Money and Capital: A Quantitative Analysis[*]

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Abstract

We study the effects of money (anticipated inflation) on capital formation. Previous papers on this topic adopt reduced-form approaches, putting money in the utility function or imposing cash in advance, but use otherwise frictionless models. We follow a literature that is more explicit about the frictions making money essential. This introduces several new elements, including a two-sector structure with centralized and decentralized markets, stochastic trading opportunities, and bargaining. We show how these elements matter qualitatively and quantitatively. Our numerical results differ from findings in the reduced-form literature. The analysis reduces the previously large gap between mainstream macro and monetary theory.

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1 Introduction

This paper studies the effects of monetary policy, in the sense of fully anticipated inflation, on capital formation. The relation between money and capital is a classic issue, going back to Tobin (1965), Sidrauski (1967a, 1967b), Stockman (1981), Cooley and Hansen (1989, 1991), Gomme (1993), Ireland (1994) and many others. All of these papers adopt reduced-form approaches: they either put money in the utility function or impose cash-in-advance constraints, in an attempt to capture implicitly the role of money in the exchange process, but in other respects use frictionless models. An alternative literature on money, going back to Kiyotaki and Wright (1989, 1993), Aiyagari and Wallace (1991), Shi (1995), Trejos and Wright (1995), Kocherlakota (1998), Wallace (2001) and others, is more explicit about the frictions that make money essential, and in doing so has introduced some new elements into monetary economics, including detailed descriptions of specialization, information, matching, pricing, etc. Our goal is to see how these elements matter for the impact of inflation on investment, and other variables, including welfare. It turns out that modeling microfoundations in more detail does indeed make a difference for the quantitative results.

We build on the two-sector model in Lagos and Wright (2005), where some economic activity takes place in centralized markets and some in decentralized markets. This is useful because, in addition to providing microfoundations for the role of a medium of exchange, decentralized markets allow us to introduce ingredients like stochastic trading opportunities and bargaining, while centralized markets allow us to incorporate capital as in standard growth theory. This contrasts sharply with other attempts to study money and capital in models with frictions, including Shi (1999), Shi and Wang (2006), and Menner (2006), who build on Shi (1997), and Molico and Zhang (2005), who build on Molico (2006). Those models have only decentralized markets. It is much easier to connect with mainstream macro and to incorporate not only capital but other ingredients, like fiscal policy, in a model with some centralized trade. In particular, as a special case, in nonmonetary equilibrium, our model reduces to the textbook growth model, while those mentioned above reduce to something quite different – autarky.

Reducing the gap between models of decentralized trade and mainstream macro has been a challenge for some time. As Azariadis (1993) put it, "Capturing the transactions motive for holding money balances in a compact and logically appealing manner has turned out to be an enormously complicated task. Logically coherent models such as those proposed by Diamond (1984) and Kiyotaki and Wright (1989) tend to be so removed from neoclassical growth theory as to seriously hinder the job of integrating rigorous monetary theory with the rest of macroeconomics." And as Kiyotaki and Moore (2001) put it, "The matching models are without doubt ingenious and beautiful. But it is quite hard to integrate them with the rest of macroeconomic theory – not least because they jettison the basic tool of our trade, competitive markets." We think the framework presented here constitutes a big step towards monetary economics with microfoundations and mainstream macroeconomics, both in terms of theory and especially in terms studying quantitative issues.[1]

Several ingredients turn out to matter for the results. First, stochastic trading opportunities are important for matching the model to the facts.[2] Second, the two-sector structure is critical: in our baseline model, capital produced in the centralized market is used in the decentralized market, where money is essential, which generates interesting feedback from decentralized to centralized markets and in particular from inflation to investment. Third, perhaps surprisingly, the quantitative results depend a lot on what one assumes about price formation in the decentralized market. If we consider bargaining between buyers and sellers, inflation has little impact on investment, although it has a sizable impact on welfare: going from 10% inflation to the Friedman rule barely changes the capital stock, but is worth around 3% of total consumption. Alternatively, if we consider Walrasian price taking, the same experiment increases the long-run capital stock between 3% and 5%, and has a welfare

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[1] A previous attempt to put capital into an explicit monetary model by Aruoba and Wright (2003) lead to some undesirable implications, including the following dichotomy: one can solve independently for allocations in the centralized and decentralized markets. This implies monetary policy has no impact on investment, employment or consumption in the centralized market, which in a sense means money and growth theory have not been integrated at all (Howitt 2003; Waller 2003). The model here does not dichotomize, in general, although it does in a special case.

[2] As an example, it is hard to match observations on velocity, especially at high frequencies, in cash-in-advance models, and the results are very sensitive to the length of a period. This is not the case in models with search or other features generating stochastic trade opportunities, like the one presented here. Telyukova and Visschers (2008) recently make a simlar point.

effect of 1.5% across steady states, or 1% when we take into account transitions.

It is worth emphasizing that these numbers are quite different from previous findings using reduced-form models, implying that the impact of including elements from modern monetary theory can be significant.[3] We also find sizable effects from fiscal policy, although not so different from the previous literature. On balance, however, we find that if we have to make up the lost revenue with distortionary instead of lump sum taxes, reducing inflation is still a good idea, and the Friedman rule is still the best policy (contrary to some earlier reports e.g. Cooley and Hansen 1991). We also demonstrate something very new: in the bargaining version of the model, holdup problems in the demands for money and capital can be quantitatively important. This is true even if we have bargaining only in the decentralized market and this accounts less than 8% of aggregate output. Again, our conclusion is that taking into account details of the microfoundations can make a big difference for policy-relevant quantitative issues.

The rest of the paper is organized as follows. In Section 2 we describe the model. In Section 3 we discuss calibration. In Section 4 we present quantitative results. In Section 5 we conclude. The Appendix contains details of the analysis and alternative specifications.

2 The Basic Model

2.1 General Assumptions

A $[0, 1]$ continuum of agents live forever in discrete time. In order to integrate elements of both mainstream macro and search theory, we adopt the sectoral structure in Lagos and Wright (2005), hereafter LW, where each period these agents engage in two types of economic activity. Some of this activity takes place in a frictionless centralized market, referred to as

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[3] Cooley and Hansen (1989, 1991) find much smaller effects on capital and welfare; e.g. their welfare numbers are substantially below 1% Gomme (1993) gets even smaller effects in an endogenous growth model. Ireland (1994) gets welfare numbers around 0.67%. Lucas (2000) has no capital, but using the approach in Bailey (1956) he gets welfare numbers below 1%; earlier efforts at this approach by Lucas (1981) and Fischer (1981) get 0.3% to 0.45%. Imorhoroglu and Prescott (1991) also get less than 1%. A few papers find somewhat larger effects, such as Dotsey and Ireland (1996), in part because even though inflation does not affect total capital very much it does affect the amount of resources used in intermediation (see also Freeman and Kydland 2000 or Aiyagari, Braun and Eckstein 1998).

the CM, and some takes place in a decentralized market, referred to as the DM, with two main frictions: a *double coincidence problem*; and *anonymity.* These frictions combine to make some medium of exchange essential in the DM. As this is not the place to go into all of the details, for formal discussions of essentiality and the role of anonymity we refer readers to Kocherlakota (1998), Wallace (2001), and Aliprantis, Camera and Puzzelo (2006, 2007).

Given that some medium of exchange is essential, one issue in monetary theory is to determine endogenously which objects serve this function (e.g. Kiyotaki and Wright 1989). In order to focus on other questions, however, other papers avoid this by assuming there is a unique storable asset that qualifies for the role. Since we obviously cannot assume a unique storable asset in a paper called "Money and Capital" we need to say a few words about the issue. What we have to offer is a story along the lines of the "worker-shopper pair" in Lucas' (1980), extended slightly based on ideas going back at least to Menger (1892) about currency having certain advantages in terms of *portability* and *recognizability.* First, in terms of portability, we assume that in the DM agents have their capital physically fixed in place at production sites. Thus, when you want to buy output from someone you must visit their location, and since you cannot bring your capital with you, it cannot be used in payment. Because currency is portable it can be used for payments.

This use of spatial separation is in the spirit of the "worker-shopper" story used to motivate cash-in-advance. But one really should go beyond this and ask why claims to capital (or claims generally) cannot overcome spatial separation. One approach is to invoke recognizability. A stark assumption that works is that agents can costlessly counterfeit other claims, but not currency, say because the monetary authority has a monopoly on the technology for producing hard-to-counterfeit notes. Given this, sellers no more accept claims to capital from anonymous buyers in the DM than they accept personal IOU's. Thus, money has a role even when capital is a storable factor of production. Although we do not consider this the last word on the coexistence of money and other assets, we do think that it is logically coherent.[4]

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[4] See Lester, Postlewaite and Wright (2008) and the references therein for attempts to formalize the role of recognizability more rigorously in monetary theory.

As in standard growth theory, we assume that in the CM there is a general good that can be used for consumption or investment, produced using labor H and capital K hired by firms in competitive markets. Profit maximization implies $r = F_K(K, H)$ and $w = F_H(K, H)$, where F is the technology, r the rental rate, and w the real wage. Constant returns implies equilibrium profits are 0. In the DM these firms do not operate, but an agent's own effort e and capital k can be used with technology $f(e, k)$ to produce a different good. Note that k appears as an input the DM, because when you go to a seller's location he has access to his capital, even though you do not have access to your capital. This is important – it is the fact that capital produced in the CM is productive in the DM that breaks the dichotomy mentioned in fn. 1 meaning money has interesting effects on investment and other CM variables. Other details and variations in the basic LW setup are much less important – it does not matter whether e.g. the CM or DM meets first.

In the DM, each period with probability σ an agent discovers he is a *buyer*, which means he wants to consume but cannot produce, so he visits the location of someone that can produce; with probability σ he is a *seller*, which means he can produce but does not want to consume, and he waits at his location for someone to visit him; and with probability $1 - 2\sigma$ he is a *nontrader*, and he neither produces nor consumes.[5] In some buyer-seller meetings, the former is able to buy on credit, paying in the next CM. We think of these as meetings that are monitored by the "authorities" or meetings between agents who know each other. Let ℓ (for loan) be the payment made in the CM, measured in dollars, and assume it is costlessly enforced; there is no strategic default. But credit is only available in meetings with probability $1 - \omega$. With probability ω, the buyer is anonymous, or the meeting not monitored, and the seller demands cash.

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[5] We also tried a version where nontraders produce and consume for themselves, which changes the quantitative results slightly, but not a lot. Also note that this taste and technology shock specification is basically equivalent to bilateral matching, where there is a probability σ of wanting to consume something produced by a random partner. We use taste and technology shocks because it fits better with our ideas about spatial separation, with buyers visiting sellers' locations. Again, see Telyukova and Visschers (2008) for more discussion of why some sort of stochastic trading are important. One more remark here is that our buyers choose which seller to visit in some possibly random but coordinated way, so that they end up bilaterally matched – i.e. we do not have the coordination problem emphasized in so-called directed search theory mentioned in Section 2.3.

Instantaneous utility for everyone in the CM is $U(x) - Ah$, where x is consumption and h labor (linearity in h reduces the complexity of the model considerably, although Rocheteau et al. 2008 show how to get the same simplifications with general preferences by assuming indivisible labor and lotteries à la Rogerson 1988). In the DM, with probability σ you are a buyer and enjoy utility $u(q)$, and with probability σ you are a seller and get disutility e, where q is consumption and e labor (normalizing the disutility of DM labor to be linear is merely a choice of units and has no implications for any results reported here). Assume u and U have the usual monotonicity and curvature properties. Solving $q = f(e, k)$ for $e = c(q, k)$, we get the utility cost of producing q given k.[6] Agents discount between the CM and DM at rate β but not between the DM and CM.

Government sets the money supply so that $M_{+1} = (1 + \tau)M$, where $+1$ denotes next period. We use τ as our policy instrument, but we could instead target inflation or nominal interest rates – in steady state, inflation equals τ and the nominal rate is defined by the Fisher equation $1 + i = (1 + \tau)/\beta$. They also consume G, levy a lump-sum tax T, labor income tax t_h, capital income tax t_k, and sales tax t_x in the CM (we omit sales taxes in the DM to ease the presentation, but when we added them it made little difference for results). Letting δ be the depreciation rate of capital, which is tax deductible, and p the CM price level, the government budget constraint is $G = T + t_h wH + (r - \delta)\, t_k K + t_x X + \tau M/p$ if we interpret M as currency. When we interpret it as currency plus inside money, as in some of the discussion below, we have to adjust the budget, but this does not matter if we can change T, since with quasi-linear utility the (wealth) effect of changing T is nil.

Let $W(m, k, \ell)$ be the value function for an agent in the CM holding m dollars and k units of capital who owes ℓ from the previous DM. Let $V(m, k)$ be the DM value function. The CM problem is

$$\begin{aligned} W(m,k,\ell) &= \max_{x,h,m_{+1},k_{+1}} \left\{ U(x) - Ah + \beta V_{+1}(m_{+1}, k_{+1}) \right\} \qquad (1) \\ \text{s.t. } (1+t_x)\, x &= w\,(1-t_h)\, h + \left[1 + (r-\delta)\,(1-t_k)\right] k - k_{+1} - T + \frac{m - m_{+1} - \ell}{p}. \end{aligned}$$

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[6] In Appendix B.1 we show $c_q > 0$, $c_k < 0$, $c_{qq} > 0$, and $c_{kk} > 0$, under the usual monotonicity and convexity assumptions, and $c_{qk} < 0$ if $f_k f_{ee} < f_e f_{ek}$, which holds if k is a normal input.

