Search-and-Matching Models of Monetary Exchange

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1 Introduction

In this article we review a class of equilibrium search (matching) models that can be used to study the trading process, and in particular to develop a formal theory of *money as a medium of exchange.* Developing such a theory is one of the longest-standing issues in economics, but it met with at best limited success prior to the development of search-based models, which provide a natural framework in which to formalize venerable stories about money helping to facilitate exchange.[1]

2 Background

Diamond (1982) introduced a framework that, although it cannot used directly, can be extended naturally to build microfoundations for monetary economics. In his model, a $[0, 1]$ continuum of infinitely-lived agents interact in an economy where activity takes place in two distinct sectors: one for production and one

[1] These stories, going back to Smith, Jevons, Menger, Wicksell, and others (many of which are reprinted in Starr 1990) concern a *double coincidence of wants problem* in bilateral exchange, as discussed below. Overlapping generations models (e.g. Wallace 1980) provide an alternative approach. Ostroy and Starr (1990) survey earlier attempts to develop microfoundations for monetary theory, including Jones (1976), which is similar in spirit if not detail to modern search models. There is not space here to discuss pros and cons of the various approaches, but it seems fair to say search and matching models now dominate the area.

for exchange. In the first sector, agents encounter potential production opportunities randomly over time according to a Poisson process with arrival rate α. Each opportunity yields a unit of output at cost $c \geq 0$, where c is random with CDF $F(c)$. Since c is observed before a production decision is made, given an opportunity, there is a reservation cost k such that agents produce iff $c \leq k$. For now, these goods are indivisible, and agents can store at most one at a time.

All goods yield utility of consumption $u > 0$, except by assumption agents cannot consume their own output; hence they must trade. Traders with goods meet bilaterally in the exchange sector according to a Poisson process with arrival rate γ. Upon meeting they trade, consume, and return to production. Since all goods are the same, and indivisible, every meeting yields trade, and every trade is a one-for-one swap. Generally, $\gamma = \gamma(N)$ depends on the measure of agents in the exchange sector N. This is based on a matching technology that gives the number of agents who meet a partner per unit time as $m(N)$, with $m'(N) > 0$, implying $\gamma(N) = m(N)/N$ for all $N > 0$.

Let V_0 and V_1 be the value functions for producers and traders. The flow Bellman equation for a producer is[2]

$$rV_0 = \alpha E \max\{V_1 - V_0 - c, 0\} = \alpha \int_0^k (k - c) dF(c),$$

where $k = V_1 - V_0$. Similarly, for a trader

$$rV_1 = \gamma(N)(u + V_0 - V_1) = \gamma(N)(u - k).$$

In words, the flow value rV_0 equals the arrival rate of opportunities times the expected option value of switching from production to exchange, while rV_1 equals the arrival rate of meetings times the gain from trading and switching back.

[2] We focus on steady states; for dynamics, see e.g. Diamond and Fudenberg (1989).

Combining these equations,

$$rk = \gamma(N)(u - k) - \alpha \int_0^k (k - c)dF(c).$$

Given N this has a unique solution for k. Given k, in steady state, the flow of agents from production to exchange must equal the flow back,

$$(1 - N)\alpha F(k) = m(N).$$

An equilibrium is a pair (N, k) satisfying these last two equations. It is simple to derive results concerning existence, comparative statics, etc. As Diamond emphasizes, under increasing returns in $m(N)$, if any nondegenerate equilibrium (one with production) exists then multiple such equilibria exist. Under constant returns, a unique nondegenerate equilibrium exists iff parameters fall in a certain range – e.g. u is not too low, r not too high, etc. To complete our review of this basic model, notice that exchange is trivial, even though it is restricted to bilateral trade, because there only one good (or, all goods are the same). To make money interesting we need to generalize this.[3]

To ease the presentation we first simplify the production process. Assume everyone is *always* in the exchange sector, and instead of carrying goods around, they can produce whenever they meet someone, at deterministic cost $C \geq 0$. Now, following Kiyotaki and Wright (1991, 1993), assume goods come in varieties, say colors. Each agent produces a particular color, but different agents like to consume different colors. The simplest specification assumes agents get $u = U > C$ from any good in some set, and $u = 0$ from other goods, and x is the probability output is in the relevant set (i.e. an agent wants what the other

[3]Diamond (1984) took a short cut to getting money into the model with a cash-in-advance constraint. By changing the environment as we do below, we see this is not only uninteresting, it is unnecessary.

agent can produce) in any random meeting. Also, since agents can produce whenever they want, to simplify things we assume goods are nonstorable.[4]

Assuming exchange requires mutual agreement, which occurs when I want to consume your good and you want to consume mine, trade now only occurs in a meeting with probability x^2, at least if the event that I want your good is independent of the event that you want mine (see below). This captures nicely the famous *double coincidence problem* with direct barter: trade requires meeting someone who produces something you like – which would be a coincidence – and also likes what you produce – a double coincidence. Payoffs are given by $rV_B = \gamma x^2(U - C)$, where the subscript on V_B stands for "barter." If x is small, which is the case if there is a lot of specialization, double coincidence meetings are rare and V_B is very low.

