# A Model with Costly-State Verification 

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## A Model with Costly-State Verification

- Tradition of financial accelerator of Bernanke, Gertler, and Gilchrist (1999), Carlstrom and Fuerst (1997), and Christiano, Motto, and Rostagno (2009).
- Elements:
(1) Information asymmetries between lenders and borrowers $\Rightarrow$ costly state verification (Townsend, 1979).
(2) Debt contracting in nominal terms: Fisher effect.
(3) Changing spreads.
- We will calibrate the model to reproduce some basic observations of the U.S. economy.


## Flowchart of the Model



## Households

- Representative household:

$$
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} e^{d_{t}}\left\{u\left(c_{t}, l_{t}\right)+v \log \left(\frac{m_{t}}{p_{t}}\right)\right\}
$$

- $d_{t}$ is an intertemporal preference shock with law of motion:

$$
d_{t}=\rho_{d} d_{t-1}+\sigma_{d} \varepsilon_{d, t}, \varepsilon_{d, t} \sim \mathcal{N}(0,1)
$$

- Why representative household? Heterogeneity?


## Asset Structure

- The household saves on three assets:
(1) Money balances, $m_{t}$.
(2) Deposits at the financial intermediary, $a_{t}$, that pay an uncontingent nominal gross interest rate $R_{t}$.
(3) Arrow securities (net zero supply in equilibrium).
- Therefore, the household's budget constraint is:

$$
c_{t}+\frac{a_{t}}{p_{t}}+\frac{m_{t+1}}{p_{t}}=w_{t} l_{t}+R_{t-1} \frac{a_{t-1}}{p_{t}}+\frac{m_{t}}{p_{t}}+T_{t}+\digamma_{t}+\text { tre }_{t}
$$

where:

$$
\operatorname{tre}_{t}=\left(1-\gamma^{e}\right) n_{t}-w^{e}
$$

## Optimality Conditions

- The first-order conditions for the household are:

$$
\begin{gathered}
e^{d_{t}} u_{1}(t)=\lambda_{t} \\
\lambda_{t}=\beta \mathbb{E}_{t}\left\{\lambda_{t+1} \frac{R_{t}}{\Pi_{t+1}}\right\} \\
-u_{2}(t)=u_{1}(t) w_{t}
\end{gathered}
$$

- Asset pricing kernel:

$$
S D F_{t}=\mathbb{E}_{t} \beta \frac{\lambda_{t+1}}{\lambda_{t}}
$$

and standard non-arbitrage conditions.

## The Final Good Producer

- Competitive final producer with technology

$$
y_{t}=\left(\int_{0}^{1} y_{i t}^{\frac{\varepsilon-1}{\varepsilon}} d i\right)^{\frac{\varepsilon}{\varepsilon-1}}
$$

- Thus, the input demand functions are:

$$
y_{i t}=\left(\frac{p_{i t}}{p_{t}}\right)^{-\varepsilon} y_{t} \quad \forall i,
$$

- Price level:

$$
p_{t}=\left(\int_{0}^{1} p_{i t}^{1-\varepsilon} d i\right)^{\frac{1}{1-\varepsilon}}
$$

## Intermediate Goods Producers

- Continuum of intermediate goods producers with market power.
- Technology:

$$
y_{i t}=e^{z_{t}} k_{i t-1}^{\alpha} 1_{i t}^{1-\alpha}
$$

where

$$
z_{t}=\rho_{z} z_{t-1}+\sigma_{z} \varepsilon_{z, t}, \varepsilon_{z, t} \sim \mathcal{N}(0,1)
$$

- Cost minimization implies:

$$
\begin{gathered}
m c_{t}=\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} \frac{w_{t}^{1-\alpha} r_{t}^{\alpha}}{e^{Z_{t}}} \\
\frac{k_{t-1}}{I_{t}}=\frac{\alpha}{1-\alpha} \frac{w_{t}}{r_{t}}
\end{gathered}
$$

## Sticky Prices

- Calvo pricing: in each period, a fraction $1-\theta$ of firms can change their prices while all other firms keep the previous price.
- Then, the relative reset price $\Pi_{t}^{*}=p_{t}^{*} / p_{t}$ satisfies:

$$
\begin{gathered}
\varepsilon g_{t}^{1}=(\varepsilon-1) g_{t}^{2} \\
g_{t}^{1}=\lambda_{t} m c_{t} y_{t}+\beta \theta \mathbb{E}_{t} \Pi_{t+1}^{\varepsilon} g_{t+1}^{1} \\
g_{t}^{2}=\lambda_{t} \Pi_{t}^{*} y_{t}+\beta \theta \mathbb{E}_{t} \Pi_{t+1}^{\varepsilon-1}\left(\frac{\Pi_{t}^{*}}{\Pi_{t+1}^{*}}\right) g_{t+1}^{2}
\end{gathered}
$$

