A Modern Equilibrium Model

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Household Problem

- Preferences:

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{l_t^{1+\gamma}}{1+\gamma} \right\}$$

- Budget constraint:

$$c_t + k_{t+1} = w_t l_t + r_t k_t + (1-\delta) k_t, \ \forall \ t > 0$$

- Complete markets and Arrow-Debreu securities.

First Order Conditions

$$
\begin{aligned}
c_t^{-\sigma} &= \lambda_t \\
\beta E_t c_{t+1}^{-\sigma} &= \lambda_{t+1} \\
\psi l_t^{\gamma} &= \lambda_t w_t \\
\lambda_t &= \lambda_{t+1} \left(r_{t+1} + 1 - \delta \right)
\end{aligned}
$$

Problem of the Firm

- Neoclassical production function:

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$$

- By profit maximization:

$$\begin{aligned} \alpha A_t k_t^{\alpha-1} l_t^{1-\alpha} &= r_t \\ (1-\alpha) A_t k_t^{\alpha} l_t^{1-\alpha} l_t^{-1} &= w_t \end{aligned}$$

- Investment x_t induces a law of motion for capital:

$$k_{t+1} = (1-\delta) k_t + x_t$$

Evolution of the technology

- A_t changes over time.

- It follows the AR(1) process:

$$\begin{aligned} \log A_t &= \rho \log A_{t-1} + z_t \\ z_t &\sim \mathcal{N}(0, \sigma_z) \end{aligned}$$

- Interpretation of ρ.

A Competitive Equilibrium

- We can define a competitive equilibrium in the standard way.

- Conditions:

$$
\begin{aligned}
c_t^{-\sigma} &= \beta E_t c_{t+1}^{-\sigma} \left(r_{t+1} + 1 - \delta \right) \\
\psi l_t^{\gamma} &= \lambda_t w_t \\
r_t &= \alpha A_t k_t^{\alpha - 1} l_t^{1-\alpha} \\
w_t &= \left(1 - \alpha \right) A_t k_t^{\alpha} l_t^{1-\alpha} l_t^{-1} \\
c_t + k_{t+1} &= A_t k_t^{\alpha - 1} l_t^{1-\alpha} + \left(1 - \delta \right) k_t \\
\log A_t &= \rho \log A_{t-1} + z_t
\end{aligned}
$$

Behavior of the Model

- We have an initial shock: productivity changes.
- We have a transmission mechanism: intertemporal substitution and capital accumulation.
- We can look at a simulation from this economy.
- Why only a simulation?

Comparison with US economy

- Simulated Economy output fluctuations are around 75% as big as observed fluctuations.

- Consumption is less volatile than output.

- Investment is much more volatile.

- Behavior of hours.

Assessment of the Basic Real Business Model

- It accounts for a substantial amount of the observed fluctuations.
- It accounts for the covariances among a number of variables.
- It has some problems accounting for the behavior of the hours worked.
- More important question: where do productivity shocks come from?

Negative Productivity Shocks I

- The model implies that half of the quarters we have negative technology shocks.

- Is this plausible? What is a negative productivity shocks?

- Role of trend.

Negative Productivity Shocks II

- s.d. of shocks is 0.01. Mean quarter productivity growth is 0.0048 (to give us a 1.9% growth per year).

- As a consequence, we would only observe negative technological shocks when $\varepsilon_t < -0.0048$.

- This happens in the model around 33% of times.

- Ways to fix it.

Some Policy Implications

- The basic model is Pareto-efficient.
- Fluctuations are the optimal response to a changing environment.
- Fluctuations are not a sufficient condition for inefficiencies or for government intervention.
- In fact in this model the government can only worsen the allocation.
- Recessions have a “cleansing” effect.

Extensions I

- We can extend our model in several directions.

- Examples we are not going to cover:

  1. Fiscal Policy shocks (McGrattan, 1994).

  2. Agents with Finite Lives (Ríos-Rull, 1996).

  3. Indivisible Labor (Rogerson, 1988, and Hansen, 1985).

  4. Home Production (Benhabib, Rogerson and Wright, 1991).

Extensions We Study (Cumulative)

- Money (Cooley and Hansen, 1989).

