The Search-Theoretic Approach to Monetary Economics: A Primer*

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Abstract

This paper presents a simple version of the basic models used in the search-theoretic approach to monetary economics. We discuss results on the existence of monetary equilibria, the potential for multiple equilibria, and welfare. We do this for models where prices are fixed, and also where prices are determined endogenously using bilateral bargaining theory. We also discuss the nature of the frictions necessary to construct a model with an essential role for money. We conclude the paper with a review of many extensions and applications in the related literature.

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1 Introduction

This paper presents some results in monetary theory derived using very simple game-theoretic models of the exchange process. The underlying model is a variant of what is called search theory, a framework that has been used extensively in a wide variety of applications. This approach is well-suited to discussing the process of exchange, and the role that money may have in this process. In addition, the approach utilized here is explicitly strategic, in the following natural sense: when I decide whether or not to accept in trade a certain object other than one I desire for my own consumption – e.g., money – I need to formulate a conjecture regarding the probability that other agents will accept it from me in the future. This evidently ought to be modeled as a game.

In search-theory, the type of game that will be considered is explicitly dynamic, and exchange takes place in real time. Also, the models allow us to focus precisely on various frictions in the exchange process that potentially give a role for money as part of an equilibrium arrangement, or an e¢cient arrangement. These frictions include the following: agents are not always in the same place at the same time; there is no way to enforce long-run commitments (unless they are dynamically incentive compatible); and that agents are anonymous in the sense that their histories are not public information. These types of frictions are crucial for a logically coherent theory of money, and the approach described here helps to make clear the role of each of these frictions.

This approach is to be contrasted with trying to model the role of money in a competitive equilibrium (Walrasian) model – a di¢cult task that has met with, at best, mixed success in the past.[1] First, in a competitive equilibrium model, the exchange process is not explicity modeled. That is, agents start with an initial allocation A, and choose a …nal allocation B so as to maximize utility, subject to the latter not costing more than the former, but there is no discussion of how they get from point A to point B. Does some unmodeled agent (maybe the auctioneer) make the necessary trades with a "pick up and delivery service"? Or do the agents trade directly with each other? Do they trade bilaterally, or multilaterally? In real time, or before the start

[1] See Ostroy and Starr (1990) for a survey.

of production and consumption activity? Do they barter directly or trade indirectly using media of exchange? Questions like these are not addressed in the standard competitive equilibrium paradigm. By contrast, search models are designed exactly with these issues in mind.

In Section 2, we present a version of the model in Kiyotaki and Wright (1991,1993) and derive results on the existence of monetary equilibria, the potential for multiple equilibria, and welfare. We also discuss in detail the nature of the frictions necessary to construct a model with an essential role for money, following the discussion in Kocherlakota (1998). This model is quite simple because the objects that agents trade (both goods and money) are indivisible, and so we can focus on the process of exchange without worrying about the determination of exchange rates. In Section 3, we present a version of the model in Shi (1995) and Trejos and Wright (1995), where prices are also determined. In Section 4, we review some of the extensions that have been analyzed in the literature.

2 The Basic Model

2.1 General Assumptions

To model anonymous trade, it is natural to start with a large number of traders – formally, we assume a $[0,1]$ continuum of agents. We assume for simplicity that these agents live forever and discount the future at rate r. There is also a $[0,1]$ continuum of indivisible consumption goods. To generate gains from trade, we need to assume agents are specialized. There are many ways to do this, but an easy one is to assume that each agent i has the ability to produce just one type of good. The unit production cost for any agent is $c \geq 0$. Also, for convenience we assume that these goods are nonstorable, so that they have to be consumed at the same date they are produced. This obviously means that consumption goods cannot serve as media of exchange, which allows us to highlight the role of fiat money.

In order to make trade interesting in the model, we need to assume that tastes are heterogeneous. Again, there are many ways to do this, and for simplicity we assume the following. First, given any two agents i and j, write iWj to mean "i wants to consume the good that j produces" – in the sense that i derives utility $u > c$ from consuming what j produces if iWj and he derives utility 0 from consuming what j produces otherwise. Then, for any agents selected at random, we assume $\text{prob}(iWi) = 0$, $\text{pr}(jWi) = x$, and $\text{pr}(jWi|iWj) = y$. The first assumption, $\text{prob}(iWi) = 0$, means that no agent ever wants to consume his own output (which is why they trade). The second assumption parameterizes the extent of the basic search friction: the smaller is x the lower is the probability that a random trader has what you want. However, it is the third assumption that is the important one, since it parameterizes Jevons' famous (1875) so-called double coincidence of wants problem: the smaller is y, the lower is the probability that a trader who has what you want also wants what you have.[2]

In addition to the above-mentioned consumption goods, there is another object called flat money. Money here consists of an exogenously fixed quan-

[2] Note that several notions of specialization in the literature are special cases of this model. For example, in Kiyotaki and Wright (1989) or Aiyagari and Wallace (1991), there are N goods and N types of agents, where type n produces good n and wants good $n + 1 (\text{mod}\, N)$. Then $x = 1/N$, and $y = 1$ if $N = 2$ while $y = 0$ if $N > 2$. Alternatively, in Kiyotaki and Wright (1991,1993) and much of the related literature, the events $\{iWj\}$ and $\{jWi\}$ are independent, and so $y = x$.

tity of $M \in [0,1]$ indivisible units of a storable object (of course, money has to be storable to be useful). Initially, one unit of money is randomly allocated to M agents. We assume that agents holding money cannot produce (one way to motivate this is to assume that after you produce you need to consume before you can produce again). Thus, no one can ever acquire more than one unit of money, and hence and all agents always hold 0 or 1 units of money. What makes it fiat money is the assumption that it is intrinsically worthless: no one can consume it and it generates no utility for an agent holding it, unless the agent can trade for something that he does consume.