Eliminating h using the budget and taking FOC, assuming interiority, we get[7]

$$\begin{aligned} x &: U'(x) = \frac{A(1+t_x)}{w(1-t_h)} \\ m_{+1} &: \frac{A}{pw(1-t_h)} = \beta V_{+1,m}(m_{+1}, k_{+1}) \\ k_{+1} &: \frac{A}{w(1-t_h)} = \beta V_{+1,k}(m_{+1}, k_{+1}). \end{aligned} \tag{2}$$

Then we have two key results. First, since (m, k, ℓ) does not appear in (2), for any distribution of (m, k, ℓ) across agents entering the CM the distribution of (m_{+1}, k_{+1}) is degenerate.[8] Second, W is linear:

$$\begin{aligned} W_m(m,k,\ell) &= \frac{A}{pw(1-t_h)} \\ W_k(m,k,\ell) &= \frac{A[1+(r-\delta)(1-t_k)]}{w(1-t_h)} \\ W_\ell(m,k,\ell) &= \frac{-A}{pw(1-t_h)} \end{aligned} \tag{3}$$

Moving to the DM, we have

$$V(m,k) = \sigma V^b(m,k) + \sigma V^s(m,k) + (1-2\sigma)W(m,k,0), \tag{4}$$

where the values to being a buyer and a seller are

$$V^b(m,k) = \omega[u(q_b) + W(m-d_b,k,0)] + (1-\omega)[u(\hat{q}_b) + W(m,k,\ell_b)] \tag{5}$$

$$V^s(m,k) = \omega[-c(q_s,k) + W(m+d_s,k)] + (1-\omega)[-c(\hat{q}_s,k) + W(m,k,-\ell_s)]. \tag{6}$$

In these expressions, q_b and d_b (q_s and d_s) denote the quantity consumed and dollars exchanged when buying (selling) for money, while $\hat{q}_b$ and ℓ_b ($\hat{q}_s$ and $-\ell_s$) denote the quantity consumed and value of the loan for the buyer (seller) in a credit meeting. Notice we have written the expressions as though no money is exchanged in credit matches; this is without loss in generality since everyone is indifferent between payments made in the DM and the next CM. Similarly, the fact that the loan ℓ is nominal is irrelevant due to perfect foresight.

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[7] One can adapt the discussion in LW to guarantee the concavity of the problem and interiority of the solution; instead, in our quantitative analysis, we can check it directly.

[8] This does *not* mean two people with very different (m, k, ℓ) will look identical after one period, since for interiority and hence for the result that the distribution is degenerate after the CM, wealth cannot be too disperse at the start of the CM. It is only if we start agents with similar (m, k, ℓ) that we get degeneracy.

Using the linearity of W, we can write V as

$$\begin{aligned} V(m,k) &= W(m,k,0) + \sigma\omega\left[u(q_b) - \frac{d_b A}{pw(1-t_h)}\right] + \sigma\omega\left[\frac{d_s A}{pw(1-t_h)} - c(q_s,k)\right] \\ &\quad + \sigma(1-\omega)\left[u(\hat{q}_b) - \frac{A\ell_b}{pw(1-t_h)}\right] + \sigma(1-\omega)\left[\frac{A\ell_s}{pw(1-t_h)} - c(\hat{q}_s,k)\right] \end{aligned} \tag{7}$$

This yields

$$\begin{aligned} V_m(m,k) &= \frac{A}{pw(1-t_h)} + \sigma\omega\left[u'\frac{\partial q_b}{\partial m} - \frac{A}{pw(1-t_h)}\frac{\partial d_b}{\partial m}\right] \\ &\quad + \sigma\omega\left[\frac{A}{pw(1-t_h)}\frac{\partial d_s}{\partial m} - c_q\frac{\partial q_s}{\partial m}\right] \end{aligned} \tag{8}$$

$$\begin{aligned} V_k(m,k) &= \frac{A\left[1+(r-\delta)(1-t_k)\right]}{w(1-t_h)} + \sigma\omega\left[u'\frac{\partial q_b}{\partial k} - \frac{A}{pw(1-t_h)}\frac{\partial d_b}{\partial k}\right] \\ &\quad + \sigma\omega\left[\frac{A}{pw(1-t_h)}\frac{\partial d_s}{\partial k} - c_q\frac{\partial q_s}{\partial k} - c_k\right] \\ &\quad + \sigma(1-\omega)\left[u'\frac{\partial \hat{q}_b}{\partial k} - \frac{A}{pw(1-t_h)}\frac{\partial \ell_b}{\partial k}\right] \\ &\quad + \sigma(1-\omega)\left[\frac{A}{pw(1-t_h)}\frac{\partial \ell_s}{\partial k} - c_q(\hat{q}_s,k)\frac{\partial \hat{q}_s}{\partial k} - c_k(\hat{q}_s,k)\right]. \end{aligned} \tag{9}$$

Once we specify how the terms of trade (q, d, $\hat{q}$ and ℓ) are determined, in equilibrium, we can substitute for their derivatives in (8) and (9), which will turn out to simplify things a lot since several of these derivatives are 0.

First, as a benchmark, consider the planner's problem when money is *not* essential, say, because $\omega = 0$ and all meetings are monitored:

$$\begin{aligned} J(K) &= \max_{X,H,K_{+1},q}\left\{U(X) - AH + \sigma\left[u(q) - c(q,K)\right] + \beta J_{+1}(K_{+1})\right\} \\ \text{s.t. } X &= F(K,H) + (1-\delta)K - K_{+1} - G \end{aligned} \tag{10}$$

Eliminating X, and again assuming interiority, we have the FOC

$$\begin{aligned} q &: u'(q) = c_q(q,K) \\ H &: A = U'(X)F_H(K,H) \\ K_{+1} &: U'(X) = \beta J'_{+1}(K_{+1}). \end{aligned} \tag{11}$$

The envelope condition $J'(K) = U'(X)[F_K(K,H) + 1 - \delta] - \sigma c_k(q,K)$ implies

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1}, H_{+1}) + 1 - \delta] - \beta\sigma c_k(q_{+1}, K_{+1}). \tag{12}$$

From the first condition in (11), $q = q^*(K)$ where $q^*(K)$ solves $u'(q) = c_q(q,K)$. Then the paths for (K_{+1}, H, X) satisfy the Euler equation (12), the second FOC in (11), and the constraint in (10). This characterizes the first best, or FB for short.[9]

Note the presence of the term $-\beta\sigma c_k(q_{+1}, K_{+1}) > 0$ in (12), which reflects the fact that investment affects DM as well as CM productivity because K is used in both sectors. If K did not appear in $c(q)$ the system would dichotomize: we could first set $q = q^*$, where q^* solves $u'(q) = c'(q)$, and then solve the other conditions independently for (K_{+1}, H, X). The fact that K is used in the DM and produced in the CM breaks this dichotomy. Here we assume it is the *same* K used in both sectors. Appendix A.1 presents a version with two *distinct* capital goods used in the CM and DM, which looks a bit more like Lucas and Stokey (1987), at least if we assume the two sectors are equally capital intensive. Also, Appendix A.2 presents a version where K is used only in the CM but produced and traded in the DM, which is reminiscent of Stockman (1981). As discussed in Section 4.3, these variations do not affect the main quantitative results too much.[10]

2.2 Bargaining

Here we assume the terms of trade in the DM are determined by bargaining. Consider first a meeting where the seller demands money. If the buyer's and seller's states are (m_b, k_b) and

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[9] To be more precise, the methods in Stokey and Lucas (1989) tell us the solution is fully characterized by the FOC and envelope condition, or equivalently, we can replace the FOC for K_{+1} and envelope condition by the Euler equation and transversality condition. This also applies when we define equilibrium. One can use standard methods to check when there is a unique steady state and the planner's solution converges to it, under the usual kind of assumptions. Things are more complicated for equilibria because we allow for fiscal, monetary, and bargaining distortions. In Appendix B.4 we show analytically that for the price-taking version there exists a unique steady state for the functional forms used in the calibration; in the bargaining version we rely on numerical results.

[10] A referee suggested that having the *same* capital used in both sectors is a big part of the reason for our sizable quantitative effects, which is why we present results for both. While it is true that the one- and two-capital stock models behave differently if we hold parameters constant, if we recalibrate the parameters to fit the same targets, in both models the net effects are similar. The same referee also points out that our setup is related to theories with endogenous capital utilization (e.g. Hayashi and Prescott 2002) since the degree to which K gets used in the DM is determined in equilibrium as a function of e.g. policy.

(m_s, k_s), we assume (q, d) solves the generalized Nash bargaining problem with bargaining power for the buyer given by θ and threat points given by continuation values. Since the buyer's payoff from trade is $u(q) + W(m_b - d, k_b, 0)$ and his threat point is $W(m_b, k_b, 0)$, by the linearity of W, his surplus is $u(q) - Ad/pw\,(1 - t_h)$. Similarly, the seller's surplus is $Ad/pw\,(1 - t_h) - c(q, k_s)$. Hence the bargaining solution is

$$\max_{q,d}\left[u(q) - \frac{Ad}{pw\,(1-t_h)}\right]^{\theta}\left[\frac{Ad}{pw\,(1-t_h)} - c(q,k_s)\right]^{1-\theta} \quad \text{s.t. } d \le m_b.$$

One can show (as in LW) that in equilibrium $d = m_b$. Inserting this and taking the FOC with respect to q,

$$\frac{m_b}{p} = \frac{g(q,k_s)w\,(1-t_h)}{A}, \tag{13}$$

where

$$g(q,k_s) \equiv \frac{\theta c(q,k_s)u'(q) + (1-\theta)u(q)c_q(q,k_s)}{\theta u'(q) + (1-\theta)c_q(q,k_s)}. \tag{14}$$

Writing $q = q(m_b, k_s)$, where $q(\cdot)$ is given by solving (13), the relevant derivatives in (8) and (9) are $\partial d/\partial m_b = 1$, $\partial q/\partial m_b = A/pw\,(1-t_h)\,g_q > 0$ and $\partial q/\partial k_s = -g_k/g_q > 0$, where

$$\begin{aligned} g_q &= \frac{u'c_q[\theta u' + (1-\theta)c_q] + \theta(1-\theta)(u-c)[(u'c_{qq} - c_q u'')}{[\theta u' + (1-\theta)c_q]^2} > 0 \\ g_k &= \frac{\theta u'c_k\,[\theta u' + (1-\theta)c_q] + \theta(1-\theta)(u-c)u'c_{qk}}{[\theta u' + (1-\theta)c_q]^2} < 0. \end{aligned}$$

Now consider a meeting where credit is available. We assume the buyer has the same bargaining power θ. Then it is easy to see that $(\hat{q}, \ell)$ are determined just like (q, d) above except there is no constraint on ℓ the way we had $d \le m_b$ in monetary trades. Hence,

$$\begin{aligned} u'(\hat{q}) &= c_q(\hat{q}, k_s) \\ \frac{A\ell}{pw\,(1-t_h)} &= (1-\theta)\,u(\hat{q}) + \theta c(\hat{q}, k_s). \end{aligned}$$

Notice that, given $k_s = K$, $\hat{q}\,(K)$ is the same as the solution to the planner's problem $q^*(K)$. Hence, $\partial \hat{q}_b/\partial k_b = \partial \ell_b/\partial k_b = 0$ and

$$\begin{aligned} \frac{\partial \hat{q}_s}{\partial k_s} &= \frac{c_{qk}(\hat{q}, k_s)}{u''(\hat{q}) - c_{qq}(\hat{q}, k_s)} > 0 \\ \frac{\partial \ell_s}{\partial k_s} &= \frac{pw\,(1-t_h)}{A}\left[\frac{u'(\hat{q})c_{qk}(\hat{q}, k_s)}{u''(\hat{q}) - c_{qq}(\hat{q}, k_s)} + \theta c_k(\hat{q}, k_s)\right] \end{aligned}$$

Inserting these results and imposing $(m, k) = (M, K)$, (8) and (9) reduce to

$$V_m(M, K) = \frac{(1-\sigma\omega)A}{pw(1-t_h)} + \frac{\sigma\omega A u'(q)}{pw(1-t_h)\, g_q(q, K)} \tag{15}$$

$$V_k(M, K) = \frac{A[1+(r-\delta)(1-t_k)]}{w(1-t_h)} - \sigma\omega\gamma(q, K) - \sigma(1-\omega)(1-\theta)\, c_k(\hat{q}, K) \tag{16}$$

where it is understood that $q = q(M, K)$ and $\hat{q} = \hat{q}(K)$, while

$$\gamma(q, K) \equiv c_k(q, K) + c_q \frac{\partial q}{\partial K} = c_k(q, K) - c_q(q, K) \frac{g_k(q, K)}{g_q(q, K)} < 0. \tag{17}$$

The last two terms in (16) capture the idea that if a seller carries an extra unit of capital, he reduces the marginal cost of producing in the DM, which augments the value of investment in capital in the CM. The expression in (17) captures the non-price-taking behavior of agents in our model: the first term reflects the cost reduction due to an extra unit of capital and the second term reflects the change in costs due to the change in the terms-of-trade the seller faces since he has more capital.

Substituting (15) and (16), as well as prices $p = AM/w(1-t_h)\, g(q, K)$, $r = F_K(K, H)$, and $w = F_H(K, H)$, into the FOC for m_{+1} and k_{+1}, we get the equilibrium conditions

$$\frac{g(q, K)}{M} = \frac{\beta g(q_{+1}, K_{+1})}{M_{+1}} \left[1 - \sigma\omega + \sigma\omega \frac{u'(q_{+1})}{g_q(q_{+1}, K_{+1})}\right] \tag{18}$$

$$\begin{aligned} U'(X) &= \beta U'(X_{+1}) \{1 + [F_K(K_{+1}, H_{+1}) - \delta](1-t_k)\} \\ &\quad - \beta(1+t_x)\sigma [\omega\gamma(q_{+1}, K_{+1}) + (1-\omega)(1-\theta)\, c_k(\hat{q}_{+1}, K_{+1})]. \end{aligned} \tag{19}$$

Two other conditions come from the FOC for X and the resource constraint,

$$U'(X) = \frac{A(1+t_x)}{(1-t_h)\, F_H(K, H)} \tag{20}$$

$$X + G = F(K, H) + (1-\delta)K - K_{+1}. \tag{21}$$

An *equilibrium with bargaining* is defined as (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (18)-(21), given policy and the initial condition K_0.

