A Model with Costly Enforcement

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A Model with Costly Enforcement

- We keep the basic structure as before, except that now the financial market friction is the cost of enforcing contracts.

- Structure:

    1. Borrower may decide to renege on debt.

    2. If that is the case, the lender can only recover the fraction $(1 - \theta)$ of the gross return $R_{t+1}^k p_t q_t k_t$ where:

    $$(1 - \theta) R_{t+1}^k < R_t$$

    and the borrower keeps the rest, $\theta R_{t+1}^k p_t q_t k_t$.

Costly Enforcement Model II

- Value of project:

$$V_t = R_{t+1}^k p_t q_t k_t - R_t \left(q_t k_t - n_t \right)$$

- Incentive constraint:

$$V_t \geq \theta R_{t+1}^k p_t q_t k_t$$

- Since the constraint must be binding:

$$R_{t+1}^k p_t q_t k_t - R_t \left(q_t k_t - n_t \right) = \theta R_{t+1}^k p_t q_t k_t \Rightarrow$$

$$q_t k_t = \frac{1}{1 - (1 - \theta) \frac{R_{t+1}^k}{R_t} p_t} n_t$$

Costly Enforcement Model III

- Advantage: much easier to handle than costly state verification model.

- Disadvantage: no default in equilibrium, no spreads.

- When to use each of them?

- Can we move from firms to banks?

An Application I: Bank Runs

- 2007-2010: run on investment funds instead of classical run on banks.

- Suggests we may want to think again about runs on financial institutions.

- Calling them or not a bank is somewhat irrelevant: any institution that engages in maturity transformation.

- Gorton (2010)'s emphasis on runs on funds during the crisis.

BD+Banks
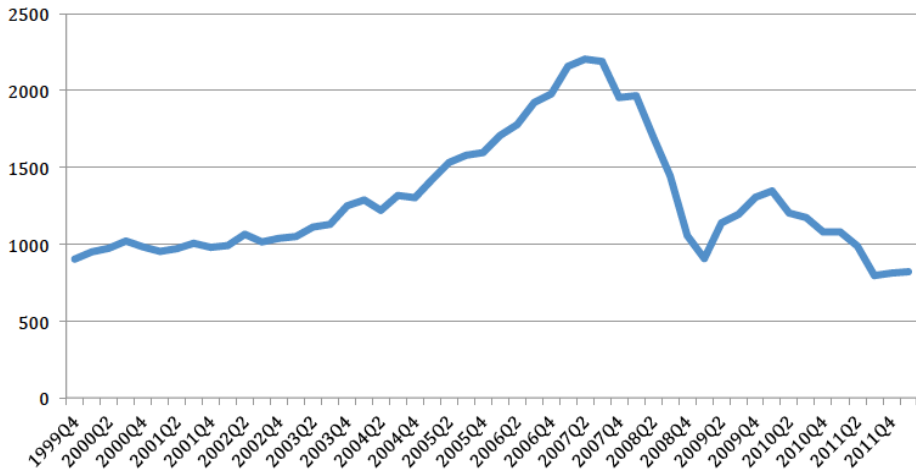
An Application II: Bank Runs

- Kiyotaki and Gertler (2012) incorporate Diamond and Dybvig (1983) into the dynamic macro model we saw in last lecture.

- Main idea: maturity mismatch.

- To keep the presentation simple, we will get rid of nominal rigidities.

- Also, this will facilitate comparison with a neoclassical framework.

- But, first, let us review Diamond and Dybvig's model.

A Review of Diamond and Dybvig: Agents

- Continuum of agents.

- Three-dates economy:

  1. $t = 0$: each agents endowed with 1 unit of good.

  2. $t = 1$: early consumption, with probability π_1 and utility $u(c_1)$.

  3. $t = 2$: late consumption, with probability $\pi_1 = 1 - \pi_2$ and utility $u(c_2)$.

- Think about the need of consumption as a liquidity shock $i.i.d.$ for each agent. Law of large numbers.

- Consumption in other date does not yield utility.

- Expected utility of agents:

$$\pi_1 u(c_1) + \pi_2 u(c_2)$$

A Review of Diamond and Dybvig: Technology

- Technology:

  1. Storage$\Rightarrow$1 good at t is transformed into 1 good $t + 1$.

  2. Long-term illiquid investment project$\Rightarrow$1 good at time $t = 0$ is transformed into $R > 1$ at $t = 2$. However, if liquidated at $t = 1$, we get $l \leq 1$.