We now describe the trading process. Rather than assuming a centralized (Walrasian) market, here the agents must trade bilaterally. The simplest way to model this is to assume that they meet according to a pairwise random matching process. Upon meeting, a pair decides whether or not to trade, after which they part company and re-enter the matching process. Let α denote the (Poisson) arrival rate in the matching process – i.e., it is the probability per unit time of meeting someone.[3] For reasons that we will discuss later,

[3] It is actually the bilateral matching and not the random matching assumption that is important. Corbae, Temzelides and Wright (2000) show how to redo the model allowing

we assume here that the history of every agent's past meetings and trades is not known to anyone else.

We want to analyze the individual trading strategies of agents. First, it is obvious that you should accept a good in trade if and only if it is a good that you consume. The less obvious issue is whether you should accept money. Let ¦ be the probability that you believe that a random agent will accept money from you, and let ¼ denote your best response (i.e., if you think others accept money with probability ¦ your utility maximizing strategy is to accept it with probability ¼). We will say that money is used as a medium of exchange, or circulates, if and only if ¼ > 0. We now determine ¼ using dynamic programming. To this end, let V_0 and V_1 be the value functions (lifetime, discounted, expected utility) of agents with 0 and with 1 unit of money, respectively. Since we only consider stationary, symmetric, equilibria in this paper, V_j does not depend on time or on the agent's name, only his money inventories.

agents to choose who they meet endogenously, rather than meeting each other at random. The basic insights discussed here go through in this version of the model, although it becomes more complicated because one has to determine equilibrium meeting patterns as well as the equilibrium trades.

If we think of time as proceeding in discrete periods of length Δ, we can calculate the payoff to holding money as follows. First, the probability of meeting anyone during this period is approximately $\alpha\Delta$ by the Poisson assumption.[4] If it is the case that the person you meet can produce (i.e., he does not have money), which occurs with probability $1 - M$, and you want what he can produce, which occurs with probability x, and he accepts your money, which occurs with probability Π, then you trade, consume and continue without money, for a total payoff of $u + V_0$. In all other events (you meet no one, you meet someone with a good you do not want, etc.) you simply continue with your money, for a total payoff of V_1. Hence,

$$V_1 = \frac{1}{1 + r\Delta}\{\alpha\Delta(1 - M)x\Pi(u + V_0) + [1 - \alpha\Delta x(1 - M)\Pi]V_1 + o(\Delta)\}$$

where $o(\Delta)$ is the approximation error associated with Poisson arrivals and hence satisfies $\lim_{\Delta \to 0} o(\Delta)/\Delta = 0$.

Rearranging, we have

$$r\Delta V_1 = \alpha\Delta(1 - M)x\Pi(u + V_0 - V_1) + o(\Delta).$$

[4] That is, the probability of meeting one person in a period of length Δ is equal to $\alpha\Delta + o(\Delta)$, where $o(\Delta)$ is the approximation error, and satisfies $o(\Delta)/\Delta \to 0$ as $\Delta \to 0$.

Dividing by Δ and taking the limit as $\Delta \to 0$, we arrive at the continuous time version of Bellman's equation:

$$rV_1 = \alpha x(1 - M)\mu(u + V_0 - V_1). \tag{1}$$

An analogous argument implies that the value function for an agent without money satisfies

$$rV_0 = \alpha xy(1 - M)(u - c) + \alpha xM\pi(V_1 - V_0 - c). \tag{2}$$

The first term in this expression represents the gain from a direct barter trade, while the second represents the gain from trading goods for money with probability π, which the agent will choose optimally.

2.2 Equilibria

Define the net gain from trading goods for money by $\Delta_0 = V_1 - V_0 - c$, and the net gain from trading money for goods by $\Delta_1 = u + V_0 - V_1$. If we normalize $\alpha x = 1$ in order to reduce notation (which we can always do with no loss of generality by redefining units of time appropriately), we have

$$\Delta_1 = \frac{[M\pi + (1 - M)y](u - c) + ru}{r + M\pi + (1 - M)\mu} \tag{3}$$

$$\Delta_0 = \frac{(1 - M)(\Pi - y)(u - c) - rc}{r + M\pi + (1 - M)\Pi}. \tag{4}$$

Given Π, the optimal strategy π for the individual satisfies:[5]

$$\pi \begin{cases} = 1 \\ \in [0;1] \\ = 0 \end{cases} \text{ as } \Delta_0 \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \tag{5}$$

Figure 1 shows the best response correspondence, $\pi = \pi(\Pi)$ (for parameters such that $r < \hat{r}$, where $\hat{r}$ is defined below). Notice that your best response is to accept money if and only if the probability that other agents accept money is above a threshold $\hat{\pi}$, where

$$\hat{\pi} = \frac{rc + (1 - M)y(u - c)}{(1 - M)(u - c)}. \tag{6}$$

A special case of interest is when production is free, $c = 0$, which implies $\hat{\pi} = y$. This says that when $c = 0$ you should trade goods for money if and only if it is at least as easy to get your consumption good with money as it is by direct barter (i.e., if and only if $\pi \geq y$). When $c > 0$, money actually has to have a strictly greater probability of being accepted than a barter

[5] Notice that we only consider explicitly the best response of the agent deciding to accept money. In principle, the agent with the money also needs to decide whether he wants to trade; but since $\Delta_1 > 0$ he always does want to trade in this model.

trade for you to be indifferent to accepting money or not, because you have to incur the production cost now to get the money.