- Given Calvo pricing, the price index evolves as:

$$
1=\theta \Pi_{t}^{\varepsilon-1}+(1-\theta) \Pi_{t}^{* 1-\varepsilon}
$$

## Capital Good Producers I

- Capital is produced by a perfectly competitive capital good producer.
- Why?
- It buys installed capital, $x_{t}$, and adds new investment, $i_{t}$, to generate new installed capital for the next period:

$$
x_{t+1}=x_{t}+\left(1-S\left[\frac{i_{t}}{i_{t-1}}\right]\right) i_{t}
$$

where $S[1]=0, S^{\prime}[1]=0$, and $S^{\prime \prime}[\cdot]>0$.

- Alternative:
(1) Adjustment cost in capital.
(2) Time to build.


## Capital Good Producers II

- Technology illiquidity.
- Importance of irreversibilities?
- The period profits of the firm are:

$$
q_{t}\left(x_{t}+\left(1-S\left[\frac{i_{t}}{i_{t-1}}\right]\right) i_{t}\right)-q_{t} x_{t}-i_{t}=q_{t}\left(1-S\left[\frac{i_{t}}{i_{t-1}}\right]\right) i_{t}-i_{t}
$$

where $q_{t}$ is the relative price of capital.

## Capital Good Producers III

- Discounted profits:

$$
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \frac{\lambda_{t}}{\lambda_{0}}\left(q_{t}\left(1-S\left[\frac{i_{t}}{i_{t-1}}\right]\right) i_{t}-i_{t}\right)
$$

Since this objective function does not depend on $x_{t}$, we can make it equal to $(1-\delta) k_{t-1}$.

- First-order condition of this problem is:
$q_{t}\left(1-S\left[\frac{i_{t}}{i_{t-1}}\right]-S^{\prime}\left[\frac{i_{t}}{i_{t-1}}\right] \frac{i_{t}}{i_{t-1}}\right)+\beta \mathbb{E}_{t} \frac{\lambda_{t+1}}{\lambda_{t}} q_{t+1} S^{\prime}\left[\frac{i_{t+1}}{i_{t}}\right]\left(\frac{i_{t+1}}{i_{t}}\right)^{2}=1$
and the law of motion for capital is:

$$
k_{t}=(1-\delta) k_{t-1}+\left(1-S\left[\frac{i_{t}}{i_{t-1}}\right]\right) i_{t}
$$

## Entrepreneurs I

- Entrepreneurs use their (end-of-period) real wealth, $n_{t}$, and a nominal bank loan $b_{t}$, to purchase new installed capital $k_{t}$ :

$$
q_{t} k_{t}=n_{t}+\frac{b_{t}}{p_{t}}
$$

- The purchased capital is shifted by a productivity shock $\omega_{t+1}$ :
(1) Lognormally distributed with CDF $F(\omega)$ and
(2) Parameters $\mu_{\omega, t}$ and $\sigma_{\omega, t}$
(3) $\mathbb{E}_{t} \omega_{t+1}=1$ for all $t$.
- Therefore:

$$
\mathbb{E}_{t} \omega_{t+1}=e^{\mu_{\omega, t+1}+\frac{1}{2} \sigma_{\omega, t+1}^{2}}=1 \Rightarrow \mu_{\omega, t+1}=-\frac{1}{2} \sigma_{\omega, t+1}^{2}
$$

- This productivity shock is a stand-in for more complicated processes such as changes in demand or the stochastic quality of projects.