- Reasons:

  1. Technological.

  2. Monitoring.

  3. Lemons...

A Review of Diamond and Dybvig: Efficient Allocation

- Social planner: perfect risk-pooling among agents.
- Invest I and store $1 - I$ in such a way that no long-term project is liquidated too early.
- We solve

$$\max \pi_1 u\left(c_1\right) + \pi_2 u\left(c_2\right)$$
$$\text{s.t. } \pi_1 c_1 = 1 - I$$
$$\pi_2 c_2 = RI$$

- Then:

$$\max \pi_1 u\left(\frac{1 - I}{\pi_1}\right) + \pi_2 u\left(\frac{RI}{\pi_2}\right)$$

- Optimality condition:

$$u'\left(\frac{1 - I}{\pi_1}\right) = R u'\left(\frac{RI}{\pi_2}\right)$$

A Review of Diamond and Dybvig: Autarky

- Each agent invests I in the long-term project at $t = 0$ and stores $1 - I$.
- If liquidity shock at $t = 1$,

$$c_1 = 1 - I + lI = 1 - (1 - l)I \leq 1$$

Otherwise

$$c_2 = RI + 1 - I = 1 + (R - 1)I \leq R$$

(at least one of the two inequalities is strict).

- Expected utility:

$$\pi_1 u \left(1 - (1 - l)I\right) + \pi_2 u \left(1 + (R - 1)I\right)$$

- I is always ex post inefficient: either too low or too high. Inferior to efficient allocation.

A Review of Diamond and Dybvig: Financial Markets I

- p unit of good at $t = 1$ can be exchanged for 1 unit at $t = 2$.

- Then

$$c_1 = pRI + 1 - I$$

  and

$$c_2 = RI + \frac{1 - I}{p}$$

- Note $c_1 = pc_2$.

- Also, utility is increasing in I if $pR > 1$ and decreasing if $pR < 1$.

- Thus, in a equilibrium where I is endogenous, $pR = 1$.

- Hence, allocation is $c_1 = 1$ and $c_2 = R$.

A Review of Diamond and Dybvig: Financial Markets II

- Expected utility:

$$\pi_1 u\left(1\right) + \pi_2 u\left(R\right)$$

  dominates autarky, but it is still not efficient because liquidity is not properly allocated.

- To see this, note that, in general

$$u'\left(1\right) = R u'\left(R\right)$$

- For instance, if $u'\left(1\right) > R u'\left(R\right)$, impatient consumers get more in the optimal allocation than in the equilibrium with financial markets (she needs to be insured against the liquidity risk better than what she can get on her own by storing all her endowment).

A Review of Diamond and Dybvig: Fractional Reserve Banking I

- A bank can offer a contract to depositors: (c_1^*, c_2^*).

- It must be the case that $c_2^* > c_1^*$ (otherwise, depositors will always cash-in at $t = 1$ regardless of the liquidity shock).

- Let us suppose that agents withdraw funds when they want to consume.

- Then, bank keeps reserves $\pi_1 c_1^*$ and invests in the long-term project $1 - \pi_1 c_1^*$.

A Review of Diamond and Dybvig: Fractional Reserve Banking II

- Payouts:

$$c_2^* = R \frac{1 - \pi_1 c_1^*}{\pi_2}$$

- By competition, $\left(c_1^*, c_2^* \right)$ should satisfy:

$$\max_{c_1^*} \pi_1 u \left(c_1^* \right) + \pi_2 u \left(R \frac{1 - \pi_1 c_1^*}{\pi_2} \right)$$

  or

$$u' \left(c_1^* \right) = R u' \left(c_2^* \right)$$

- This is the same optimality condition than the social planner!

- Intuition: coalition.

A Review of Diamond and Dybvig: Fractional Reserve Banking II

- Problem: what if the depositors show up at $t = 1$?

- Bank run (self-fulling prophecy).

- Sequential service constraint.

- It is a Nash, regardless of the investors beliefs about the soundness of the portfolio of the bank.

- Inefficient allocation where the bank has to liquidate early the long-run project.

A Review of Diamond and Dybvig: Solutions

1. Narrow banking Wallace (1996):

    1. Pay in all events: even worse than autarky.

    2. Pay if liquidation: same than autarky.

    3. Securitization: same than equilibrium with financial markets.

2. Suspension of Convertibility.

3. Equity: Jacklin (1986).

4. Deposit insurance.

Environment

- We deal now with a more general model.

- Households and bankers.