A symmetric equilibrium is defined, in the usual way, as a fixed point of this correspondence: taking Π as given, the individual's choice is $\pi = \Pi$. In order to characterize the set of equilibria, first notice that there is always an equilibrium where $\pi = 0$, since $\pi = 0$ implies $\Delta_0 < 0$. Let

$$\bar{r} = \frac{(1 - M)(1 - y)(u - c)}{c}. \qquad (7)$$

We claim the following: if $r > \bar{r}$ then $\pi = 0$ is the only equilibrium; if $r < \bar{r}$ there are two other equilibria, $\pi = 1$ and $\pi = \hat{\pi} \in (0, 1)$; and these are the only (symmetric, stationary) equilibria. To check $\pi = 1$, note that $\pi = 1$ implies $\Delta_0 \geq 0$ if and only if $r \leq \bar{r}$. To check for $\pi \in (0, 1)$, set $\Delta_0 = 0$, solve for $\pi = \hat{\pi}$, and verify that $\hat{\pi} \in (0, 1)$ if and only if $r < \bar{r}$.

Several comments are in order concerning these results. First, since the money is fiat money, there is always a nonmonetary equilibrium where it does not circulate. This illustrates that the value of fiat money is always somewhat tenuous – it depends on beliefs, in the sense that if agents think it will not be valued then it will not be valued. Second, notice that whenever monetary equilibria exist, multiple equilibria exist, where π can be either 0, 1, or between 0 and 1. Also notice that for monetary equilibria to exist at all agents need to be relatively patient (we need r to be small), since they pay the cost of producing to get money today but only get the benefits in the future when they meet someone who produces a good they like and accepts money. Alternatively, we can rearrange $r < \hat{r}$ to say that monetary equilibria exist if and only if $M \leq \hat{M}$ where

$$\hat{M} = 1 - \frac{rc}{(1-y)(u-c)}. \tag{8}$$

This says that monetary equilibria only exist if money is not too plentiful. The intuition for this is as follows: as the fraction of agents with money becomes larger, the probability of finding an agent without money, and hence the value of holding money, decreases. Consequently, money will be accepted with positive probability only if the amount of money in the economy is sufficiently small.

2.3 Welfare

One question that can be addressed in this framework is how money affects the efficiency of exchange and welfare. We define welfare as $W = MV_1 + (1 - M)V_0$. After simplification, this implies that in equilibrium (i.e., when $\Pi = \pi$) we have

$$rW = (1 - M)[(1 - M)y + M\pi](u - c). \tag{9}$$

The first thing to note is that, given M, welfare is increasing in π; in particular, agents are better off when money circulates than when it does not. This is due to a liquidity effect: money facilitates exchange and so increases the frequency of consumption.

We now ask how welfare depends on M. This is complicated by the fact that, while money does play a useful role in exchange, the way we set up the model implies that money also crowds out production because money holders cannot produce. While one can also proceed under the alternative specification that allows money holders to produce, for our purposes the current version will suffice. In particular, we will now show that W can be increasing in M over some region, due to the liquidity effect, even though money crowds out production.

To proceed, first consider the pure strategy monetary equilibrium, $\pi = 1$, and suppose we maximize W by choosing M subject to the constraint $M \leq \bar{M}$, where $\bar{M}$ was defined in (8), which is necessary for a monetary equilibrium to exist. Let use assume $rc < (1 - y)(u - c)$, so that $\bar{M} > 0$. Then the solution is as follows: $M^* = 0$ if $y \geq 1/2$, while $M^* = \min\left\{\frac{1-2y}{2-2y}; \bar{M}\right\}$ if $y < 1/2$. Notice that $M^* = 1/2$ if $y = 0$ and the constraint is not binding, and that M^* is decreasing in y.[6] To understand this, note that the role of money here is to allow trade to take place when agent i wants what agent j can produce but not vice-versa, and i has money but j does not. Maximizing the probability of this event implies there should be the same number of agents with and without money: $M = 1/2$. But one also has to take into account the fact that money crowds out barter when $y > 0$ (if $y = 0$ there is no barter to crowd out). Therefore, if $y > 0$ we have $M^* < 1/2$. If $y \geq 1/2$ the

[6] The following generalized version of this result holds in a model where agents with money are allowed to produce, and thereby allowed to accumulate more than one unit of money: If $\hat{M}$, say, is any upper bound on the number of units of money an agent can store, then with $y = 0$ the optimal money supply is $\hat{M}/2$. See Berensten (1999b).

liquidity effect is completely dominated by the crowding out effect so that $M^* = 0$.

For completeness, consider the other equilibria. In a non-monetary equilibrium ($\pi = 0$), obviously we have $M^* = 0$. Again, as the only role of money is to serve as a medium of exchange, if it is never accepted it cannot provide liquidity and merely crowds out trade. For the mixed strategy monetary equilibrium, after inserting $\pi = \hat{\pi}$, we have

$$rW = y(u - c) + [rc - y(u - c)]M.$$

Hence, welfare is linear in M. If $rc > y(u - c)$, we have $M^* = \bar{M}$, and if $rc < y(u - c)$, we have $M^* = 0$. Figure 2 shows W as a function of M, where the labels W_1, W_0 and W_π denote welfare in equilibria with $\pi = 1$, $\pi = 0$, and $\pi \in (0,1)$, respectively. The curves for W_1 and W_π are only drawn for $M \leq \bar{M}$, since these equilibria exist only in this range. In the figure, $M^* < \bar{M}$ for the chosen parameter values. Figure 3 shows W as a function of y, given that M is set to the welfare maximizing M^* for each value of y, as well as M^*.

2.4 The Essentiality of Money

Following Kocherlakota (1998), at this stage, it is instructive to highlight the role played by various frictions in the model, so as to understand what it is that makes money essential.[7] First, we need some sort of double coincidence of wants problem. For this it is important that not everybody can trade mutilaterally. For example, consider an economy with three agents, where each agent of type $n \in \{1, 2, 3\}$ produces good n and wants good $n+1 (\text{mod } 3)$. If all agents meet at the same place and are able to make multilateral trades, it is feasible for each agent to produce and consume in the period: agent 2 produces for agent 1, agent 1 produces for agent 3, and agent 3 produces for agent 2. Our restriction to bilateral meetings, given specialized tastes and technology, is merely a convenient way to generate a double coincidence problem. In the 3-agent example, in every bilateral meeting one agent wants what the other produces while the other does not.