Money

- Money in the Utility function (MIU).
- Comparison with Cash-in-Advance.
- You can show both approaches are equivalent.
- Are we doing the right thing (Wallace, 2001)?

Households

- Utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\upsilon}{1-\xi} \left(\frac{m_t}{p_t} \right)^{1-\xi} - \psi \frac{l_t^{1+\gamma}}{1+\gamma} \right\}.$$

- Budget constraint:

$$c_t + x_t + \frac{m_t}{p_t} + \frac{b_{t+1}}{p_t} = w_t l_t + r_t k_t + \frac{m_{t-1}}{p_t} + R_t \frac{b_t}{p_t} + T_t + \Pi_t$$

First Order Conditions I

$$
\begin{aligned}
c_t^{-\sigma} &= \lambda_t \\
\beta E_t c_{t+1}^{-\sigma} &= \lambda_{t+1} \\
\psi l_t^{\gamma} &= \lambda_t w_t \\
\upsilon \left(\frac{m_t}{p_t} \right)^{-\xi} &= \lambda_t \frac{1}{p_t} + \lambda_{t+1} \frac{1}{p_{t+1}} \\
\lambda_t \frac{1}{p_t} &= \lambda_{t+1} R_{t+1} \frac{1}{p_{t+1}} \\
\lambda_t &= \lambda_{t+1} \left(r_{t+1} + 1 - \delta \right)
\end{aligned}
$$

First Order Conditions II

$$c_t^{-\sigma} = \beta E_t\{c_{t+1}^{-\sigma}(r_{t+1} + 1 - \delta)\}$$

$$c_t^{-\sigma} = \beta E_t\{c_{t+1}^{-\sigma} R_{t+1} \frac{p_t}{p_{t+1}}\}$$

$$\psi l_t^{\gamma} = c_t^{-\sigma} w_t$$

$$\upsilon \left(\frac{m_t}{p_t}\right)^{-\xi} = c_t^{-\sigma} + E_t\{c_{t+1}^{-\sigma} \frac{p_t}{p_{t+1}}\}$$

The Government Problem

- The government sets the nominal interest rates according to the Taylor rule:

$$\frac{R_{t+1}}{R} = \left(\frac{R_t}{R}\right)^{\gamma_R} \left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y}\right)^{\gamma_y} e^{\varphi_t}$$

- How? Through open market operations.

- How are those operations financed? Lump-sum transfers T_t such that the deficit are equal to zero:

$$T_t = \frac{m_t}{p_t} - \frac{m_{t-1}}{p_t} + \frac{b_{t+1}}{p_t} - R_t \frac{b_t}{p_t}$$

Interpretation

- π: target level of inflation (equal to inflation in the steady state),
- y: the steady state output
- R: steady state gross return of capital.
- φ_t: random shock to monetary policy distributed according to $\mathcal{N}(0, \sigma_\varphi)$.
- The presence of the previous period interest rate, R_t, is justified because we want to match the smooth profile of the interest rate over time observed in U.S. data.

Advantages and Disadvantages of Taylor Rules

- Advantages:

  1. Simplicity.

  2. Empirical foundation.

- Problems:

  1. Normative versus positive.

  2. Empirical specification.

Behavior of the Model

- We have a second shock: interest shock.

- We have a transmission mechanism: intertemporal substitution and capital accumulation.

- For a standard calibration, the second shock buys us very little.

Extensions We Study (Cumulative)

- Money (Cooley and Hansen, 1989).

- Monopolistic Competition (Blanchard and Kiyotaki, 1987, and Horstein, 1993).

Monopolistic Competition

- Final good producer. Competitive behavior.

- Continuum of intermediate good producers with market power.

- Alternative formulations: continuum of goods in the utility function.

- Otherwise, the model is the same as the model with money.

The Final Good Producer

- Production function:

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon - 1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$

  where ε controls the elasticity of substitution.