Several remarks are in order here. First, we do not explicitly include $\hat{q} = \hat{q}(K)$ in the definition of equilibrium since it is a known function of K. Also, a *monetary* equilibrium requires $q > 0$ at every date. A nonmonetary equilibrium satisfies $q = 0$ at all dates, while (K_{+1}, H, X) solves (19)-(21) with $\gamma = 0$; this is exactly the equilibrium for a standard nonmonetary model. Although we are interested in dynamics, as a reference point, when $M_{+1} = (1+\tau) M$ for fixed τ we can define a *steady state* as a constant solution (q, K, H, X) to (18)-(21). In steady state inflation is τ and the nominal interest rate is $i = (1+\rho)(1+\tau) - 1$, where $\rho = 1/\beta - 1$ is the real rate. Then (18)-(19) simplify to

$$\frac{i}{\sigma\omega} = \frac{u'(q)}{g_q(q, K)} - 1 \tag{22}$$

$$\rho = [F_K(K, H) - \delta](1 - t_k) - \frac{(1 + t_x)\sigma}{U'(X)}[\omega\gamma(q, K) + (1 - \omega)(1 - \theta) c_k(\hat{q}, K)]. \tag{23}$$

If capital is not used in the DM, then $c(q, K) = c(q)$ and $\gamma(q, K) = c_k(q, K) = 0$. This version dichotomizes, and since M appears in (18) but not (19)-(21), monetary policy affects q but not (K_{+1}, H, X), or therefore, $\hat{q}$. Equilibrium does not dichotomize when K enters $c(q, K)$; as in the planner's problem, this implies we cannot solve independently for q and the CM variables. Notice however that if $\theta = 1$ then, although K enters $c(q, K)$, (19)-(21) can be solved for (K_{+1}, H, X), then (18) determines q since $\gamma(q, K) = 0$. So if $\theta = 1$ money still does not influence CM variables, even though anything that affects the CM (e.g. taxes) influences q. Intuitively, when $\theta = 1$ sellers do not get any of the surplus from DM trade, and so investment decisions are based solely on returns to K that accrue in the CM. Looking at (17), when $\theta = 1$, the cost reduction due to having more capital is exactly matched by the increase in cost due to higher production.

This is an extreme version of a holdup problem in the demand for capital. More generally, for any $\theta > 0$, sellers do not get the full return on capital from DM trade, and hence they underinvest relative to the efficient outcome. This holdup problem is not present in standard models, and constitutes a distortion over and above those from taxes and monetary inefficiencies.[11] If we run the Friedman Rule (FR) by setting $i = 0$ and levy only lump-sum taxes, we are left with the holdup problem on capital and a related problem on money emphasized in LW. In some models all holdup problems can be resolved if one sets bargaining power θ correctly (Hosios 1990). This is *not* possible here: $\theta = 1$ resolves the problem in the

---

[11] Capital holdup problems may have been neglected in macro, but see Caballero and Hamour (1998) or Caballero (1999). Kurmann (2008) provides a more recent discussion.

demand for money, but this is the worst case for investment; and $\theta = 0$ resolves the problem in the demand for capital, but this this is the worst case for money. Under bargaining there is no θ that can eliminate this double holdup problem, which has implications for both the empirical performance of bargaining models and their welfare implications.

2.3 Price Taking

While our holdup problems cannot simultaneously be solved by bargaining, some other solution concepts work much better. For example, it is by now well known that *competitive search equilibrium*, based on directed search and price posting, rather than bargaining, resolves multiple holdup problems (Shimer 1995; Moen 1997; Acemoglu and Shimer 1999). And *competitive equilibrium* with Walrasian price taking also does the job here, even though this is not true in all models (e.g. Rocheteau and Wright 2005 show that competitive search equilibrium can do better than competitive equilibrium in environments with search externalities, but we have no such externalities). Since it is easier to present, relative to price posting with directed search, in this section we consider price taking in the DM.[12]

We assume for simplicity that there are two distinct markets – one for anonymous traders where cash is needed, and where credit is available. The DM value function has the same form as (4), but now, in the market with anonymous buyers,

$$\begin{aligned} V^s(m,k) &= \max_q \{-c(q,k) + W(m + \tilde{p}q, k, 0)\} \\ V^b(m,k) &= \max_q \{u(q) + W(m - \tilde{p}q, k, 0)\} \text{ s.t. } \tilde{p}q \le m \end{aligned}$$

where $\tilde{p}$ is the price (which generally differs from the CM price level p). Similarly, in the market where agents can buy on credit,

$$\begin{aligned} \hat{V}^s(m,k) &= \max_{\hat{q}} \{-c(\hat{q},k) + W(m, k, -\hat{p}\hat{q})\} \\ \hat{V}^b(m,k) &= \max_{\hat{q}} \{u(\hat{q}) + W(m, k, \hat{p}\hat{q})\}. \end{aligned}$$

---

[12] One can interpret this, if one likes, as a monetary version of the labor-search model in Lucas and Prescott (1974), where large numbers of buyers visit "islands" with large numbers of sellers, and on each "island" prices are taken as given. Combined with our assumptions about locations and taste-technology shocks, the difference between bargaining and price-taking can be interpreted as saying that in one version buyers and sellers meet bilaterally and in other they meet in large groups.

The FOC for the sellers in the two DM markets are

$$
\begin{aligned}
c_q(q,k) &= \tilde{p}W_m = \tilde{p}A/pw\,(1-t_h) \\
c_q(\hat{q},k) &= -\hat{p}W_\ell = \hat{p}A/pw\,(1-t_h)\,.
\end{aligned}
$$

Market clearing implies buyers and sellers choose the same q and $\hat{q}$ As with bargaining, in the anonymous market, buyers spend all their money so $q = M/\tilde{p}$. Inserting $\tilde{p} = M/q$, we get the analog to (13) from the bargaining model

$$
\frac{M}{p} = \frac{qc_q(q,k)w\,(1-t_h)}{A}. \tag{24}
$$

Similarly, in the market where credit is available, it is easy to see $\hat{q} = \hat{q}(K)$, the same as in the bargaining model (or planner's problem) but now $\ell = pw\,(1-t_h)\,u'(\hat{q})\hat{q}/A$. Then the analogs to (15) and (16) are

$$
\begin{aligned}
V_m(M,K) &= \frac{(1-\sigma\omega)A}{pw\,(1-t_h)} + \frac{\sigma\omega u'(q)}{\tilde{p}} \\
V_k(M,K) &= \frac{A + A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)} - \sigma\omega c_k(q,K) - \sigma\,(1-\omega)\,c_k(\hat{q},K).
\end{aligned}
$$

Inserting these into (2) yields the analogs to equilibrium conditions (18) and (19)

$$
\begin{aligned}
\frac{c_q(q,K)q}{M} &= \frac{\beta c_q(q_{+1},K_{+1})q_{+1}}{M_{+1}}\left[1-\sigma\omega+\sigma\omega\frac{u'(q_{+1})}{c_q(q_{+1},K_{+1})}\right] && (25) \\
U'(X) &= \beta U'(X_{+1})\left\{1+\left[F_K(K_{+1},H_{+1})-\delta\right](1-t_k)\right\} && (26) \\
&\quad -\beta\,(1+t_x)\,\sigma\left[\omega c_k(q_{+1},K_{+1})+(1-\omega)\,c_k(\hat{q}_{+1},K_{+1})\right].
\end{aligned}
$$

The other equilibrium conditions (20)-(21) are the same as in the previous model. Then an *equilibrium with price taking* is given by (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (25)-(26) and (20)-(21), given policy and K_0. The difference between bargaining and price taking is the difference between (18)-(19) and (25)-(26). Notice the equilibrium condition for q here looks like the one from the bargaining model when $\theta = 1$, and the equilibrium condition for K looks like the one from the bargaining model when $\theta = 0$. This suggests that price taking avoids both holdup problems in this environment, and it it not hard to show formally that this is correct.[13]

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[13] Set $t_k = t_h = t_x = 0$. Then under price taking the equilibrium conditions for (K_{+1}, H, X) are the same

2.4 A Digression on Banking

At first blush it might seem the relevant notion of money in the model is $M0$. This is one interpretation, but not the only one. Without going into too much detail we mention that one can introduce banks into the framework following the approach in Berentsen, Camera and Waller (2007) or He, Huang and Wright (2005, 2008). Berentsen et al. assume that after trading stops in the the CM, so that agents have decided on their m_{+1}, it is revealed which ones will want to consume and which ones will be able to produce while banks are still open but before agents go to the DM (Chiu and Meh 2007 do something very similar). As the sellers have no use for money, they deposit it in banks, who then lend it to buyers. One can imagine them lending out the same physical currency, or as keeping it in the vault and issuing bank-backed securities that can be used in payments, at least as long as these securities are not easily counterfeitable.

Introducing banks explicitly as in Berentsen et al. changes some details, but the basic structure remains fairly close to what we have here. He et al. study a similar environment, and add the assumption that currency can be stolen while bank-backed securities cannot – e.g., it may be safer to go to the DM with your checkbook or debit card than a big bundle of cash. They further allow fractional reserves, and derive a money multiplier in one version of the model, with $M1$ determined endogenously as a function of $M0$ and the reserve ratio. If the resource cost of banking goes to zero, e.g., in that model cash may stop circulating, to be replaced in all DM transactions by bank liabilities. More generally, outside and inside money can both circulate in the DM. Otherwise, that model is also quite close to what we have here.

We think more needs to be done to address many interesting issues related to financial intermediation in these kinds of models, but we mention banks here for the following reasons.

---

as those for the planner problem. Hence the equilibrium coincides with the FB iff $u'(q) = c_q(q, K)$. From (25), this means $c_q(q, K)q/M = \beta c_q(q_{+1}, K_{+1})q_{+1}/M_{+1}$. Using (24) this reduces to $1/pw = \beta/p_{+1}w_{+1}$. Since $w = A/U'(X)$, it further reduces to $p/p_{+1} = U'(x)/\beta U'(X_{+1})$. Since in any equilibrium the slope of the indifference curve $U'(x)/\beta U'(X_{+1})$ equals the slope of the budget line $1 + \rho$, with ρ equal to the real interest rate, the relation in question finally reduces to $p_{+1}/p = 1/(1 + \rho)$. Using the Fisher equation, this holds and hence $q = q^*(K)$ solves (25) iff we set the nominal rate to $i = 0$. We conclude that under price taking with lump-sum taxes, setting $i = 0$ yields the FB.

First, we do not necessarily want to take M to be currency – we present results below for several measures of money, including $M0$, $M1$ etc., and hence it seems worth arguing that not only $M0$ fits with the physical environment. In fact, although we report results for several measures, we use $M1$ in the baseline calibration. Second, consistent with this, when we appeal to micro data to calibrate the fraction ω of DM trades where credit is not available, we aggregate cash, check and debit card, but not credit card, purchases into money trades. This is based on two criteria: we think of checks and debit cards as simply convenient ways to access demand deposits, which like cash are liquid and pay virtually 0 interest; and we think the key distinction with credit cards is that they allow you to consume now and work later, while with either money or demand deposits you have to work first, as recently emphasized e.g. in Dong (2008).

3 Quantitative Analysis

3.1 Preliminaries

We begin with some accounting. The price levels in the CM and DM are p and $\tilde{p} = M/q$, respectively, where p satisfies

$$p = \frac{AM}{(1 - t_h)\, g\,(q, K)\, F_H(K, H)} \tag{27}$$

in the bargaining version of the model by (13), and

$$p = \frac{AM}{(1 - t_h)\, qc_q\,(q, K)\, F_H(K, H)} \tag{28}$$

in the price-taking version by (24). Nominal output is $pF\,(K, H)$ in the CM, and $\sigma\omega M + \sigma(1-\omega)\ell$ in the DM. Using p as the unit of account, real output in the CM is $Y_C = F\,(K, H)$ and in the DM is $Y_D = \sigma\omega M/p + \sigma(1-\omega)\sigma\ell/p$. Total real output is $Y = Y_C + Y_D$.

Define the share of output produced in the DM by $s_D = Y_D/Y$, the share of output where money is essential by $s_M = Y_M/Y$ where $Y_M = \sigma\omega M/p$, and the share where credit is used by $s_\ell = Y_\ell/Y$ where $Y_\ell = \sigma(1-\omega)\ell/p$. We do not calibrate these shares, but compute them indirectly from other variables. Notice that, by definition, velocity is $v = pY/M =$

$\sigma\omega Y/Y_M$. Hence, $s_M = Y_M/Y = \sigma\omega/v$. The maximum σ can be is 1/2, and the maximum ω can be is 1, so given $M1$ velocity is around 5, s_M is bounded above by 10%. With our benchmark calibrated parameters, s_M is actually closer to 8%. There are two points we want to emphasize. First, to think about the size of the different sectors, one does not necessarily have to take a stand on which goods are traded in which sector. Second, the results presented below do not depend on having a large amount of monetary trade – around 92% of economic activity looks like what one sees in a standard nonmonetary model.

We will also discuss the *markup* μ, defined by equating $1+\mu$ to the ratio of price to marginal cost. The markup in the CM market is 0, since it is competitive. The markup in the DM under price taking is also 0. With bargaining, the markup in the DM is derived as follows. First consider monetary trades. Marginal cost in terms of utility is $c_q(q,K)$. Since a dollar is worth $A/p(1-t_h)w$ utils, marginal cost in dollars is $c_q(q,K)p(1-t_h)w/A$. Since the price is $\tilde{p} = M/q$, the markup in monetary trade is given by

$$1+\mu_M = \frac{M/q}{c_q(q,K)p(1-t_h)w/A} = \frac{g(q,K)}{qc_q(q,K)},$$

after eliminating M using (27). Similarly, the markup in credit trade in the DM is

$$1+\mu_\ell = \frac{\ell/\hat{q}}{c_q(\hat{q},K)p(1-t_h)w/A} = \frac{(1-\theta)u(\hat{q})+\theta c(\hat{q},k_s)}{\hat{q}c_q(\hat{q},K)}.$$

The average markup in the DM is then $\mu_D = \omega\mu_M + (1-\omega)\mu_\ell$, and the aggregate markup is $\mu = s_D\mu_D$.

Finally, we will also discuss certain elasticities, including the interest elasticity of money demand $\xi = \frac{\partial M/p}{\partial i}\frac{i}{M/p}$. Actually, we could do everything in terms of the elasticity with respect to the inflation rate, and the results will be the same to the extent that the Fisher equation holds in the data. We use the interest rate when we look at money mainly because this is what most people use. When we discuss the elasticity of investment we do use the inflation rate. In any event, consider ξ under bargaining (price taking is similar). Using (27) and differentiating, we get

$$\xi = \left(g_q\frac{\partial q}{\partial i} + g_k\frac{\partial K}{\partial i}\right)\frac{i}{g} + \left(F_{HH}\frac{\partial H}{\partial i} + F_{HK}\frac{\partial K}{\partial i}\right)\frac{i}{F_H}. \tag{29}$$

It is now a matter of substituting $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$ (see Appendix B.2) to yield ξ as a function of the allocation and parameters.