Even given a double coincidence problem, however, for money to have an

[7] Hahn (1965) said that money was essential if there are (desirable) outcomes that can be supported as equilibria with money but not without money.

essential role, it is also important that agents cannot commit to long term agreements. Consider the following credit arrangement: "produce for anyone you meet that wants your good." This arrangement resembles credit in the sense that agents receive consumption today in exchange for nothing but an implicit "promise" to repay (someone) in the future in kind. It is also obviously an efficient arrangement – i.e., it generates the maximum possible expected utility, let us call it $W_c = (u - c)/r$, where the subscript on W_c stands for credit. If agents could commit to this arrangement ex ante they would do so, and there would be no need for money. So clearly an imperfect ability to commit to future actions is essential if money is going to have a role.

However, even in the absence of explicit commitments, cooperative agreements like the credit arrangement can sometimes be enforced by reputational considerations if individual actions are public information. Thus, consider the arrangement "produce for anyone who wants your production good as long as everyone else has done so in the past; as soon as someone deviates from this, trigger to plan X," where plan X is to be determined. Of course, we need plan X to be self-enforcing (i.e., it has to be an equilibrium), and we want the outcome of plan X to be sufficiently unpleasant that it keeps people from deviating from the efficient arrangement. We will assume here that plan X is to trade if and only if there is a double coincidence of wants, which generates expected utility $W_b = y(u - c)/r$, where the subscript stands for barter.[8]

It is in individual's self interest to not deviate from the credit arrangement when they are supposed to produce if and only if $-c + W_c \geq W_b$, which after simplification reduces to $r \leq \hat{r} = (1 - y)(u - c)/c$. As always, if agents are sufficiently patient, the threat of triggering to pure barter supports the efficient outcome, and again money is not essential. Of course, this assumes that agents' trading histories are publically observed, since otherwise it is not possible for agents to use trigger strategies.[9] When trading histories are

[8] We do not trigger to autarchy because we assume that if two agents want to barter without it being observed, they can; hence, the worst possible equilibrium is the one where double coincidence trades occur but no other trades occur. Nothing much hinges on this – a similar message holds if we can trigger to autarchy, say, and in fact it is easier to support credit-like arrangements by triggering to autarchy.

[9] If there was a small number of agents then even if agents did not observe all other agents' histories, but only their own history, we could potentially support the efficient arrangement by the following strategies: if ever you directly observe someone deviate (by

private information, without money the only sustainable outcome is pure barter. With money, however, we can do better than pure barter even with private trading histories. We saw above that monetary equilibria exist as long as $r \leq r_1 = (1 - M)(1 - y)(u - c)/c$. Notice that monetary exchange generates lower welfare than the credit arrangement, although more than pure barter.

Money does not do as well as credit because of the random meeting technology and the fact that money holdings are bounded, as this leads to some meetings where I want your good and you do not want my good, but either I have no money or you already have money. In these meetings, monetary exchange will not work, while credit could work as long as we have publically observed trading histories. Even if we relax the upper bound on money hold-

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not producing for you when you would like him to), stop producing for anyone else. This would set off a chain of agents who observe deviations and would eventually lead the economy into autarchy. With a large number of agents, however, if I fail to produce for you, there is zero probability that in the future I will meet you or I will meet someone who has met you, and so the chain will never get back to me. Hence, with a large number of agents, it does not suffice in terms of supporting credit to have agents observe only their own histories. See Araujo (2000) or Corbae, Temzelides and Wright (2000) for more discussion.

ings, the fact that money holdings must be bounded below by zero means that money cannot do as well as credit in a random matching environment. In any case, we conclude that there is an essential role for money in the model because of three things: the double coincidence problem, the lack of commitment, and private information concerning trading histories. For an extended discussion of these issues, see Kocherlakota (1998, 2000).

3 Prices

In this section, we provide an extension where the assumption of indivisible goods is relaxed, although money is still indivisible and so agents will always have either 0 or 1 unit of money. Based on Shi (1995) and Trejos and Wright (1995), we will use bargaining theory to endogenously determine prices. For simplicity, here we set $y = 0$ so that there is no direct barter.[10]

Given goods are perfectly divisible, let $u(q)$ be the utility of consuming q units of one's consumption good and $c(q)$ the disutility of producing q units of

[10] Shi (1995) and Trejos and Wright (1995) also consider the case with barter. In Rupert, Schindler and Wright (2000) we present the general case, and also use a more general bargaining solution.

one's production good. We assume $u(0) = c(0)$, $u'(0) > c'(0) = 0$, $u'(q) > 0$, $c'(q) > 0$, $u''(q) \leq 0$, and $c''(q) \geq 0$, for $q > 0$, with at least one of the weak inequalities strict. For future reference, we define q^* by $u'(q^*) = c'(q^*)$. Also, there is a $\hat{q} > 0$ such that $u(\hat{q}) = c(\hat{q})$. (See Figure 4 for an illustration.) When a buyer meets a seller who can produce the right good, they bargain over how much q will be exchanged for the buyer's unit of money, implying a nominal price $p = 1/q$. Otherwise the model is exactly the same as in the previous section.