- Final good producer is perfectly competitive and maximize profits, taking as given all intermediate goods prices p_{ti} and the final good price p_t.

Maximization Problem

- Thus, its maximization problem is:

$$\max_{y_{it}} p_t y_t - \int_0^1 p_{it} y_{it} di$$

- First order conditions are:

$$p_t \frac{\varepsilon}{\varepsilon - 1} \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1} - 1} \frac{\varepsilon - 1}{\varepsilon} y_{it}^{\frac{\varepsilon-1}{\varepsilon} - 1} - p_{it} = 0 \qquad \forall i$$

Working with the First Order Conditions I

- Dividing the first order conditions for two intermediate goods i and j, we get:

$$\frac{p_{it}}{p_{jt}} = \left(\frac{y_{it}}{y_{jt}}\right)^{-\frac{1}{\varepsilon}}$$

or:

$$p_{jt} = \left(\frac{y_{it}}{y_{jt}}\right)^{\frac{1}{\varepsilon}} p_{it}$$

- Interpretation.

Working with the First Order Conditions II

- Hence:

$$p_{jt}y_{jt} = p_{it}y_{it}^{\frac{1}{\varepsilon}}y_{jt}^{\frac{\varepsilon-1}{\varepsilon}}$$

- Integrating out:

$$\int_0^1 p_{jt}y_{jt}dj = p_{it}y_{it}^{\frac{1}{\varepsilon}}\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}}dj = p_{it}y_{it}^{\frac{1}{\varepsilon}}y_t^{\frac{\varepsilon-1}{\varepsilon}}$$

Input Demand Function

- By zero profits ($p_t y_t = \int_0^1 p_{jt} y_{jt} dj$), we get:

$$p_t y_t = p_{it} y_{it}^{\frac{1}{\varepsilon}} y_t^{\frac{\varepsilon-1}{\varepsilon}} \Rightarrow p_t = p_{it} y_{it}^{\frac{1}{\varepsilon}} y_t^{-\frac{1}{\varepsilon}}$$

- Consequently, the input demand functions associated with this problem are:

$$y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} y_t \qquad \forall i$$

- Interpretation.

Price Level

- By the zero profit condition $p_t y_t = \int_0^1 p_{it} y_{it} di$ and plug-in the input demand functions:

$$p_t y_t = \int_0^1 p_{it} \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t di \Rightarrow p_t^{1-\varepsilon} = \int_0^1 p_{it}^{1-\varepsilon} di$$

- Thus:

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

Intermediate Good Producers

- Continuum of intermediate goods producers.

- No entry/exit.

- Each intermediate good producer i has a production function

$$y_{it} = A_t k_{it}^{\alpha} l_{it}^{1-\alpha}$$

- A_t follows the AR(1) process:

$$\begin{aligned} \log A_t &= \rho \log A_{t-1} + z_t \\ z_t &\sim \mathcal{N}(0, \sigma_z) \end{aligned}$$

Maximization Problem I

- Intermediate goods producers solve a two-stages problem.

- First, given w_t and r_t, they rent l_{it} and k_{it} in perfectly competitive factor markets in order to minimize real cost:

$$\min_{l_{it},k_{it}} \{w_t l_{it} + r_t k_{it}\}$$

subject to their supply curve:

$$y_{it} = A_t k_{it}^{\alpha} l_{it}^{1-\alpha}$$

First Order Conditions

- The first order conditions for this problem are:

$$
\begin{aligned}
w_t &= \varrho \left(1-\alpha\right) A_t k_{it}^{\alpha} l_{it}^{-\alpha} \\
r_t &= \varrho \alpha A_t k_{it}^{\alpha-1} l_{it}^{1-\alpha}
\end{aligned}
$$

where ϱ is the Lagrangian multiplier or:

$$k_{it} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t} l_{it}$$

- Note that ratio capital-labor only is the same for all firms i.

Real Cost

- The real cost of optimally using l_{it} is:

$$\left(w_t l_{it} + \frac{\alpha}{1-\alpha} w_t l_{it}\right)$$

- Simplifying:

$$\left(\frac{1}{1-\alpha}\right) w_t l_{it}$$

Marginal Cost I

- The firm has constant returns to scale.