But is it really necessarily the case that trade occurs iff both parties want to consume what the other produces? Following ideas in Kocherlakota (1998), suppose agents get together at the start of time and discuss when to trade. Clearly, they agree that whenever *either* agent wants what the other produces he should get it, since this maximizes ex ante welfare

$$rV_C = \gamma \left[x^2(U - C) + x(1 - x)U - (1 - x)xC \right] = \gamma x(U - C),$$

where the subscript on V_C stands for "cooperation" (or perhaps "commitment" or "credit"). As long as $x < 1$, $V_C > V_B$. However, suppose agents cannot

[4] When goods *are* storable, Kiyotaki and Wright (1989) determine endogenously which objects serve as media of exchange, potentially including commodity plus fiat money. That model illustrates the trade off between fundamental properties like storability, and equilibrium properties like acceptability. It has many implications – e.g. there can be multiple equilibria with different monies, objects with bad fundamental properties may end up as money, and so on. Generalizations and applications of the model include Marimon et al. (1990), Aiyagari and Wallace (1991, 1992), Kehoe et al. (1993), Wright (1995) and Duffy and Ochs (1999). Here, by making goods nonstorable, we focus on determining how an economy operates when there is a single candidate medium of exchange, fiat money.

commit now to do things when they meet later that are not in their interest at that time. Then trades must satisfy IC (incentive compatibility), the binding condition being that you should be willing to produce in meetings where you do not consume.

If we can keep a public record of all agents' behavior, we can try to use *trigger strategies* to support cooperative trade as follows: instruct agents to cooperate as long as everyone else does; but if anyone deviates, trigger to ... "something bad." One can argue the worst trigger is "autarky" which yields $V_A = 0$; or it may be "barter" which yields V_B. In the former case the relevant IC condition is $-C + V_C \geq V_A$, which simplifies to $rC \leq \gamma x(U - C)$; in the latter case it is $-C+V_C \geq V_B$, which simplifies to $rC \leq \gamma x(1-x)(U-C)$. In either case, if r is small we can sustain cooperative trade. Moreover, one can prove formally that money has no role here (Kocherlakota 1998; Wallace 2001); instead of proving this here we move to models where money does have a role.

3 First-Generation Search Models of Money

Suppose it is difficult to use triggers because, say, there is incomplete monitoring or record keeping, or to take the simplest situation, suppose agents have no memory – they just cannot recall what happened in previous meetings! Kocherlakota (1998), Kocherlakota and Wallace (1998), Wallace (2001), Temzilides et al. (2003), Araujo (2004), and Aliprantis et al. (2006) explore less extreme variations, but our assumption allows us to make the point more easily. In our "memoryless" world, your continuation payoff V_M cannot depend on what you do in a given meeting. Hence, the relevant constraint to get you to produce without consuming is $-C + V_M \geq V_M$, which is violated for any $C > 0$. There is no scope for using threats to sustain cooperation without memory (generally, there is limited scope when memory is imperfect, which is what we need; we use the starkest case merely for tractability).

Suppose we introduce into this world a new object called fiat *money.*[5] At the start of time, we endow a fraction M of the population each with $m = 1$ unit of money and the rest with $m = 0$. Initially, those with $m = 0$ can produce; after this, agents can produce after they consume but not before. This implies agents with money cannot produce, and at any point in time everyone either has $m = 1$ or $m = 0$. Now, even without memory, agents have an option other than pure barter: offer money for goods. Let Π be the probability a random producer accepts such an offer, and let π be your best response.

