Navigating by Falling Stars: Monetary Policy with Fiscally Driven Natural Rates *

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Abstract

We study a new type of monetary-fiscal interaction in a heterogeneous-agent New Keynesian model with a fiscal block. Due to household heterogeneity, the stock of public debt affects the natural interest rate, forcing the central bank to adapt its monetary policy rule to the fiscal stance to guarantee that inflation remains at its target. There is, however, a minimum level of debt below which the steady-state inflation deviates from its target due to the zero lower bound on nominal rates. We analyze the response to a debt-financed fiscal expansion and quantify the impact of different timings in the adaptation of the monetary policy rule, as well as the performance of alternative monetary policy rules that do not require an assessment of the natural rates. We validate our findings with a series of empirical estimates.

JEL Classification: E32, E58, E63.

Keywords: HANK models, natural rates, fiscal shocks.

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1 Introduction

After the demise of the Bretton Woods system in 1971, a consensus emerged among academics and policymakers in which central banks were entrusted with the task of controlling inflation, while treasuries had to guarantee debt sustainability. This separation of tasks was enshrined in the Maastricht Treaty in 1992. The European Union member states gave birth to the European Central Bank with the primary objective of preserving price stability. At the same time, national treasuries were subject to a set of fiscal rules aimed at preventing debt and deficits from increasing above certain thresholds.

Economics provided intellectual support to this arrangement. In the basic representative-agent New Keynesian model, or RANK (Woodford, 2003; Galí, 2008), the central bank can always deliver on its price stability mandate if certain conditions are satisfied. First, the treasury prevents debt from exploding. Second, the central bank responds forcefully enough to changes in inflation, i.e., it follows the so-called Taylor principle (or a variation of it). Third, the central bank tracks the natural rate r^* in the long run, and the latter is high enough to prevent the zero lower bound (ZLB) on nominal interest rates from becoming binding.[1]

The key result in the RANK framework is that the natural rate depends only on structural parameters, such as the household discount factor or productivity growth. Many economists feel confident in assuming that such parameters remain constant or evolve slowly according to secular trends.

Interestingly, this key result breaks down once we deviate from the complete-market representative-agent framework. Instead, consider a heterogeneous-agent New Keynesian (HANK) model, as popularized by Kaplan et al. (2018) and Auclert et al. (2018), among many others. These models incorporate a continuum of atomistic households subject to idiosyncratic risk and that can save only by using non-state-contingent instruments. One important feature of HANK models is the fact that the natural rate depends on the stock of public debt, as originally pointed out by Aiyagari and McGrattan (1998), and more recently by Rachel and Summers (2019). The intuition is simple: given market incompleteness, the stock of public debt determines how much households can self-insure against negative idiosyncratic shocks and, therefore, the interest rate at which the savings market clears.

The link between debt and natural rates opens the door to a form of monetary-fiscal interaction that has, so far, been unexplored. If the treasury changes the long-run stock

[1] In this paper, the natural rate is the real interest rate in the deterministic steady state of the economy. This concept is different from (although often confused with) the neutral interest rate, which is the real interest rate that would exist in a counterfactual economy with flexible prices. The neutral interest rate evolves stochastically in response to the different shocks that hit the economy. See Platzer et al. (2022) for a careful comparison of the natural vs. the neutral interest rate. The authors also call the natural rate the long-run r^* and the neutral rate the short-run r^*.

of public debt, this decision necessarily moves the natural rate. The central bank should then either incorporate the new natural rate into its Taylor rule or ignore it, which will bias long-term inflation expectations.

Our main argument goes beyond HANK models. Overlapping generation (OLG) models, for instance, also share that feature, as in Eggertsson et al. (2019) and Aguiar et al. (2023). The latter authors introduce a tractable New Keynesian OLG model to analyze the global equilibrium dynamics of inflation, interest rates, and labor earnings in response to changes in the stock of public debt. Furthermore, the empirical literature has documented that this channel is significant in the data. See, for example, Rachel and Summers (2019).

Fiscally Driven natural rates vs. the FTPL. The reader should note that the interaction we focus on is conceptually different from the one analyzed under the fiscal theory of the price level (FTPL). The core insight of the FTPL, dating back to Sargent and Wallace (1981), is that if the treasury is not committed to guaranteeing debt sustainability, the central bank may be forced to accommodate fiscal expansions to prevent debt from exploding. In these circumstances, inflation is determined by the need for stabilizing public debt. This is the kind of logic behind the Maastricht Treaty discussed above. In our argument, instead, the treasury collects taxes, spends, and funds deficits by issuing debt according to a fiscal rule that stabilizes the long-run real debt level. Fiscal policy affects the conduct of monetary policy not by refusing to stabilize long-term debt, but by shifting natural rates.

Our analysis. We postulate a HANK model with a central bank that sets nominal interest rates following a standard Taylor rule and a treasury. The economy has a unique steady state in which the debt target of the treasury pins down the natural rate: the higher this target, the higher the natural rate is. Steady-state inflation deviates from the central bank's target in proportion to the difference between the natural rate and the intercept of the Taylor rule. Thus, to ensure price stability, the central bank should change the Taylor rule intercept depending on the treasury's long-run debt target. In comparison, the RANK version of the model (when we shut down household heterogeneity) has a natural rate that depends only on household preferences (as in the conventional New Keynesian framework), and the central bank can ignore the treasury's long-run debt target.

There is nevertheless a situation in which the central bank in a HANK world cannot deliver on its long-run mandate even if it is willing to adapt its rule to the fiscal position. If the debt target is low enough, the natural interest rate becomes negative. If the natural rate is so negative that its sum with the inflation target is below zero, then the ZLB will be binding in the long term, and inflation will be equal to the opposite of the natural rate. Thus, there is a minimum debt level compatible with the inflation target. For any debt target below that minimum level, the central bank fails to deliver on its mandate.

Among many possible experiments, we focus on the response of the economy to a debt-financed fiscal expansion because we find it illustrates our argument better than other exercises. At time zero, the treasury announces an increase in its debt target, which temporarily expands the room for government spending. The economy then converges to a new steady state with a higher debt and natural rate. To achieve its inflation target in the long run, the central bank must increase the intercept in its Taylor rule, which raises the steady-state level of real and nominal rates. If it fails to do so, inflation increases both in the long and in the short term, though the impact on real variables is negligible. Another way to think about this result is that, by not adjusting the intercept in its Taylor rule, the central bank is implicitly changing its inflation target, which does not have much of a long-run effect on real variables.

The increase in inflation after the fiscal expansion is larger than in the counterfactual RANK model. The different inflation dynamics between HANK and RANK are mainly due to the different paths for employment and consumption. In the RANK model, consumption declines entirely due to intertemporal substitution, as households save more to profit from the increase in the real interest rate. This channel is also present in the HANK model, especially for wealthy households. However, it is partially compensated by an income effect that pushes consumption up, as poor low-income households can now increase their consumption due to an increase in labor income.

We explore a series of extensions of our model. We first extend the model to explore a proposal by Orphanides and Williams (2002), who argue that a differential rule in which deviations of inflation from its target affect the change in nominal interest rates avoids the need for an assessment of the natural rate. We show how such a rule produces much less volatility in inflation while the dynamics of real variables are relatively similar. This suggests that such a rule may be a good candidate for dealing with fiscally driven natural rates.

Our second extension analyzes different types of fiscal expansions. While our baseline exercise studies an increase in government consumption, here we consider two alternative options that lead to the same path for public debt: a reduction in income taxes and an increase in lump-sum transfers to households. These alternatives imply differences for nominal and real variables, both in the steady state and in the transition. Their impact on the natural interest rate is relatively similar as the determination of r^* mainly depends on the stock of debt. It is not identical, however, because alternative fiscal policies differ in their implications for household insurance, thus affecting the demand for debt.

Our third extension is to analyze the impact of timing decisions regarding the change in the policy rule. We modify the previous experiments and consider the case where the fiscal expansion is announced 12 quarters in advance. We then study the impact of three different scenarios: (i) changing the monetary policy rule on the announcement date, (ii) changing the rule on the date of the actual change, and (iii) changing it 12 quarters later than the actual change. While the three scenarios yield similar dynamics for inflation after the change in the rule, the later the announcement, the higher the inflation prior to the change.

Our fourth extension is to incorporate long-term public debt into the model. The muted impact on real variables in the short run of sticking to the old monetary policy rule after a debt-financed fiscal change is an artifact of considering short-term debt. If we extend the model to include long-term public bonds, the impact becomes significant in the short run. This is a consequence of the Fisher effect, as the surprise increase in inflation reduces the real value of debt. This reduction leads to a redistribution from wealthy households to the treasury, which in turn allows it to engage in a more aggressive fiscal expansion.[2]

In Section 7, we present empirical evidence validating the mechanisms in our model. First, we show how a standard estimate of the natural rate (Lubik and Matthes, 2015) increases in response to a rise in the debt-to-GDP ratio in a magnitude similar to the one predicted by the model. Second, we develop an analytical expression linking deviations of long-term inflation from the central bank's inflation target to the policy gap between the natural rate and the central bank's long-term rate implicit in its reaction function. We evaluate this expression using market data on long-term interest rates and inflation expectations and find significant policy gaps, especially in the post-pandemic period.

Literature review. This paper is related to the large literature on monetary-fiscal interactions, particularly under the FTPL (see Leeper, 1991; Sims, 1994; Cochrane, 1999; Woodford, 1995; or Schmitt-Grohe and Uribe, 2000; among others). More recently, Bianchi et al. (2022) analyze a model in which a monetary-led and a fiscally-led policy mix coexist at the same time, as the central bank accommodates unfunded fiscal shocks causing persistent movements in inflation. Bigio et al. (2023) study the expectation of a monetary-fiscal reform. Under the reform, monetary policy is temporarily obliged to provoke inflation to aid the treasury in making its debt sustainable. After the reform, debt and inflation stabilize again.

This paper also contributes to the literature analyzing fiscal and monetary policies in HANK models, including Oh and Reis (2012), Kaplan et al. (2018), Auclert et al. (2018), Hagedorn et al. (2019), McKay and Reis (2021), Wolf (2021), or Ferriere and Navarro (2018). Bayer et al. (2023) analyze how a permanent increase in the ratio of public debt to GDP increases the real public bond yield in the long run. Hagedorn (2016) shows that prices and inflation are jointly and uniquely determined by fiscal and monetary policy in a HANK model with nominal debt. Kaplan et al. (2023) analyze the FTPL in the context of a heterogeneous-agent model with flexible prices. In their model, a permanently higher deficit is associated

[2]The empirical relevance of the Fisher effect has been documented by Doepke and Schneider (2006), Adam and Zhu (2016), and Ferreira et al. (2023), among others.

with a lower steady-state real interest rate and less real public debt, as well as a higher long-run inflation rate for a given setting of monetary policy. Independently from us, Hansel (2024) analyzed how government debt expansions can increase the natural rate of interest and create inflation in a two-asset HANK economy.

This paper is also related to the literature on the estimation of the natural rate and its implications for monetary policy. There are different methods for estimating the natural rates, including semi-parametric methods (such as Laubach and Williams, 2003, or Holston et al., 2017), nonstructural time series methods (Lubik and Matthes, 2015), or methods based on extracting information on the expected long-run real interest rate from bond prices (e.g., Christensen and Rudebusch, 2019, or Davis et al., 2023). Since these methods do not always yield similar results, there is a significant degree of uncertainty in the estimation of the natural rate. This may lead to misperceptions about the level of natural rates, with implications for the monetary policy stance, as discussed by Ajello et al. (2020). Chortareas et al. (2023) estimate a time-varying Taylor rule for the U.S. and document how the Federal Reserve has occasionally misread the natural rate of interest. Daudignon and Tristani (2023) analyze the optimal monetary policy response to stochastic changes in the natural rate in a New Keynesian model. Finally, Bauer and Rudebusch (2020) show how accounting for time variation in natural rates is crucial for understanding the dynamics of the yield curve, a key object in monetary policy transmission.