- Two goods:

  1. Durable asset, capital, which does not depreciate and it is in fixed supply $\overline{k} = 1$.

  2. A nondurable good.

- Relative price of capital: q_t.

Capital

- Capital is hold by banks and households:

$$k_t^b + k_t^h = \overline{k} = 1$$

- When capital k_t^b is hold by a bank at period t, it produces $z_{t+1}k_t^b$ of nondurable good at period $t+1$.
- When capital k_t^h is hold by a household at period t, it requires $f\left(k_t^h\right)$ to produce $z_{t+1}k_t^b$ of nondurable good at period $t+1$.
- Interpretation as management cost.
- Assumption:

$$f\left(k_t^h\right) = \left\{ \begin{array}{l} \frac{\alpha}{2}\left(k_t^h\right)^2 \text{ for } k_t^h \leq \overline{k}_t^h \in (0,1) \\ \alpha\overline{k}_t^h\left(k_t^h - \frac{\overline{k}_t^h}{2}\right) \text{ for } k_t^h > \overline{k}_t^h \end{array} \right.$$

- Kink in management costs allows the household to absorb all the capital in case of a banking collapse.

Representative Household

- Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log c_t^h$$

- Endowment of nondurable goods $z_t w^h$.

- Deposits in a bank that pay R_{t+1} if no bank run.

- If bank run, a depositor receives either the full payment or nothing, depending on the timing of the withdrawal.

- We assume that, ex ante, the household gives zero probability to bank run.

- Hence, budget constraint:

$$c_t^h + d_t + q_t k_t^h + f\left(k_t^h\right) = z_t w^h + R_t d_{t-1} + (q_t + z_t) k_{t-1}^h$$

Optimality Conditions

- The first-order conditions for the household are:

$$\frac{1}{c_t} = \lambda_t$$

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} R_{t+1}$$

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} R_{t+1}^h$$

$$R_{t+1} = \frac{q_{t+1} + z_{t+1}}{q_t + f'\left(k_t^h\right)}$$

- Asset pricing kernel:

$$SDF_t = \beta \frac{\lambda_t}{\lambda_{t-1}}$$

and standard non-arbitrage conditions.

Expression for Consumption

- Define:

$$F_t = z_t w^h + f'\left(k_t^h\right) k_t^h - f\left(k_t^h\right) + \mathbb{E}_t \lambda_{t+1} F_{t+1}$$

as the sum of the value of the endowment $(z_t w^h)$ plus the returns from holding capital $(f'\left(k_t^h\right) k_t^h - f\left(k_t^h\right))$ plus the continuation value.

- Then, from budget constraint and optimality conditions, we get:

$$c_t^h = (1 - \beta)\left(R_t d_{t-1} + (q_t + z_t) k_{t-1}^h + F_t\right)$$

Bankers I

- Continuum of bankers.

- Perfect competition.

- Risk neutral.

- Survival rate σ, with expected life $\frac{1}{1-\sigma}$ (replaced by new bankers), and consumption at terminal date.

- Equity n_t such that:
$$q_t k_t^b = n_t + d_t$$

- Initial wealth w^b: take it as an exogenous endowment to simplify algebra.

Bankers II

- Evolution of equity:

$$n_t = (q_t + z_t) k_{t-1}^b - R_t d_{t-1}$$

with $c_t^b = n_t$.

- Then:

$$
\begin{aligned}
V_t &= \mathbb{E}_t \sum_{i=1}^{\infty} (1-\sigma) \sigma^{i-1} \beta^i c_{t+i}^b \\
&= \mathbb{E}_t \left[(1-\sigma) \beta n_{t+1} + \beta \sigma V_{t+1} \right]
\end{aligned}
$$

Note the recursive structure.

Moral Hazard Problem I

- A banker can divert a fraction θ of assets for personal use and depositors can only recover $1 - \theta$ of assets.

- Hence, following arguments we have already presented, the incentive constraint (IC) is:
$$V_t \geq \theta q_t k_t^b$$

- Problem of the banker
$$\max V_t = \mathbb{E}_t \sum_{i=1}^{\infty} (1 - \sigma) \sigma^{i-1} \beta^i c_{t+i}^b$$
$$\text{s.t. } n_t = (q_t + z_t) k_{t-1}^b - R_t d_{t-1}$$
$$V_t \geq \theta q_t k_t^b$$

Moral Hazard Problem II

- Guess:

$$V_t = \nu_{kt} k_t^b - \nu_t d_t$$

Why this structure? Linear preferences+expected utility.