3.2 Calibration

Consider the following functional forms for preferences and technology:

$$\text{CM:} \qquad U(x) = B\frac{x^{1-\varepsilon}-1}{1-\varepsilon} \text{ and } F(K,H) = K^{\alpha}H^{1-\alpha}$$

$$\text{DM:} \qquad u(q) = C\frac{(q+b)^{1-\eta}-b^{1-\eta}}{1-\eta} \text{ and } c(q,k) = q^{\psi}k^{1-\psi}$$

The cost function $c(\cdot)$ comes from the technology $q = e^{\chi}k^{1-\chi}$ with $\psi = 1/\chi$; if $\psi = \chi = 1$ then k does not appear in $c(\cdot)$ and the model dichotomizes. The parameter b in $u(q)$ is introduced merely so that $u(0) = 0$, which is useful for technical reasons (it keeps threat points in bargaining well defined for all η). This means relative risk aversion is not constant, but if $b \approx 0$, it is approximately constant at $\eta q/(q+b) \approx \eta$. We set $b = 0.0001$ and $\varepsilon = \eta = 1$ as a benchmark, but we show that the results are robust to these choices in Section 4.3. We normalize $C = 1$ with no loss in generality.[14]

To describe the calibration strategy for the remaining parameters, we start with a heuristic description, then provide details. The first point we make is that our approach is a natural extension of standard methods. To pick a typical application, Christiano and Eichenbaum (1992) study a one-sector growth model, parameterized by

$$U = \log(x) + A(1-h) \text{ and } Y = K^{\alpha}h^{1-\alpha}$$

for their indivisible-labor version; for their divisible-labor version replace $A(1-h)$ by $A\log(1-h)$. There are four parameters, calibrated as follows. First, set the discount factor $\beta = 1/(1+\rho)$ where ρ is some observed average interest rate. Then set the depreciation rate

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[14] Consider the following argument for the case of $b = 0$ – i.e. for log utility. Before normalizing, the underlying utility function is $\mathcal{U} = B\log(x) - Ah + C\log(q) - De$. Write the DM production function as $q = Ze^{\chi}k^{1-\chi}$. Then making a change of variable $\tilde{q} = qD/Z^{1/\chi}$ we have $\mathcal{U} = B\log(x) - Ah + C\log(\tilde{q}Z^{1/\chi}/D) - \tilde{q}k^{(\chi-1)/\chi}$ which is basically what we use. Obviously we can normailize $C = 1$ without loss of generality, and with log utility the choice of constants Z and D has no effect on individual decisions (it affects only the units of DM output). This argument is literally true only for $b = 0$, but for the values of b used in practice it provides an excellent approximation.

$\delta = I/K$ to match the investment-capital ratio. Then set α to match *either* labor's share of income LS or the capital-output ratio K/Y, since these yield the same result, given there are no taxes in the model (see below). Finally, set A to match observed average hours worked h.

This method can be adapted to many scenarios. For example, Greenwood et al. (1995) calibrate a two-sector model, with home production, as follows. Consider

$$U = \log(x) + A(1 - h_m - h_n),\ Y_m = K_m^{\alpha_m} h_m^{1-\alpha_m} \text{ and } Y_n = K_n^{\alpha_n} h_n^{1-\alpha_n},$$

where $x = [Dx_m^\kappa + (1-D)x_n^\kappa]^{1/\kappa}$, and x_m, h_m and k_m are consumption, hours and capital in the market while x_n, h_n and k_n are consumption, hours and capital in the nonmarket or home sector. After setting $\beta = 1/(1+\rho)$, as above, the two-sector version of the standard method is this: set δ_m and and δ_n to match I_m/K_m and I_n/K_n; set α_m and α_n to match K_m/Y_m and K_n/Y_m; and set A and D to match h_m and h_n. We are left with κ, which is hard to pin down based on steady state observations, and is therefore typically set based on direct estimates of relevant elasticities.

Since we also have a two-sector model, we use a variant of the home-production method. Thus, first set β, δ and A as above. Then set α and ψ to match *both* K/Y and LS. As we said, in the standard one-sector model, without taxes, it does not matter if one calibrates α to LS or K/Y, but with taxes calibrating α to LS yields a value for K/Y that is too low (Greenwood et al. 1995; Gomme and Rupert 2005). The idea here is to set α to match LS, then try to use ψ to match K/Y, since DM production provides an extra kick to the return on K. Given this, we set the utility parameter B and probabilities σ and ω to match some money demand observations, as discussed below, which is the analog of picking κ in home production framework, and is similar to what is done in any calibrated monetary model. This completes the heuristic description.

We now go into detail. Our benchmark model is annual to facilitate comparison with the literature, but as we discuss below, the results are basically the same for quarterly and monthly calibrations. We pin down $\beta = 1/(1+\rho)$ with $\rho = 0.025$.[15] We set $t_h = 0.242$ and

---

[15] This is the annual after-tax real interest rate in the 1951-2004 U.S. data, based on an average pre-tax

$t_k = 0.548$, the average effective marginal tax rates in McGrattan et al. (1997) (Gomme, Ravikumar and Rupert 2006 and Gomme and Rupert 2005 report similar numbers). We compute $t_x = 0.069$ directly as the average of excise plus sales tax revenue divided by consumption. We set $G/Y = 0.25$. We set $\delta = I/K = 0.070$, using residential and nonresidential structures plus producer equipment and software for K. We set $\alpha = 0.288$ to get $LS = 0.712$, which we determine using the method in Prescott (1986).

In order to pin down the fraction ω of DM trades where credit is not available, we look at two sources. First, Klee (2008) finds that shoppers use credit cards in 12% of total transactions in the supermarket scanner data. The remaining transactions use cash, checks and debit cards, which we recall from our digression on banking fit with our notion of money. We do not necessarily want to say that our DM corresponds literally to supermarket shopping, but since this is the best available data we take it as representative of shopping more generally. Second, using consumer survey data, Cooley and Hansen (1991) come up with a similar measure of around 16%. We take 15% to be a good compromise, and set $\omega = 0.85$, but we discuss robustness with respect to this particular choice below. Obviously it has to matter somewhat, since e.g. $\omega = 0$ means money is not valued and hence monetary policy is neutral, but over a reasonable range we find ω does not matter too much.

This pins down all but five parameters, A and B from preferences, the cost parameter ψ, the probability σ, and, in the bargaining model, θ. These parameters are determined simultaneously to match the following targets. First, hours worked as a fraction of discretionary time, $H = 1/3$ (from standard time-use data, as discussed e.g. in Greenwood et al. 1995). Second, average velocity, $v = 5.29$ which we measure directly using $M1$. Third, $K/Y = 2.32$ when we measure K as defined above. Fourth, a money demand elasticity of $\xi = -0.226$, which we as discussed in Appendix B.3. Fifth, in models with bargaining we target the DM markup, which we set to $\mu = 0.30$, as discussed below. We choose these parameters simultaneously to minimize the squared percentage distance between the targets in the data

---

nominal rate on Aaa-rated corporate bonds of 7.2%, an inflation rate from the GDP deflator of 3.6%, and a tax on bond returns of 30% from the NBER TAXSIM model. As is standard, although we do not explicitly incorporate a bond market in the discussion of equilibrium, obviously we can price bonds that trade between meetings of the CM in the standard way and use interest rates to pin down β.

and model. Sometimes we add the long-run elasticity of investment with respect to inflation as a target, which we estimate on quarterly data as $\zeta = -0.023$, using same method used for money demand.[16]

The parameters we fix and the calibration targets are summarized in Table 1. These are all fairly standard with the possible exception of our DM markup of 30%. To measure this, we consider the evidence discussed by Faig and Jerez (2005) from the Annual Retail Trade Survey on markups across various retail establishments. At the low end, Warehouse Clubs and Superstores come in around 17%, Automotive Dealers 18%, and Gas Stations 21%. At the high end are Specialty Foods come in at 42%, Clothing and Footware 43%, and Furniture 44%. These retailers do not deal in cash only, although some like Gas Stations, Specialty Foods, and Clothing and Footware do tend to be somewhat cash intensive, but they constitute examples of what we have in mind for the DM. We pick $\mu_D = 0.3$, right in the middle of these data. We also describe results for other values of μ_D in the robustness discussion.

3.3 Decision Rules

We first scale all nominal variables by M, so that $\hat{m} = m/M$, $\hat{p} = p/M$ etc. Then the individual state becomes $(\hat{m}, k, K)$, where in equilibrium $\hat{m} = 1$ and $k = K$. Although the above presentation was more general, now we are interested in recursive equilibrium, given by time-invariant decision rules $[q(K), K_{+1}(K), H(K), X(K)]$ and value functions $[W(K), V(K)]$ solving the relevant equations – e.g. (18)-(21), (1) and (7) in the bargaining model. We solve these equations numerically using a nonlinear global approximation, which is important for accurate welfare computations.[17] Figure 1 plots the decision rules and value function for two preferred parameterizations (Models 3 and 5 as described in the next

---

[16] Obviously, we can have more targets than parameters when we minimize deviations. Adding ζ as a target is something like adding the empirical labor supply elasticity in calibrating a standard business cycle model, which one may or may not like, but in any case we report results with and without targeting ζ in what follows. Although the estimated value of -0.023 may appear small, it is statistically significant and economically relevant: for example raising inflation from our benchmark value to 7% reduces investment by 2.3%, which is nothing to scoff at.

[17] Specifically, we use the Weighted Residual Method with Chebyshev Polynomials and Orthogonal Collocation. See Judd (1992) for details, and Aruoba et al. (2006) for a recent comparison of solution methods.

section) for four scenarios: the planner's problem; monetary equilibrium at the FR; monetary equilibrium at 10% inflation; and nonmonetary equilibrium. We discuss the economic content of these pictures below.

4 Results

4.1 Model 'Fit'

The basic calibration results are in Table 2. One column lists the relevant moments in the data, while the others list the moments from five specifications of the model. Model 1 uses bargaining in the DM with bargaining power $\theta = 1$, giving up on the DM markup μ_D as a target; it is presented mainly as a benchmark since we already proved that $\theta = 1$ implies money cannot affect the CM variables at all. Models 2 and 3 use bargaining with θ calibrated along with the other parameters; the difference between Models 2 and 3 is that the latter adds the investment elasticity ζ as a target while the former does not. Models 4 and 5 use price taking in the DM, so there is no θ, calibrating the rest of parameters to match the targets other than the DM markup; the difference between Models 4 and 5 is again that the latter adds ζ as a target while the former does not.

We do well matching the targets with two exceptions. First, we match the DM markup μ_D only if we assume bargaining and calibrate θ, rather than fixing it at 1 or assuming price taking, for obvious reasons. Second, we do a good job matching K/Y and ζ only in the price-taking model, for reasons that we now explain. Intuitively, our calibration sets the CM technology parameter α to match LS and then tries to hit K/Y using the DM technology parameter ψ (although we think this way of looking at things is instructive, it is meant only to be suggestive, since in fact we pick all of the parameters simultaneously). When $\psi = 1$, K is not used in the DM, and K/Y is too low, as in the standard model once taxes are introduced. As we increase ψ above 1 the return on K from its use in the DM increases and hence so does K/Y. But, in practice, with bargaining, this effect is tiny because the holdup problem eats up most of the DM return on K. Of course, this depends on bargaining power, but even if we pick θ to maximize K/Y we cannot get it big enough.

Intuitively, this result is due to the double holdup problem: if θ is big then buyers have all the bargaining power, which makes q big, other things being equal, but gives little return from DM trade to sellers; and if θ is small then sellers have all the bargaining power, which gives them a big share of the return, but only on a very small q. There is no way around it with bargaining. With price taking, however, the holdup problems vanish, and we can pick ψ to match K/Y exactly. The same intuition about how holdup problems affect the level of K/Y also explains how they affect the elasticity ζ: with bargaining, any extra return on K from DM production due to lower inflation will not increase aggregate K much, since the DM return is a small fraction of the overall return to investment. Again, this is not a problem with price taking, and we can hit ζ perfectly.

Earlier we alluded to the fact that we back out a DM share s_D of only around 8%, as seen in Table 2. We think this is reasonable, since it means we are not so far (in this metric) from the standard growth model. Because s_D is relatively small, however, our aggregate markup is only around 2%. This is lower than the numbers one might take away from Basu and Fernald (1997), e.g., but as a referee pointed out, it is not clear one wants to match those numbers for several reasons. In particular, we completely abstract here from any markups in the CM.[18] In any case, as we will see in the robustness analysis below, the results do not hinge much on μ_D – e.g. we can recalibrate to match an aggregate markup of 10%, which requires a much bigger DM markup, and the key numbers are quite similar (which may be surprising, but will be explained in Section 4.3).

Finally, one might ask how we match the empirical money demand curve, which is often taken to be a measure of 'fit' in monetary calibration exercises, usually using $M1$ (e.g. Lucas 2000). A plot of i versus M/pY from the data and our model looks similar to what sees with other models in the literature. As with all those approaches, it is not easy to match both the observations with very low i and high M/pY from the first decade and those with low M/pY from the last decade in the sample. But we do not put too much weight on plots of i versus M/pY, anyway, since this specification for money demand assumes a unit income

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[18] In ongoing work, Aruoba and Schorfheide (2008) introduce a markup in the CM by incorporating monopolistic competition, calibrating to around 15% in each sector.

elasticity, which is rejected in the regression results reported in Appendix B.3. From those regression results, we would have to say that our money demand curve fits quite well.

4.2 Experiments

Here we consider experiments where, starting in a steady state, we make a once-and-for-all change in the growth rate of money τ and track the behavior of the economy over time. Since inflation in steady state equals τ, we abuse language slightly and describe our experiments as a change in inflation, but note that it actually does not jump to the new steady state level in the short run (i.e. inflation may not equal τ during the transition). Table 3 contains results for each of the five models when we perform a common experiment in the literature and change $\tau^1 = 0.1$ to the FR, which is $\tau^2 = -0.0239$ in the baseline calibration. For now, we make up any change in government revenue with the lump-sum tax T, and consider other fiscal options below. Table 3 presents ratios of equilibrium values of several variables at the two inflation rates.[19]

The first thing to note is that q^1/q^2 is considerably less than 1, varying between 0.67 and 0.84, depending on the model. Inflation is a tax on DM activity, and these results show that this tax is quantitatively very important for q. In Model 1 this is the only effect, since $\theta = 1$ implies monetary policy has no impact on the CM. In Models 2 and 3, monetary policy does affect the CM, in principle, but the impact is tiny, as one should expect from the discussion in Section 4.1. Models 1-3 predict that going to the FR increases aggregate output Y by 2%, essentially all due to the change in q. In Models 4 and 5 the effects are very different. First, q actually changes by more; and second, now K changes and by quite a lot – either 3% or 5%, according to Model 4 or 5. This makes CM consumption X change by about 1%, and the net impact on Y is now 3%.