The rest of the paper is structured as follows. Section 2 introduces our HANK model and Section 3 its calibration and computation. Section 4 explores the monetary-fiscal interactions in our model in the long run, while Section 5 analyzes the dynamics of such interaction. Section 6 discusses several extensions of our model. Section 7 presents empirical evidence validating the main elements of our analysis, and Section 8 concludes.

2 A HANK model with monetary and fiscal policy

To investigate monetary policy with fiscally driven natural interest rates, we introduce a baseline discrete-time HANK model with monetary and fiscal policy. We follow Auclert et al. (2023) and assume that wages are subject to nominal rigidities and hours are determined by a union on behalf of the workers.[3] Firms produce the final good with the labor supplied by the union. The model is closed by a monetary policy authority, which determines the nominal

[3] Auclert et al. (2023) show that HANK models with wage rigidities and flexible prices can simultaneously match plausible estimates of marginal propensities to consume (MPCs), marginal propensities to earn (MPEs), and fiscal multipliers. In contrast, HANK models with price rigidities and frictionless labor markets fail at doing so. The drawback of sticky prices is that they induce countercyclical mark-ups. Thus, the MPCs and MPEs of the model cannot match the data unless we distribute the corresponding profits in an *ad-hoc* way.

interest rate, and a treasury, which taxes, spends, and issues public debt.

Households. There is a continuum of households indexed by $i \in [0,1]$. Households derive utility from consumption, $c_{i,t}$, and disutility from working $n_{i,t}$ hours. They can only save in a nominal public bond. Given a discount factor β, the intertemporal problem solved by each household is:

$$
\begin{aligned}
V(a_{i,t}, z_{i,t}) = \max_{c_{i,t}, a_{i,t+1}} & \; u(c_{i,t}) - v(n_{i,t}) + \beta \mathbb{E}_t [V(a_{i,t+1}, z_{i,t+1})] \\
\text{s.t. } \; c_{i,t} + a_{i,t+1} &= (1+r_t) a_{i,t} + (1-\tau) \frac{W_t}{P_t} z_{i,t} n_{i,t} + T_t, \\
a_{i,t+1} &\geq 0,
\end{aligned}
$$

where $a_{i,t}$ is the household's asset position in real terms at the start of the period, $z_{i,t}$ is the idiosyncratic labor productivity, r_t denotes the *ex-post* real return of bonds in period t, W_t is the nominal wage, and P_t is the price level. Labor income is taxed at a constant rate τ. Households receive real net lump-sum transfers T_t from the treasury. Households cannot short bonds, i.e., $a_{i,t+1} \geq 0$.

At time t, household i works $n_{i,t}$ hours. A union chooses these hours on behalf of households. Each hour provides $z_{i,t}$ units of effective labor, so that aggregate hours are $N_t = \int_0^1 z_{i,t} n_{i,t} di$. The idiosyncratic shock $z_{i,t}$ follows a first-order Markov chain with mean $\mathbb{E}_t z_{i,t+1} = 1$. Agents take their hours $n_{i,t}$ as given. We assume a proportional allocation rule for labor hours, with $n_{i,t} = N_t$. The nominal wage W_t is determined by union bargaining as specified below.

Unions. We follow a standard formulation for sticky wages with heterogeneous agents, similar to Auclert et al. (2018) and Auclert et al. (2021b). Suffice it to say that the union aggregates different labor tasks provided by the households into a homogeneous labor service. Appendix A describes the details. The union employs all households for the same number of hours N_t and sets nominal wages by maximizing the welfare of the average household subject to a penalty term on the deviation of nominal wages from the central bank's inflation target $\overline{\pi}$.

Solving this problem leads to a wage Phillips curve:

$$
\begin{aligned}
\log\left(\frac{1+\pi_t^w}{1+\overline{\pi}}\right) = \kappa_w \left[-\frac{\epsilon_w - 1}{\epsilon_w} (1-\tau) \frac{W_t}{P_t} \int u'(c_{it}) z_{it} di + v'(N_t) \right] N_t + \\
\beta \log\left(\frac{1+\pi_{t+1}^w}{1+\overline{\pi}}\right),
\end{aligned}
\tag{1}
$$

where ϵ_w is the elasticity of substitution between different labor tasks, κ_w is the slope of the Phillips curve (itself a nonlinear function of other parameters of the model), and $\pi_t^w \equiv \frac{W_t}{W_{t-1}} - 1$ is the nominal wage inflation rate.

Firms. There is a continuum of identical firms. Firms produce final goods using a constant return-to-scale technology $Y_t = \Theta N_t$, where N_t is aggregate labor and $\Theta > 0$ is a constant productivity parameter. The real wage is given by $\frac{W_t}{P_t} = \Theta$. From these equations, wage inflation is equal to goods inflation, $\pi_t^w = \pi_t$, where the latter is defined as $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$.

Monetary policy. The central bank sets the nominal interest rate on nominal bonds i_t according to a standard monetary policy rule that responds to inflation, and it is subject to a ZLB (below, we will explore some of the consequences of this bound being binding):

$$\log(1+i_t) = \max\left\{\log(1+\overline{r}) + \log(1+\overline{\pi}) + \phi_\pi \log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right), 0\right\}, \tag{2}$$

where $\phi_\pi \geq 1$ is the slope of the Taylor rule, $\overline{\pi}$ is the inflation target, and $\overline{r}$ is the real rate intercept.

Fiscal policy. The treasury collects labor taxes from households and spends on real government consumption G_t and real lump-sum transfers to households T_t (a negative real transfer can be interpreted as a lump-sum tax). Also, the treasury can issue one-period nominal bonds (later, in Section 6, we will introduce long-term debt). Government consumption does not enter into the utility of the households (or, equivalently, it enters in a separable way from private consumption and labor supply, and we can drop the term).

Given a level of real tax collection $\mathcal{T}_t$, the public debt accumulates according to:

$$P_t B_t = (1+i_{t-1})P_{t-1}B_{t-1} + P_t(G_t + T_t - \mathcal{T}_t),$$

where B_t denotes the stock of bonds in real terms. If we define *ex-post* real rates as $(1+r_t) \equiv (1+i_{t-1})\frac{P_{t-1}}{P_t}$, we can express the government's budget constraint as

$$B_t = (1+r_t)B_{t-1} + T_t + G_t - \mathcal{T}_t.$$

Tax collection is given by:

$$\mathcal{T}_t = \int_0^1 \tau \frac{W_t}{P_t} z_{i,t} n_{i,t} di,$$

where τ is a constant. Nonetheless, $\mathcal{T}_t$ follows an endogenous process determined by the evolution of its underlying component variables. Similarly to τ, transfers $T_t = T$ will be a constant in the baseline calibration.

In comparison, government consumption, G_t, follows a fiscal rule depending on the expenditure $\overline{G}$ and the debt target $\overline{B}$:

$$G_t = \overline{G} - \phi_G(B_{t-1} - \overline{B}), \tag{3}$$

where $0 < \phi_G < 1$ controls the speed of fiscal adjustment when debt is not at its target. This fiscal rule is studied by Auclert et al. (2020, Section 5.3). It is also closely related to the fiscal rules studied by Auclert and Rognlie (2018).[4]

Why do we have a law of motion for G_t while keeping τ and T constant? As we will show later, in the steady state of the model, $G_t = \overline{G}$, making government consumption a constant determined by a single policy parameter (similar to how $\mathcal{T}_t$ is determined by τ and T_t by T). The difference between how we treat G, τ, and T is not relevant for the steady state.

The main aim of our paper is to analyze the effects of unexpected shocks to $\overline{B}$. The law of motion for G_t (which adjusts temporarily as B_t converges to the new $\overline{B}$) allows us to measure the response of G_t and other aggregate variables to such a shock, independently from changes in τ and T_t. In later variations of the model, we will also consider cases in which taxes and transfers adjust. In these variations, we will replicate the path for B_t that results from the baseline exercise but will induce it by adjusting either taxes or transfers (keeping $G_t = \overline{G}$ constant). Because we want to match in all exercises the same path of B_t, we cannot force τ and T_t to follow a predetermined law of motion, as we do with G_t.

By being able to freely adjust τ and T_t to match the path of B_t, we can effectively gauge the economy's response to unexpected shocks to $\overline{B}$ using different combinations of changes in G_t, τ, and T_t. Our model, though not linear, can be reasonably approximated by a simple linear combination of the three exercises we conduct. This analytical strategy allows us to cover a wide range of scenarios without unnecessarily lengthening the paper.

Aggregation and market clearing. In equilibrium, the labor, bond, and good markets clear:

$$
\begin{aligned}
N_t &= \int_0^1 z_{i,t} n_{i,t} di, \\
B_t &= \int_0^1 a_{i,t+1} di, \\
C_t &= \int_0^1 c_{i,t} di,
\end{aligned}
$$

and the aggregate resource constraint holds: $G_t + C_t = Y_t$.

3 Calibration and computation

We calibrate our model at quarterly frequency by borrowing standard parameter values from the literature and matching observations of the U.S. economy. Table 1 summarizes the

[4] Kaplan et al. (2023), on the other hand, consider a rule in which the fiscal deficit adjusts instead of government consumption.

calibration, which we explain below in more detail.

Table 1: Calibration parameters, Baseline model

	Parameter	Value	Target/Sources
	Preferences		
σ	Elasticity of intertemporal substitution	1	Standard
φ	Frisch elasticity of labor supply	0.5	Standard
ν_φ	Disutility of labor parameter	0.791	$N_{ss} = 1$
β	Quarterly discount factor	0.992	1% real interest rate in DSS
	Income process		
ρ_e	Persistence income process (annual)	0.91	Floden and Lindé (2001)
σ_e	Std. dev. idiosyncratic shock (annual)	0.92	Floden and Lindé (2001)
	Production		
Y	Quarterly output	1	Normalization
Θ	Constant level of TFP	1	Normalization
κ_w	Slope of the wage Phillips curve	0.1	Aggarwal et al. (2023)
ϵ_w	Elasticity of substitution	10	Standard
	Fiscal policy		
r	Real interest rate (annual)	0.01	Baseline case
$\overline{B}$	Debt target	2.8	Debt-to-GDP 70%
$\overline{G}$	Government spending target	0.2	Spending-to-GDP 20%
τ	Tax rate	0.277	Tax to GDP ratio in 2022
T	Net transfers	0.07	Constant debt level in DSS
ϕ_G	Coefficient in the fiscal rule	0.1	Baseline case
	Monetary policy		
ϕ_π	Taylor rule coefficient	1.25	Standard
$\overline{\pi}$	Inflation target (annual)	0.02	Standard

Preferences. We assume log utility over consumption: $u(c) = \log(c)$. The disutility over hours is parameterized using a function with a constant Frisch elasticity: $v(n) = \nu_\varphi n^{1+\frac{1}{\varphi}}/(1+\frac{1}{\varphi})$. We set the Frisch elasticity φ to 0.5. The preference shifter $\nu_\varphi = 0.791$ is calibrated so that, given all other parameters, total employment is 1 in the steady state. This is an immaterial normalization. We calibrate the discount factor to match a real interest rate of 1% annually. This implies a quarterly discount factor of $\beta = 0.992$.

Income process and borrowing limit. The persistence and standard deviation of income shocks are taken from estimates by Floden and Lindé (2001) of the U.S. wage process. We set the persistence of the income process to match a persistence of 0.91 yearly and the standard deviation of innovations to match the standard deviation of log gross earnings of 0.92. We first convert these values to quarterly frequency and then approximate the income process with a Markov chain with 11 discrete states calculated as in Rouwenhorst (1995).[5]

We discretize the asset space using a double-exponential transformation of a uniformly spaced grid using 500 grid points, with a minimum asset level of $\underline{a} = 0$ (the borrowing limit) and a maximum asset level of $\overline{a} = 150$. We solve the household problem using the endogenous grid method (Carroll, 2006; Barillas and Fernández-Villaverde, 2007).