- With some simple algebra:

$$V_t = \mu_t q_t k_t^b + \nu_t n_t$$

where

$$\mu_t = \frac{\nu_{kt}}{q_t} - \nu_t$$

is the excess marginal value of assets over deposits.

Moral Hazard Problem III

- Now, IC is:

$$\mu_t q_t k_t^b - \nu_t n_t \geq \theta q_t k_t^b \Rightarrow$$
$$\nu_t n_t \geq (\theta - \mu_t) q_t k_t^b$$

Hence, IC is binding if and only if $\theta > \mu_t > 0$.

- If IC is not binding, $\mu_t = 0$, that is, competition forces down the excess marginal value of assets over deposits to zero.

- When $\mu_t > 0$, there is excess return to induce the right behavior and the price of capital is low.

- From now on, I will assume that IC is binding.

Moral Hazard Problem IV

- Then:

$$\frac{q_t k_t^b}{n_t} = \frac{\nu_t}{\theta - \mu_t} = \phi_t$$

  where ϕ_t is the maximum leverage ratio.

- Interpretation.

- Note linearity in bank assets. Too big to fail?

- Alternatives?

Value of Franchise I

- With some algebra, we get

$$
\begin{aligned}
V_t &= \mu_t q_t k_t^b + v_t n_t \\
&= \mathbb{E}_t \left[(1 - \sigma) \beta n_{t+1} + \beta \sigma V_{t+1} \right] \\
&= \beta \mathbb{E}_t \left\{ \begin{array}{l} \left[1 - \sigma + \sigma \left(v_{t+1} + \phi_{t+1} \mu_{t+1} \right) \right] * \\ \left[\left(R_{t+1}^b - R_{t+1} \right) q_t k_t^b + R_{t+1} n_t \right] \end{array} \right\}
\end{aligned}
$$

where

$$
R_{t+1}^b = \frac{q_{t+1} + z_{t+1}}{q_{t+1}}
$$

is the realized rate of return on bank assets and

$$
R_{t+1}^b - R_{t+1}
$$

is the realized excess return.

Value of Franchise II

- Then, by matching coefficients

$$\mu_t = \beta \mathbb{E}_t \Omega_{t+1} \left(R^b_{t+1} - R_{t+1} \right)$$
$$\nu_t = \beta \mathbb{E}_t \Omega_{t+1} R_{t+1}$$

where

$$
\begin{aligned}
\Omega_{t+1} &= 1 - \sigma + \sigma \left(\nu_{t+1} + \phi_{t+1} \mu_{t+1} \right) \\
&= 1 - \sigma + \sigma \left(\nu_{t+1} + \frac{q_{t+1} k^b_{t+1}}{n_{t+1}} \mu_{t+1} \right)
\end{aligned}
$$

is the (probability weighted) marginal value of net bank worth at time $t+1$.

Aggregation

- Total bank net worth times leverage is equal to capital owned by banks:

$$q_t k_t^b = \phi_t n_t$$

- Total net worth of banks:

$$n_t = \sigma \left\{ (z_t + q_t) \, k_{t-1}^b - R_t d_{t-1} \right\} + (1 - \sigma) \, w^b$$

- Consumption of bankers:

$$c_t^b = (1 - \sigma) \left\{ (z_t + q_t) \, k_{t-1}^b - R_t d_{t-1} \right\}$$

- Total output:

$$
\begin{aligned}
y_t &= z_t + z_t w^h + (1 - \sigma) \, w^b \\
&= c_t + c_t^b + f \left(k_t^h \right)
\end{aligned}
$$

Equilibrium Conditions I

- The first-order conditions of the household:

$$\frac{1}{c_t} = \beta \mathbb{E}_t \frac{1}{c_{t+1}} R_{t+1}$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t \frac{1}{c_{t+1}} \frac{q_{t+1} + z_{t+1}}{q_t + f'\left(k_t^h\right)}$$

- Consumption of bankers:

$$c_t^b = (1 - \sigma) \left\{ (z_t + q_t) k_{t-1}^b - R_t d_{t-1} \right\}$$

- Evolution of wealth of bankers:

$$n_t = \sigma \left\{ (z_t + q_t) k_{t-1}^b - R_t d_{t-1} \right\} + (1 - \sigma) w^b$$

Equilibrium Conditions II

- IC:

$$\frac{q_t k_t^b}{n_t} = \beta \frac{\mathbb{E}_t \Omega_{t+1} R_{t+1}}{\theta - \mu_t}$$

- The coefficients of the value of the franchise:

$$\mu_t = \beta \mathbb{E}_t \Omega_{t+1} \left(\frac{q_{t+1} + z_{t+1}}{q_{t+1}} - R_{t+1} \right)$$

$$\Omega_t = 1 - \sigma + \sigma \left(\beta \mathbb{E}_t \Omega_{t+1} R_{t+1} + \frac{q_t k_t^b}{n_t} \mu_t \right)$$

- Market clearing:

$$q_t k_t^b = n_t + d_t$$

$$k_t^b + k_t^h = 1$$

$$z_t + z_t w^h + (1 - \sigma) w^b = c_t + c_t^b + f \left(k_t^h \right)$$

Bank Run I

- Sequential service obligation.