Before discussing the intuition for these results, consider welfare. As is standard, we solve for Δ such that agents are indifferent between reducing τ and increasing total consumption (X, q and $\hat{q}$) by a factor Δ. We report the answer comparing across steady states – jumping

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[19] When a *1* appears in italics, the true number is not exactly unity but shows up this way due to rounding, to distinguish effects that are theoretically 0 from those that not exactly 0 but numerically very small.

instantly from τ^1 and K^1 to τ^2 and K^2 – as well as the cost of the transition from K^1 to K^2 and the net gain to changing τ starting at K^1. This net gain is the true benefit of the policy change, although we think the steady state comparison is also interesting (it tells us how much an agent facing τ^1 and K^1 would pay to trade places with someone facing τ^2 and K^2). In Model 1 there is no transition since τ does not affect K, and in Models 2 and 3 we expect it to be unimportant, since τ does not affect K much, but in Models 4 and 5 the transition is significant. We also report the net gain to reducing τ to 0, instead of all the way to FR, to check how much of the gain comes from eliminating inflation and how much comes from deflation (most comes from the former).

In Model 1, with $\theta = 1$, going from 10% inflation to the FR is worth around 3/4 of 1% of consumption, commensurate with the findings surveyed in Craig and Rocheteau (2008). In Models 2 and 3, with $\theta \approx 0.9$, this policy is worth just under 3% of consumption. Intuitively, at $\theta \approx 0.9$ the money holdup problem makes q very low, so any additional reduction is very costly. In Models 4 and 5 the steady state gain is about half that in Models 2 and 3, since the economy is closer to the first best with price taking. In Models 4 and 5 inflation has a sizable impact on K and X, but since much of the gain accrues in the long run, and agents work more and consume less during the transition, the net gain is closer to 1%. Figure 2 shows the transitions for Models 3 and 5. In Model 5, e.g., in the short run H increases around 1.5% and X falls slightly before settling down to the new steady state, while DM output jumps on impact around 35% and quickly settles down. The difference between the two panels of Figure 2 is the size of the adjustment in CM variables: with bargaining, holdup problems mean K changes only about 0.4% in in the long run, while with price-taking K changes over 4%, and this makes all CM effects bigger.

Table 4 compares the FR and FB allocations. The differences are big, mainly due to taxation (McGrattan et al. 1997 find similar results in standard nonmonetary models). We also report the gain to moving from the FR to the FB after setting $t_h = t_k = t_x = 0$ and recalibrating parameters. In Models 4 and 5, the gain in this case is 0 because as we showed the FR implements the FB. In Model 1, with capital holdup but no money holdup, the steady state gain is around 4%, although much is lost in transition. In Models 2 and 3, with both holdup problems, the steady state gain is around 16% and 22%.[20] These calculations provide measures of the impact of holdup problems: based on the steady state comparisons, e.g., one could say 4% of consumption is the cost of capital holdup and an additional $12\% - 18\%$ is the cost of money holdup. Although there is no single 'correct' way to decompose these effects, this suggests holdup may be quantitatively important, even though bargaining occurs only in the DM and s_D is only around 8%.[21]

Table 5 reports the actual allocations, not just the ratios of the allocations, at different τ, to facilitate comparisons across models. Notice that q is considerably lower in Models 2 and 3 than in other models, showing the impact of the money holdup problem. The table also reports the allocation in the nonmonetary equilibrium, which can be considered the limit as inflation goes to ∞. Although we can of course compute the cost of very high inflation – e.g., going from 100% inflation to the FR is worth around 13% in Model 3 and 9% in Model 5 – one should take these calculations cautiously for (at least) two reasons. First, agents may well devise other ways to trade in the DM at very high inflation – e.g. ω may vary with policy, as in Dong (2008), even if one thinks it might not vary much for τ below 10%. Second, our numerical results are more sensitive to parameter choices at very high inflation rates.

At the risk of redundancy, we also discuss results using the decision rules. In Figure 1, for Model 5 we see that as we lower τ the decision rule for K_{+1} shifts up, and steady state K increases, although it is still far from the FB even at the FR (the symbols on each curve show the location of the steady state, but the FB steady state $K = 3.59$ is off the chart).

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[20] When Model 2 is recalibrated with no taxes, we set ψ equal to its calibrated value in the benchmark calibration as the calibration routine increases it without bound.

[21] The calibrated parameters differ across the columns in Table 4. Suppose we instead fix the parameters as in Model 3, and consider three cases: (i) $\theta = 1$; (ii) θ calibrated; and (iii) price taking. With taxes, going from the FR to FB in these three scenarios is worth, in terms of steady state (net) comparisons: (i) 39.52 (22.30); (ii) 43.95 (26.53); and (iii) 7.62 (4.19). With taxes set to 0 we get: (i) 11.32 (3.99); (ii) 15.86 (8.42); and (iii) 0 (0). Looking at the results without taxes, one could say the cost of capital holdup in terms of steady state is 11.32, or 3.99 with transition, and the cost of money holdup is 4.54, or 4.43 with transition. With taxes the cost of capital holdup including transitions is 18.11 while the cost of money holdup is 4.23. Again, there is no single way to measure these effects, but all of this indicates that holdup problems may well be important.

Also, the decision rule for q shifts up, increasing q in the short run and more in the longer run as we move along the decision rule for q with the growth in K. The latter effect is important here, since K grows a lot. In Model 3 the decision rule for K_{+1} and hence steady state K change little. The decision rule for q shifts, giving a short-run effect, but there is little additional long-run effect. Still, inflation is very costly in Model 3 because the decision rule for q at the FR is quite far from the decision rule at the FB, so any change in q matters a lot, while in Model 5 the decision rules for q at the FR and FB are virtually coincident.

One can also consider lowering τ and making up the revenue shortfall with proportional rather than lump-sum taxes. Table 6 reports results for the case where we make up revenue with lump-sum taxes, reproducing Table 3, and with labor or consumption taxes; we could not solve the case where we make it up with capital taxation, since increasing t_k lowered K by so much that sufficient revenue was not forthcoming (presumably the current tax rate is on the wrong side of the Laffer curve). Since we are using $M1$, in this baseline calibration, we realistically assume government seigniorage revenue is only 10% of τ times the change in M in a given period.[22] Going to the FR and making up the revenue with labor taxes e.g. requires raising t_h from 24.2% to between 24.4 and 24.7%, which is a small change. On net, even though higher taxes are distortionary, the overall impact of an inflation reduction is to improve welfare in all models.

In the last two columns of Table 6, mainly for the sake of comparison, we report results for the extreme assumption that the government is able to collect seigniorage revenue from all of $M1$, as in e.g. Cooley and Hansen (1991). For Model 4, the new labor income tax required to make up for lost revenue is now 29.3% and the net effect of the policy is a welfare loss worth about 1.2%. However, for Model 5, which has a higher cost of inflation due to holdup problems, even though the labor income tax increases to 30.3%, there is still a welfare gain of 0.01%. Using sales taxes, the welfare gain is 0.74%. In general, these results are somewhat sensitive to the details of the calculation, but we still think they are interesting. For instance, we find that going to FR is may be the best policy, even if we cannot make

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[22] For these experiments, we allow the government to issue a bond paying interest equal to the discount rate so that we do not have to adjust taxes each period during the transition.

up the revenue with lump sum taxes, while e.g. Cooley and Hansen (1991) report this can never be beneficial.[23]

4.3 Robustness

We redid all the calculations for many alternative specifications, but in the interest of space, in Table 7 we report the results in terms of one statistic: the net welfare gain of going from 10% inflation to the FR. The first row is the benchmark model. The first robustness check involves shutting down the distorting taxes, both for the case where other parameters are kept at benchmark values, and when they are recalibrated. Most of the results are similar to the benchmark calibration, although the cost of inflation is somewhat lower, especially under price taking. This is because the FR achieves the FB under price taking without distortionary taxes, and hence the cost of moderate inflation is low, by the envelope theorem. It is no surprise that some results depend on what one assumes about taxation, and since taxes are a fact of life, we trust the benchmark calibration. We then varied the preference parameters b, ε and η. One can look at the numbers for oneself, but we conclude the results are not overly sensitive.[24]

We then consider changing our target for the DM markup. It simply does not matter much – e.g. lowering μ_D to 10%, only reduces the welfare cost from 2.70% to 2.43% in Model 2 and from 2.95% to 2.83% in Model 3. This may be surprising, but it can be understood as follows. First, note that when $\theta = 1$ the markup is actually negative in Table 2, because take-it-or-leave-it offers by buyers means $p = AC < MC$. Thus, just to get $\mu_D > 0$ we need θ significantly below 1: e.g. $\mu_D = 0.01$ requires $\theta = 0.927$, which implies the money holdup problem is already important enough to generate a sizable welfare cost of around 2.43%. We also show results where we increase μ_D to 50% and 100% and those where we target an aggregate markup of 10%. The results are similar, although the numbers increase slightly as

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[23] Note that we are not saying anything here about optimal monetary policy when fiscal policy is *also* set to maximize utility, since here we are taking the existing tax rates as given from the data; see Aruoba and Chugh (2008).

[24] Lowering η generally increases the welfare cost of inflation as it makes money demand more responsive to changes in inflation, but otherwise results are in line with the benchmark calibration. One can also vary β, δ etc. over reasonable ranges without affecting things too much (not reported).

markup increases and hence the money holdup problem gets worse.

The table also shows that the results are not very sensitive to using different time periods for the calibration, and not at all sensitive to assuming a different length for a period – a quarterly or monthly instead of an annual model delivers very similar predictions. This is easy to understand: to go from an annual to a quarterly or monthly model, we simply adjust inflation, velocity, interest rates, K/Y and I/K by the relevant factor. The calibrated σ declines, because a shorter period reduces the probability of consuming in any given DM, but the welfare conclusions do not change. We find this important because changing frequency typically *does* change the results in some models, including standard cash-in-advance models, where agents generally spend all their money every period. This is not the case here.[25]

We also consider different values for the payment parameter ω. Perhaps surprisingly, the results are robust to this choice within a wide range. Even when only 25% of DM trades require cash, e.g., the welfare costs are similar, and in fact somewhat higher than the benchmark case. To understand this, first note that it is certainly true that a reduction in ω reduces the cost of inflation when we fix all other parameters. But when we recalibrate parameters, as we change ω, in order to match the calibration targets, σ increases and B falls. On net, this renders DM activity just about as important for welfare as before. Obviously, $\omega = 0$ means money is not valued and hence inflation is irrelevant, but if $\omega = 0$ then we could not match our calibration targets. For values of ω in a reasonable range, as long as we match the same targets, the net effects are very similar.

What does matter is the empirical measure of money, $M0$, $M1$, $M2$ or $M3$. These alternatives imply different values for average velocity, and given our calibration method, this changes the cost of inflation. Intuitively, consider the traditional method of computing the cost of inflation by the area under the money demand curve.[26] With a broader definition of M (i.e. lower velocity), the curve shifts up and increases the estimated cost. An apparent

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[25] We are not saying that one could not somehow introduce randomness and a precautionary demand into cash-in-advance or other models of money; we are saying it is important to do so.

[26] This discussion is heuristic and not meant as an endorsement of that method. Craig and Rocheteau (2008) discuss the relationship between the reduced-form approach and model-based welfare calculations, and show that when $\theta < 1$ the former underestimates the true welfare cost.

puzzle is that using $M3$ yields a smaller welfare cost than using $M2$. This is due to the fact that, in going from $M2$ to $M3$, while velocity is falling, the calibrated money demand elasticity ξ is also falling; the latter effect, which makes the money demand curve flatter, happens to dominate. Overall, the results indicate that the measure of money does matter, as it should, and as it will in any monetary theory. If forced to choose, we think $M1$ is most appropriate for reasons discussed in Section 2.4, but this is open to further discussion.

One can go beyond these parameter or measurement issues and consider robustness with respect to larger modeling choices. As we mentioned earlier, we also studied a version of the model with two capital stocks, K_C and K_D (see Appendix A.1). Tables 8 and 9 report results for this model with bargaining and with price taking, called Models 6 and 7. These two-capital analogs of Models 3 and 5 do about as well as in matching the targets. In Models 6 and 7, q actually increases by more than in the baseline models when we reduce τ, tending to make inflation more costly. However, there are also other effects, and the net cost of inflation is slightly lower in Model 7 than 5. These other effects occur because in Model 5 the same K is used to produce all output,while in Model 7 q is produced with K_D while X is produced with K_C. Despite these details, the overall picture from the two-capital-stock version is similar to the base case.

Tables 8 and 9 also report results from another extension where K is used in the CM only, but produced and traded in the DM (see Appendix A.2). The bargaining and price-taking versions are called Models 8 and 9. These are potentially interesting because now inflation taxes capital accumulation directly (as in earlier models by Stockman 1981 and Shi 1999), and not only indirectly via q. Now τ has a sizable effect on K under bargaining, not only under price taking. Overall, the results are not so different from the base case, however, even if the welfare cost estimates are affected somewhat. It may be worth studying these alternative models in more detail in the future, although to do so one might want to rethink the calibration strategy. We presented them here mainly to show that the basic ideas carry over to alternative formulations, and that many of the numerical results do not hinge overly critically on details.

4.4 Summary of Results

Here is what we think we learn from all of this:

- One can integrate elements from models with explicit trading frictions into capital theory in a way that in principle generates interesting effects of money on investment.
- One can use standard methods to calibrate the model, even though it contains some parameters like σ or θ that are not in standard models.
- We do a good job matching most of the targets, although obviously with price taking we cannot match the markup, and, due to holdup problems, with bargaining we cannot match K/Y or the elasticity of K with respect to i that well.
- From the calibration, one can back out the relative size of the DM and CM from observables, and our DM accounts for only around 8% of total output.
- Inflation is a tax on DM consumption q, and its impact is big.
- Qualitatively, given K is useful for producing q, inflation reduces investment; quantitatively, this effect is tiny under bargaining but big (3 to 5%) under price taking.
- The holdup problems for both money and investment are is important, even if the DM is relatively small.
- Under price taking, reducing inflation from 10% to the FR is worth 1.5% across steady states, and 1% taking into account the transition; it is worth around 3% under bargaining.
- With either price taking or bargaining, much of the gain is achieved by reducing inflation to 0 rather than going all the way to the FR.
- The costs of fiscal distortions are big, but it is still possible to argue it is desirable to replace inflation with proportional taxation.