Production. We normalize the steady-state quarterly output and total factor productivity Θ to one. This implies that total hours equal output in the steady state. We set the value of elasticity between labor tasks ϵ_w to 10, which is a standard value in the literature (e.g., Wolf, 2021). We take the slope of the wage Phillips curve $\kappa_w = 0.1$ from Aggarwal et al. (2023).

Fiscal policy. We assume a conventional target of government consumption of 20% of GDP, which is close to the U.S. data and is also the number used by Auclert et al. (2018). Because we have normalized quarterly output to one, this implies that $\overline{G} = 0.2$.

In our simulations, we consider different levels of public debt in the steady state. We take as a benchmark the case in which the annual real interest rate is 1%, and public debt stands at 70% of annual GDP. In our quarterly model, the stock of public debt at which the real interest rate is at its steady-state value is, therefore, $\overline{B} = 4 \times 0.7 = 2.8$. The values of the tax rate and net transfers are chosen to balance the budget in the steady state and ensure that public debt remains constant, i.e., total tax revenue exactly covers government spending plus net transfers and interest payments on public debt. The OECD reports a tax-to-GDP ratio of 0.277 for the United States in 2022. We set transfers to 7% of annual GDP, so that $\mathcal{T}_{ss} = G_{ss} + T_{ss} + r_{ss} \times B_{ss} = 0.2 + 0.07 + 0.01/4 \times 2.8 = 0.277$. Because $\mathcal{T}_{ss} = \tau Y_{ss}$ and $Y_{ss} = 1$, this implies that $\tau = 0.277$. Finally, for the baseline case, we set the parameter ϕ_G, which governs how quickly G_t responds to deviations from the debt target, to 0.1.

Monetary policy. We parameterize the Taylor rule to achieve an inflation target of 2% annually and set the Taylor rule coefficient to 1.25.

A RANK version of the model. In order to gain further insight into our model, we also consider for comparison purposes RANK version of our economy. The version eliminates the idiosyncratic income shock by setting $z_{i,t} = 1$. All the other structural parameters are the same as in the baseline HANK model except for two preference parameters that need to be adjusted for the RANK model to replicate the steady state of the HANK model. First, the value of the discount factor β is re-calibrated to 0.9975, so that the real interest rate of the steady state of the RANK model coincides with that of the HANK. Second, we specify preferences

[5]The quarterly persistence is calculated as $\rho_Q = 0.92^{1/4} \approx 0.977$ and the quarterly standard deviation of innovations as $\sqrt{(0.91^2/(\sum_{t=0}^{3} \rho_Q^{2t})} \approx 0.476$.

over consumption in the RANK model by $u(c) = \alpha \log(c)$. The parameter α is a certainty equivalence adjustment that is needed to replicate the marginal rate of substitution between average consumption and leisure in the wage Phillips curve. We calibrate α to 0.9721, so that the marginal utility of consumption in the RANK model replicates the productivity-weighted average marginal utility of consumption in the HANK model.

Computation. Given the set of exercises we want to undertake with our HANK model (e.g., explore the effects of unanticipated long-term changes in fiscal policy), we follow the recent literature by solving nonlinearly for impulse responses to one-time, unanticipated aggregate shocks using the sequence-space method. More concretely, we rely on the computational toolkit developed by Auclert et al. (2021a) for both computation and the decomposition of impulse responses.

4 Monetary-fiscal interactions in the long run

We begin by analyzing the deterministic steady state (DSS) of the model. In the DSS, there are no aggregate shocks, but there are idiosyncratic shocks at the household level. First we characterize the demand for bonds and the supply of bonds and then combine supply and demand with the monetary policy rule to obtain the real interest rate and the inflation rate. We denote all steady-state variables with the subindex "ss," except for the real interest rate, for which we use the standard r^*, since, in our model, this variable coincides with what is usually called the long-run natural rate. In addition, we retain the t subindex for variables that relate to choices made by households because they still face idiosyncratic shocks in the DSS.

Demand for bonds. The demand for bonds aggregates the individual savings decision of households, which accumulate bonds to smooth their consumption across time and idiosyncratic states of the world. We express the aggregate demand for bonds in the DSS by

$$A_{ss}(r^*) = \int_0^1 a_{i,t+1}(r^*)\, di,$$

where we make explicit that both the aggregate and the individual demands for bonds are a function of r^*. We show in Appendix B that the demand for bonds is a continuous and monotonically increasing function of r^*, with $\lim_{r^* \to \frac{1-\beta}{\beta}} A_{ss}(r^*) = \infty$ and $\lim_{r^* \to -\infty} A_{ss}(r^*) = 0$. The solid red line in Figure 1 displays the demand for bonds in our baseline calibration.

Supply of bonds. The supply of bonds in the DSS results from combining the treasury's budget constraint, $B_{ss} = (1 + r^*)B_{ss} + G_{ss} - T_{ss}$, with the fiscal rule $G_{ss} = \overline{G} - \phi_G(B_{ss} - \overline{B})$. In Appendix C, we show that if the reaction to the debt level in the fiscal rule is large relative

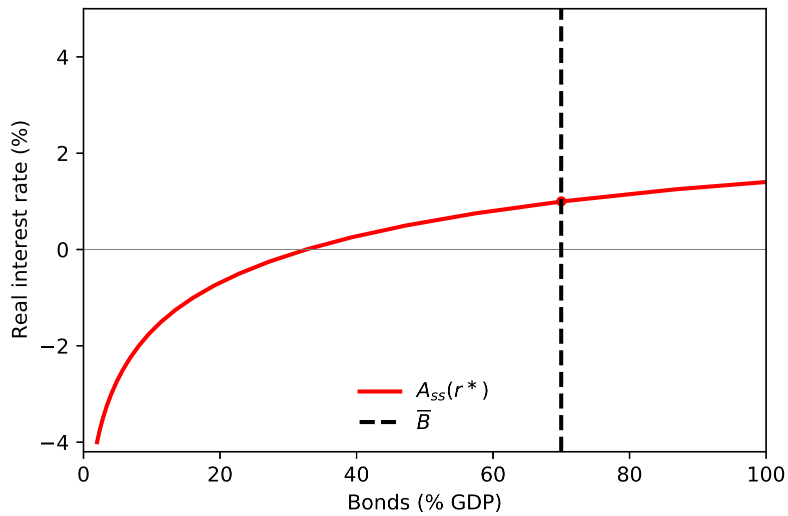


Figure 1: Determination of the steady-state real interest rate

to the rate of time preference, then the supply of bonds in the DSS does not depend on r^* and has the form $B_{ss} = \overline{B}$ (the technical condition is $\phi_G > (1-\beta)/\beta$, which holds in our baseline calibration). The supply of bonds in this case is depicted as a vertical line in Figure 1.

Monetary policy rule. For this discussion, we approximate the monetary policy rule (2) in the DSS by $i_{ss} \approx \max\{\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), 0\}$. This rule can be combined with the long-run Fisher equation $i_{ss} = r^* + \pi_{ss}$ to get:

$$r^* + \pi_{ss} \approx \max\{\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), 0\}. \tag{4}$$

It is well known that equation (4) has two solutions (Benhabib et al., 2002). The first solution, which we term a *non-binding ZLB* scenario, corresponds to the case in which $\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}) > 0$, and results in the solution $r^* + \pi_{ss} \approx \overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi})$, or equivalently

$$\pi_{ss} \approx \overline{\pi} + \frac{r^* - \overline{r}}{\phi_\pi - 1}. \tag{5}$$

That is, steady-state inflation equals the central bank inflation target plus a term that accounts for the deviation of the intercept in the Taylor rule from the steady-state natural rate. Consequently, if the central bank wishes to ensure that long-run inflation remains at its target, it must equate the intercept in its Taylor rule to the natural rate. This is the standard prescription in the New Keynesian model.

The second solution to equation (4) is a *binding ZLB* scenario, in which $\overline{r}+\overline{\pi}+\phi_\pi(\pi_{ss}-\overline{\pi}) \leq 0$, so that the maximum of the right-hand side in the equation is zero. In this case, the nominal interest rate is zero, and $\pi_{ss} = -r^*$.

The DSS. We combine the supply and demand of bonds and the monetary policy rule to characterize the DSS of the model. In Appendix C, we prove that, as long as the condition $\phi_G > (1-\beta)/\beta$ holds, so that the supply of bonds in a DSS is completely inelastic with respect to r^*, there exists a unique DSS. In our baseline calibration, this condition on the model's parameters is satisfied.

When we equate supply and demand, we find that $A_{ss}(r^*) = \overline{B}$. The vertical supply of bonds allows us to pin down the range of steady-state real interest rates that align with the steady-state debt target. Because the demand for bonds slopes upward, an increase in the steady-state debt level will result in a higher steady-state real interest rate. By inverting the function $A_{ss}(r^*)$, we can establish that the relationship $r^*\left(\overline{B}\right)$ is an increasing function.

The minimum debt level compatible with an inflation target. Next, we combine the real interest rate that clears the bond market in the DSS with the Taylor rule. When the ZLB does not bind, we assume that the central bank picks this real interest rate as the intercept in the Taylor rule to ensure that inflation remains on target. That is, $\overline{r} = r^*\left(\overline{B}\right)$ and, hence, $\pi_{ss}\left(\overline{B}\right) = \overline{\pi}$. This can always be achieved if real interest rates are high enough. There is, however, a level of the debt target $\overline{B}^*$, defined as

$$r^*\left(\overline{B}^*\right) + \overline{\pi} = 0,$$

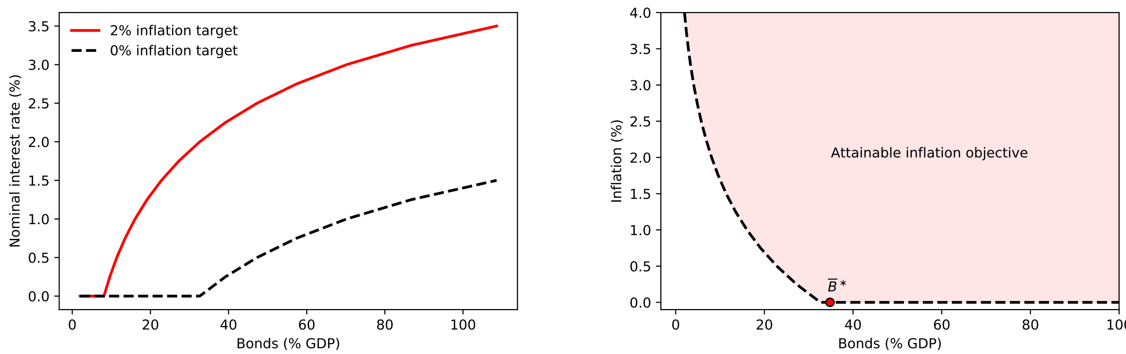
for which the nominal interest rate is zero. For $\overline{B} < \overline{B}^*$, the central bank is forced to accept steady-state inflation levels above its target due to the inability of nominal rates to become negative. In this case, the ZLB is binding, $i_{ss} = 0$, and inflation is determined by $\pi_{ss} = -r^*\left(\overline{B}\right)$. We call $\overline{B}^*$ the *minimum debt level compatible with the inflation target* $\overline{\pi}$ because the central bank can deliver on its inflation target in the DSS only if $\overline{B} > \overline{B}^*$.