- At the start of period t, before realization of returns, depositors run on the bank: they do not roll over their deposits.

- Small ε cost of not rolling over the deposit (to avoid runs without foundation).

- Bank liquidates its capital by selling it to households at price q_t^*.

- Therefore, a run is possible if some depositors will lose their assets in a run:
$$(q_t^* + z_t)\, k_{t-1}^b < \gamma R_t d_{t-1}$$
where γ is the percentage of depositors who run.

Bank Run II

- With some algebra

$$(q_t^* + z_t - \gamma R_t q_{t-1}) k_{t-1}^b + \gamma R_t n_{t-1} < 0$$

  or

$$R_t^{b*} = \frac{q_t^* + z_t}{q_{t-1}} < \gamma R_t \left(1 - \frac{1}{\phi_{t-1}}\right)$$

- Interpretation: a bank run is possible when the realized rate of return of bank assets, R_t^{b*}, in case of a liquidation, is low in comparison with the rate of return on deposits R_t and the leverage ratio, ϕ_{t-1}.

- R_t^{b*}, R_t, and ϕ_{t-1} are all equilibrium objects and, hence, they depend on the state of the economy.

- Calibration: regular shocks do not push the economy to the bank-run region, but large shocks do.

Liquidation Price I

- When a run occurs, banks liquidate and no new banks enter into the economy (new potential bankers just eat their initial endowment): $n_t = d_t = 0$ and R_{t+1} is not defined.

- Alternative: slow recovery. Nothing too important for what we have to discuss today.

- Households get all the capital: $k_t^h = 1$.

Liquidation Price II

- Then, after a run:

$$c_t^b = (1 - \sigma) w^b$$

$$c_t = z_t + z_t w^h - f(1)$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t \frac{1}{c_{t+1}} \frac{q_{t+1}^* + z_{t+1}}{q_t^* + \alpha \overline{k}^h}$$

(note: capital holdings are heterogeneous, but we assume that fees are the same).

- Rearranging terms and solving forward:

$$q_t^* = \mathbb{E}_t \left(\sum_{i=1}^{\infty} \beta^i \frac{c_t}{c_{t+i}} \left(z_{t+i} - \alpha \overline{k}^h \right) \right) - \alpha \overline{k}^h$$

- Difference between illiquidity and insolvency: depends on the relation between q_t and q_t^*.

An Extension

- Why would banks issue short-term liquid deposits instead of long-term bonds?

- Incorporate liquidity risk by households.

- After all the decisions are made, some members of the household must undertake emergency consumption c_t^m.

- They will withdraw it from deposits.

- With minor changes, all the equilibrium conditions go through.

Table 1: Parameters

	Baseline Model	
β	0.99	Discount rate
σ	0.95	Bankers survival probability
θ	0.35	Seizure rate
α	0.1	Household managerial cost
$\bar{K}^h$	0.096	Threshold capital for managerial cost
γ	0.72	Fraction of depositors that can run
ρ	0.95	Serial correlation of productivity shock
Z	0.0161	Steady state productivity
ω^b	0.0019	Bankers endowment
ω^h	0.045	Household endowment
	Additional Parameters for Liquidity Model	
κ	62.67	Preference weight on c_m
$\bar{c}^m$	0.01	Threshold for c_m
π	0.03	Probability of a liquidity shock
γ_L	0.67	Fraction of depositors that can run