- Most of these results are fairly robust, although the empirical measure of M does matter.
- Many of these results differ from findings in the literature (recall fn. 4).
- A key element of the framework is the explicit two-sector structure, although it does not matter much (as long as we recalibrate parameters) whether the same or different capital stocks are used in the different sectors, whether capital is traded in one setor or the other, and so on.

Perhaps the most surprising results to us are that the effects of inflation are so different under bargaining and price taking in the DM, and that in the latter case the effects on capital and output are really quite big, while in the former case the effects on capital are small, but the effects on output are still sizable. The welfare costs also differ with the DM pricing mechanism, but a distinction is that the output and investment predictions are potentially testable. We close this section by briefly discussing their plausibility. Under price taking, the model predicts that going from the FR to 10% inflation decreases output by 2 - 3% depending on the pricing mechanism, and decreases investment by 0.4% - 5% depending on the mechanism. How does this compare with data?

When one looks at time-series for the US, the relationships do not appear very strong: the correlation between real output and inflation or investment and inflation are essentially zero or slightly negative. Our empirical estimate of the elasticity of investment with respect to inflation e.g. was reported in Table 2 as $\zeta = -0.023$. Although this looks small, as argued in fn. 16 it is statistically and economically significant. In any case, our models are consistent with this evidence, since at least in some of the calibration exercises we used $\zeta = -0.023$ as a target, and at least with price taking we hit it. Hence we are in a crude sense consistent with the time series. But it may be fair to say that our analysis is more about long- than short-run effects, in the sense that although we do study the transitional dynamics, we focus on responses to fairly large one-time changes in inflation. So rather than time series, it was suggested (by referees) that we consider the cross section evidence.

Figure 3 plots average real GDP and investment expenditures versus inflation for 22 developed countries over the period 1950-1999 (GDP and investment are from Penn World Tables 6.1; inflation is from IFS). This is not meant to be a rigorous econometric analysis, and we realize there are issues of endogeneity here, but nevertheless we think these data are striking. The correlation between real GDP and inflation is -0.81 and between investment and inflation is -0.74, very strong negative relationships. While more, and more sophisticated, econometric analyses are welcome, this is not the place to get into that endeavor. The point we want to make here is that there is nothing obvious in a quick look at the data to suggest we are way off in our predictions of potentially big effects from inflation on output and capital formation.

5 Conclusion

We studied a framework with trading frictions in some markets, as in monetary theory, and capital, as in growth theory. We already summarized the findings. Our overall conclusion is that it is quantitatively relevant to incorporate elements from the literature on the microfoundations of money – including bargaining, alternating centralized and decentralized markets, and stochastic trading opportunities – into theories of investment. In terms of future work, it may be interesting to consider more general preferences, still quasi-linear but nonseperable between x and q. We explored this case, and think it is interesting because, among other things, it allows one to flexibly parameterize the degree of substitutability between CM and DM consumption. Also, this nonseperability breaks the dichotomy between the CM and DM even if K is not used in the latter. We do not go into the details here in order to focus the channel between money and investment coming from the fact that K is used in DM production, and also because it requires a totally different approach to calibration. In terms of other ideas, one could take more seriously financial intermediation, study optimal monetary and fiscal policy, examine the business cycle properties of the model, etc. All of this is left to other research.

A Appendix: Alternative Specifications

Here we sketch two extensions mentioned in the text and discussed in the robustness section. First, we relax the assumption that the same capital stock is used in both markets. Second, we assume new capital is acquired in the DM rather than the CM. To reduce notation, in this Appendix, we set $\omega = 1$.

A.1 Two Capital Goods

Suppose k_C is used in production in the CM and k_D is used in production in the DM, and they depreciate at rates δ_C and δ_D. Both k_C and k_D are produced in the CM. Following the baseline model, the same portability and recognizability assumptions imply neither k_C nor k_D can be used as a medium of exchange in the DM. For the sake of illustration, there is no tax on k_D, and we present only the bargaining version (price taking and the planner problem are similar).

The problem in the CM is now

$$\begin{aligned} W(m, k_C, k_D) &= \max_{x,h,m_{+1},k_{C+1},k_{D+1}} \left\{ U(x) - Ah + \beta V(m_{+1}, k_{C+1}, k_{D+1}) \right\} \\ \text{s.t. } (1+t_x)\, x &= w\,(1-t_h)\, h + \left[1 + (r-\delta_C)(1-t_k)\right] k_C - k_{C+1} - T + \frac{m - m_{+1}}{p} \\ & + (1-\delta_D)\, k_D - k_{D+1}. \end{aligned}$$

Eliminating h using the budget equation, we have the FOC:

$$\begin{aligned} x &: \; U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \\ m_{+1} &: \; \frac{A\,(1+t_x)}{pw\,(1-t_h)} = \beta V_m(m_{+1}, k_{C+1}, k_{D+1}) \\ k_{+1} &: \; \frac{A}{w\,(1-t_h)} = \beta V_k(m_{+1}, k_{C+1}, k_{D+1}) \\ z_{+1} &: \; \frac{A}{w\,(1-t_h)} = \beta V_z(m_{+1}, k_{C+1}, k_{D+1}). \end{aligned}$$

The envelope conditions for W_m, W_k and W_z are derived in the obvious way. The usual logic implies the distribution of (m, k_C, k_D) is degenerate for agents leaving the CM.

The DM is exactly as before, except we replace $c(q,k)$ with $c(q,k_D)$ and $g(q,k)$ with $g(q,k_D)$. The value function in the DM and the envelope conditions for V_m, V_k and V_z are derived in the obvious way. This leads to

$$\begin{aligned}
\frac{g(q,K_D)}{M} &= \frac{\beta g(q_{+1},K_{D+1})}{M_{+1}}\left[1-\sigma+\sigma\frac{u'(q_{+1})}{g_q(q_{+1},K_{D+1})}\right] &(30)\\
U'(X) &= \beta U'(X_{+1})\left\{1+\left[F_K(K_{C+1},H_{+1})-\delta_C\right](1-t_k)\right\} &(31)\\
U'(X) &= \beta U'(X_{+1})\left[1-\delta_D-\frac{(1+t_x)\,\sigma\gamma(q_{+1},K_{D+1})}{U'(x_{+1})}\right] &(32)\\
U'(X) &= \frac{A\,(1+t_x)}{F_H(K_C,H)\,(1-t_h)} &(33)\\
X+G &= F(K_C,H)+(1-\delta_C)\,K_C-K_{C+1}+(1-\delta_D)\,K_D-K_{D+1} &(34)
\end{aligned}$$

where $\gamma(\cdot)$ is defined in (17). An *equilibrium* is now given by (positive, bounded) paths for (q,K_{C+1},K_{D+1},H,X) satisfying (30)-(34). It does not dichotomize, in general, since K_D is used in the DM and produced in the CM.

A.2 Capital Acquired in the DM

Suppose new k is acquired in the DM. Following Shi (1999), agents do not consume DM output q, but use it as an intermediate input that is transformed one-for-one into k, which is an input to CM production. Each period a fraction σ of agents in the DM can produce the intermediate input, and a fraction σ can transform it into capital. Although agents cannot acquire new capital in the CM, they are allowed to trade used capital. We again present only the bargaining version.

Let k be the amount of capital held by an agent entering the CM and k'_{+1} the amount of capital taken out, and hence into the next DM. We show how to construct equilibrium where the distribution of (m,k') coming out of the CM is degenerate, even though the distribution going in is not. To begin, the the CM problem is

$$\begin{aligned}
W(m,k) &= \max_{x,h,m_{+1},k'_{+1}} U(x)-Ah+\beta V_{+1}(m_{+1},k'_{+1})\\
\text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h+[r-(r-\delta)\,t_k]k+(1-\delta)\,\phi k-\phi k'_{+1}-T+\frac{m-m_{+1}}{p}
\end{aligned}$$

where ϕ is the goods price of used capital in terms of x. The FOC are:

$$\begin{aligned} x &: U'(x)=\frac{A\left(1+t_x\right)}{w\left(1-t_h\right)} \\ m_{+1} &: \frac{A}{pw\left(1-t_h\right)}=\beta V_{+1,m}(m_{+1},k'_{+1}) \\ k'_{+1} &: \frac{A\phi}{w\left(1-t_h\right)}=\beta V_{+1,k}(m_{+1},k'_{+1}) \end{aligned} \tag{35}$$

The envelope conditions are obtained in the obvious way.

Buyers in the DM spend all their money, and bring $k=k'+q$ to the CM. The bargaining solution now implies that q solves $m_b/p=g(q,r,w,\phi)$ where

$$g(q,r,w,\phi)\equiv\frac{\left(1-t_h\right)w\left[\theta c\left(q\right)+\left(1-\theta\right)c'\left(q\right)q\right]\left[r-\left(r-\delta\right)t_k+\left(1-\delta\right)\phi\right]}{\theta A[r-\left(r-\delta\right)t_k+\left(1-\delta\right)\phi]+\left(1-\theta\right)\left(1-t_h\right)wc'\left(q\right)}.$$

In the DM, we have

$$V(m,k')=W(m,k')+\sigma\left\{\tfrac{A[r-(r-\delta)t_k+(1-\delta)\phi]q(m)}{w(1-t_h)}-\tfrac{Am}{pw(1-t_h)}\right\}+\sigma E\left\{\tfrac{A\tilde{m}}{pw(1-t_h)}-c\left[q\left(\tilde{m}\right)\right]\right\},$$

where the expectation is with respect to the money holdings $\tilde{m}$ of agents and we assume you visit one at random (we will establish, but have not yet established, that $\tilde{m}=M$ is degenerate). Then

$$\begin{aligned} V_m(m,k') &= \frac{\left(1-\sigma\right)A}{pw\left(1-t_h\right)}+\frac{\sigma\left[r-\left(r-\delta\right)t_k+\left(1-\delta\right)\phi\right]}{pw\left(1-t_h\right)g_q(q,r,w,\phi)} \\ V_k(m,k') &= \frac{A\left[r-\left(r-\delta\right)t_k+\left(1-\delta\right)\phi\right]}{\left(1-t_h\right)w}. \end{aligned}$$

Since V_m is independent of k', the FOC for m_{+1} in (35) implies m_{+1} is independent of k'_{+1} and hence degenerate.

The analog to (18) is

$$\frac{\hat{g}\left(q,K,H,\phi\right)}{F_H\left(K,H\right)M}=\frac{\beta\hat{g}\left(q_{+1},K_{+1},H_{+1},\phi_{+1}\right)}{F_H\left(K_{+1},H_{+1}\right)M_{+1}}\left[1-\sigma+\sigma\Xi(q_{+1},K_{+1},H_{+1},\phi_{+1})\right] \tag{36}$$

where

$$\begin{aligned} \hat{g}(q,K,H,\phi) &\equiv g\left[q,F_K(K,H),F_H(K,H),\phi\right] \\ \Xi(q,K,H,\phi) &\equiv \frac{F_K\left(K,H\right)\left(1-t_k\right)+\delta t_k+\left(1-\delta\right)\phi}{\hat{g}\left(q,K,H,\phi\right)}. \end{aligned}$$

The FOC for k'_{+1} is

$$\frac{\phi}{F_H(K,H)} = \frac{\beta\left[F_K(K_{+1},H_{+1})(1-t_k)+\delta t_k+(1-\delta)\phi_{+1}\right]}{F_H(K_{+1},H_{+1})},$$

which is an arbitrage condition that implies the demand for k'_{+1} is indeterminate. Hence we can set $k'_{+1}=(1-\delta)K$ for all agents, so (m_{+1},k'_{+1}) is degenerate. The other conditions are

$$\begin{aligned} K_{+1} &= (1-\delta)K+\sigma q_{+1} && (37)\\ U'(X) &= \frac{A(1+t_x)}{(1-t_h)F_K(K,H)} && (38)\\ X+G &= F(K,H) && (39)\end{aligned}$$

An *equilibrium* is now given by paths for (q,ϕ,K_{+1},H,X) satisfying (36)-(39). Again, it obviously does not dichotomize, in general.

B Appendix: Details

B.1 The Cost Function

Here we verify the properties of the DM cost function $c(q,k)$, derived from a production function $q=f(k,e)$ that is strictly increasing and concave, and a disutility of effort $\ell(e)$ that is is strictly increasing and convex. By definition, saying k is a normal input means that, in the problem $\min\{we+rk\}$ s.t. $f(k,e)\geq q$, the solution satisfies $\partial k/\partial q = f_e f_{ek}-f_k f_{ee}>0$. To proceed, rewrite $q=f(k,e)$ as $e=\xi(q,k)$ Then $\partial e/\partial q=\xi_q=1/f_e>0$ and $\partial e/\partial k=\xi_k=-f_k/f_e<0$. Also $\xi_{qq}=-f_{ee}/f_e^3>0$, $\xi_{kk}=-\left(f_e^2 f_{kk}-2f_e f_k f_{ke}+f_k^2 f_{ee}\right)/f_e^3>0$, and $\xi_{kq}=-\left(f_{ek}f_e-f_{ee}f_k\right)/f_e^3$. Hence, $c_q=\ell'/f_e>0$, $c_k=-\ell' f_k/f_e<0$, $c_{qq}=\left[\ell''\ell'^2 f_e-\ell' f_{ee}\right]/f_e^3>0$, $c_{kk}=-\left[\ell'\left(f_e f_{kk}-2f_e f_k f_{ke}+f_k^2 f_{ee}\right)-f_e f_k^2\ell''\right]/f_e^3>0$ and $c_{qk}=-\left[\ell'' f_e f_k-\ell'\left(f_k f_{ee}-f_e f_{ek}\right)\right]/f_e^3$. These results establish that c is increasing and convex in q and decreasing and convex in k, and that $c_{qk}<0$ if k is a normal input, as claimed.