We illustrate this result in Figure 2. Panel (a) displays the nominal interest rate for two different inflation targets, 2% (solid red) and 0% (dashed black). The graph shows that, for an inflation target of 2%, the minimum debt level $\overline{B}^*$ is 8% of GDP. In contrast, for 0%, it is 33%.[6] The dashed line in Panel (b) displays a frontier $\pi\left(\overline{B}^*\right)$ for positive inflation targets. The shaded area above this frontier contains the set of (non-negative) inflation targets that can be achieved in equilibrium for varying levels of debt. The level of debt $\overline{B}^*$ shown in this graph is the lowest level of debt compatible with an inflation objective of zero.[7]

Additional assets. At first sight, the existence of a minimum level of debt compatible

[6] In these exercises, we let $\overline{G}$ adjust to ensure that the treasury satisfies its budget constraint.

[7] Our fiscal rule delivers different values for the primary surplus $T_{ss} - G_{ss}$ as a function of the debt target (see Figure 14 in Appendix D). Thus, as discussed by Kaplan et al. (2023), our fiscal rule avoids the problems of multiplicity of steady states even with persistent deficits.



(a) Steady-state nominal interest rate

(b) Steady-state inflation

Figure 2: Steady-state nominal interest rate and inflation for different inflation targets

Note: The graphs are based on the baseline calibration but allow the level of bonds $\overline{B}$ to vary. In Panel (a), the dashed line shows results for an inflation target of 0% and the red solid line for 2%. The area in Panel (b) shows the combinations of (non-negative) inflation objectives that can be achieved in equilibrium for a varying level of debt. The level of debt B^* is the lowest level of debt compatible with an inflation objective of zero.

with an inflation target seems to depend on the absence of other assets that households can use to save, like private productive capital (for a model with public debt and capital, see Bayer et al., 2023). This is not the case, however. Consider, for instance, an environment in which we add productive physical capital using a standard Cobb-Douglas production function. In this case, the net return on capital is determined by subtracting depreciation from its marginal productivity. Suppose marginal productivity is sufficiently low (indicating substantial capital accumulation due to insufficient public debt for households to save on). In that case, the net return on capital is negative (see Barro et al., 2022, for a quantitative model that delivers negative net returns on capital). Agents will exhaust arbitrage opportunities until the net return on capital equals the real return on bonds. If the inflation target is not high enough and the ZLB binds, then the central bank will not be able to implement its inflation target, as in the model without capital. Hence, while the presence of other assets might change Figure 2 quantitatively, the main message conveyed by this figure carries over: there needs to be enough outstanding public debt in order for the central bank to achieve its inflation target.

Optimal monetary policy. A natural question is how the results in this section impact the design of optimal monetary policy, either under discretion or commitment. While a complete analysis of the optimal monetary policy response to fiscal policy in the context of HANK models goes beyond the aim of this paper, we provide some initial insights.[8] Let π^*_{ss} denote the inflation rate in the DSS of the optimal problem, be that under commitment or

[8] Optimal monetary policy in HANK economies requires dealing with infinite or high-dimensional problems, as discussed in Nuño and Thomas (2022), Bhandari et al. (2021), or Dávila and Schaab (2022).

discretion. Because the long-run Fisher equation $i_{ss} = r^* + \pi^*_{ss}$ needs to hold, two implications follow. First, given any optimal inflation level, a change in the long-run stock of debt will necessarily imply a change in the optimal long-run nominal interest rate. Second, there exists a minimum debt level compatible with the optimal inflation target. Our analysis highlights that these two conditions bind optimal monetary policy and that the design of monetary policy cannot be separated from fiscal policy.

5 A surprise debt-financed fiscal expansion

In this section, we analyze the dynamic effects of a debt-financed fiscal expansion. As mentioned in the introduction, this approach is an effective way to showcase the mechanisms we emphasize in this paper. The economy is initially at the DSS that corresponds to the calibration described in Section 3. Then, at $t = 0$, the treasury announces an unexpected and permanent increase in its debt target $\overline{B}$ of 10 percentage points (p.p.) in terms of the initial GDP. All agents and the central bank know this increase to be permanent.

5.1 Comparison of steady states

We show the values of key variables that differ across the initial and final DSS in Table 2. The second line in Table 2 shows that the natural interest rate r^* experiences a 16 basis point (bps) increase in the new DSS of the HANK model. This value is close to the 25 bps rise computed by Bayer et al. (2023) in response to a very persistent 10 p.p. increase in debt. In terms of Figure 1, the vertical black line shifts to the right, intersecting the demand curve (red line) at a higher interest rate. The increased supply of bonds lowers their price, resulting in a higher yield.

In order for the treasury to meet its intertemporal budget constraint and maintain a constant level of public debt in the new DSS, the increased $\overline{B}$ must be funded through a higher tax rate τ, reduced spending, or both. To simplify the analysis, we assume that the treasury reduces government consumption in the new DSS and that it sets the intercept of the fiscal rule (3) immediately to this new level. As a result, in the new DSS, the increase in interest payments on public debt is completely counteracted by an increase in the primary surplus.[9] The required change in the intercept of the fiscal rule can be approximated as $\Delta\overline{G} \approx \Delta\mathcal{T}_{ss} - (\Delta r^* \overline{B}_{ss} + r^* \Delta\overline{B}_{ss}) = -0.03 - (0.16\% \times 70 + 1\% \times 10) \approx -0.26$. On the other hand, in the RANK version of the model, a similar calculation implies that the intercept of the fiscal rule only needs to decrease by 0.11.

[9] We later consider alternative scenarios in which the treasury raises taxes or reduces net transfers to households.

	Initial steady state	New steady state HANK	New steady state RANK	Difference HANK	Difference RANK
Bonds (% GDP)	70.00	80.00	80.00	10.00	10.00
Real interest rate	1.00	1.16	1.00	0.16	0.00
Nominal interest rate	3.02	3.19	3.02	0.17	0.00
Output	100.00	99.90	99.96	-0.10	-0.04
Consumption	80.00	80.16	80.07	0.16	0.07
Govt. consumption	20.00	19.74	19.89	-0.26	-0.11
Tax revenue	27.70	27.67	27.69	-0.03	-0.01
Primary surplus (% GDP)	0.70	0.93	0.80	0.23	0.10

Table 2: DSS in the baseline HANK model and the RANK model
Note: The nominal interest rate in the initial DSS is 3.02% and not 3.00% because it satisfies the non-linear version of the Fisher equation $i_{ss} = 1.01 \times 1.02 - 1 \approx 3.02\%$.

To preserve price stability, the central bank must increase the intercept in its monetary policy rule $\overline{r}$ by the same amount as r^*. If the central bank did not adapt the intercept, then it would be forced to accept higher steady-state inflation, as shown in equation (5) for the case in which the ZLB does not bind. The change in government consumption $\Delta\overline{G}$ must also react to the increased interest rate in the new DSS. Not only does $\overline{G}$ need to decrease to accommodate the additional debt servicing at the initial interest rate, but it also needs to decrease to account for the higher interest rate, resulting in a total reduction of government consumption of 26 bps.

After an increase in $\overline{B}$, real variables move as well. Households receive higher interest payments and experience a positive wealth effect due to lower government spending, resulting in an increase of 16 bps in consumption. This higher consumption leads households to work less, causing an output loss of 10 bps. This effect on labor supply through wealth effects from changes in government spending has been known since Christiano and Eichenbaum (1992).

We also show the results for the RANK version of the model in Table 2. The initial DSS is the same for both versions of the model by design, and only the new steady states are different. The main difference between both versions of the model is that the DSS natural interest rate r^* remains unchanged in the RANK case, whereas it does change in the HANK version. This is because the long-run demand for bonds in the RANK version is perfectly elastic, and the DSS real interest rate is pinned down by $r^*_{RANK} = 1/\beta$. Thus, in a RANK model, the central bank does *not* need to adjust its Taylor rule in response to the new fiscal circumstances.

The treasury must still adjust the fiscal rule in the RANK version of the model, although to a lesser degree than in the HANK version. There are two reasons for this difference. First, the real interest rate does not increase in the RANK version, and hence, the primary surplus needed to compensate for higher interest payments is smaller. Second, in the RANK version, output does not drop as much as in the HANK version, and tax revenues hold up better.

5.2 Dynamics after the announcement

We now turn to the dynamic response of the economy after the unexpected increase in the debt target $\overline{B}$. The solid red lines in Figure 3 display the transitional dynamics in the baseline HANK economy.

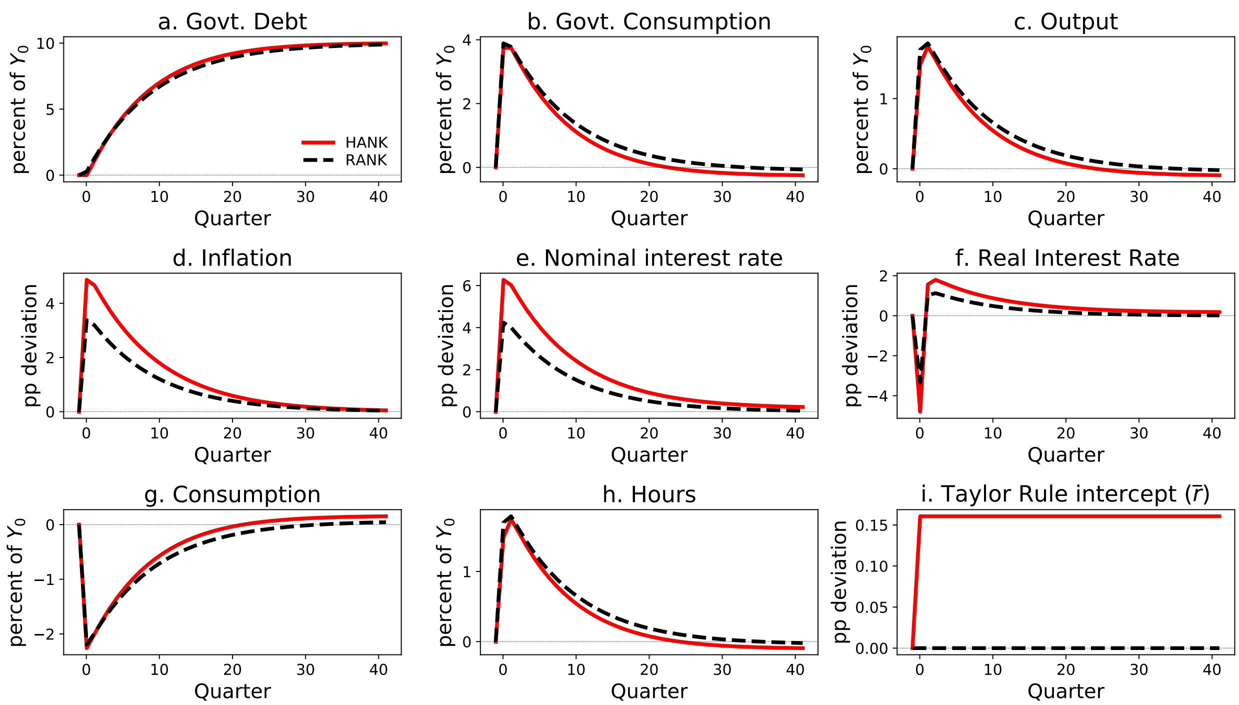


Figure 3: Dynamics after a surprise debt-financed fiscal expansion

As shown in Panel a, public debt is very close to its new steady-state value after 40 quarters. The treasury reaches a higher $\overline{B}$ through a temporary increase in spending that lasts for approximately 20 quarters, as shown in Panel b. Afterwards, government consumption is permanently lower because the treasury needs to service its higher debt and output and tax receipts are lower. This transitory increase in government consumption leads to an expansion in output (as depicted in Panel c) and inflation (as shown in Panel d) through the standard New Keynesian transmission of demand shocks (discussed below). Annualized inflation peaks at 6.9%, which is 4.9 p.p. above the 2% target, and GDP increases by 1.7 p.p.