If V_m and V_p are the value functions of agents with and without money,

$$\begin{aligned} rV_p &= \gamma(1-M)x^2(U-C) + \gamma M x \pi(V_m - V_p - C) \\ rV_m &= \gamma(1-M)x\Pi(U + V_p - V_m). \end{aligned}$$

For example, rV_p equals the arrival rate of agents with goods $\gamma(1-M)$, times the double coincidence probability x^2, times the gain from barter $U-C$; plus the arrival rate of agents with money γM, times the probability of trade $x\pi$, times the gain $V_m - V_p - C$. We restrict attention to pure strategies (mixed strategy equilibria are not robust here; see Schevchenko and Wright 2004). Then the best response condition is $\pi = 0$ if $V_m - V_p < C$ and $\pi = 1$ if $V_m - V_p > C$. It is easy to see $\pi = 0$ is always an equilibrium, and $\pi = 1$ is an equilibrium iff

$$rC \leq \gamma(1-M)x(1-x)(U-C).$$

[5] By definition, a medium of exchange is an object that is accepted in trade not to be used for consumption (or production), but to be traded again later for something else. When an object serving as a medium of exchange is for some people at some times a consumption good, it is called commodity money. When an object with no consumption value serves as a medium of exchange it is fiat money.

Naturally, $\pi = 0$ is an equilibrium – if no one else accepts money, why would you? It is more interesting that $\pi = 1$ can be an equilibrium, since then intrinsically worthless money is valued, as a medium of exchange. Given M, one can check $\pi = 1$ yields higher payoffs than $\pi = 0$. Alternatively, if we choose M to maximize welfare, one can check $M > 0$ iff x is not too big. Hence, introducing money can improve welfare, even given the assumption that money holders cannot produce. The convention of money as a medium of exchange is good because it eases trade. Now, $\pi = 1$ is only an equilibrium when r is not too high, and one can check the cutoff for r here is more stringent (and payoffs lower) than when we had memory and triggers – i.e. money is not a perfect mechanism.[6] Still, money can do pretty well here, and if we cannot use triggers it is the only way to improve on pure barter.

The model is obviously crude, yet it gets at the essence of money. To recap, the results assume the following explicit frictions: 1. A double coincidence problem (generated here by random bilateral matching, although there are other devices in the related literature); 2. imperfect commitment; and 3. imperfect memory (or anything else that makes it difficult to use triggers). These frictions are severe – but no one said it was going to be easy to get money into economic theory in an interesting way. There are many extensions and applications of this model (some of which are surveyed in Rupert et al. 2000), but in the interest of space, we now move on to models where prices are endogenous.[7]

[6] One reason money is not as good as memory is the *random* nature of matching. The problem is that you might e.g. have two meetings in a row where you want a good from someone who does not want your good, and in the second one you will have run out of money (this can also happens with positive probability when we relax the upper bound of 1 on money holdings). However, in an *endogenous* (rather than random) matching model this never happens – when you have no money you do not go to someone whose good you like, but to someone who likes your good. See Temzilides et al. (2003).

[7] We mention one extension to endogenous specialization in Kiyotaki and Wright (1993), based on ideas in Adam Smith. Consider the case where the probability that someone accepts

4 Second-Generation Models

Suppose that goods are no longer indivisible, but can be consumed and produced in any amount $q \geq 0$, which yields utility $U(q)$ and disutility $-C(q)$, respectively. These functions have all the usual properties, plus $C(0) = U(0) = 0$. We maintain for now the assumptions that money is indivisible, money holders cannot produce, and everyone holds $m \in \{0, 1\}$. But we relax the assumption of independence in generating the double coincidence problem: the probability that I like your good is x, but now the probability that I like your good and you like mine is y, and not necessarily x^2, in general.[8]

Conditional on money being accepted ($\pi = 1$), we have

$$\begin{aligned} rV_p &= \gamma(1-M)y\left[U(\hat{q}) - C\hat{q})\right] + \gamma Mx\left[V_m - V_p - C(\bar{q})\right] \\ rV_m &= \gamma(1-M)x\left[U(\bar{q}) + V_p - V_m\right], \end{aligned}$$

where $\hat{q}$ is the amount traded in barter and $\bar{q}$ the amount traded for money. It facilitates the presentation to start with the case $y = 0$ and then give general results. Now, to determine the equilibrium value of money, as in Shi (1995) or Trejos and Wright (1995), we say the following: when I meet you and want your good, if I have $m = 1$ while you $m = 0$, we *bargain* over the q you produce for my money, taking as given $\bar{q}$ in all other meetings. Equilibrium is a fixed point, $q = \bar{q}$, and the price level is $p = 1/q$.

your good x is a *choice variable*: if you want a large fraction of the population to like your output, you cannot specialize too much, which reduces productivity. Thus, the arrival rate in the production sector (going back to Diamond's two sector setup) is $\alpha(x)$, with $\alpha' < 0$. When choosing x, you take the average X as given, and in equilibrium $x = X$. Two results follow. First, monetary equilibria have lower x than nonmonetary equilibria, so the use of money enhances specialization and productivity. Second, $x \to 0$ as $\gamma \to \infty$, so when fricitons vanish, agents specialize completely, and since the double coincidence probability is x^2, barter completely disappears.

