2. Homework in Econ 682 - Game Theory and Applications

The homework is due on March 3rd at the beginning of class. Answers have to be typed, with the exception of graphs which may be drawn by hand.

Question 1: Consider the extensive form games in Figue 1, Figure 2, and Figure 3 at the end of this homework. For each of these:

1. List the strategy set for each player.
2. List all subgames.
3. Find the subgame perfect Nash Equilibria for each game. (Give the equilibrium and the outcome. Recall that the outcome are the actions taken on the equilibrium path, while the equilibrium lists the entire strategy of each player.)
4. Find all Nash equilibria (in pure strategies) for each game. (Give the equilibrium and the outcome. Recall that the outcome describes the actions taken on the equilibrium path, while the equilibrium describes the entire strategy of each player. You do not have to go through the entire normal form, as long as in the end there is no doubt that you have considered and checked all possible equilibria.)

Question 2: Consider Figure 4 at the end of this homework. The figure has two "mistakes". Can you spot them? (Please explain what is not right.)

Question 3: There are three firms. Each firm sets a quantity $q_i \in \mathbb{R}_+$. The resulting price at which they can sell their quantity is $P(q_1, q_2, q_3) = 10 - (q_1 + q_2 + q_3)$. Find the subgame-perfect outcome for each of the following extended Stackelberg games:

1. Firm 1 moves first, firm 2 moves second, firm 3 moves last. Each firm observes the choices made earlier.
2. Firm 1 moves first, firm 2 moves second, firm 3 moves last. Firm 2 and 3 see the choice of firm 1, but firm 3 does not see the choice of firm 2.

Question 4: There are two firms. The game has two stages. Each firm first chooses a non-negative capacity c_i. No firm can sell more than its capacity. Capacity is costless. After observing the capacity choices, the subsequent competition is a la Bertrand: Each of the firms chooses a non-negative price p_i. The number of consumers that would like to buy at price p is given by the demand curve $D(p) = 30 - p$. The firm with the strictly lower price (call this price p_L) can sell up to capacity if there is enough demand at price p_L. If the demand is less than its capacity, it sells up to the demand. Call the amount sold by the low-price firm Q_L. The firm with the strictly higher price (call its price p_H) can sell to the remaining demand $D(p_H) - Q_L$ if that is positive (but not more than its capacity). In case both firms offer the same price, then a fair coin toss determines which firm is the low price one that can sell first.

In this environment, is it possible to sustain the cournot quantities and prices as a subgame perfect equilibrium outcome. (Note that the cournot quantities of each firm are 10, and so the price is 10)? Say first "YES" or "NO", and then explain why.

Question 4: Firm 1 wants to outsource a project to firm 2. The project will only be a success if firm 2 puts in effort. Assume that the probability that the project is a success is equal to the effort that firm 2 puts in. The game proceeds in the following stages:

- First, firm 1 proposes a fixed payment w and a bonus b. The wage w is paid independently of the outcome of the project, while b is only paid if the project is a success.
- Second, firm 2 decides to accept the proposal or not. If it rejects both parties obtain a payoff of zero.
- If firm 2 accepts, it decides on a level of effort $e \in [0, 1]$.
- If the project is a success, firm 1 obtains $1 - w - b$, where 1 is the output that is generated. Firm 2 obtains $w + b - c(e)$, where $c(e)$ is the cost of effort.
- If the project is a failure, firm 1 obtains $-w$ and firm 2 obtains $w - c(e)$.

Choose any strictly convex cost function $c(e)$ that you like, and solve for the subgame perfect equilibrium outcome of this game.

Question 5: Consider the Rubinstein Bargaining Game that we discussed in class, but assume that player 1 has discount factor δ_1 and player 2 has a potentially different discount factor δ_2.[1] Show that as the number of rounds goes to infinity, the payoffs converge to $(1 - \delta_2)/(1 - \delta_1\delta_2)$ for player 1 and $\delta_2(1 - \delta_1)/(1 - \delta_1\delta_2)$ for player 2.

[1] The game with two periods of offers can be found on page 69 in the book. The question is what would happen if stages (1a) to (2b) get repeated many times before (3) is realized.

Question 6: Solve question 2.8 in the book.

Question 7: Solve question 2.11 in the book.