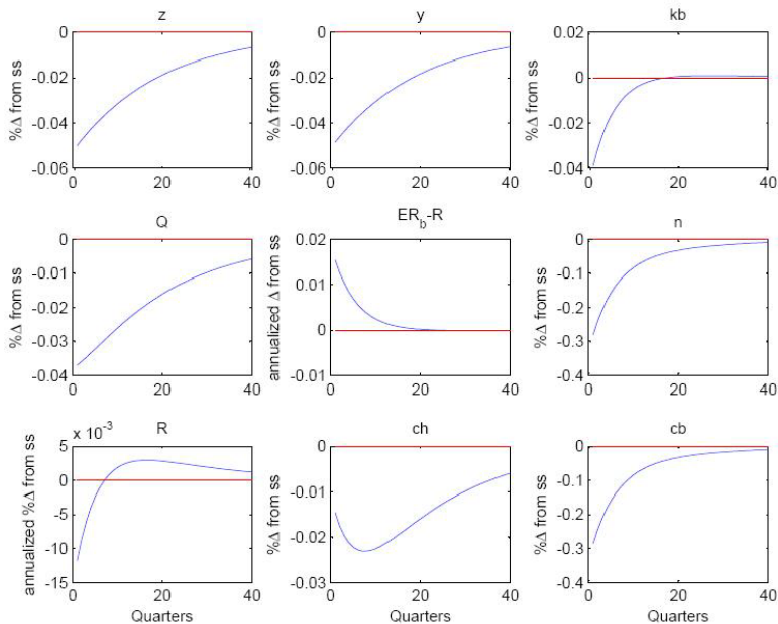Figure 1: A Recession in the Baseline Model: No Bank Run Case

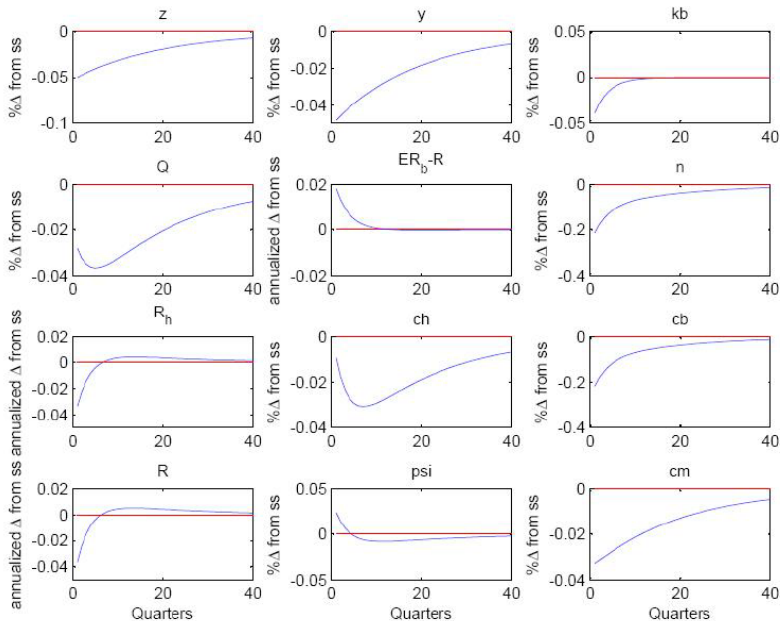Figure 2: A Recession in the Liquidity Risk Model: No Bank Run Case
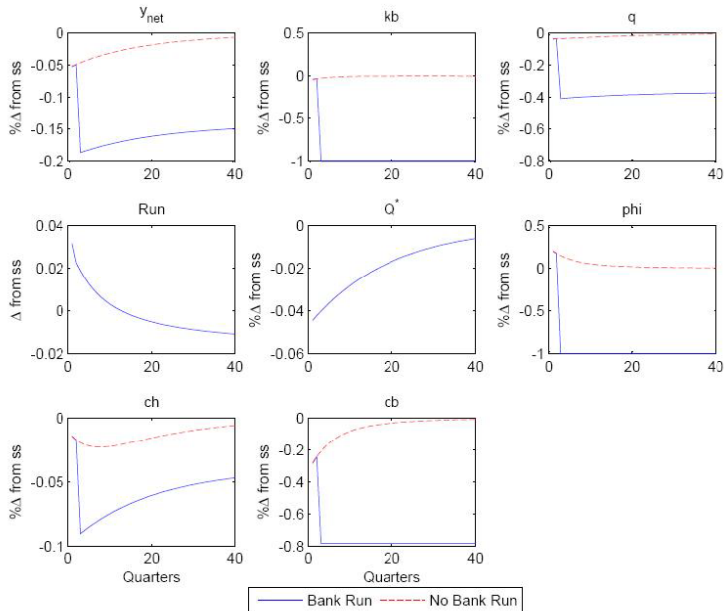
Figure 3: Ex Post Bank Run in the Baseline Model

Figure 4: Ex Post Bank Run in the Liquidity Risk Model