## Entrepreneurs II

- The standard deviation of this productivity shock evolves:

$$
\log \sigma_{\omega, t}=\left(1-\rho_{\sigma}\right) \log \sigma_{\omega}+\rho_{\sigma} \log \sigma_{\omega, t-1}+\eta_{\sigma} \varepsilon_{\sigma, t}, \varepsilon_{\sigma, t} \sim \mathcal{N}(0,1)
$$

- The shock $t+1$ is revealed at the end of period $t$ right before investment decisions are made. Then:

$$
\begin{gathered}
\log \sigma_{\omega, t}-\log \sigma_{\omega}=\rho_{\sigma}\left(\log \sigma_{\omega, t-1}-\log \sigma_{\omega}\right)+\eta_{\sigma} \varepsilon_{\sigma, t} \\
\Rightarrow \widehat{\sigma}_{\omega, t}=\rho_{\sigma} \widehat{\sigma}_{\omega, t-1}+\eta_{\sigma} \varepsilon_{\sigma, t}
\end{gathered}
$$

- More general point: stochastic volatility.


## Entrepreneurs III

- The entrepreneur rents the capital to intermediate goods producers, who pay a rental price $r_{t+1}$.
- Also, at the end of the period, the entrepreneur sells the undepreciated capital to the capital goods producer at price $q_{t+1}$.
- Therefore, the average return of the entrepreneur per nominal unit invested in period $t$ is:

$$
R_{t+1}^{k}=\frac{p_{t+1}}{p_{t}} \frac{r_{t+1}+q_{t+1}(1-\delta)}{q_{t}}
$$

## Debt Contract

- Costly state verification framework.
- For every state with associated $R_{t+1}^{k}$, entrepreneurs have to either:
(1) Pay a state-contingent gross nominal interest rate $R_{t+1}^{\prime}$ on the loan.
(2) Or default.
- If the entrepreneur defaults, it gets nothing: the bank seizes its revenue, although a portion $\mu$ of that revenue is lost in bankruptcy.
- Hence, the entrepreneur will always pay if it $\omega_{t+1} \geq \bar{\omega}_{t+1}$ where:

$$
R_{t+1}^{\prime} b_{t}=\bar{\omega}_{t+1} R_{t+1}^{k} p_{t} q_{t} k_{t}
$$

- If $\omega_{t+1}<\bar{\omega}_{t+1}$, the entrepreneur defaults, the bank monitors the entrepreneur and gets $(1-\mu)$ of the entrepreneur's revenue.


## Zero Profit Condition

- The debt contract determines $R_{t+1}^{\prime}$ to be the return such that banks satisfy its zero profit condition in all states of the world:

$$
\begin{gathered}
\underbrace{\left[1-F\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right] R_{t+1}^{\prime} b_{t}}_{\text {Revenue if loan pays }} \\
+\underbrace{(1-\mu) \int_{0}^{\bar{\omega}_{t+1}} \omega d F\left(\omega, \sigma_{\omega, t+1}\right) R_{t+1}^{k} p_{t} q_{t} k_{t}}_{\text {Revenue if loan defaults }}=\underbrace{s_{t} R_{t} b_{t}}_{\text {Cost of funds }}
\end{gathered}
$$

- $s_{t}=1+e^{\bar{s}+\widetilde{s}_{t}}$ is a spread caused by the cost of intermediation such that:

$$
\widetilde{s}_{t}=\rho_{s} \widetilde{s}_{t-1}+\sigma_{s} \varepsilon_{s, t}, \varepsilon_{s, t} \sim \mathcal{N}(0,1)
$$

- For simplicity, intermediation costs are rebated to the households in a lump-sum fashion.
- External finance premium.


## Optimality of the Contract

- This debt contract is not necessarily optimal.
- However, it is a plausible representation for a number of nominal debt contracts that we observe in the data.
- Also, the nominal structure of the contract creates a Fisher effect through which changes in the price level have an impact on real investment decisions.
- Importance of working out the optimal contract.


## Characterizing the Contract I

- Define share of entrepreneurial earnings accrued to the bank:

$$
\Gamma\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)=\bar{\omega}_{t+1}\left(1-F\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right)+G\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)
$$

where:

$$
G\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)=\int_{0}^{\bar{\omega}_{t+1}} \omega d F\left(\omega, \sigma_{\omega, t+1}\right)
$$

- Thus, we can rewrite the zero profit condition of the bank as:

$$
\frac{R_{t+1}^{k}}{s_{t} R_{t}}\left[\Gamma\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)-\mu G\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right] q_{t} k_{t}=\frac{b_{t}}{p_{t}}
$$

which gives a schedule relating $R_{t+1}^{k}$ and $\bar{\omega}_{t+1}$.