B.2 Money Demand Elasticity

The interest elasticity of money demand is $\xi=\frac{\partial(M/P)}{\partial i}\frac{i}{M/P}$. To compute this in the bargaining model (price taking is similar) we need to determine $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$ and substitute them into (29). Eliminating X, we can write the steady state as 3 equations in (q, K, H):

$$\frac{i}{\sigma\omega} = \frac{u'(q)}{g_q(q,K)} - 1$$
$$\rho = \left[F_K(K,H) - \delta\right](1-t_k) - \frac{\sigma(1+t_x)}{U'\left[F(K,H)-\delta K - G\right]}\left[\omega\gamma(q,K) + (1-\omega)(1-\theta)c_k(\hat{q},K)\right]$$
$$U'\left[F(K,H) - \delta K - G\right]F_H(K,H) = \frac{A(1+t_x)}{(1-t_h)}$$

where $\hat{q}$ solves $u'(\hat{q}) - c_q(\hat{q},K) = 0$ with $d\hat{q}/dK = c_{qk}(\hat{q},K)/\left[u''(\hat{q}) - c_{qq}(\hat{q},K)\right]$.

We take the total derivative of this system to obtain

$$B\begin{bmatrix} dq \\ dK \\ dH \end{bmatrix} = \begin{bmatrix} di \\ 0 \\ 0 \end{bmatrix}$$

where

$$B = \begin{bmatrix} \frac{\sigma\omega(g_q u'' - u' g_{qq})}{g_q^2} & -\frac{\sigma\omega u' g_{qk}}{g_q^2} & 0 \\ -\frac{\sigma\omega(1+t_x)\gamma_q U'}{U'^2} & \Theta & \frac{(1-t_k)U'^2 F_{KH} + (1+t_x)U'' F_H[\sigma\omega\gamma + \sigma(1-\varepsilon)(1-\theta)c_k(\hat{q},K)]}{U'^2} \\ 0 & (F_K - \delta)F_H U'' + F_{KH}U' & F_H^2 U'' + F_{HH}U' \end{bmatrix}$$

and

$$\begin{aligned} \Theta &= (1-t_k)F_{KK} - \frac{(1+t_x)}{(U')^2}\left\{\sigma\omega\gamma_k U' + \sigma(1-\omega)(1-\theta)\left[c_{qk}(\hat{q},K)\frac{d\hat{q}}{dK} + c_{kk}(\hat{q},K)\right]\right. \\ &\quad \left. - (F_K - \delta)\left[\sigma\omega\gamma + \sigma(1-\varepsilon)(1-\theta)c_k(\hat{q},K)\right]U''\right\} \end{aligned}$$

and all DM objects without an explicit argument refer to those with q. We can now compute the partials as

$$\frac{\partial q}{\partial i} = B_{11}^{-1}\ \frac{\partial K}{\partial i} = B_{21}^{-1}\ \frac{\partial H}{\partial i} = B_{31}^{-1}$$

where B_{ij}^{-1} refers to the (i,j) element of B^{-1}.

B.3 Money Demand Estimation

Here we clarify how we get our empirical elasticity of money demand with respect to the nominal rate, ξ. Following a common specification in the literature (e.g. Goldfeld and Sichel 1990), we write the log of real money ($\tilde{m}_t$) as a linear function of log nominal interest ($\tilde{\imath}_t$) and log real output ($\tilde{y}_t$), allowing for first-order autocorrelation in the residuals. We estimated this using levels and first differences, but since the relevant results are statistically identical we report only the latter:

$$\Delta\tilde{m}_t = \beta_y\Delta\tilde{y}_t + \beta_i\Delta\tilde{\imath}_t - \rho\beta_y\Delta\tilde{y}_{t-1} - \rho\beta_i\Delta\tilde{\imath}_{t-1} + \rho\Delta\tilde{m}_{t-1} + \nu_t$$

$$\beta_y = 0.369\ \ (0.124)\,,\ \beta_i = -0.226\ \ (0.045)\,,\ \rho = 0.347\ \ (0.131)\,,\ R^2 = 0.423$$

Here ρ is the AR(1) coefficient for the residuals in the original equation in levels and the numbers in parentheses are standard errors. The long-run interest elasticity is $\xi = -0.226$, with a relatively small standard error of 0.05. By the standards of the relevant literature, this specification fits quite well.

B.4 Existence and Uniqueness

Here we show that for the functional forms we use in the calibrated model, under pricing taking and $\omega = 1$, a steady state exists and under certain conditions is unique. With the functional forms in question, (25), (26), (20) and (21) can be written:

$$\frac{K^{1-\psi}}{q^{-\psi}} = \frac{\beta}{1+\pi}\left[(1-\sigma)\frac{K_{+1}^{1-\psi}}{q_{+1}^{-\psi}} + \sigma\psi(q_{+1}+b)^{-\eta}q_{+1}\right] \tag{40}$$

$$\frac{X_{+1}^{\varepsilon}}{X^{\varepsilon}} = \beta(1-t_k)\left[\alpha\left(\frac{K_{+1}}{H_{+1}}\right)^{\alpha-1} + 1 - \delta\right] - \frac{\sigma\beta(1+t_x)(1-\psi)}{B}\frac{X_{+1}^{\varepsilon}K_{+1}^{-\psi}}{q_{+1}^{-\psi}} \tag{41}$$

$$X = \left[\frac{B(1-\alpha)(1-t_h)}{A(1+t_x)}\frac{K^{\alpha}}{H^{\alpha}}\right]^{1/\varepsilon} \tag{42}$$

$$X = K^{\alpha}H^{1-\alpha} + (1-\delta)K - K_{+1} - G \tag{43}$$

Let $\mathbb{k} = K/H$, and combine (43) and (42) to get

$$\frac{\mathbb{k}}{K}\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}\mathbb{k}^{\alpha}\right]^{1/\varepsilon} = \mathbb{k}^{\alpha} + (1-\delta)\mathbb{k} + \frac{H_{+1}}{H}\mathbb{k}_{+1} - \frac{G}{K}\mathbb{k}.$$

Hence, in steady state,

$$K = \frac{\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}B\mathbb{k}^{\alpha}\right]^{1/\varepsilon} + G}{\mathbb{k}^{\alpha-1} - \delta}. \tag{44}$$

Given $b \approx 0$, (40)-(42) reduce to:

$$\begin{aligned} q &= \left(\frac{\sigma\psi}{i+\sigma}\right)^{\frac{1}{\psi+\eta-1}} K^{\frac{\psi-1}{\psi+\eta-1}} && (45) \\ X &= \left[\frac{(1-\alpha)(1-t_h)B}{A(1+t_x)}\mathbb{k}^{\alpha}\right]^{1/\varepsilon} && (46) \\ 1 &= \beta\left[1+(\alpha\mathbb{k}^{\alpha-1}-\delta)(1-t_k)\right] && (47) \\ &\quad + \tfrac{(\psi-1)\sigma\beta(1-\alpha)(1-t_h)}{A}\left(\tfrac{\sigma\psi}{i+\sigma)}\right)^{\frac{\psi}{\psi+\eta-1}} \mathbb{k}^{\frac{\alpha(\psi+\eta-1)-(1-\alpha)\psi\eta}{\psi+\eta-1}} \left\{\frac{1-\delta\mathbb{k}^{1-\alpha}}{\left[\frac{(1-\alpha)(1-t_h)B}{A(1+t_x)}\mathbb{k}\right]^{1/\varepsilon}+G}\right\}^{\frac{\psi\eta}{\psi+\eta-1}} \end{aligned}$$

Notice (47) is one equation in $\mathbb{k}$. The RHS approaches ∞ as $\mathbb{k} \to 0$ and approaches a value less than 1 as $\mathbb{k}^{1-\alpha} \to 1/\delta$. Hence a solution to (47) exists. The solution is unique if we assume $\alpha(\psi+\eta-1) < (1-\alpha)\,\psi\eta$, since then the RHS is strictly decreasing in $\mathbb{k}$. Given $\mathbb{k}$, (44) yields K, (45) yields q, (46) yields X, and $H = \mathbb{k}/K$. So we have existence, and uniqueness under a simple restriction.

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Table 1 - Benchmark Calibration

(a) ‘Simple’ Parameters

Parameters	b	$\varepsilon = \eta$	β	t_h	t_k	t_x	G/Y	δ	α	ω
Targets	0.0001	1	0.976	0.242	0.548	0.069	0.25	0.070	0.288	0.85

(b) Remaining Parameters

Parameters	A	B	ψ	σ	θ
Targets	H	v	K/Y	$-\xi$	μ_D
Target Values	0.33	5.29	2.32	0.23	0.30

Table 2 - Calibration Results

	Data	Model 1 $\theta = 1$	Model 2 calibrate θ	Model 3 calibrate θ calibrate ζ	Model 4 price taking	Model 5 price taking calibrate ζ
Calibrated Parameters						
σ		0.25	0.26	0.27	0.25	0.26
B		1.28	0.99	0.80	2.35	2.32
ψ		1.87	2.35	2.60	1.16	1.29
A		3.41	2.67	2.18	6.38	6.37
θ		–	0.90	0.90	–	–
Calibration Targets						
μ_D	30.00	-46.48 (*)	29.99	29.72	0.00 (*)	0.00 (*)
K/Y	2.32	2.16	2.19	2.18	2.32	2.40
H	0.33	0.33	0.33	0.33	0.33	0.33
v	5.29	5.30	5.31	4.85	5.29	5.28
ξ	-0.23	-0.23	-0.23	-0.22	-0.23	-0.23
ζ	-0.023	0 (*)	-0.002(*)	-0.002	-0.014 (*)	-0.023
Miscellaneous						
s_D		6.88	6.88	8.08	5.21	5.39
s_M		3.95	4.10	4.64	4.06	4.12
μ		-3.20	2.06	2.40	0.00	0.00
$q/\hat{q}$		0.87	0.56	0.57	0.80	0.83
Sq. Error		0.0024	0.0030	0.8541	0.0000	0.0011

Note: The calibration targets marked with (*) are not targeted in the corresponding model and is not included in the computation of the squared error.

Table 3 - $\tau = 0.1$ vs. FR

	Model 1	Model 2	Model 3	Model 4	Model 5
Allocation					
q^1/q^2	0.78	0.82	0.84	0.67	0.69
$\hat{q}^1/\hat{q}^2$	1.00	*1.00*	*1.00*	*1.00*	0.99
K^1/K^2	1.00	*1.00*	*1.00*	0.97	0.95
H^1/H^2	1.00	*1.00*	*1.00*	*1.00*	*1.00*
X^1/X^2	1.00	*1.00*	*1.00*	0.99	0.99
Y_C^1/Y_C^2	1.00	*1.00*	*1.00*	0.99	0.98
Y^1/Y^2	0.98	0.98	0.98	0.97	0.97
Welfare					
ss gain	0.71	2.74	2.99	1.32	1.67
transition	0.00	-0.04	-0.04	-0.30	-0.49
net gain	0.71	2.70	2.95	1.02	1.17
net gain to 0	0.67	2.10	2.29	0.89	0.98

Table 4 - FR vs. FB

	Model 1	Model 2	Model 3	Model 4	Model 5
Allocation					
q^1/q^2	0.65	0.34	0.30	0.92	0.88
$\hat{q}^1/\hat{q}^2$	0.65	0.54	0.48	0.92	0.88
K^1/K^2	0.39	0.35	0.31	0.54	0.56
H^1/H^2	0.73	0.71	0.70	0.76	0.76
X^1/X^2	0.59	0.58	0.56	0.64	0.65
Y_C^1/Y_C^2	0.61	0.58	0.55	0.69	0.69
Y^1/Y^2	0.58	0.55	0.51	0.69	0.70
Welfare					
ss gain	25.84	36.41	43.95	15.31	14.80
transition	-11.57	-14.69	-17.42	-6.95	-6.78
net gain	14.27	21.72	26.53	8.36	8.02
Welfare with no Taxes					
ss gain	3.65	16.19	21.74	0.00	0.00
transition	-2.96	-7.75	-10.13	0.00	0.00
net gain	0.69	8.45	11.61	0.00	0.00

Table 5 - Allocations

	Model 1	Model 2	Model 3	Model 4	Model 5
First Best					
$q = \hat{q}$	1.15	1.35	1.52	0.98	0.98
Y_C	0.76	0.81	0.85	0.70	0.70
Y	0.85	0.93	1.00	0.74	0.75
K	2.75	3.18	3.59	2.18	2.24
H	0.45	0.46	0.47	0.44	0.44
X	0.45	0.46	0.47	0.42	0.42
K/Y	3.24	3.44	3.58	2.95	3.01
Equilibrium at FR					
q	0.74	0.46	0.46	0.90	0.86
$\hat{q}$	0.74	0.73	0.74	0.90	0.86
Y_C	0.46	0.47	0.47	0.48	0.49
Y	0.49	0.51	0.52	0.51	0.52
K	1.08	1.10	1.11	1.18	1.25
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.26	0.26	0.27	0.27
K/Y	2.18	2.17	2.15	2.32	2.42
Equilibrium at $\tau = 0$					
q	0.70	0.43	0.44	0.82	0.79
$\hat{q}$	0.74	0.73	0.74	0.90	0.86
Y_C	0.46	0.47	0.47	0.48	0.48
Y	0.49	0.50	0.51	0.50	0.51
K	1.08	1.10	1.11	1.17	1.24
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.26	0.26	0.27	0.27
K/Y	2.19	2.18	2.16	2.32	2.41
Equilibrium at $\tau = 0.1$					
q	0.57	0.37	0.39	0.60	0.60
$\hat{q}$	0.74	0.73	0.74	0.90	0.85
Y_C	0.46	0.47	0.47	0.47	0.48
Y	0.48	0.50	0.50	0.49	0.50
K	1.08	1.10	1.10	1.15	1.19
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.26	0.26	0.27	0.27
K/Y	2.22	2.21	2.19	2.32	2.38
Nonmonetary Equilibrium					
$\hat{q}$	0.74	0.72	0.72	0.89	0.83
Y_C	0.46	0.46	0.46	0.46	0.46
Y	0.47	0.48	0.48	0.47	0.47
K	1.08	1.07	1.07	1.07	1.07
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.26	0.26	0.26	0.26
K/Y	2.30	2.25	2.24	2.29	2.29

Table 6 - $\tau = 0.1$ vs FR and...