Letting V_1 and V_0 denote the value functions and taking $q = Q$ as given, the generalizations of the dynamic programming equations can be expressed as

$$rV_1 = (1 - M)[u(Q) + V_0 - V_1] \quad (10)$$

$$rV_0 = M[V_1 - V_0 - c(Q)]. \quad (11)$$

These can be easily solved for $V_1 = V_1(Q)$ and $V_0 = V_0(Q)$. Taking $V_1(Q)$ and $V_0(Q)$ as given, q will solve a bargaining problem. In a (symmetric) equilibrium, of course, $q = Q$. The bargaining model can be formulated in several different ways, but for simplicity here we adopt the symmetric Nash bargaining solution with zero threat points,

$$q = \arg\max \, [u(q) + V_0(Q)][V_1(Q) - c(q)]; \qquad (12)$$

where the maximization is subject to $u(q) + V_0 \geq V_1$ and $V_1 - c(q) \geq V_0$.[11]

The bargaining solution (12) defines a mapping $q = q(Q)$ from $[0; \hat{q}]$ into itself. That is, if other agents are giving Q units of output for one unit of money, then a particular pair bargaining bilaterally will agree to $q = q(Q)$. An equilibrium is a fixed point, $q = q(Q)$. In general, we have to be careful with the constraints on the bargaining problem. It turns out that when $y = 0$, the constraints are never binding in equilibrium. However, if $y > 0$ the constraints may bind, and it is therefore instructive to proceed allowing for the possibility of binding constraints. The constraints can be rewritten $c(q) \leq D(Q)$ and $u(q) \geq D(Q)$, where $D(Q) = V_1(Q) - V_0(Q)$. The former constraint is satisfied iff $q \leq f(Q)$ and the latter is satisfied iff $q \geq g(Q)$, for increasing functions f and g. As shown in Figure 5, both f and g go through the origin in the $(Q; q)$ plane, and g lies below f and below the 45^o line for all $Q \in [0; \hat{q}]$. Also, f crosses the 45^o line at a unique $q_1 \in (0; \hat{q}]$. Hence, our

[11] See Osborne and Rubinstein (1990) for the bargaining theory.

search for equilibria can be constrained to the interval $[0; q_1]$.

The first order condition for an interior solution to (12), taking $V_1 = V_0(Q)$ and $V_0 = V_0(Q)$ as given, is

$$[V_1(Q) - c(q)]u'(q) - [u(q) + V_0(Q)]c'(q) = 0:$$

This defines a function $q = e(Q)$, also shown in the figure. It also goes through the origin and intersects the 45° line at a unique point q^e. Hence, $q = q(Q)$ can be written as follows: $q(Q) = \min\{e(Q); f(Q)\}$ for all $q \in [0; q^e]$, and $q(Q) = \max\{e(Q); g(Q)\}$ for all $q \in [q^e; q_1]$. For $q > q_1$, it does not really matter how we define $q(Q)$, as it necessarily is below the 45° line, and we set it equal to $q(Q) = 0$. From this it is clear that for all parameter values, $q = q(Q)$ has exactly two fixed points: a nonmonetary equilibrium $q = 0$, and a unique monetary equilibrium $q = q^e > 0$.[12]

One important property of monetary equilibrium (that continues to hold even if $y > 0$) is the following. Recall that q^* is defined by $u'(q^*) = c'(q^*)$.

[12] If $y > 0$, one can show that for big r there are no monetary equilibria, while for small r there are multiple monetary equilibria. Also, in this paper we consider only stationary equilibria. More generally, there can exist dynamic equilibria, where q changes over time, including cyclical and sunspot equilibria; see Wright (1994), Coles and Wright (1998), and Ennis (1999).

Then it is easy to show that $e(q^*) < q^*$ and, therefore, $q^e < q^*$, as seen in Figure 5. This is significant because q^* is the efficient outcome. More precisely, if we define welfare as before, $W = MV_1 + (1 - M)V_0$, after simplification we have

$$rW = M(1 - M)[u(q) - c(q)]. \quad (13)$$

Hence, W is maximized with respect to q at q^*. The result $q^e < q^*$ says that in equilibrium q^e is too low – or, equivalently, the price level is too high.[13]

The economic intuition for this result is straightforward. If a seller could turn the proceeds from his production into immediate consumption, as in a static or frictionless model, then he would produce until $u'(q) = c'(q)$. But in a monetary exchange, the proceeds from production consist of cash that can only be spent in the future when an opportunity to buy comes along.

[13] This is true even though bargaining is bilaterally efficient in the sense that the agreement is on the Pareto frontier in each exchange, taking as given the value of Q that prevails in other exchanges. The point is that all agents would be better off (in an ex ante sense) if they could get everyone to commit to increasing q. A stronger result is actually true: not only is q^e too low according to the ex ante criterion W, it is also too low according to the ex post criteria V_0 and V_1. That is, buyers and even sellers would be better off if q were bigger. The result that $q^e < q^*$ also can be shown to hold in models where agents can hold any amount of money; see Trejos and Wright (1995, pp. 133-4)

Since he discounts the future, a seller is only willing to produce less than the amount that satisfies $u'(q) = c'(q)$. Indeed, to verify that it is frictions that are driving the result, observe that when $r \to 0$ or $\alpha \to \infty$ we have $q^e \to q^*$.

Another question to ask is how q depends on M. One might expect $\partial q^e/\partial M < 0$. It turns out that it is actually possible to have $\partial q^e/\partial M > 0$ for small M (at least if r is also small). The explanation is that when M is close to zero, there is very little trade. In this case, increasing M increases the frequency of productive meetings between buyers and sellers, which increases both V_1 and V_0. The net effect on the bargaining solution can be a higher q. However, there is some threshold $\hat{M} < \frac{1}{2}$ such that $\partial q^e/\partial M < 0$ for all $M > \hat{M}$. Hence, we can be sure that the value of money eventually begins to fall as M increases.