To stabilize prices, the central bank raises the nominal interest rate by approximately 6.3 p.p., exceeding the rise in the inflation rate (Panel e). This increase is primarily caused by the term $\phi_\pi \log\left(\frac{1+\pi_t}{1+\bar{\pi}}\right)$ in the Taylor rule, accounting for roughly 6.1 p.p., whereas the intercept increases by 16 bps (Panel i). While this increase in the intercept plays a minor role in the initial periods, its importance increases as the short-term dynamics fade out. The ex-post real interest rate increases by 1.8 p.p. (Panel f). An interesting aspect of this experiment is that the ex-post real interest rate initially declines because of the surprise increase in inflation. The increase in the ex-ante real interest rate prompts households to reduce consumption (Panel g) and increase labor supply (Panel h). Consequently, households save more (and, in this way, absorb the additional public debt). Due to the increase in labor supply (and thus output), the decline in private consumption is less than the increase in government consumption.

As time progresses and the short-run dynamics fade out, the economy converges to the new DSS with higher debt, lower government spending, higher private consumption, and higher real interest rates.

The dashed black lines in Figure 3 show the response of the RANK version of the model. The transmission of the debt-financed fiscal expansion is more muted relative to the HANK version. Inflation increases by 3.5 p.p. on impact in the RANK model, compared to 5 p.p. in the HANK (see Panel d in Figure 3).

To gain a better understanding of the difference between the two versions of the model, we express the wage Phillips curve (1) as an infinite discounted sum:

$$\log\left(\frac{1+\pi_0}{1+\overline{\pi}}\right)=\sum_{t=0}^{\infty}\beta^t\kappa_w\left[-\frac{(\epsilon_w-1)}{\epsilon_w}(1-\tau)\int u'(c_{i,t})z_{it}di+v'(N_t)\right]N_t,$$

that is, inflation is a function of the entire future evolution of three endogenous quantities: the cross-sectional average of marginal utilities $\int u'(c_{i,t})z_{it}di$, labor disutility $v'(N_t)$, and hours worked N_t.

Using this formula, Figure 4 decomposes inflation in the first period in terms of aggregate consumption, consumption heterogeneity, and employment.[10] The higher increase in inflation in the HANK model is the combination of three factors. First, the stronger rebound in aggregate consumption in the HANK model (Panel g in Figure 3) reduces the deflationary effect of the higher marginal utility of aggregate consumption (light gray bar in Figure 4). Second, the decrease in the dispersion of consumption provides an additional upward push to inflation (dark gray bar). Third, these two factors trump the lower increase in hours (red bar and Panel h in Figure 3).

The different responses of consumption and employment are a consequence of the differences

[10]We employ the Jacobians of the wage Phillips curve and combine them with the general equilibrium response of aggregate consumption C_t, a consumption heterogeneity measure $\int u'(c_{i,t})z_{it}di - u'(C_t)$, and employment N_t in each model. The impact of aggregate consumption for the HANK model is obtained by replacing the average of marginal utilities of consumption with the marginal utility of aggregate consumption in the wage Phillips curve. The part explained by heterogeneity is the difference between these two measures.

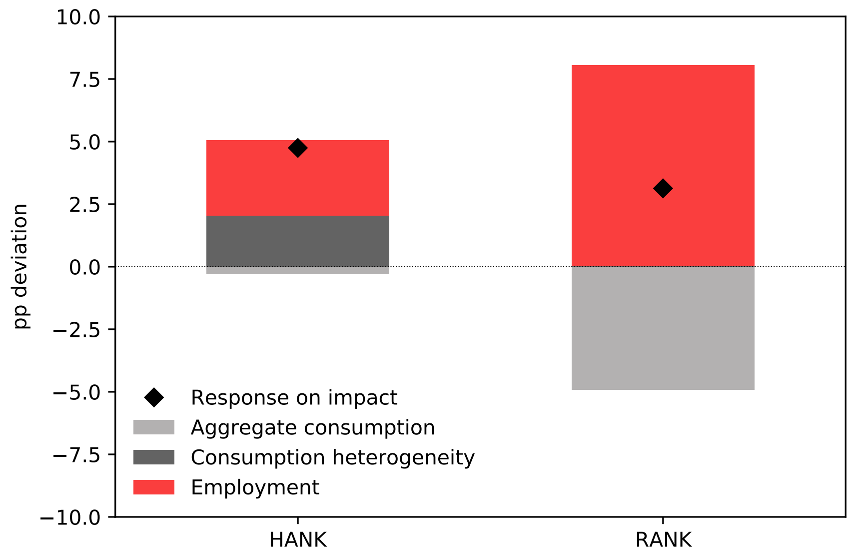
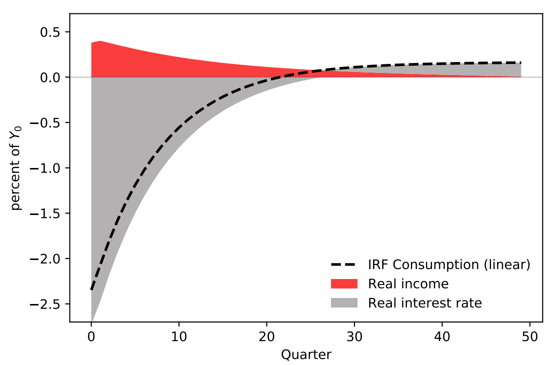
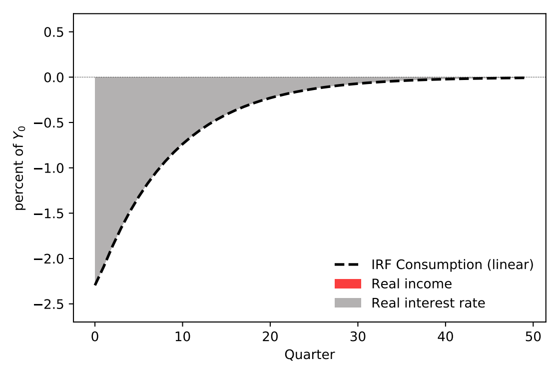


Figure 4: Decomposition of the response of inflation on impact
Note: The decomposition uses the Jacobians of the wage New Keynesian Phillips curve and combines them with the general equilibrium response of aggregate consumption, a consumption heterogeneity measure, and employment in each version of the model.

in shock propagation when markets are incomplete. In the RANK model, intertemporal substitution prompts households to reduce their consumption and to increase their labor supply, so that they can accumulate more assets when real rates are increasing. In the HANK model, however, there is an additional income effect. The increase in labor and capital income leads to higher consumption.



(a) HANK model



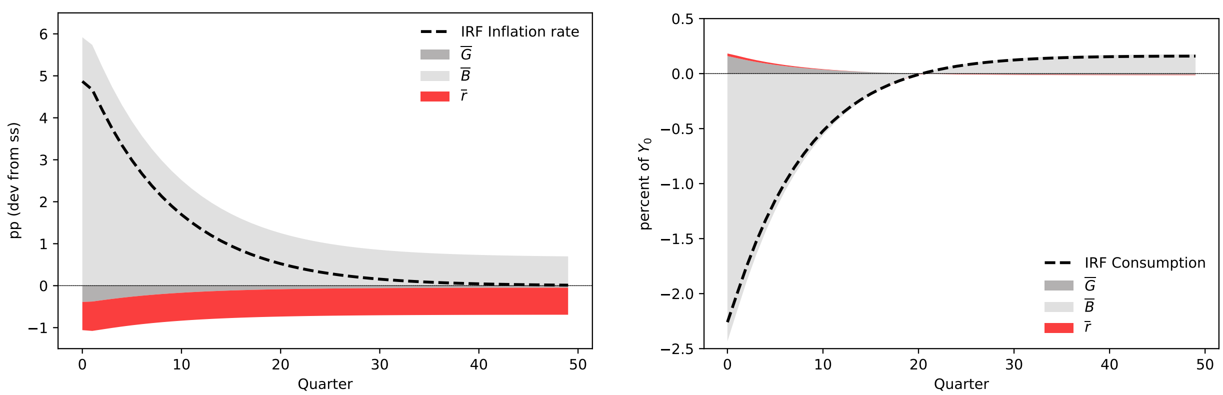
(b) RANK model

Figure 5: Decomposition of the response of aggregate consumption
Note: The decomposition uses the Jacobians of the household block in each model and then combines them with the general equilibrium response of real after-tax income and the ex-post real interest rate in each model. The decomposition is exact for impulse response functions (IRFs) obtained with linear solution methods. The vertical axis is the same in both graphs.

Figure 5 breaks down the aggregate response of consumption into two components: the influence of real interest rates (which combines intertemporal substitution and income effects associated with real interest rates) and after-tax labor income, which reflects the increase in hours worked. The figure illustrates that although the impact of real rates is more pronounced in HANK, suggesting a lower path of consumption, the income effect partially compensates for this in the short and medium run. In the long run, the higher income from higher real rates becomes dominant. Conversely, the response in hours exhibits the opposite pattern compared to consumption, as real wages remain unchanged.

5.3 The effects of changes in policy variables

As previously discussed, three policy parameters change simultaneously in our policy experiment: (i) the debt target $\overline{B}$, which increases by 10 p.p. of initial output; (ii) the expenditure target $\overline{G}$, which drops to ensure fiscal balance in the long run; and (iii) the intercept in the monetary policy rule $\overline{r}$, which increases to respond to changes in r^*. To understand the role played by each change separately, we decompose the IRFs of inflation and consumption.



(a) Inflation (b) Consumption

Figure 6: Decomposition of the IRFs of inflation and consumption in terms of policy variables

Panel (a) in Figure 6 breaks down the response of inflation into the individual effects of each of these changes. This decomposition utilizes the sequence-space representation of the model and follows the methodology of Auclert et al. (2021a) to split the IRF of inflation into these three components. To a first-order approximation, the general equilibrium response of inflation can be decomposed as $d\pi = \mathcal{J}_{\pi,\overline{G}} \times d\overline{G} + \mathcal{J}_{\pi,\overline{B}} \times d\overline{B} + \mathcal{J}_{\pi,\overline{r}} \times d\overline{r}$. Here, $\mathcal{J}_{\pi,x}$ represents a matrix consisting of the Jacobians of inflation with respect to the three policy variables $x = [\overline{G}, \overline{B}, \overline{r}]$ at different horizons, and dx are column vectors that contain the complete temporal paths followed by these three variables in their transition to the new DSS.

Three important results emerge from this exercise. First, the bulk of the inflationary response can be attributed to the fiscal expansion (light gray area). We discuss the particular mechanisms behind this result below. Second, the negative impact of the expenditure target $\overline{G}$ on inflation is only primarily in the short run and is quantitatively relatively small. Third, and most importantly, failure to update the monetary policy rule to reflect the new natural rate would result in significant inflationary effects. This is illustrated by the red area in Panel (a), which attributes a deflationary impact of approximately 0.8 p.p. in the long term to the change in the value of the Taylor rule intercept. Interestingly, this impact is roughly the same in the short term, at around 0.7 p.p. In other words, if the central bank had not updated the intercept in the Taylor rule, inflation in the short term would have been approximately 0.7 p.p. higher. This value is consistent with the analytical formula (5) derived for the DSS. The discrepancy between the natural rate and the intercept in the monetary policy rule $r^* - \overline{r}$ is 16 bps, and with a slope of ϕ_π at 1.25, the resulting increase in steady-state inflation would be approximately $0.16 \times 4 \approx 0.7$ p.p.

In Panel (b), we apply the same decomposition to the consumption response. In this case, the intercept $\overline{r}$ plays a negligible role. The same is true for other real variables in the model. The conclusion is that while $\overline{r}$ is a significant driver of inflation dynamics (and nominal variables more in general), it does not affect real variables. In the next section, we will see that the introduction of long-term debt changes this result, opening the door to a significant impact on real variables.