## Characterizing the Contract II

- Now, define the ratio of loan over wealth:

$$
\varrho_{t}=\frac{b_{t} / p_{t}}{n_{t}}=\frac{q_{t} k_{t}-n_{t}}{n_{t}}=\frac{q_{t} k_{t}}{n_{t}}-1
$$

- and we get

$$
\frac{R_{t+1}^{k}}{s_{t} R_{t}}\left[\Gamma\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)-\mu G\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right]\left(1+\varrho_{t}\right)=\varrho_{t}
$$

that is, all the entrepreneurs, regardless of their level of wealth, will have the same leverage, $\varrho_{t}$.

- A most convenient feature for aggregation.
- Balance sheet effects.


## Problem of the Entrepreneur

- Maximize its expected net worth given the zero-profit condition of the bank:
$\max _{\varrho_{t}, \bar{\omega}_{t+1}} \mathbb{E}_{t}\left\{\begin{array}{c}\frac{R_{t+1}^{k}}{R_{t}}\left(1-\Gamma\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right)+ \\ \eta_{t}\left[\frac{R_{t+1}^{k}}{s_{t} R_{t}}\left[\Gamma\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)-\mu G\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right]-\frac{\varrho_{t}}{1+\varrho_{t}}\right]\end{array}\right\}$
- After a fair amount of algebra:

$$
\mathbb{E}_{t} \frac{R_{t+1}^{k}}{R_{t}}\left(1-\Gamma\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right)=\mathbb{E}_{t} \eta_{t} \frac{n_{t}}{q_{t} k_{t}}
$$

where the Lagrangian multiplier is:

$$
\eta_{t}=\frac{s_{t} \Gamma_{\omega}\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)}{\Gamma_{\omega}\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)-\mu G_{\omega}\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)}
$$

- This expression shows how changes in net wealth have an effect on the level of investment and output in the economy.


## Death and Resurrection

- At the end of each period, a fraction $\gamma^{e}$ of entrepreneurs survive to the next period and the rest die and their capital is fully taxed.
- They are replaced by a new cohort of entrepreneurs that enter with initial real net wealth $w^{e}$ (a transfer that also goes to surviving entrepreneurs).
- Therefore, the average net wealth $n_{t}$ is:

$$
n_{t}=\gamma^{e} \frac{1}{\Pi_{t}}\left[\left(1-\mu G\left(\bar{\omega}_{t}, \sigma_{\omega, t}\right)\right) R_{t}^{k} q_{t-1} k_{t-1}-s_{t-1} R_{t-1} \frac{b_{t-1}}{p_{t-1}}\right]+w^{e}
$$

- The death process ensures that entrepreneurs do not accumulate enough wealth so as to make the financing problem irrelevant.


## The Financial Intermediary

- A representative competitive financial intermediary.
- We can think of it as a bank but it may include other financial firms.
- Intermediates between households and entrepreneurs.
- The bank:
(1) Lends to entrepreneurs a nominal amount $b_{t}$ at rate $R_{t+1}^{\prime}$,
(2) But recovers only an (uncontingent) rate $R_{t}$ because of default and the (stochastic) intermediation costs.
(3) Thus, the bank pays interest $R_{t}$ to households.


## The Monetary Authority Problem

- Conventional Taylor rule:

$$
\frac{R_{t}}{R}=\left(\frac{R_{t-1}}{R}\right)^{\gamma_{R}}\left(\frac{\Pi_{t}}{\Pi}\right)^{\gamma_{\Pi}\left(1-\gamma_{R}\right)}\left(\frac{y_{t}}{y}\right)^{\gamma_{y}\left(1-\gamma_{R}\right)} \exp \left(\sigma_{m} m_{t}\right)
$$

through open market operations that are financed through lump-sum transfers $T_{t}$.

- The variable $\Pi$ represents the target level of inflation (equal to inflation in the steady-state), $y$ is the steady state level of output, and $R=\frac{\Pi}{\beta}$ the steady state nominal gross return of capital.
- The term $\varepsilon_{m t}$ is a random shock to monetary policy distributed according to $\mathcal{N}(0,1)$.


## Aggregation

- Using conventional arguments, we find expressions for aggregate demand and supply:

$$
\begin{gathered}
y_{t}=c_{t}+i_{t}+\mu G\left(\bar{\omega}_{t}, \sigma_{\omega, t}\right)\left(r_{t}+q_{t}(1-\delta)\right) k_{t-1} \\
y_{t}=\frac{1}{v_{t}} e^{z_{t}} k_{t-1}^{\alpha} l_{t}^{1-\alpha}
\end{gathered}
$$

where $v_{t}=\int_{0}^{1}\left(\frac{p_{i t}}{p_{t}}\right)^{-\varepsilon} d i$ is the inefficiency created by price dispersion.