We can also ask how M affects welfare. It is clear that if a planner can choose both M and q to maximize W, he will choose $M = \frac{1}{2}$ and $q = q^*$. This is because $M = \frac{1}{2}$ maximizes the number of trades (just as in the previous section when $y = 0$), and $q = q^*$ maximizes the surplus that results from each trade. However, suppose the planner can choose only M, and q is determined in equilibrium. Then the value of M that maximizes W will satisfy the first order condition

$$\frac{\partial W}{\partial M} \stackrel{s}{=} (1 - 2M)[u(q) - c(q)] + M(1 - M)[u'(q) - c'(q)]\frac{\partial q^e}{\partial M} = 0.$$

As the second term is negative at $M = \frac{1}{2}$, the solution is $M^o < \frac{1}{2}$. This illustrates the trade-off between providing liquidity (making trade easier), and reducing the value of money (lowering the surplus from each trade). Reducing the value of money is welfare-reducing here because, as we have already established, q is too low in equilibrium.

Many other applications and results can be derived in this model, some of which are reviewed below. Also, alternative bargaining solutions generate similar although of course quantitatively different outcomes. A very simple example of such a bargaining solution is to assume that agents with money get to make take-it-or-leave-it offers to producers. This implies they will demand the quantity that satisfies $V_1 - V_0 = c(q)$ since this is the most that a producer would give to acquire currency. Notice that this immediately implies $V_0 = 0$ by virtue of (11). Hence, we have $V_1 = c(q)$, and inserting this into (10) we have

$$rc(q) = (1 - M)[u(q) - c(q)]. \tag{14}$$

An equilibrium is a q that solves (14) – one equation in one unknown. In any case, the main point we want to make here is that the basic search-theoretic model of monetary exchange can be easily generalized to endogenize prices.

4 Extensions and Related Literature

In this section, we provide a short overview of a few of the extensions and applications of the above models in the literature. Some papers we will discuss briefly; others we will merely mention. The intent here is to provide a bibliography more than a review, so that the reader who is interested at least knows where to look.[14]

[14] There is a long list of work in search theory that is tangentially related to the approach to monetary economics discussed here. Not all such work can be discussed in this brief review, but we do want to mention Diamond (1982), for although there was no money in that model, it was in some regards quite similar to the model in Section 2. Actually, the version in Diamond (1984) does have money, but it is imposed via a cash-in-advance constraint; hence, although it looked in some ways similar to the framework presented here, the spirit was quite different. See also Diamond and Yellin (1985). We also want to mention Jones (1976), and the extensions by Oh (1989) and Iwai (1996), which attempt to

There are several dimensions along which the basic search-theoretic monetary model can be generalized. Specialization is endogenized in more detail in Kiyotaki and Wright (1993), Burdett et al. (1995), Shi (1997b) and Reed (1999), for example. More general production structures are incorporated in Kiyotaki and Wright (1991) and Johri (1999). Long term partnerships, in addition to one-time exchanges, are considered in Siandra (1995) and Corbae and Ritter (1997). Various extensions of bargaining are considered by Engineer and Shi (1998, 1999), Berensten, Molico and Wright (1999) and Jafarey and Masters (1999). We already mentioned credit in Section 2, and there are several papers that try to get both money and credit into the model at the same time: Kocherlakota and Wallace (1998), assume histories are imperfectly observed over time; Cavalcanti and Wallace (1999a, 1999b) and Cavalcanti, Erosa and Temzelides (1999) assume that the histories of only some agents are observed; and Jin and Temzelides (1999) assume only histories of local neighbors are observed. Also, following Diamond (1990), there

build a model along similar lines to the ones presented here; see Ostroy and Starr (1990) for a review of this and related work. Other general discussions that concentrate more on models like the ones in this paper include Wallace (1996, 1997).

are papers with bilateral credit and money, and repayment (explicitly or implicitly) enforced by collateral. These include Hendry (1992), Shi (1996), Schindler (1998) and Li (2000).

There are many papers on commodity, as opposed to fiat, money. The basic idea here is to try to determine endogenously which of many possible goods ends up as media of exchange. The model in Kiyotaki and Wright (1989) considers a version of the model where type i consumes good i and produces $i + 1$, with $N = 3$ types. The goods are all storable, although they have different storage costs. It is shown that goods with low storage costs may or may not end up serving as money, depending on parameter values, and also depending on which equilibrium we end up in – that is, there can be equilibria where high storage costs goods are used as money. Aiyagari and Wallace (1991, 1992) generalize this to N types and consider several applications. Wright (1995) extends the model to allow agents to choose their type. Renero (1994, 1997, 1998a) considers several extensions of the framework. Among other things, he shows that equilibria where goods with high storage costs serve as money can have good welfare properties, perhaps surprisingly (the intuition is that in such equilibria there is more trade). Other related papers include Kehoe, Kiyotaki and Wright (1993), Cuadras-Morato and Wright (1997) and Renero (1998b).

There is also a literature on search models with private information. In Williamson and Wright (1994), there is uncertainty concerning the quality of goods. In such an environment, a generally recognizable money has the potential role of mitigating the informational frictions and inducing agents to adopt strategies that increase the probability of acquiring high-quality output; hence, there is a role for money even if there is no double coincidence parameter (i.e., even if $y = 1$). Trejos (1997) presents a simplified version of the model (essentially by setting $y = 0$), which allows him to get analytical solutions to the model. Kim (1996) endogenizes the extent of the private information problem. Cuadras-Morato (1994) and Yiting Li (1995) use a version of this model to study commodity money. All of the above papers assume indivisible goods. Trejos (1999) combines private information with divisible goods and bargaining. Velde, Weber and Wright (1999) and Burdett, Trejos and Wright (1999) use commodity money models with private information to study some issues in monetary history, including Gresham's law. Other related papers include Wallace (1998), Williamson (1998) and Katzman, Kennan and Wallace (1999).

