3. Homework in Econ 682 - Game Theory and Applications

The homework is due on April 2nd at the beginning of class. Answers have to be typed, with the exception of graphs which may be drawn by hand.

Dynamic Games:

Question 1: (Moral Hazard) Consider the following game between a health insurance company (player 1) and a person who wants to obtain health insurance (player 2). Player 2 is risk-averse while player 1 is risk-neutral, which provides the rationale for insurance.

Player 2 faces the following problem: He has 400 in wealth. If he gets sick, the cost of health care is 300. He consumes all money he has left after paying for health care and/or insurance. His utility function over consumption is $u(c) = \sqrt{c}$. The only problem of sickness is the monetary cost of health care.[1] Player 2 can prevent getting sick by putting in effort (no smoking, excercising,...). If he wants to obtain a probability p of remaining healthy, that yields an additional utility term $-5/(1-p)$, i.e. $p = 0$ induces zero additional utility while any positive probability of remaining healthy is costly (and staying health with probability 1 is infinitely costly).

1. (Decision Theory) What is the optimal effort (i.e. the optimal p) that player 2 chooses?

2. (Social Optimum) Assume that effort is perfectly observable. Player 1 can insurance player 2 by paying an amount B if player 2 becomes sick but collects a fee of F if the player 2 remains healthy.

   (a) Determine the level of insurance and of effort that maximizes the sum of utility for the agents.

   (b) Assume player 2 can make a binding offer regarding (B, F, p) to player 1, which player 1 can accept or reject (player 1 gets zero after rejection). Solve for the subgame perfect equilibrium. Show how much profit player 1 makes.[2]

3. (Moral Hazard) Consider exactly the same setup as in part 2b) but assume that p is not observable. So player 2 makes offer (B, F), player 1 accepts or rejects, then player 2 chooses p, and then payments are made dependent on whether player 2 got sick or not. Without solving the model: Give an informed argument whether the optimum can be reached. If not, why not?

[1] We could also include "pain" of player 2 from being sick, but leave this out due to analytical complexity.

[2] Note for yourself that the outcome is the same as if insurers made the offer but there is perfect competition between insurers.

Question 2: (Philipp's research agenda - directed labor search) There are two firms A and B with productivity levels X_A and X_B and two identical workers 1 and 2. First, firms simultaneously post wages w_A and w_B. The workers observe the wages and simultaneously decide at which firm they want to apply for a job (they can only apply to one firm). A firm that has one applicant hires him and pays the wage. A firm that has two applicant randomly hires one of them and pays the wage to him. Firms that cannot hire and workers that do not get hired gets zero. If firm $i \in \{A, B\}$ hires a worker at wage w_i it makes profits $X_i - w_i$ and the worker gets utility w_i. Firms maximize expected profits, workers maximize expected utility. We are looking for subgame perfect equilibria in which workers play a symmetric strategy in every subgame (i.e. both workers in the subgame apply to each firm with the same probability).

1. Consider the subgame after wage announcements w_A and w_B. In the symmetric equilibrium of the subgame, find the probability with which the workers apply to firm 1 and firm 2. What is the probability that a worker remains unemployed? How does that vary with the difference $w_A - w_B$? (Be careful with boundaries.)

2. Assume both firms are identical, i.e. $X_A = X_B = 1$. Solve for the equilibrium of the entire game.

3. Assume $X_A = 2/3$ and $X_B = 1/3$.

   (a) If workers apply to firm A with probability p_A and to firm B with probability $1 - p_A$, which application probability p_A^* induces the largest expected output?

   (b) Find the equilibrium application probability? Compare it (and the resulting unemployment rate) with 3a.

Question 3: Answer questions 2.13 and 2.15 in the book for an intercept $a = 10$.

Games of incomplete information:

Question 4:

1. Answer question 3.4 in the book.

2. Answer question 3.5 in the book (Read section 3.2.A to get an idea how to do this.)

3. Answer question 3.6 in the book.