[8] Consider N goods and N types, where type n produces good n, but likes good $n+1$ modulo N. If $N = 2$ then $x = y = 1/2$ (if I like your good you must like mine), while if $N \geq 3$ then $x = 1/N$ and $y = 0$ (if I like your good you cannot lke mine). It is only under independence, which does not hold in these examples, that we necessarily have $y = x^2$.

One can use any bargaining solution, including generalized Nash

$$\max\left[U(q)+V_p-V_m\right]^{\theta}\left[V_m-V_p-C(q)\right]^{1-\theta},$$

for any $\theta \in (0,1)$; we use $\theta = 1$ because it is so easy.[9] When $\theta = 1$, agents with $m = 0$ get no gains from trade since they have no bargaining power (and $y = 0$). Hence $V_p = 0$, $V_m = C(q)$, and the Bellman equation for V_m reduces to

$$rC(q) = \gamma(1-M)x\left[U(q)-C(q)\right].$$

This is 1 equation in q, with two solutions: 0 and a unique $q > 0$. Again, we get equilibrium where an intrinsically worthless object is valued as a medium of exchange. It is easy to do comparative statics, welfare analysis, etc. in this model (e.g. it is immediate that q falls and p rises when M or r increase).

Once we reintroduce some barter – i.e. once we allow $y > 0$ – one can show that, in addition to $q = 0$, generically there either exists 2 equilibria with $q > 0$ or no equilibrium with $q > 0$. If y is small then equilibrium with $q > 0$ always exists. It is not much harder to analyze the general case $\theta \in (0,1)$. This model has a large number of variations, extensions and applications – too many to review here (again see Rupert et al. 2000). Suffice it to say that the basic results of first-generation models more or less go through, with additional insights concerning prices.

5 Third-Generation Models

The approach sketched above provides a compelling microfoundation for monetary theory: it is based on sound economic thinking going back to some very

[9] See Rupert et al. (2001) for $\theta \in (0,1)$ and other generalizations. Alternatives to bargaining studied in versions of this model include posting (e.g. Curtis and Wright 2004) and auctions (Julien et al. 2007). Or, instead of imposing a particular pricing mechanism, one can study the entire set of incentive-fesaible trades (Wallace 2001).

famous economists, brought up to date with modern and rigorous methods and ideas. Still, obviously those first- and second-generation models are quite abstract and quite special. In particular, the assumption that agents hold $m \in \{0, 1\}$ is severe, and precludes using the models for much quantitative and policy analysis. The difficult part of relaxing this and allowing any $m \geq 0$ is that we need to keep track of the distribution of m across agents, which is complicated by the random nature of matching and the endogenous amount of money spent in each match. There are several ways to deal with this problem. Some analytic results are available in Green and Zhou (1998) and Camera and Corbae (1999) e.g., while computation methods are used by Molico (2006).

Another approach is to amend the environment to get around this problem while hopefully maintaining the spirit and essence of the matching models outlined above. There are two main ways to do this, following either Shi (1997) or Lagos and Wright (2005). The Shi model assumes the fundamental decision makers are not individuals, but families, each with a large number of members. If the individual members experience independent random meetings, when they return to the household at the end of each period the total amount of money in the family is pinned down by the law of large numbers. Hence, each household starts the next period with the same (deterministic) amount of money. There are many extensions and applications of this framework; see Shi (2006) for some references.

The Lagos-Wright model alternatively assume that at the end of each round of decentralized trade agents go to a centralized market where they can (among other things) rebalance their money holdings. Assuming quasi-linear utility, all agents choose the same m for next period, independent of the amount with which they start. Again, agents enter each round of decentralized trade with the same m here, just as in the family model (although there are several interesting differences between the approaches). Versions of either model are easily used for quantitative and policy analysis. These models are perhaps still special, since they use "tricks" to harness the distribution of m, but this is merely for technical convenience in deriving analytic results. If one is willing to use a computer, most of the special assumptions can be avoided (see e.g. Chiu and Molico 2006).

6 Conclusion

We have reviewed several generations of search-and-matching models of the exchange process that can be used to provide microfoundations for monetary economics. While the literature is big, and growing fast, hopefully this short essay conveys some of the main ideas and models in an accessible fashion.

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