in the probability of trade (hence the first term on the right-hand side) plus, if meeting a seller posting  $p'$ , the discounted value of the real balances  $\varepsilon$  which will be carried over to the next period (hence the second term on the right-hand side). The inequality reduces to  $\varepsilon \leq \sigma_b(u - p') / (iN + \sigma_b)$ . For any finite  $N$ , we can find  $\varepsilon$  such that the above inequality holds and  $p' + \varepsilon < p^*$ . So all sellers charging  $p' < p^*$  cannot be an equilibrium. Once again we get a unique single-price equilibrium with all sellers charging  $p^*$ .

Two points deserve emphasis. First, in the finite case, sellers realize that their choices affect buyers'

choices in the CM, but all agents still act competitively. That is, after the sellers post prices, agents choose their CM consumption, output, and real balances taking those prices as given; however, a seller understands that if he posts a slightly higher price in terms of real balances it is feasible for buyers to bring these real balances to the DM. Notice this does not quite work if prices are quoted in nominal terms, since then any candidate equilibrium has all sellers posting a nominal price of  $p = M$  and all the buyers taking out  $m = M$ , and so when a seller deviates to a higher nominal price, this will not affect the resulting equilibrium money holdings of the buyers. This illustrates that whether the model is formulated in nominal or real terms can matter.

Second, an important if perhaps not surprising message is that the results are sensitive to assumptions about timing. From the baseline analysis, when buyers and sellers move simultaneously, there is a continuum of equilibria, even in the finite case, since there is still a coordination issue. If sellers move first, at least with a finite number of agents, they extract the maximum surplus across the equilibria in the simultaneous-move game. And if we instead assume that buyers move first, a straightforward modification of the above argument implies they extract all the surplus, and the unique equilibrium is  $p = c$ . As is often the case, the timing matters.

## 4 Conclusion

Comparison with Diamond (1970) tells us that making agents trade with money can have big effects on the set of equilibria. It is by now well known how to get around the result in Diamond, that there must be a single price, using a variety of technical devices, including multilateral matching or heterogeneous preferences (Burdett and Judd 1983; Albrecht and Axell 1984). Already related monetary economies have been shown to generate endogenous distributions of money or prices with similar devices (Head and Kumar 2005; Dutu, Julien and King 2007; Galenianos and Kircher 2007; Curtis and Wright 2004). It is interesting to conjecture that there might exist equilibria in economies like the one considered here with dispersion in money or prices *without* heterogeneous preferences or multilateral matching. This is a subject of ongoing research.

We finish this Conclusion by commenting on a few additional observations made by the referee and associate editor. In their analysis of the indeterminacy of single-price stationary monetary equilibria along the lines of Green-Zhou, Kamiya and Shimizu (2006) argue that there are two essential elements. First, they show that a continuum of prices are consistent with a constant  $G(m)$  distribution iff  $G$  has no mass points; second, they show that a nonempty set of these prices are consistent with equilibrium behavior, or are self-fulfilling, due to coordination issues. We highlight the latter element, but our analysis is silent on the former element: we not only do not need  $G$  to have no mass points, but due to the presence of the CM, in our equilibria  $G$  is degenerate, as is typical in Lagos-Wright-style models. Again, it is an open question whether equilibria with nondegenerate  $G$  exist.

Kamiya and Shimizu also argue that Green-Zhou indeterminacy is robust to at least some changes in the environment, including making goods divisible and altering how the terms of trade are determined. This is not the case here, in the following sense. First, if we have divisible goods and replace price posting by bargaining, as in the basic Lagos-Wright model, there is a unique stationary monetary equilibria (Wright 2008). In nonmonetary models, price posting is often equivalent to bargaining when sellers make take-it-or-leave-it offers. This is not true in monetary models, where a buyer makes an ex ante choice of  $m$ , which limits how much he is able (if not willing) to pay ex post in a meeting. However, even if the buyer can commit to not paying more than  $m$ , by not bringing more than  $m$ , a take-it-or-leave-it offer by the seller can still take all of the ex post surplus by reducing the quantity a buyer gets for  $m$  (in fact, even if quantity is fixed, he can do something similar by offering a lottery as in Berentsen et al. 2002).

We do not allow this: we force a seller to post a price  $p$ , such that any buyer can get a unit of output for  $p$ . We are comfortable with this scenario in the sense that it seems quite realistic for at least some markets, but it is also worth further consideration – e.g. one question concerns when sellers are better off by sticking to this kind of posting, as opposed to bargaining. We leave further exploration of this to future research. The point here is to show that some, if not all, of the interesting results in

the Green-Zhou framework can be studied in a much more tractable way in the Lagos-Wright model. It is also hoped that the discussion aids understanding of the relationship between different models in monetary economies, more generally. For additional progress on this dimension, see Zhu (2008) and Hu, Kennan and Wallace (2008).

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