Question 8: A repeated game $G(T)$ is a game where in each round players simultaneously play some normal-form game G. There are T rounds, so that the stage-game G gets repeated T times. Players observe the choices made in earlier rounds. The payoffs are just the sum of the stage-game payoffs. (Think of the stage-game G for example as a simultaneous prisoner's dilemma game. If $T = 2$, then the prisoner's dilemma is played twice, and in the second period the players can observe what everyone did in the first period. The payoffs are the sum of the payoffs of the two rounds.) Prove the following

1. If the normal-form game G has a unique Nash Equilibrium (unrepeated), then for any finite number of rounds T the repeated game $G(T)$ has a unique subgame-perfect equilibrium outcome.
2. Assume the normal-form game G is a Prisoner's Dilemma game. Then for any finite number of rounds T the repeated game $G(T)$ has a unique equilibrium outcome (even if we do not require subgame-perfection).

Figure 1

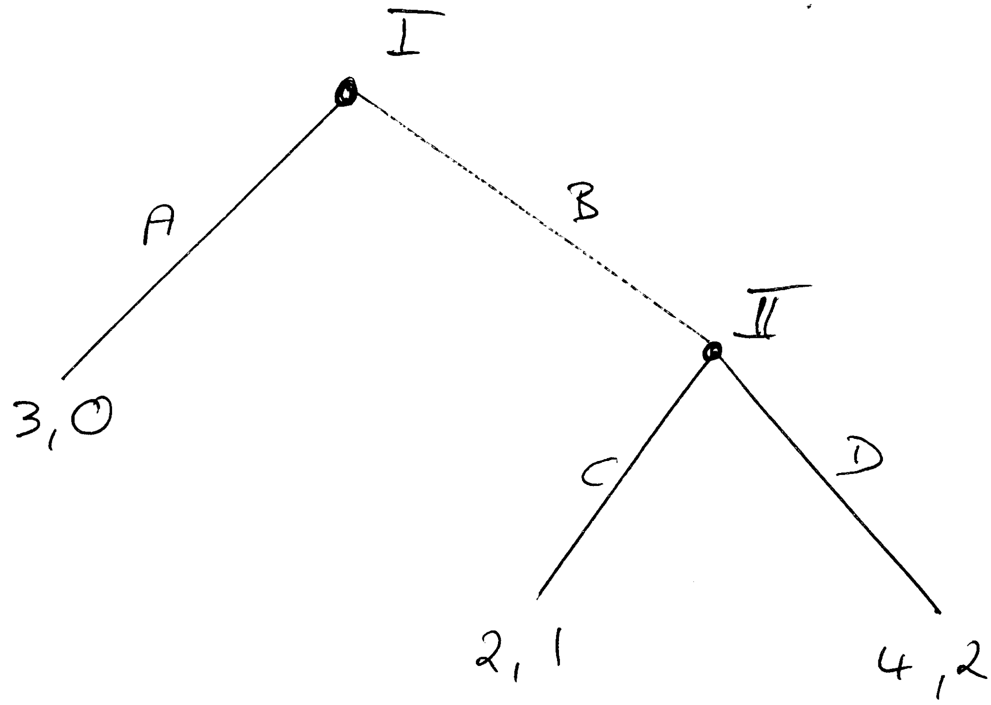

Figure 2:

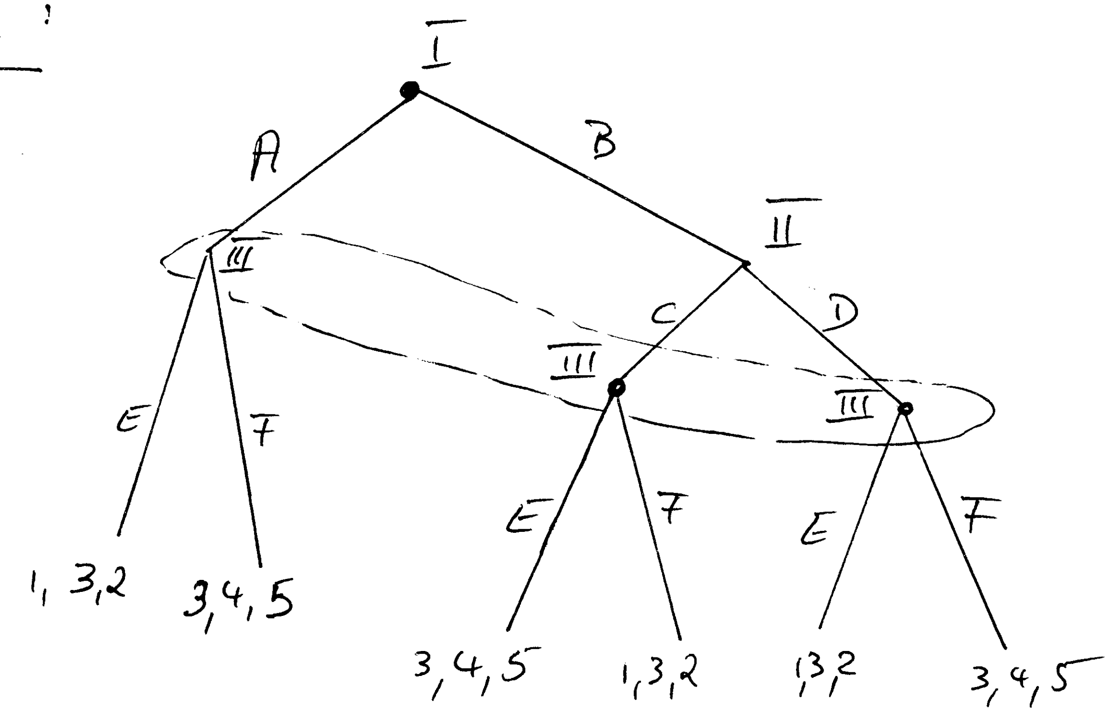

Figure 3:

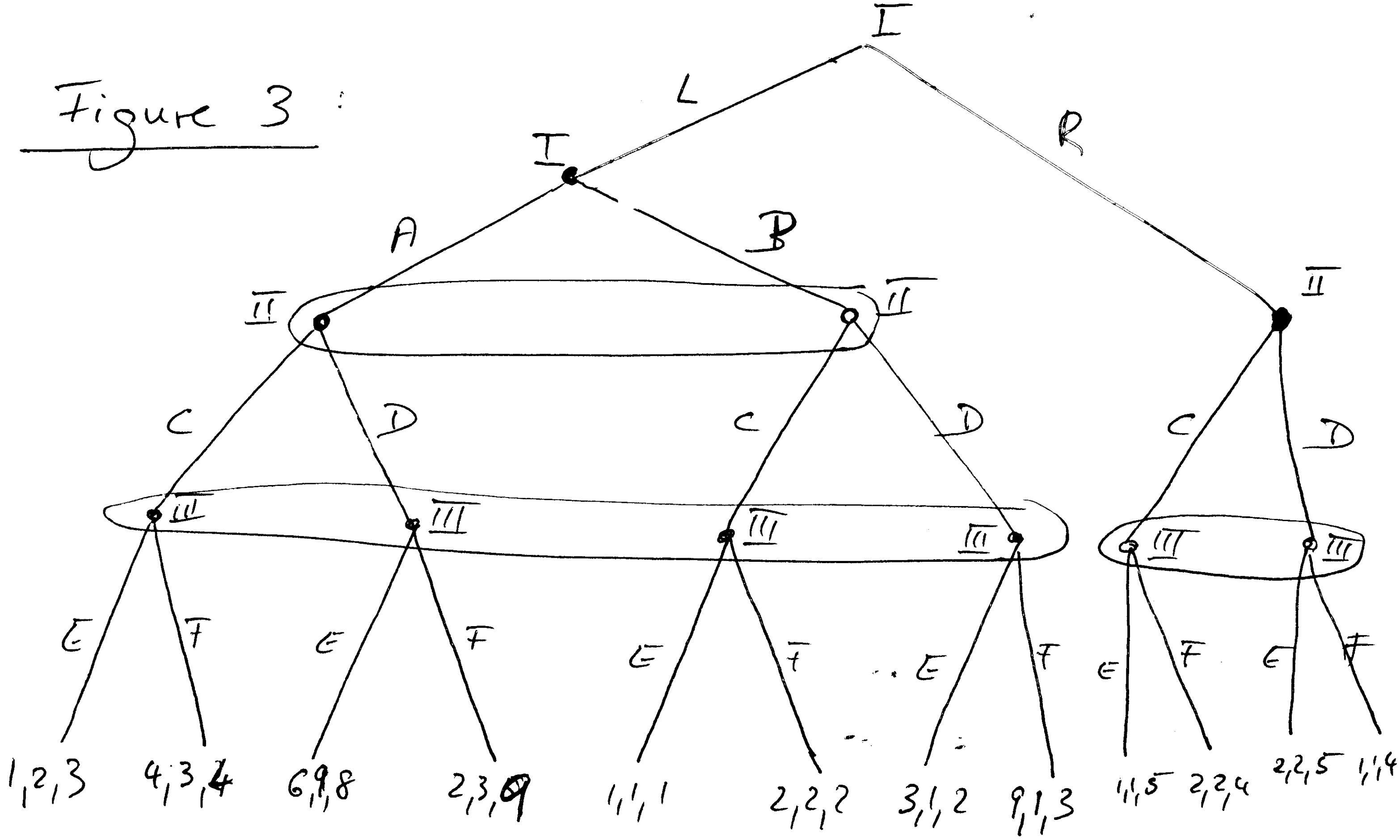

Figure 4: