

When is Money Essential?

A Comment on Aliprantis, Camera and Puzzello

Ricardo Lagos
New York University

Randall Wright
University of Pennsylvania

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Monetary theorists are concerned with, perhaps more than anything else, the essentiality of money. Money is said to be *essential* when the set of allocations that can be supported as equilibria is larger (or, sometimes, better) with money than without it.¹ Modern theory shows how various ingredients combine to make money essential: There must be some kind of a double-coincidence problem, which means that there are gains from trade between agents which cannot be exhausted by pure barter transactions; there must be imperfect commitment/enforcement, which means that agents cannot use credit to make these trades because they cannot credibly promise to honor their obligations; and there must be imperfect record keeping, or incomplete memory, or something similar that makes it difficult to use trigger strategies as a way to support cooperation. Models where these ingredients are present and hence a medium of exchange is essential include search-based monetary theory, where random bilateral matching among a large number of specialized agents generates a double-coincidence problem, and also motivates the difficulty of using punishment strategies.²

It is clear that there is no role for money as a medium of exchange without a motive for trade or with perfect enforcement or commitment to honor debt obligations. The role of memory is a bit more subtle. Suppose we try to sustain cooperative behavior, which in these models means that when I meet you and you want a good that I can produce, I should give it to you even

¹The notion of essentiality seems to go back to Hahn (1973). See Kocherlakota (1998) and Wallace (2001) for recent discussions.

²Some early papers in this now large literature include Kiyotaki and Wright (1989, 1993), Aiyagari and Wallace (1991), Kiyotaki, Matsuyama and Matsui (1993), Trejos and Wright (1995), and Shi (1995).

if you can produce nothing I want in return. Given that production is costly, what incentive do I have to behave in this way? Suppose that I deviate and fail to deliver the goods. If—and this is a big if—you can somehow report my deviation, you may be able to get other agents to punish me by not delivering goods to me in future meetings. Indeed, even if you do not know my name, and hence cannot target me specifically, you may be able to trigger the whole economy to autarky (given this is an equilibrium, which it typically is) and thereby punish me along with everyone else. If I am sufficiently patient, the threat of such punishments can give me the incentive to cooperate.

The search literature assumes, explicitly or implicitly, that these social enforcement schemes cannot work because there is no “public announcement technology” that allows you to report my deviation to the rest of the agents. Also, in these models, even if you could somehow signal through your actions to people you meet that I have failed to cooperate, and they in turn can signal this to people they meet and so on, the social punishment never comes back to haunt me because the cardinality of the set of agents is large.³ Sometimes these ideas are described by saying there is imperfect record keeping or limited memory, which can be interpreted to mean that agents are unable to keep track of the whole history of play, and in particular of other agents’ deviations. Sometimes these ideas are summarized by stating that agents are anonymous, which following various strands of existing literature (see below) can be formally interpreted to mean that agents do not observe the whole profile of individual actions. In any case, the bottom line is that deviators cannot be punished, so agents will not deliver goods unless they receive something tangible in return. Given the double-coincidence problem and the fact that goods are nonstorable, money is essential as a medium of exchange in these environments.

Early search models like those mentioned in fn. 2 are relatively simple due to extreme assumptions about the amount of money agents can carry: money holdings, m , are restricted to lie in $\{0, 1\}$. It is desirable to relax this for many reasons, including the fact that $m \in \{0, 1\}$ makes it difficult to analyze monetary policy or to do quantitative work. But when one relaxes this restriction, the distribution of m across agents becomes an endogenous state variable, and the analysis becomes very difficult.⁴ This has led some economists to construct variations

³Thus, not only is there no technology to make public announcements, but social norms along the lines of Kandori (1992) or Ellison (1994) do not work. See Araujo (2004) for a proof of this claim.

⁴See Green and Zhou (1997, 2002), Camera and Corbae (1998), Zhou (1998), Zhu (2003, 2005) and Molico (2006).

on these environments where we allow agents to hold any $m \in \mathbb{R}_+$ and yet somehow harness the distribution. One such environment is developed in Lagos and Wright (2005), henceforth LW. Agents in LW alternate over time between decentralized (bilateral) trade and centralized (Walrasian) trade. This setup is appealing because decentralized trade can make money essential, as in earlier search models, while centralized trade allows agents to rebalance their money holdings. In fact, with quasi-linear preferences, the distribution collapses: all agents take the same m out of the centralized market. This makes the framework tractable, and is one reason LW has recently become a workhorse in monetary economics.⁵

Aliprantis, Camera and Puzzello (2007), henceforth ACP, describe an environment with periodic meetings that resemble the centralized and decentralized markets in LW. In particular, they define a *match* as a partition of the set of agents, and assume in odd periods this partition consists of sets containing at most two agents (like the bilateral meetings in our decentralized markets), while in even periods it consists of a single set containing the whole population (like our centralized markets). They then argue that money is *not* essential in their model. This has led to some confusion—e.g., people keep asking us what we think of their demonstration that money is not essential in LW.⁶ After studying ACP carefully we understand how even the most conscientious reader might be led to that conclusion. But that conclusion is incorrect. To be clear, our claim is not that the analysis in ACP is wrong, but that it has no bearing on the essentiality of money in LW.