- By the properties of Calvo pricing, $v_{t}$ evolves as:

$$
v_{t}=\theta \Pi_{t}^{\varepsilon} v_{t-1}+(1-\theta) \Pi_{t}^{*-\varepsilon} .
$$

- We have steady state inflation $\Pi$. Hence, $\widehat{v}_{t} \neq 0$ and monetary policy has an impact on the level and evolution of measured productivity.


## Equilibrium Conditions I

- The first-order conditions of the household:

$$
\begin{gathered}
e^{d_{t}} u_{1}(t)=\lambda_{t} \\
\lambda_{t}=\beta \mathbb{E}_{t}\left\{\lambda_{t+1} \frac{R_{t}}{\Pi_{t+1}}\right\} \\
-u_{2}(t)=u_{1}(t) w_{t}
\end{gathered}
$$

## Equilibrium Conditions II

- The first-order conditions of the intermediate firms:

$$
\begin{gathered}
\varepsilon g_{t}^{1}=(\varepsilon-1) g_{t}^{2} \\
g_{t}^{1}=\lambda_{t} m c_{t} y_{t}+\beta \theta \mathbb{E}_{t} \Pi_{t+1}^{\varepsilon} g_{t+1}^{1} \\
g_{t}^{2}=\lambda_{t} \Pi_{t}^{*} y_{t}+\beta \theta \mathbb{E}_{t} \Pi_{t+1}^{\varepsilon-1}\left(\frac{\Pi_{t}^{*}}{\Pi_{t+1}^{*}}\right) g_{t+1}^{2} \\
k_{t-1}=\frac{\alpha}{1-\alpha} \frac{w_{t}}{r_{t}} l_{t} \\
m c_{t}=\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} \frac{w_{t}^{1-\alpha} r_{t}^{\alpha}}{e^{z_{t}}}
\end{gathered}
$$

## Equilibrium Conditions III

- Price index evolves:

$$
1=\theta \Pi_{t}^{\varepsilon-1}+(1-\theta) \Pi_{t}^{* 1-\varepsilon}
$$

- Capital good producers:

$$
\begin{aligned}
& q_{t}\left(1-S\left[\frac{i_{t}}{i_{t-1}}\right]-S^{\prime}\left[\frac{i_{t}}{i_{t-1}}\right] \frac{i_{t}}{i_{t-1}}\right) \\
& +\beta \mathbb{E}_{t} \frac{\lambda_{t+1}}{\lambda_{t}} q_{t+1} S^{\prime}\left[\frac{i_{t+1}}{i_{t}}\right]\left(\frac{i_{t+1}}{i_{t}}\right)^{2}=1 \\
& k_{t}=(1-\delta) k_{t-1}+\left(1-S\left[\frac{i_{t}}{i_{t-1}}\right]\right) i_{t}
\end{aligned}
$$

## Equilibrium Conditions IV

- Entrepreneur problem:

$$
\begin{gathered}
R_{t+1}^{k}=\Pi_{t+1} \frac{r_{t+1}+q_{t+1}(1-\delta)}{q_{t}} \\
\frac{R_{t+1}^{k}}{s_{t} R_{t}}\left[\Gamma\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)-\mu G\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right]=\frac{q_{t} k_{t}-n_{t}}{q_{t} k_{t}} \\
\mathbb{E}_{t} \frac{R_{t+1}^{k}}{R_{t}}\left(1-\Gamma\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)\right)= \\
\left(\mathbb{E}_{t} s_{t} \frac{1-F\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)}{1-F\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)-\mu \bar{\omega}_{t+1} F_{\omega}\left(\bar{\omega}_{t+1}, \sigma_{\omega, t+1}\right)}\right) \frac{n_{t}}{q_{t} k_{t}} \\
R_{t+1}^{\prime} b_{t}=\bar{\omega}_{t+1} R_{t+1}^{k} p_{t} q_{t} k_{t} \\
q_{t} k_{t}=n_{t}+\frac{b_{t}}{p_{t}} \\
n_{t}=\gamma^{e} \frac{1}{\Pi_{t}}\left[\left(1-\mu G\left(\bar{\omega}_{t}, \sigma_{\omega, t}\right)\right) R_{t}^{k} q_{t-1} k_{t-1}-s_{t-1} R_{t-1} \frac{b_{t-1}}{p_{t-1}}\right]+w^{e}
\end{gathered}
$$