5.4 Nonlinearities

The nonlinear nature of the steady-state mapping from debt targets $\overline{B}$ to natural rates r^* is illustrated in Figure 1. It is determined by the demand for bonds, which is inherently nonlinear. Although the nonlinearity is more pronounced at lower debt levels, it can still be observed in the region surrounding the debt level used in our calibration. This nonlinearity implies that increases and decreases in public debt have asymmetric consequences. Evidence of this asymmetry is shown in Table 4 in Appendix D, where we compare the variables in the new DSS resulting from a 10 p.p. increase or fall in $\overline{B}$. The new steady states are clearly asymmetric. Similarly, Table 5 in Appendix D shows the asymmetric response one quarter after the announcement (we select one quarter to avoid the effects associated with the Fisher effect on impact). Nonlinearities are smaller in the short run than in the long run, but they are still present.

6 Extensions

Next, we present several extensions of our analysis. Among the many possible exercises of interest, we will focus on the possibility of designing robust monetary policy, alternative fiscal policies, the role of anticipated effects of changes in fiscal policy, and introducing long-term bonds.

6.1 Robust monetary rules

Up to this point, we have studied the case in which the central bank endogenously reacts to fiscal changes affecting the natural rate by adapting the intercept in its Taylor rule. If these changes occur frequently, the central bank would need to adjust its monetary policy rule often.

An alternative to adapting the Taylor rule continuously would be to use a monetary policy rule that does not require knowing the value of the natural rate. A rule with this property was proposed by Orphanides and Williams (2002), although its origins trace back to early work by Phillips (1954). The key feature of this rule is that it links the *change* in nominal interest rates $i_t - i_{t-1}$ to the deviation of inflation from its target $\pi_t - \overline{\pi}$:

$$\log(1 + i_t) = \log(1 + i_{t-1}) + \phi_\pi \log\left(\frac{1 + \pi_t}{1 + \overline{\pi}}\right). \tag{6}$$

The main advantage of the rule (6) is that the natural rate does not appear explicitly in the formulation. This rule is, hence, robust to changes in the natural rate. This may also be a disadvantage under certain circumstances if the natural rate happens to reflect valuable information about the economy that is not already contained in the evolution of inflation.

Figure 7 compares a model with this robust Orphanides-Williams rule (OW, dashed black line) with the baseline model with the standard Taylor rule (solid red line) in which the intercept $\overline{r}$ is updated on impact. The experiment is the same as above, namely, a permanent increase in the debt target $\overline{B}$ of 10 p.p. of annual GDP (with an associated change in r^*). Under both rules, inflation converges back to the inflation target $\overline{\pi}$ in the new DSS.[11]

The OW rule with the same slope ϕ_π reduces the volatility of both inflation and output (Panels a and c) compared to the Taylor rule at the cost of a marginally more persistent decline in consumption. In order to better interpret these results, we express the OW rule as:

$$\log(1 + i_t) = \log(1 + i_0) + \sum_{i=0}^{t-1}\left[\phi_\pi \log\left(\frac{1 + \pi_{t-i}}{1 + \overline{\pi}}\right)\right],$$

[11] For clarity, in this section we abstract from the zero lower bound, the two inflationary regimes, and the minimum debt level compatible with the inflation target.

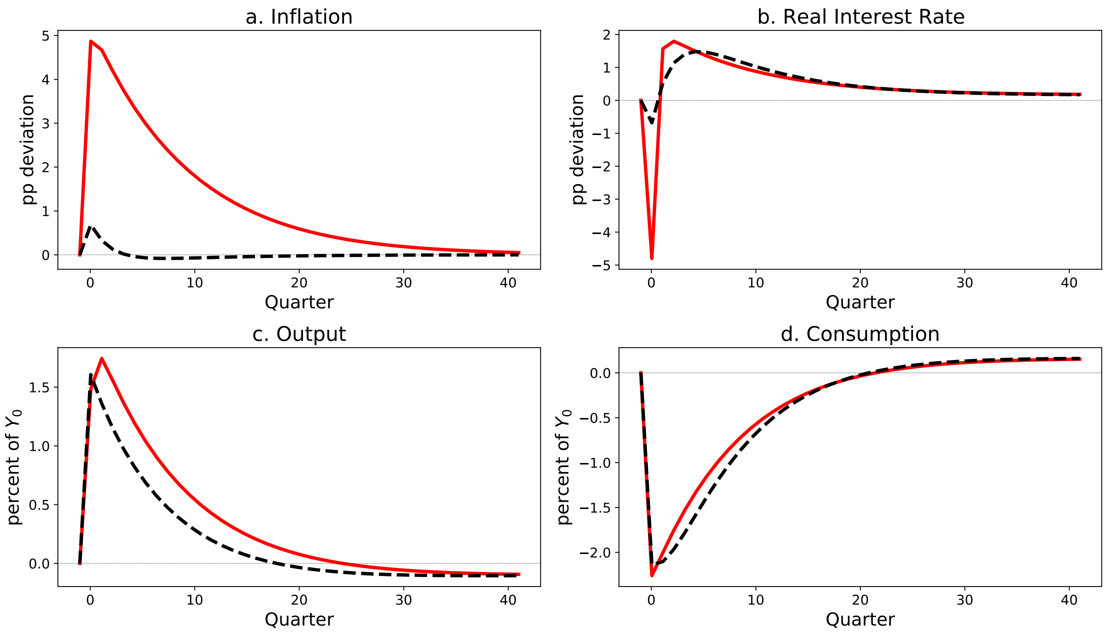


Figure 7: Comparison of a standard Taylor rule and the Orphanides-Williams rule in the HANK model (deviation with respect to the initial DSS)

where the initial nominal rate is given by $\log(1+i_0) = \log[(1+\bar{r})(1+\overline{\pi})]$ and $\bar{r}$ is the intercept of the Taylor rule before the fiscal announcement, which coincides with the natural rate in the old DSS. The ex-post real interest rate is thus

$$\log(1+r_t) = \log(1+\bar{r}) + (\phi_\pi - 1)\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right) + \sum_{i=1}^{t-1}\left[\phi_\pi \log\left(\frac{1+\pi_{t-i}}{1+\overline{\pi}}\right)\right],$$

whereas, in the baseline Taylor rule, it was

$$\log(1+r_t) = \log(1+r^*) + (\phi_\pi - 1)\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right).$$

In this equation, we have used the fact that the central bank updates the intercept of the Taylor rule to r^*.

The comparison between the two rules makes evident their differences: while the Taylor rule focuses on contemporaneous inflation deviations, the OW rule includes an additional term that averages all past inflation deviations. That makes the real rate a slow-moving object, more persistent than under the Taylor rule (Panel b), which explains the dampened reaction of inflation. Even if the central bank does not explicitly target the new natural rate, it "learns it" by accumulating inflation deviations.

6.2 Alternative fiscal policies

So far, we have assumed that the fiscal expansion is totally driven by increases in government spending. We now consider two alternative ways of raising public debt in the long run: higher transfers to households and lower income taxes.

Endogenous tax rate. We consider the case in which government consumption remains constant, but the treasury adjusts the tax rate each period so that the evolution of public debt replicates the evolution in our baseline analysis. We do not change the behavior of the central bank with respect to our baseline analysis; the central bank adjusts the intercept of the Taylor rule immediately in response to higher r^*.

The budget constraint of households remains the same as in the baseline case, except that τ_t is now time-varying:

$$c_{i,t} + a_{i,t+1} = (1 + r_t)a_{i,t} + (1 - \tau_t)\frac{W_t}{P_t}z_{i,t}n_{i,t} + T_t$$

Public debt accumulates as in the baseline case, which implies that the treasury must choose the tax rate so that tax revenue evolves according to:

$$\mathcal{T}_t = (1 + r_t)B_{t-1} + G_t + T_t - B_t.$$

Tax revenue is endogenous and is given by:

$$\mathcal{T}_t = \int_0^1 \tau_t \frac{W_t}{P_t} z_{i,t}n_{i,t}di = \tau_t \Phi N_t \int_0^1 z_t di = \tau_t Y_t.$$

This derivation follows from the equality between the real wage rate and constant productivity and from the assumption that all workers work the same number of hours $n_{i,t} = N_t$. The last equality in the expression uses the fact that idiosyncratic productivity has a mean of one and the production function is $Y_t = \Phi N_t$. Therefore, the tax rate chosen by the government in each period is $\tau_t = \mathcal{T}_t/Y_t$. In the final DSS, the tax rate is given by $\tau_{ss} = (0.2+0.07+r^* B_{ss})/Y_{ss} = 27.93\%$ because government consumption stays constant at $G_{ss} = 0.2$ and transfers at $T_{ss} = 0.07$.

Compared to the baseline model, three differences emerge in this case. First, steady-state consumption is now lower, not higher, than in the initial DSS. In the baseline case, wealthier households could consume a larger share of output due to the decline in government consumption. In the case of higher taxation, government consumption remains constant and lower household consumption reflects one-to-one the decline in output. Second, the decline in output (and consumption) is slightly higher in this case, as the reduction in after-tax wages

	Initial DSS	New DSS Endog. variable:			Difference Endog. variable:		
		G	τ	T	G	τ	T
Bonds (% GDP)	70.00	80.00	80.00	80.00	10.00	10.00	10.00
Real interest rate	1.00	1.16	1.17	1.15	0.16	0.17	0.15
Nominal interest rate	3.02	3.19	3.19	3.18	0.17	0.17	0.16
Output	100.00	99.90	99.89	100.01	-0.10	-0.11	0.01
Consumption	80.00	80.16	79.89	80.01	0.16	-0.11	0.01
Govt. consumption	20.00	19.74	20.00	20.00	-0.26	0.00	0.00
Tax revenue	27.70	27.67	27.93	27.70	-0.03	0.23	0.00
Transfer	7.00	7.00	7.00	6.78	0.00	0.00	-0.22
Primary surplus (% GDP)	0.70	0.93	0.93	0.92	0.23	0.23	0.22

Table 3: Initial and final DSS under alternative fiscal policies

Note: Initial and final DSS in the baseline HANK model and two models with alternative fiscal rules. The column labeled G refers to the baseline case in which government consumption adjusts endogenously via the fiscal rule. The column labeled τ refers to the case y in which the tax rate adjusts endogenously to replicate the evolution of public debt in the baseline model. The column labeled T refers to the case in which net transfers adjust endogenously to replicate the evolution of public debt in the baseline model.

due to higher taxation reduces the incentive to work. Third, the increase in natural rates is slightly higher (17 bps) as the lower wage increases precautionary savings.

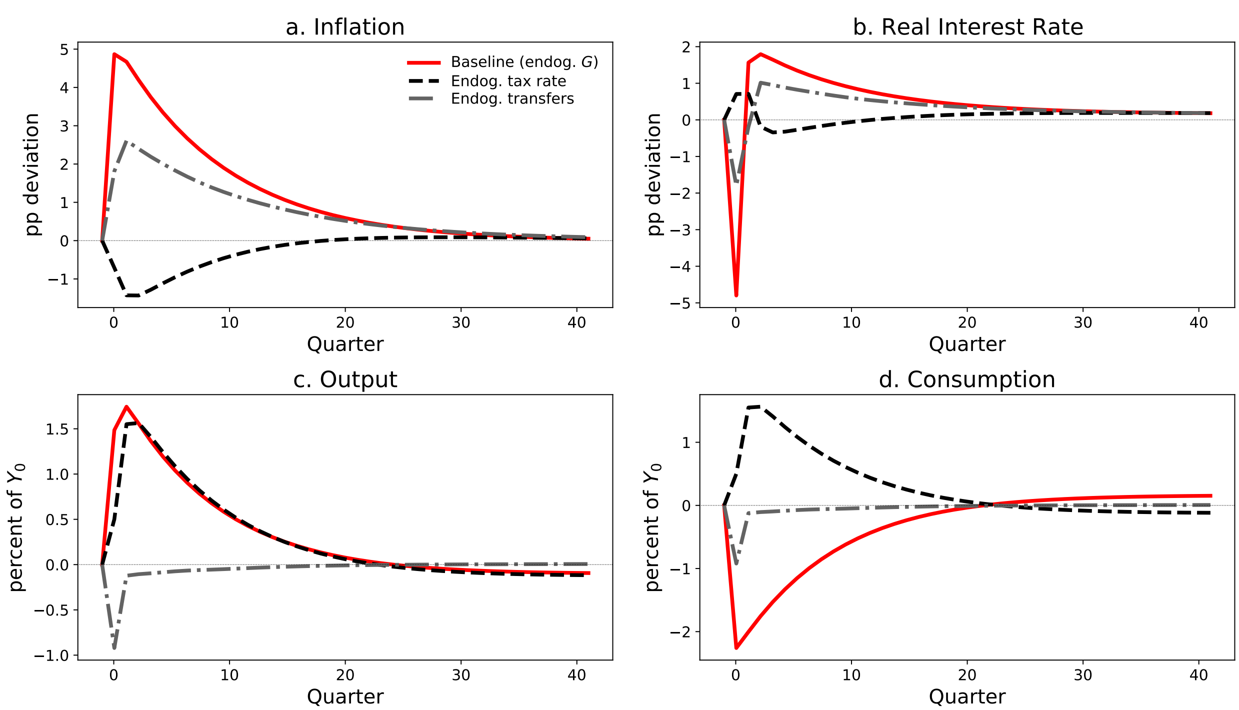


Figure 8: Dynamics after a surprise debt-financed fiscal expansion.