- Then, we can find the real marginal cost mc_t by setting the level of labor and capital equal to the requirements of producing one unit of good $A_t k_{it}^{\alpha} l_{it}^{1-\alpha} = 1$

- Thus:

$$A_t k_{it}^{\alpha} l_{it}^{1-\alpha} = A_t \left(\frac{\alpha}{1-\alpha} \frac{w_t}{r_t} l_{it} \right)^{\alpha} l_{it}^{1-\alpha} = A_t \left(\frac{\alpha}{1-\alpha} \frac{w_t}{r_t} \right)^{\alpha} l_{it} = 1$$

Marginal Cost II

- Then:

$$mc_t = \left(\frac{1}{1-\alpha}\right) w_t \frac{1}{A_t} \left(\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{-\alpha} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{1}{A_t} w_t^{1-\alpha} r_t^{\alpha}$$

- Note that the marginal cost does not depend on i.

- Also, from the optimality conditions of input demand, input prices must satisfy:

$$k_t = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t} l_t$$

Maximization Problem II

- The second part of the problem is to choose price that maximizes discounted real profits, i.e.,

$$\max_{p_{it}} \left\{ \left(\frac{p_{it}}{p_t} - mc_t \right) y_{it}^* \right\}$$

subject to

$$y_{it+\tau}^* = \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon} y_{t+\tau},$$

- First order condition:

$$\frac{1}{p_t} \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon} y_{t+\tau} - \varepsilon \left(\frac{p_{it}}{p_t} - mc_t \right) \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon-1} \frac{1}{p_t} y_{t+\tau} = 0$$

Mark-Up Condition

- From the fist order condition:

$$1 - \varepsilon \left(\frac{p_{it}}{p_t} - mc_t \right) \left(\frac{p_{it}}{p_t} \right)^{-1} = 0 \Rightarrow$$
$$p_{it} = \varepsilon \left(p_{it} - mc_t p_t \right) \Rightarrow$$
$$p_{it} = \frac{\varepsilon}{\varepsilon - 1} mc_t p_t$$

- Mark-up condition.

- Reasonable values for ε.

Behavior of the Model

- Presence of monopolistic competition is, by itself, pretty irrelevant.
- Why? Constant mark-up.
- Similar to a tax.
- Solutions:
    1. Shocks to mark-up (maybe endogenous changes).
    2. Price rigidities.

Extensions We Study (Cumulative)

- Money (Cooley and Hansen, 1989).

- Monopolistic Competition (Blanchard and Kiyotaki, 1987, and Horstein, 1993).

- Price rigidities (Calvo, 1983).

The Baseline Sticky Price Model

- The basic structure of the economy is as before:

  1. A representative household consumes, saves, holds money, and works.

  2. There is a monetary authority that fixes the one-period nominal interest rate through open market operations with public debt.

  3. Final output is manufactured by a final good producer, which uses as inputs a continuum of intermediate goods manufactured by monopolistic competitors.

- Additional constraint: the intermediate good producers face the constraint that they can only change prices following a Calvo's rule.

Maximization Problem II

- The second part of the problem is to choose price that maximizes discounted real profits, i.e.,

$$\max_{p_{it}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \, v_{t+\tau} \left\{ \left(\frac{p_{it}}{p_{t+\tau}} - mc_{t+\tau} \right) y^{*}_{it+\tau} \right\}$$

  subject to

$$y^{*}_{it+\tau} = \left(\frac{p_{ti}}{p_{t+\tau}} \right)^{-\varepsilon} y_{t+\tau},$$

- Think about different elements of the problem.

Pricing I

- Calvo pricing.

- Parameter θ_p.

- What if $\theta_p = 0$?

Pricing II

- Alternative pricing mechanisms:

  1. Time-dependent pricing: Taylor and Calvo.

  2. State-dependent pricing.

- Advantages and disadvantages:

  1. Klenow and Kryvstov (2005): “intensive” versus “extensive” margin of price changes.

  2. Caplin and Spulber (1987), Dotsey *et al.* (1999).

Discounting

- v_t is the marginal value of a dollar to the household, which is treated as exogenous by the firm.