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 3	Model 4
% of Seignorage	10%	10%	10%	10%	10%	100%	100%
Making up Revenue by T							
q^1/q^2	0.78	0.82	0.84	0.67	0.69	0.84	0.67
K^1/K^2	1.00	*1.00*	*1.00*	0.97	0.95	*1.00*	0.97
H^1/H^2	1.00	1.00	1.00	*1.00*	*1.00*	*1.00*	*1.00*
X^1/X^2	1.00	*1.00*	*1.00*	0.99	0.99	*1.00*	0.99
Y^1/Y^2	0.98	0.98	0.98	0.97	0.97	0.98	0.97
T^1/Y^1	−1.78	−1.02	−0.81	−1.13	−0.71	−2.33%	−2.51
T^2/Y^2	−1.54	−0.80	−0.57	−0.95	−0.57	−0.01%	−0.44
ss gain	0.71	2.74	3.00	1.32	1.67	3.00	1.32
transition	0.00	−0.04	−0.05	−0.30	−0.50	−0.05	−0.30
net gain	0.71	2.70	2.95	1.02	1.17	2.95	1.02
Making up Revenue by t_h							
q^1/q^2	0.78	0.82	0.84	0.67	0.69	0.87	0.67
K^1/K^2	1.01	1.00	*1.00*	0.97	0.95	1.05	1.02
H^1/H^2	1.01	1.00	1.01	*1.00*	*1.00*	1.06	1.05
X^1/X^2	1.01	1.01	1.01	*1.00*	0.99	1.09	1.06
Y^1/Y^2	0.99	0.98	0.98	0.97	0.97	1.04	1.02
New t_h	0.247	0.247	0.247	0.245	0.244	0.303	0.293
ss gain	0.45	2.48	2.70	1.18	1.58	−0.54	−1.36
transition	0.03	0.01	0.01	−0.27	−0.48	0.55	0.18
net gain	0.48	2.49	2.71	0.91	1.11	0.01	−1.18
Making up Revenue by t_x							
q^1/q^2	0.78	0.82	0.84	0.67	0.69	0.86	0.67
K^1/K^2	*1.00*	*1.00*	*1.00*	0.97	0.95	1.04	*1.00*
H^1/H^2	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	1.04	1.04
X^1/X^2	1.01	*1.00*	1.01	*1.00*	0.99	1.06	1.05
Y^1/Y^2	0.98	0.98	0.98	0.97	0.97	1.02	*1.00*
New t_x	0.075	0.075	0.075	0.072	0.071	0.140	0.129
ss gain	0.50	2.53	2.75	1.21	1.60	0.34	−0.65
transition	0.04	0.00	0.00	−0.28	−0.48	0.40	0.03
net gain	0.53	2.53	2.75	0.93	1.12	0.74	−0.62

Table 7 - Robustness

	Model 1	Model 2	Model 3	Model 4	Model 5
Benchmark	0.71	2.70	2.95	1.02	1.17
Only Lump-sum Tax					
Recalibrated	0.82	3.08	3.20	0.88	0.59
Not	0.71	2.67	2.91	0.67	0.61
Utility Parameters ε and η (Benchmark $\varepsilon = \eta = 1$)					
$\varepsilon = 2$, $\eta = 1$	0.77	2.07	3.46	0.92	1.05
$\varepsilon = 5$, $\eta = 1$	0.78	1.55	3.03	0.88	0.95
$\varepsilon = 1$, $\eta = 1/2$	0.54	3.65	6.27	1.41	1.27
$\varepsilon = 2$, $\eta = 1/2$	0.74	3.67	6.69	1.22	1.14
$\varepsilon = 5$, $\eta = 1/2$	0.75	3.81	7.38	1.11	1.07
$\varepsilon = 1$, $\eta = 2$	0.75	2.38	3.91	0.81	1.04
$\varepsilon = 2$, $\eta = 2$	0.75	1.55	4.22	0.76	0.90
$\varepsilon = 5$, $\eta = 2$	0.76	1.82	4.55	0.74	0.80
Utility Parameter b (Benchmark $b = 0.0001$)					
$b = 0.00001$	0.71	2.42	2.87	1.02	1.17
$b = 0.01$	0.71	2.35	3.61	1.03	1.18
$b = 0.1$	0.71	2.46	5.17	1.10	1.20
Markup Target (Benchmark $\mu_D = 30\%$)					
$\mu_D = 10\%$	–	2.43	2.83	–	–
$\mu_D = 50\%$	–	2.49	3.06	–	–
$\mu_D = 100\%$	–	2.86	3.26	–	–
$\mu = 10\%$	–	3.13	3.33	–	–
Measures of Money (Benchmark M1)					
$M0$	0.05	0.36	0.36	0.05	0.05
$M2$	2.05	4.36	8.32	2.62	2.36
$M3$	1.46	4.15	7.58	2.07	1.78
Frequency (Benchmark Annual)					
Quarterly	0.73	1.84	2.74	0.94	1.14
Monthly	0.72	1.51	2.66	0.95	1.18
Period (Benchmark 1951-2004)					
1961-2004	0.62	2.21	1.37	1.18	0.64
1951-1998	0.71	2.49	3.13	0.93	1.19
1986-2004	0.73	2.08	2.79	1.54	0.96
Payment Parameter ω (Benchmark $\omega = 0.85$)					
$\omega = 1$	0.72	2.52	2.91	1.02	1.19
$\omega = 0.5$	0.68	1.51	3.11	1.02	1.11
$\omega = 0.25$	0.94	1.78	3.40	1.18	1.33

Table 8 - More Robustness : Calibration Results

	Data	Model 6	Model 7	Model 8	Model 9
Calibrated Parameters					
σ		0.23	0.21	0.25	0.19
B		0.83	2.35	0.42	0.13
ψ		2.77	1.96	3.45	7.71
A		2.20	6.41	0.87	0.28
G		0.12	0.12	0.13	0.15
θ		0.84	$-$	0.42	$-$
Calibration Targets					
μ_D	30.00	30.00	0.00 (*)	30.06	0.00 (*)
K/Y	2.32	2.22	2.23	2.38	2.69
G/Y	0.25	0.25	0.25	0.25	0.25
H	0.33	0.33	0.33	0.33	0.33
v	5.29	5.07	5.28	5.51	1.53
ξ	-0.23	-0.22	-0.23	-0.08	-0.20
ζ	-0.023	-0.001	-0.023	-0.025	-0.025
Miscellaneous					
s_D		4.48	3.98	4.56	12.45
μ		1.34	0.00	1.37	0.00
Sq. Error		0.902	0.002	0.454	0.552

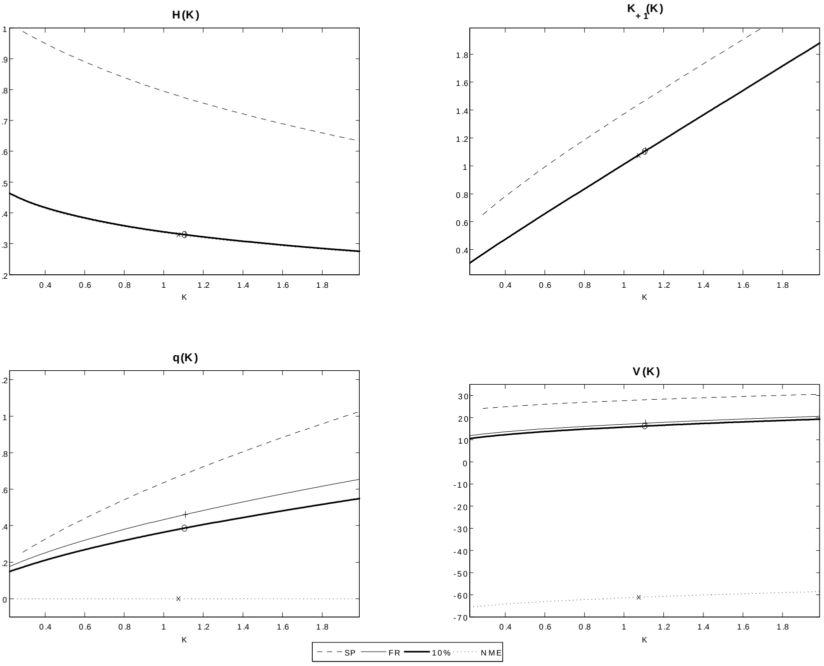
Note: The calibration targets marked with (*) are not targeted in the corresponding model and not included in the computation of the squared error.

Table 9 - More Robustness : $\tau = 0.1$ vs. FR

	Model 6	Model 7	Model 8	Model 9
Allocation				
q^1/q^2	0.66	0.62	0.81	0.93
K^1/K^2	1.00	0.99	0.81	0.93
Z^1/Z^2	0.67	0.62	$-$	$-$
ϕ^1/ϕ^2	$-$	$-$	1.10	1.03
H^1/H^2	1.00	0.99	1.02	1.01
X^1/X^2	1.00	1.00	0.94	0.98
Y_C^1/Y_C^2	1.00	0.99	0.95	0.99
Y^1/Y^2	0.98	0.97	0.93	0.92
Welfare				
ss gain	8.35	1.59	6.67	1.41
transition	-0.16	-0.49	-1.11	-0.38
net gain	8.19	1.10	5.57	1.03
net gain to 0	6.24	0.96	4.15	0.83

Figure 1 - Decision Rules and Value Functions

(a) Model 3



(b) Model 5

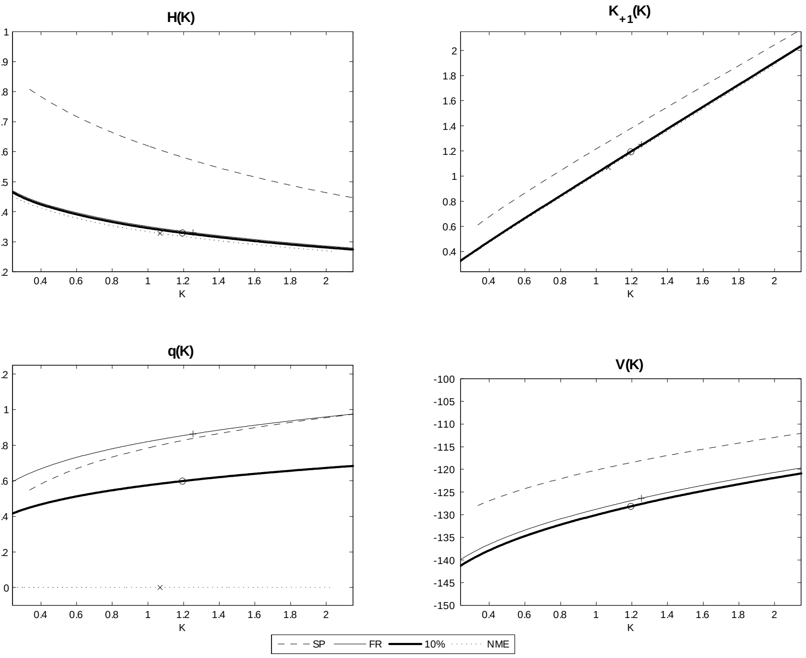
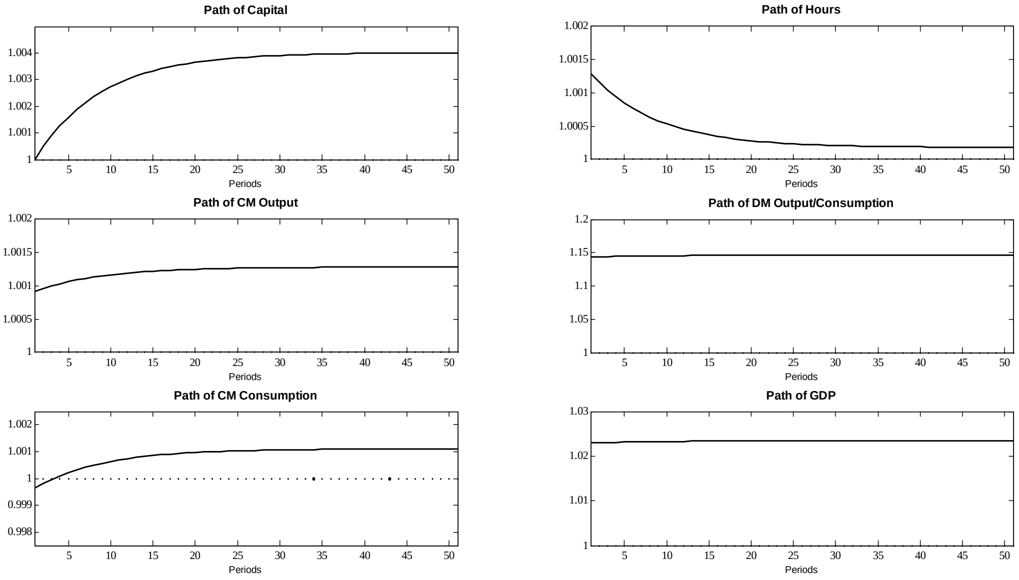


Figure 2 -10% to FR: Transitions

(a) Model 3



(b) Model 5

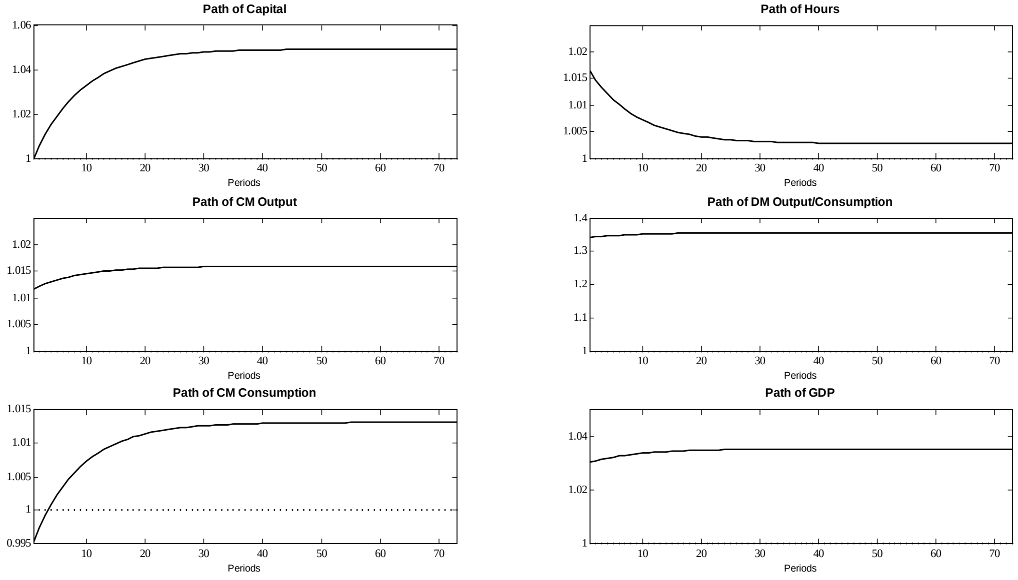


Figure 3 - Average Real GDP and Investment vs. Inflation for 22 Developed Countries (1950-1999)