The dynamics after the shock are also quite different (dashed black line in Figure 8). The increase in debt initially allows a reduction in the tax rate, which increases output as households decide to work more to enjoy the temporary increase in after-tax wages (Panel c). Consumption increases one-to-one with output, as government consumption remains unchanged (Panel d). The decrease in the marginal tax rate increases the after-tax real wage and reduces inflation because it more than compensates for the effect of consumption in the wage Phillips curve (Panel a). The fall in inflation leads to a decrease in real rates, which provides incentives for the increase in consumption (Panel b). This decline in real rates allows households to save in order to absorb the issuance of new debt.

Lump-sum transfers. Next, we solve the model so that lump-sum transfers respond endogenously in a way that replicates the evolution of public debt when government consumption and the tax rate remain constant.[12] Again, we do not change the behavior of the central bank with respect to our baseline analysis: it adjusts the intercept of the Taylor rule immediately to the higher r^*. Transfers in the new DSS are given by $T_{ss} = \tau Y_{ss} - \bar{G} - r^* B_{ss} = 6.78\%$, where government consumption remains constant at $\bar{G} = 0.2$.

The model has Ricardian properties, with minimal changes in aggregate variables in the new DSS compared to the initial DSS (see Table 3). The only notable difference is that the higher level of debt raises the natural rate by 15 bps, one bp less than in the baseline scenario. This is because lump-sum transfers do not cause distortions in the economy, and the increase in available assets for saving enhances households' ability to self-insure. As a result, there is a slight increase in output and consumption.

In a RANK model, increasing public debt through lump sum transfers does not impact consumption, output, inflation, or the real interest rate due to Ricardian effects.[13] In contrast, the HANK model shows a slight increase in inflation, driven by expectations of higher consumption and hours worked in the long run (dashed-dotted gray line in Figure 8, Panel a). The increase in inflation leads to a Fisherian fall in real bond prices in the first period. This results in a decrease in wealth of approximately 1.3% of initial output. This decrease is partially offset by an increase in net transfers from new debt issuance. The rise in transfers amounts to approximately 1% of output in the initial steady state. Consequently, aggregate non-labor income decreases by 0.3% of the initial GDP. This results in a reduction in consumption (Panel d), leading in turn to a corresponding decrease in output and hours worked (Panel c).

The conclusion of this analysis is that the fiscal policy funded through the increase in debt has important implications for the dynamics and the new DSS, while its differential impact on the natural rate is more muted.

[12]Tax revenues are still endogenous, as they are the product of the tax rate and output. In this exercise, the tax rate remains constant, but output does not.

[13]In Figure 15 in the appendix, we show the IRFs for the RANK model.

6.3 Anticipated effects

So far, we have analyzed the case in which the central bank updates its monetary policy rule immediately once the fiscal shock becomes known. We now explore the consequences of changes in the timing of the update as well as cases in which the central bank knows beforehand that a change in fiscal policy will occur.

To this end, Figure 9 shows the results of simulations in which, at time zero, the treasury announces a debt-financed fiscal expansion that will start 12 quarters in the future. Expressing this in terms of the fiscal rule, the debt target $\overline{B}$ remains at its current value for $t < 12$ and then changes to its new permanent value at $t = 12$. This change in the fiscal rule is anticipated and known by all agents, including the central bank, at $t = 0$.

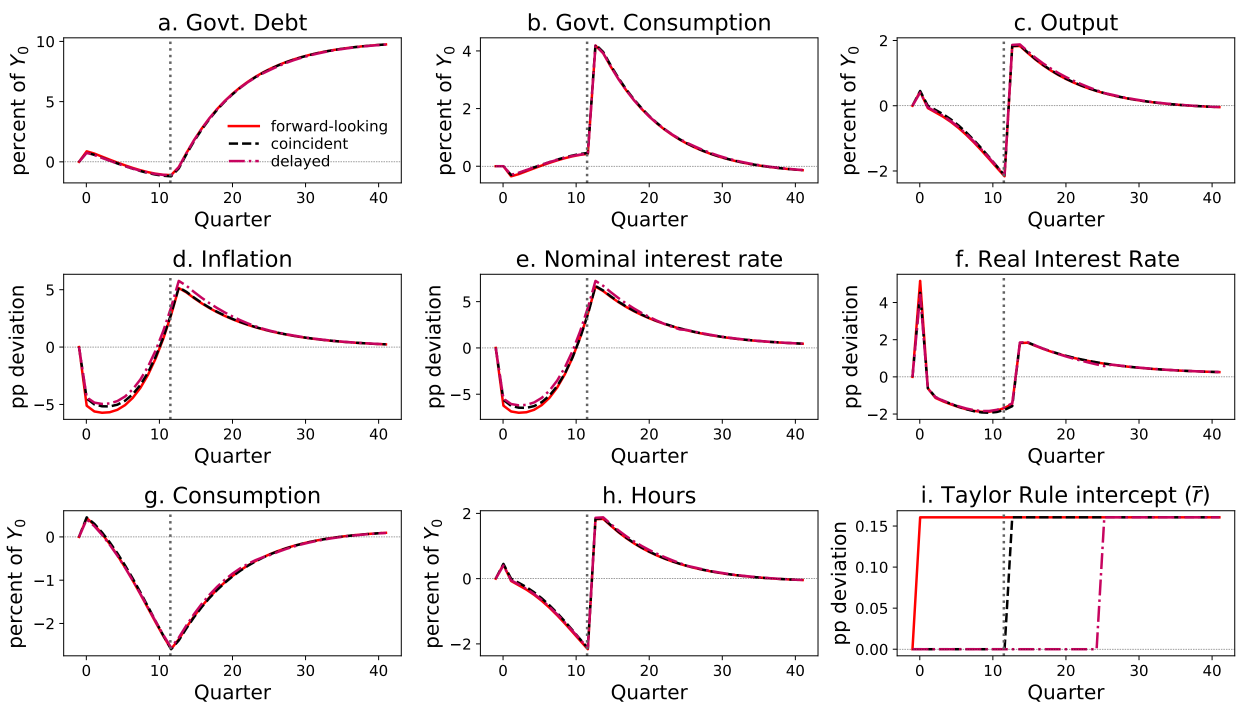


Figure 9: Dynamics of an anticipated debt-financed fiscal expansion

Note: Deviation with respect to the initial DSS after a shock that increases the value of the debt target $\overline{B}$ in the future, after 12 quarters. The figures compare three cases. In the first case, the central bank reacts immediately, anticipating the change in fiscal policy (solid red line). In the second case, it reacts after 12 quarters (dashed black line), at the same time that fiscal policy changes. In the last one, it delays the monetary policy response by another 12 quarters (dotted-dashed red line). Agents have perfect foresight in all cases.

We first consider the case in which the central bank updates the intercept in the monetary policy rule $\bar{r}$ as soon as the announcement is made (solid red line). The dynamics after the actual change in $\overline{B}$ are similar to those in the baseline model described in Figure 3. Inflation increases by 5.1 p.p., GDP increases by 1.8 p.p., and the real interest rate by 1.8 p.p. What is new are the dynamics *before* the actual change in $\overline{B}$. As households anticipate future higher real interest rates, they start saving in advance, which explains the downward trend in consumption. The increase in the demand for savings reduces real interest rates in the short run and has deflationary effects. The lower real interest rates reduce the fiscal burden, which allows for both a persistent reduction in the stock debt and an increase in government spending. The first period deserves special attention. The steep decline in inflation in the period in which the shock becomes known lowers the price level, which leads to a sudden increase in the real value of debt through the Fisher effect. This leads to an initial increase in consumption due to the rise in real wealth. But, given that bonds have a maturity of one period, this effect is short-lived (we will revisit this issue in a few paragraphs).

Next, we consider the case in which the central bank decides to maintain its monetary policy rule constant until the *actual change* in $\overline{B}$ takes place (dashed black line). Starting in $t = 12$, the evolution of all variables is very similar to the case just described. The only differences of importance between cases are limited to nominal variables up to $t = 11$. The lack of anticipatory behavior by the central bank leads to a less marked deflationary path and higher nominal interest rates. The path of initial higher nominal interest rates emerges despite the fact that $\overline{r}$ is now lower. The path of real variables is almost identical in both cases, in line with the result discussed in Figure 6.

Finally, we consider the case of a *delayed* change in the Taylor rule, which happens 12 quarters after the change in the debt target $\overline{B}$. As we observed above, the dynamics of inflation are quantitatively identical after $t = 24$, but this delayed case exhibits higher inflation and nominal interest rates compared to the previous rules. The path of real interest rates and real variables is again indistinguishable from the two-period cases.

We draw three conclusions from this exercise. First, the delays in updating the intercept in the Taylor rule lead to higher inflation. Second, once the update of the Taylor rule occurs, the ensuing dynamics are similar to those in the baseline. Third, and in line with previous results, the intercept affects inflation and nominal rates but is mostly irrelevant for real variables.

6.4 Introducing long-term debt

In our final extension, we replace one-period bonds with long-term bonds that pay a geometrically decaying coupon. This is a common modeling choice (e.g., Woodford, 2001). Long-term bonds B^L pay a nominal dividend of 1 in the first period, δ in the second period, δ^2 in the third period, and so on, with $0 \leq \delta < 1$. If $\delta = 0$, then long-term debt is effectively one-period debt. Arbitrage considerations imply that in a rational expectations equilibrium, the short-term ex-ante nominal interest rate must satisfy:

$$1 + i_t = \frac{1 + \delta Q_{t+1}}{Q_t}.$$

Now, the budget constraint (2) of the treasury becomes:

$$P_t Q_t B_t^L = (1 + \delta Q_t) P_{t-1} B_{t-1}^L + P_t (G_t + T_t - \mathcal{T}_t),$$

where the left-hand side is the nominal value of the stock of bonds at the end of period t, and the right-hand side is the nominal value of bonds at the start of period t plus debt dividends and the primary deficit.

We define the value of bonds expressed in date-t consumption goods (the real market value of debt) as $\tilde{B}_t \equiv Q_t B_t^L$. Then, $\tilde{B}_t = (1 + r_t)\tilde{B}_{t-1} + G_t - T_t$, with the ex-post real return on bonds given by $1 + r_t \equiv \frac{(1+\delta Q_t)}{Q_{t-1}} \frac{P_{t-1}}{P_t} = (1 - i_{t-1}) \frac{P_{t-1}}{P_t}$.[14] Market clearing with long-term bonds requires that:

$$\int_0^1 a_{i,t} di = Q_t B_t^L = \tilde{B}_t,$$

that is, household wealth is now held exclusively in the form of long-term debt.