## Equilibrium Conditions V

- The government follows its Taylor rule:

$$
\frac{R_{t}}{R}=\left(\frac{R_{t-1}}{R}\right)^{\gamma_{R}}\left(\frac{\Pi_{t}}{\Pi}\right)^{\gamma_{\Pi}\left(1-\gamma_{R}\right)}\left(\frac{y_{t}}{y}\right)^{\gamma_{y}\left(1-\gamma_{R}\right)} \exp \left(\sigma_{m} m_{t}\right)
$$

- Market clearing

$$
\begin{gathered}
y_{t}=c_{t}+i_{t}+\mu G\left(\bar{\omega}_{t}, \sigma_{\omega, t}\right)\left(r_{t}+q_{t}(1-\delta)\right) k_{t-1} \\
y_{t}=\frac{1}{v_{t}} e^{z_{t}} k_{t-1}^{\alpha} l_{t}^{1-\alpha} \\
v_{t}=\theta \Pi_{t}^{\varepsilon} v_{t-1}+(1-\theta) \Pi_{t}^{*-\varepsilon}
\end{gathered}
$$

## Equilibrium Conditions VI

- Stochastic processes:

$$
\begin{gathered}
d_{t}=\rho_{d} d_{t-1}+\sigma_{d} \varepsilon_{d, t} \\
z_{t}=\rho_{z} z_{t-1}+\sigma_{z} \varepsilon_{z, t} \\
s_{t}=1+e^{\bar{s}+\widetilde{s}_{t}} \\
\widetilde{s}_{t}=\rho_{s} \widetilde{s}_{t-1}+\sigma_{s} \varepsilon_{s, t} \\
\log \sigma_{\omega, t}=\left(1-\rho_{\sigma}\right) \log \sigma_{\omega}+\rho_{\sigma} \log \sigma_{\omega, t-1}+\eta_{\sigma} \varepsilon_{\sigma, t}
\end{gathered}
$$

## Calibration

- Utility function:

$$
u\left(c_{t}, l_{t}\right)=\log c_{t}-\psi \frac{l_{t}^{1+\vartheta}}{1+\vartheta}
$$

$\psi$ : households work one-third of their available time in the steady state and $\vartheta=0.5$, inverse of Frisch elasticity.

- Technology:

| $\alpha$ | $\delta$ | $\varepsilon$ | $S^{\prime \prime}[1]$ |
| :---: | :---: | :---: | :---: |
| 0.33 | 0.023 | 8.577 | 14.477 |

- Entrepreneur:

| $\mu$ | $\sigma_{\omega}$ | $w^{e}$ | $\bar{s}$ |
| :---: | :---: | :---: | :---: |
| 0.15 | 2.528 | $\frac{n}{n-k} \approx 2$ | $25 b p$. |

- For the Taylor rule, $\Pi=1.005, \gamma_{R}=0.95, \gamma_{\Pi}=1.5$, and $\gamma_{y}=0.1$ are conventional values.
- For the stochastic processes, all the autoregressive are 0.95 .


## Computation

- We can find the deterministic steady state.
- We linearize around this steady state.
- We solve using standard procedures.
- Alternatives:
(1) Non-linear solutions.
(2) Estimation using likelihood methods.

Figure 3.1: Shock to Preferences, 1










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Figure 3.2: Shock to Preferences, 2









Figure 3.3: Shock to Productivity, 1








Figure 3.4: Shock to Productivity, 2


Figure 3.5: Shock to Volatility, 1









Figure 3.6: Shock to Volatility, 2







Figure 3.7: Shock to Spread, 1





Figure 3.8: Shock to Spread, 2










Figure 3.9: Shock to Survival, 1










Figure 3.10: Shock to Survival, 2









Figure 3.11: Shock to Monetary Policy, 1










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Figure 3.12: Shock to Monetary Policy, 2


## How Can We Use the Model?

- Christiano, Motto, and Rostagno (2003): Great depression.
- Christiano, Motto, and Rostagno (2008): Business cycle fluctuations.
- Fernández-Villaverde and Ohanian (2009): Spanish crisis of 2008-2010.
- Fernández-Villaverde (2010): fiscal policy.


Figure: IRFs of Output to Different Fiscal Policy Shocks