There are several papers that attempt to model policy as follows. There is a subset of agents who are subject to the same search and information frictions as everyone else but act collectively. Let us call these agents government agents. The idea is to see how exogenously specified trading rules of government agents affect the endogenously determined equilibrium behavior of other (private) agents. Papers in this group include Victor Li (1994, 1995), Ritter (1995), Aiyagari, Wallace and Wright (1996), Aiyagari and Wallace (1997), Li and Wright (1998), Green and Weber (1996), Wallace and Zhou (1997), and Berensten (2000). For example, in Victor Li (1994, 1995, 1997), government agents can tax money holdings when they meet private agents. A key result is that it can be efficient to tax money holdings. The reason is that in his model (which also endogenizes search intensity) there is too little search by agents holding money, for standard reasons. Taxing them increases their search effort, and this can improve welfare.

The matching model seems a natural one for studying issues related to international monetary economics, because, for example, one can think about parameterizing differences in the efficiency of economic activity as well as the degrees of openness across countries in terms of the arrival rates. The first paper to analyze this in a model that includes multiple currencies and multiple countries is contained in Matsuyama, Kiyotaki and Matsumi (1993). They find several types of equilibria can arise, including some in which some money circulates only locally while another emerges as an international currency, and others where all monies are universally accepted. They compare these in terms of welfare. Zhou (1997) extends their model to study currency exchange. These models assume indivisible goods. Trejos and Wright (1999) endogenize prices using divisible goods and bargaining. Other examples of models with multiple currencies include Kultti (1996), Green and Weber (1996), Craig and Waller (1999), Peterson (1999) and Soller and Waller (2000).

There are also papers that consider intermediation (e.g., middlemen) as an alternative to, or sometimes in addition to, money. An early paper to explicitly consider intermediation in a search model (without money) is Rubinstein and Wolinski (1987). They basically generate a role for middlemen by specifying exogenously a set of agents who may have a more efficient technology for finding buyers than sellers have for finding buyers directly. A very different model is the one in Li (1998), in which private information about the quality of consumption goods together with the existence of a costly quality verification technology give rise to a role for intermediation. In Shevchenko's (2000) model, intermediation arises from inventory-theoretic considerations: middlemen keep a stock of several goods on hand in order to increase the probability that a random buyer will find something they like. The Shevchenko and Li papers also endogenize the number of intermediaries in the economy by a free entry condition. See also Camera (1999), Camera and Winkler (2000), and Hellwig (2000).

Perhaps the most important recent extension to the framework is to relax the strong assumptions on the amount of money that agents can hold – typically, zero or one unit of money, as we assumed above. Models that consider such an extension can be an order of magnitude more complicated, but are obviously more realistic, and generate many interesting new results. They are also capable of addressing some more traditional policy questions, such as the optimal rate of in‡ation, which are not easy to study in the models where agents hold only zero or one unit of money. An example of such a model is contained in Molico (1999), who allows agents to hold any nonnegative amount of money and has agents bargaining over both the quantity of goods and the amount of money they trade in each bilateral meeting. Because of the model's complexity, it can only be solved numerically, however. The numerical analysis generates interesting results concerning policy, welfare, the equilibrium distribution of prices, and other things.

Green and Zhou (1998a) and Zhou (1999) also present a model with divisible money, where several results can be derived analytically. Di¤erent from Molico, they assume that sellers set prices and cannot observe buyers' money holdings. Although in such an environment there could still exist equilibria with a distribution of prices, they only look for equilibria where all sellers set the same price. Several interesting results emerge, including the fact that there exist multiple (indeed, a continuum) of steady states, indexed by the nominal price level. Also, it is shown that there can be an endogenous upper bound for money holdings: agents with sufficient cash will not accept more (Molico can also generate this in his model). Other related references are Green and Zhou (1998b), Zhou (1998), Camera and Corbae (1999), Taber and Wallace (1999), Berensten (1999a, 1999b), and Rocheteau (1999). A different approach is taken by Shi (1997a), who presents an analytically solvable model with perfectly divisible money. However, his model is quite different in some dimensions from the rest of the papers in the literature.[15]

There are other applications and extensions, but we cannot go into them all in this brief review. However, we want to mention some examples of papers that study evolution or learning in this framework, including Marimon, McGrattan, and Sargent (1990), Sethi (1996), Staudinger (1998) and Basçi (1999). They are interested in determining which of the equilibria are more

[15] In Shi's model, the decision-making unit consists of a family with a large number of members (formally, a continuum), rather than a single individual. In this framework, members of the family get to share money holdings between periods, and so every family starts the next period with the same amount of money by the law of large numbers. Applications and extensions of this model are contained in Shi (1998, 1999).

robust; e.g., can agents learn to use money? Brown (1996), Du¤y and Ochs (1998, 1999), and Du¤y (2000) ask the same kind of questions, but using laboratory methods, with paid human subjects to test this experimentally. Although the results are by no means de…nitive, they are interesting to the extent that they point to certain areas where subjects in the lab do not behave as theory predicts. However, in the most recent experiments (Du¤y 2000), the results seem encouraging from the perspective of the theory.

5 Conclusion

In this paper, we have presented relatively simple versions of the basic search-theoretic models of monetary exchange. A variety of questions can be addressed even within these simple models, and there are a wide range of extensions and applications. We hope this illustrates the usefulness of the framework for monetary economics and will encourage the reader to pursue these issues further.