- We have complete markets in securities,

- Then, this marginal value is constant across households.

- Consequently, $\beta^\tau v_{t+\tau}$ is the correct valuation on future profits.

First Order Condition

- Substituting the demand curve in the objective function, we get:

$$\max_{p_{it}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} v_{t+\tau} \left\{ \left(\left(\frac{p_{it}}{p_{t+\tau}} \right)^{1-\varepsilon} - \left(\frac{p_{ti}}{p_{t+\tau}} \right)^{-\varepsilon} mc_{t+\tau} \right) y^*_{it+\tau} \right\}$$

- The solution p^*_{it} implies the first order condition:

$$E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} v_{t+\tau} \left\{ \left(\begin{array}{c} (1-\varepsilon) \left(\frac{p^*_{it}}{p_{t+\tau}} \right)^{1-\varepsilon} p^{*-1}_{it} \\ +\varepsilon \left(\frac{p^*_{it}}{p_{t+\tau}} \right)^{-\varepsilon} p^{*-1}_{it} mc_{t+\tau} \end{array} \right) y^*_{it+\tau} \right\} = 0$$

Working on the Expression I

- Then:

$$
\begin{aligned}
E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^\tau \, v_{t+\tau} \left\{ \left((1-\varepsilon) \frac{p_{it}^*}{p_{t+\tau}} p_{it}^{*-1} + \varepsilon p_{it}^{*-1} mc_{t+\tau} \right) y_{it+\tau} \right\} &= 0 \Rightarrow \\
E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^\tau \, v_{t+\tau} \left\{ \left(\frac{p_{it}^*}{p_{t+\tau}} - \frac{\varepsilon}{\varepsilon - 1} mc_{t+\tau} \right) y_{it+\tau} \right\} &= 0
\end{aligned}
$$

- Thus, in a symmetric equilibrium, in every period, $1-\theta_p$ of the intermediate good producers set p_t^* as their price policy function, while the remaining θ_p do not reset their price at all.

Working on the Expression II

- Note, if $\theta_p = 0$ (all firms move), all terms $\tau > 0$ are zero:

$$v_t \left\{ \left(\frac{p_{it}^*}{p_t} - \frac{\varepsilon}{\varepsilon - 1} mc_t \right) y_{it} \right\} = 0$$

- Then

$$p_{it}^* = \frac{\varepsilon}{\varepsilon - 1} mc_t p_t$$

the result we had from the basic monopolistic case.

Price Level

- The price index evolves:

$$p_t = \left[\theta_p p_{t-1}^{1-\varepsilon} + (1-\theta_p)\, p_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

- Dividing by p_{t-1}:

$$\pi_t = \frac{p_t}{p_{t-1}} = \left[\theta_p + (1-\theta_p)\, \pi_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

where $\pi_t^* = \frac{p_t^*}{p_{t-1}}$.

Aggregation I

- Now, we derive an expression for aggregate output.

- Remember that:

$$\frac{k_{it}}{l_{it}} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t}$$

- Since this ratio is equivalent for all intermediate firms, it must also be the case that:

$$\frac{k_t}{l_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t}$$

Aggregation II

- We get:

$$\frac{k_{it}}{l_{it}} = \frac{k_t}{l_t}$$

- If we substitute this condition in the production function of the intermediate good firm $y_{it} = A_t k_{it}^{\alpha} l_{it}^{1-\alpha}$we derive:

$$y_{it} = A_t \left(\frac{k_{it}}{l_{it}} \right)^{\alpha} l_{it} = A_t \left(\frac{k_t}{l_t} \right)^{\alpha} l_{it}$$

Aggregation III

- The demand function for the firm is:

$$y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} y_t \qquad \forall i,$$

- Thus, we find the equality:

$$\left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} y_t = A_t \left(\frac{k_t}{l_t}\right)^{\alpha} l_{it}$$