ACP and LW differ along several dimensions, but the one that is critical for the issue at hand is that ACP assumes “Agents can only trade and observe actions and outcomes in their match” (p. 278). The assumption that each agent can observe the actions of all other agents in the centralized market is not part of LW. As phrased in ACP, the new assumption may seem natural and innocuous. It is neither. It is not natural because our notion of a centralized market is the standard one of a Walrasian market, which means that agents observe prices, they maximize utility taking these as given, and markets clear; it does not mean agents can observe the actions of others. It is not innocuous because it is precisely the assumption that

⁵See Shi (1997) for an alternative approach that also generates a degenerate distribution of money holdings.

⁶Recently, an anonymous referee on a paper using a version of LW unknowingly provided us with a passage that epitomizes this confusion: “we know from Aliprantis et al. (2007, *Econometrica*) that anonymity in the environment described in this paper is not enough to give money an essential role: it is possible to achieve the first best allocation without the use of money.”

individual actions are observable that allows the use of trigger strategies and renders money inessential in ACP. The argument is just like the one we sketched earlier: if I deviate by failing to deliver goods to you in a bilateral meeting, you can take an action in the next centralized market that everyone else observes, which triggers the economy to autarky, and if agents are sufficiently patient, this threat supports cooperation in the decentralized market.

What needs to be stressed, and what most likely underlies the confusion surrounding the issue, is that the fact that agents trade in a centralized market is neither necessary nor sufficient for the observability of individual actions. As we have already mentioned, multilateral trade does not require individual actions to be observable, as standard Walrasian theory makes clear; neither is it implied that individual actions are unobservable when we assume agents trade bilaterally. Since individual actions are not observable in the centralized market in LW, that model simply does not admit the trigger strategies that ACP use to support cooperation. Hence the result in ACP does not contradict the fact that money is essential in LW. It is also worth emphasizing that we are not here arguing whether in general one ought to assume the profile of individual actions is observable or is not observable in centralized—or for that matter in decentralized—meetings. We are merely pointing out it is not observable in the centralized market in LW.⁷

Although this could be the end of the story regarding the difference between ACP and LW, we want to make one other point. An interesting part of the ACP working paper that is not part of the published version shows that *even if* one assumes agents in the centralized market can observe all actions—which we reiterate is not the case in LW—one can perturb the model in a simple way so that money is still essential. The idea is to assume that there is not one but many centralized markets. This works even with the observability of actions because the set

⁷We realize, however, that economists familiar with the work of Levine and Pesendorfer (1995), Fudenberg, Levine and Pesendorfer (1998), and Al-Najjar and Smorodinsky (2001) may find the construction of punishment strategies in ACP unappealing because, as ACP themselves note, it is not robust to adding even a small amount of noise in the observation of individual behavior. Intuitively, with some noise, no matter how small, in a large population the punishment will be triggered with probability near 1, no matter what agents do. Thus cooperation cannot be sustained in a large economy with noise in the observation of actions, just as it cannot be sustained in our formulation with unobservable actions. We also suspect that some macroeconomists will find the assumption that the entire action profile is observable to be rather unnatural, as it stands in contrast not only with Walrasian theory, but also with much existing work that casts macroeconomic environments as anonymous games along the lines of Green (1984) or Jovanovic and Rosenthal (1988). For example, in the optimal policy literature (Chari and Kehoe 1990; Phelan and Stacchetti 2001), the standard assumption is that agents are anonymous in the sense that only their average behavior is observable, not individual actions.

of agents can be partitioned across different centralized markets so as to guarantee that after I deviate on you, you never meet me again, nor meet anyone who will meet me, and so on, in each of the centralized markets you visit. So my deviation will never come back to haunt me even if you can reveal it in all the centralized markets you visit. And we still get a degenerate distribution of money holdings coming out of all centralized markets as long as we select the right mix of people going in. This type of construction shows how the basic idea in LW is actually quite robust: although we did not have observable actions in our original formulation, with a little care, one can allow observable actions without compromising either tractability or the essentiality of money.⁸

To conclude, note that the same sophism that led some readers of ACP to incorrectly conclude money is inessential in LW would imply money is inessential in *all* search-based models. The fact that LW has some centralized meetings while earlier models did not is beside the point since, as we have explained, centralized meetings are neither necessary nor sufficient for agents to have available publicly observable actions or announcements. In an environment similar to Kiyotaki and Wright (1989), one *could* assume that agents observe the actions of others even when they are not physically matched, and then use triggers to support cooperation. Money would be inessential in that scenario, but this observability assumption is not part of the original specification—just as it is not part of LW. We hope this sets the record straight.

⁸In fact, one can go further and conjecture that we do not really need centralized markets: it suffices to have agents meet bilaterally, as long as each agent who just spent his money in the previous decentralized market is matched with one who just acquired money, since this allows them both to rebalance m to the same target level. In this context it may be useful to make these bilateral meetings endogenous and directed (rather than random and undirected), as in Corbae et al. (2003). Of course, it might be less natural to have agents act like perfect competitors in bilateral meetings, but this is not a problem, since we can let them bargain instead. Again, this suggests the basic idea is quite robust, in the sense that one can harness the distribution in a variety of ways.

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