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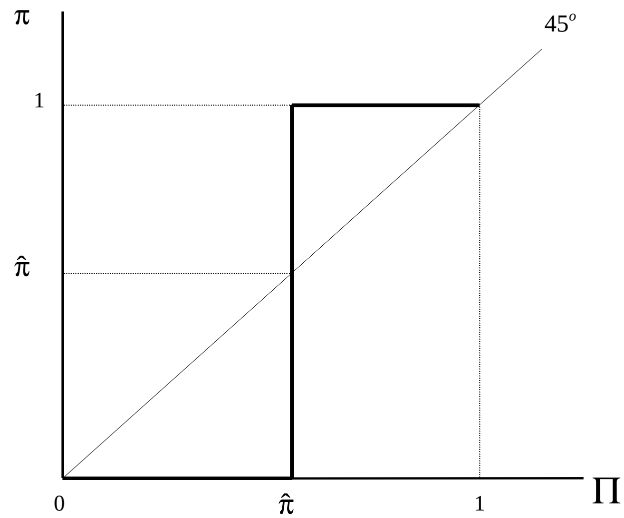

Figure 1: Best-Response Correspondence

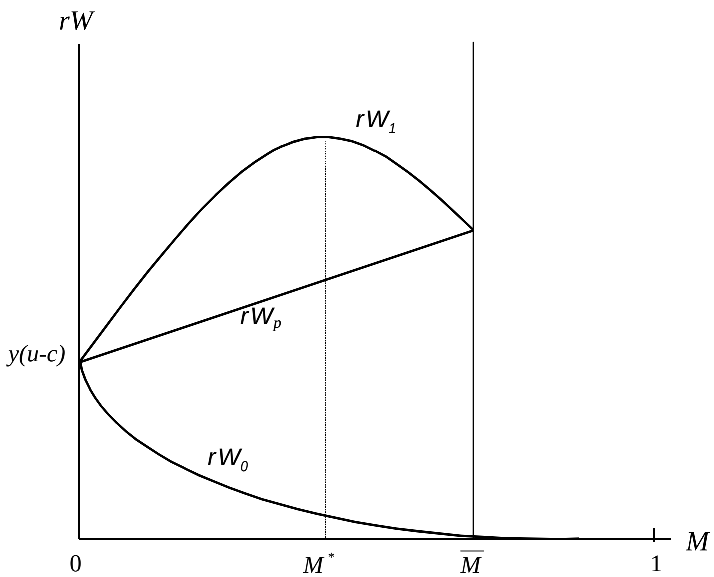

Figure 2: Welfare as a function of M

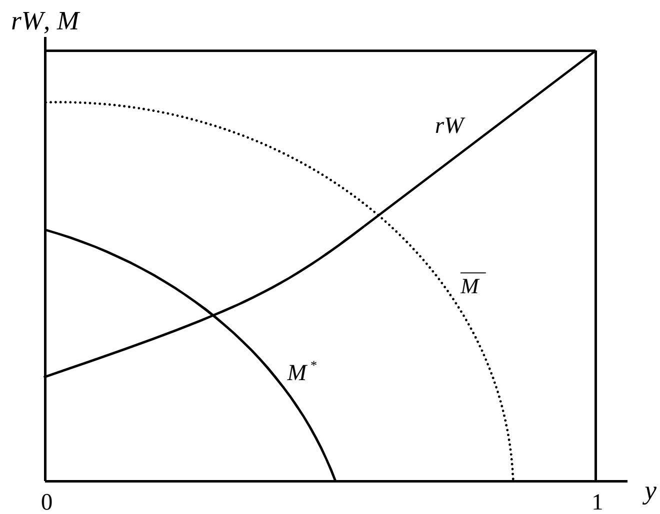

Figure 3: Welfare as a function of y (optimal M)

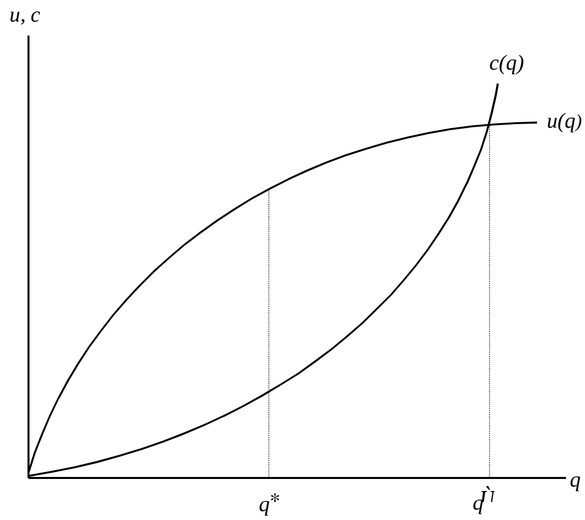

Figure 4: Functions u(q) and c(q)

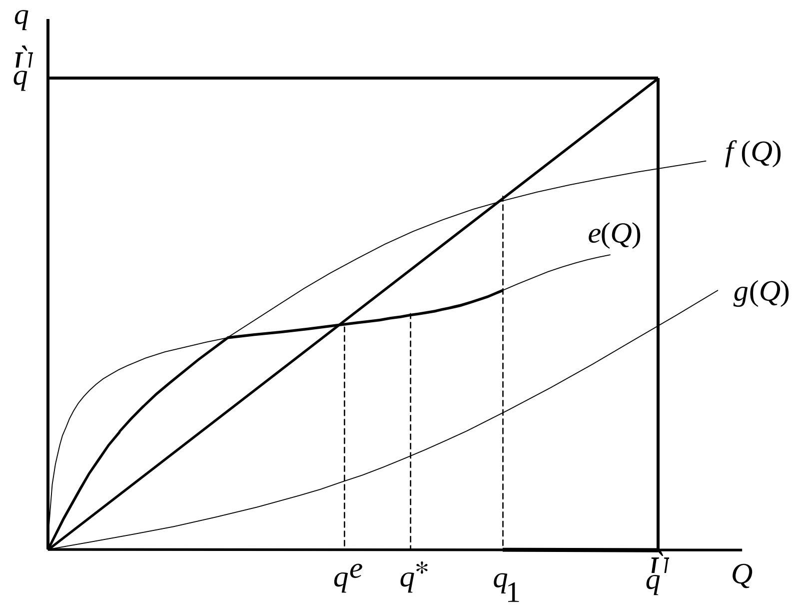

Figure 5: Monetary Equilibrium in the Divisible Goods Model