Aggregation IV

- If we integrate in both sides of this equation:

$$y_t \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di = A_t \left(\frac{k_t}{l_t}\right)^{\alpha} \int_0^1 l_{it} di = A_t k_t^{\alpha} l_t^{1-\alpha}$$

- Then:

$$y_t = \frac{A_t}{v_t} k_t^{\alpha} l_t^{1-\alpha}$$

where

$$v_t = \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di = \frac{j_t^{-\varepsilon}}{p_t^{-\varepsilon}} = j_t^{-\varepsilon} p_t^{\varepsilon}$$

and $j_t = \left(\int_0^1 p_{it}^{-\varepsilon} di\right)^{-\frac{1}{\varepsilon}}$.

Interpretation of j_t

- What is j_t?
- Measure of price disperssion.
- Measure of efficiency losses implied by inflation.
- Important to remenber for welfare analysis.

Equilibrium

- A definition of equilibrium in this economy is standard.

- The symmetric equilibrium policy functions are determined by the following blocks:

  1. The first order conditions of the household.

The First Order Conditions of the Household

$$c_t^{-\sigma} = \beta E_t\{c_{t+1}^{-\sigma}(r_{t+1} + 1 - \delta)\}$$

$$c_t^{-\sigma} = \beta E_t\{c_{t+1}^{-\sigma}\frac{R_{t+1}}{\pi_{t+1}}\}$$

$$\psi l_t^{\gamma} = c_t^{-\sigma} w_t$$

$$\upsilon\left(\frac{m_t}{p_t}\right)^{-\xi} = c_t^{-\sigma} + E_t\{c_{t+1}^{-\sigma}\frac{1}{\pi_{t+1}}\}$$

$$c_t + x_t + \frac{m_t}{p_t} + \frac{b_{t+1}}{p_t} = w_t l_t + r_t k_t + \frac{m_{t-1}}{p_t} + R_t\frac{b_t}{p_t} + T_t + \Pi_t$$

Equilibrium

- A definition of equilibrium in this economy is standard.

- The symmetric equilibrium policy functions are determined by the following blocks:

  1. The first order conditions of the household.

  2. Pricing decisions by firms.

Pricing Decisions by Firms.

The firms that can change prices set them to satisfy

$$E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \, v_{t+\tau} \left\{ \left(\frac{p_t^*}{p_{t+\tau}} - \frac{\varepsilon}{\varepsilon - 1} mc_{t+\tau} \right) y_{it+\tau} \right\} = 0$$

where:

$$y_{it+\tau}^* = \left(\frac{p_{ti}}{p_{t+\tau}} \right)^{-\varepsilon} y_{t+\tau},$$

and

$$mc_t = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \frac{1}{A_t} w_t^{1-\alpha} r_t^{\alpha}$$

Equilibrium

- A definition of equilibrium in this economy is standard.

- The symmetric equilibrium policy functions are determined by the following blocks:

  1. The first order conditions of the household.

  2. Pricing decisions by firms.

  3. Pricing of inputs and outputs.

Input and Output Prices

- Input prices satisfy:

$$k_t = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t} l_t$$

- The price level evolves:

$$\pi_t = \frac{p_t}{p_{t-1}} = \left[\theta_p + (1-\theta_p)\, \pi_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Equilibrium

- A definition of equilibrium in this economy is standard.

- The symmetric equilibrium policy functions are determined by the following blocks:

  1. The first order conditions of the household.

  2. Pricing decisions by firms.

  3. Pricing of inputs and outputs.

  4. Taylor rule and market clearing.

Taylor Rule and Market Clearing

- Taylor rule

$$\frac{R_{t+1}}{R} = \left(\frac{R_t}{R}\right)^{\gamma_R} \left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y}\right)^{\gamma_y} e^{\varphi_t}$$

- Markets clear:

$$c_t + x_t = y_t = \frac{A_t}{v_t} k_t^\alpha l_t^{1-\alpha}$$

where $v_t = \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di$ and $k_{t+1} = (1-\delta) k_t + x_t$.

Further Extensions

- Investment-specific technological change.
- Sticky wages.
- Open economy.