We modify the fiscal rule so that it reacts to the real market value of debt $\tilde{B}$:

$$G_t = \overline{G} - \phi_G (\tilde{B}_{t-1} - \overline{B}).$$

We choose the same parameter values for $\overline{G}$ and $\overline{B}$ as in the model with one-period debt. This implies that $\tilde{B}_{ss} = Q_{ss} B_{ss}^L = \overline{B}$. From market clearing, we again have that $A_{ss} = \overline{B}$ and the DSS one-period real interest rate is the same as in the model with one-period public debt. We calibrate the parameter δ to 0.95 so that the steady-state duration of bonds is 18 quarters (4.5 years), similar to the duration of assets and liabilities in the U.S. economy estimated by Doepke and Schneider (2006).[15]

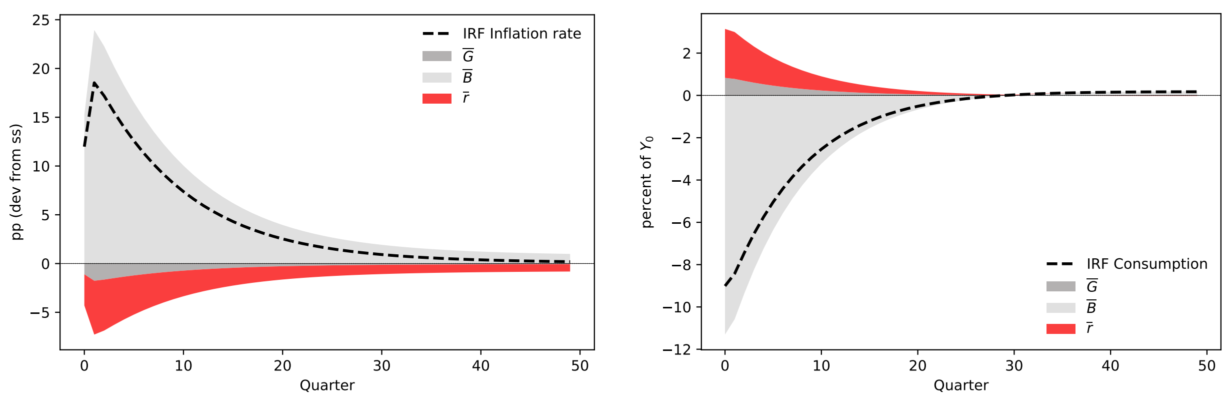
The introduction of long-term bonds has two main consequences. First, it greatly amplifies the (transitory) responses of nominal and real variables to a debt-financed fiscal expansion. Second, it changes our conclusion, discussed in the context of Figure 6, that if the central bank sticks to the old monetary policy rule, it affects inflation but not real variables such as consumption or output. With long-term debt, instead, real variables are also significantly

[14] This ex-post real return will not coincide with the ex-ante real return that was expected by bondholders in the period prior to the arrival of an unexpected shock. The reason is twofold. First, the price level in period t will jump, as occurs with short-term debt. Second, Q_t will also jump, leading to a discrepancy between the nominal return of the bond that was expected in the prior period and the realized ex-post nominal return of holding the bond.

[15] The formula for the duration is $D_{ss} = (1 + i_{ss})/(1 + i_{ss} - \delta)$.

affected along the transition path.

Both consequences are due to the Fisher effect: with long-term debt, the previously unanticipated increase in the path of inflation following the fiscal announcement reduces the real price of bonds Q at time zero. This implies that the real market value of debt plummets on impact, reducing the fiscal burden of the treasury. According to the fiscal rule, this allows the treasury to engage in a much larger fiscal expansion than with short-term debt.[16] Interestingly, this mechanism implies a larger wealth redistribution from wealthy households, which initially owned most of the bonds, to the treasury.



(a) Inflation (b) Consumption

Figure 10: Decomposition of the response of inflation and consumption in terms of policy variables with long-term bonds

Figure 10 displays the decomposition of the response of inflation and consumption into the separate effects of changes in the debt target $\overline{B}$, the expenditure target $\overline{G}$, and the intercept in the monetary policy rule $\overline{r}$. Contrary to what happened in Figure 6, the intercept now plays a significant positive role in the consumption response in the first 20 quarters. If the central bank did not adjust its rule, there would be higher inflation and this would lead to even lower initial market values of debt and a larger fiscal expansion.

7 Validating evidence

In this section, we provide empirical support to validate the two main claims of the paper: (i) that debt-financed fiscal expansions increase the natural rate; and (ii) that central banks adjust their reaction function in response to high- and medium-frequency changes in natural rates.

[16] The response is qualitatively similar to that displayed in Figure 3, but quantitatively much larger. The key difference is the decline in the stock of debt on impact, as displayed in Figure 16 in Appendix D.

7.1 Response of the natural rate to an increase in public debt

First, we estimate the response of the natural interest rate, r^*, to an increase in the debt-to-GDP ratio. We use the natural rate estimated by Lubik and Matthes (2015) using a time-varying parameter vector autoregressive model (TVP-VAR) as our measure of r^*.[17]

We tackle the estimation from two complementary approaches. In our first approach, we estimate an IRF using local projections (LP; Jordà, 2005). We specify our estimating equation as:

$$r^*_{t+h} = \alpha_h + \beta_h D_{t-1} + \mathbf{x}_t \gamma_h + u_{t+h},$$

for $h = 0, \ldots, H$, where r^*_{t+h} is the outcome variable, the natural rate r^* observed h periods from today, D_{t-1} refers to the lagged debt-to-GDP ratio, β_h makes up the elements of the IRF of the outcome variable at horizon h relative to its lagged value today; $\mathbf{x}_t$ collects all additional control variables including lags of the outcome variable, and the debt-to-GDP ratio, as well as lagged values of the federal funds rate, inflation, and the unemployment rate. The sample is from 1967:Q1 to 2023:Q2.

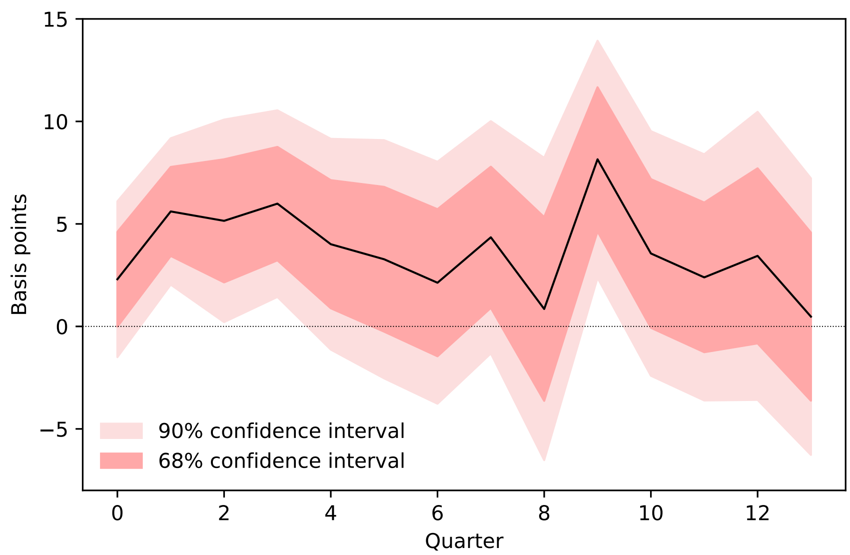
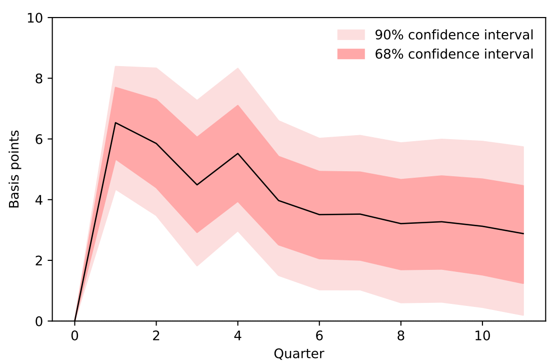


Figure 11: IRF of r^* to a 1% increase in the public debt-to-GDP ratio, LP
Note: We estimate an LP with $r^*_{t+h} = \alpha_h + \beta_h D_{t-1} + \mathbf{x}_t \gamma_h + u_{t+h}$ and plot the regression coefficient β_h (the solid line) associated with the lagged public debt-to-GDP ratio D_{t-1}. The control variables $\mathbf{x}_t$ include four lags of the change in r^*, three additional lags of the public debt-to-GDP ratio, and four lags of the federal funds rate, the GDP deflator, and the unemployment rate. The shaded areas represent the 68% and 90% confidence intervals using Eicker–Huber–White standard errors following Montiel Olea and Plagborg-Møller (2021).

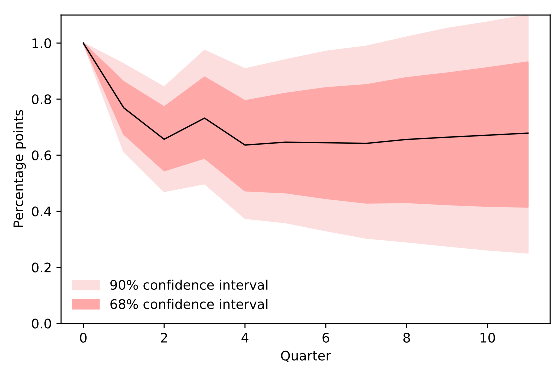
[17] A continuously updated dataset is available at `https://www.richmondfed.org/research/national_economy/natural_rate_interest`.

Figure 11 shows the IRF of r^* to a one p.p. increase in the public debt-to-GDP ratio. We estimate that r^* increases by between two and five bps immediately after this one p.p. increase in debt. This scales up to a 20-50 bps increase in response to a 10 p.p. fiscal expansion. Recall that in our model a 10 p.p. fiscal expansion causes a rise in r^* of 18 bps (Table 2). Thus, our model roughly matches the lower range of these empirical estimates. The empirical estimate is also similar to the results reported by Rachel and Summers (2019, Table 2). Averaging the findings in the literature, these authors report that a 10 p.p. increase in the debt-to-GDP ratio leads to an increase in r^* of 35 bps.

In our second approach, we estimate a structural vector autoregression (SVAR) model with r^*, the debt-to-GDP ratio, the federal funds rate, inflation, and the unemployment rate as variables. We identify a structural shock using a recursive (Cholesky) identification, assuming that r^* responds to the lagged debt-to-GDP ratio, which is consistent with the specification used in the exercise using LPs and with an interpretation of our model where the central bank has a rapid but not instantaneous response to the fiscal shock (perhaps due to delays in the reporting of fiscal spending and tax revenues).



(a) Response of r^*



(b) Response of debt as percentage of GDP

Figure 12: IRF of r^* to a 1% increase in the public debt-to-GDP ratio, SVAR
Note: IRF of r^* and the debt-to-GDP ratio to a one p.p. shock to the public debt-to-GDP ratio. We use a Cholesky identification with the order: public debt-to-GDP, r^*, inflation, unemployment rate, and the federal funds rate. The lag length is $p = 4$. The solid lines are the point estimates. The shaded areas represent the 68% and 90% confidence intervals computed using a wild bootstrap with 10,000 replications.

Figure 12 plots the response to a structural shock that leads to a one p.p. increase in the debt-to-GDP ratio. The findings indicate similar effects on r^*, ranging between three and six bps. Moreover, the impact on the debt level appears to be highly persistent. Again, these results roughly agree with the findings of our model. Even if the latter might be underestimating the total impact of fiscal shocks on r^*, it lies within a reasonable distance of the empirical findings, and it is in stark contrast with the RANK model, where r^* remains unchanged.

7.2 Natural rates and the central bank's reaction function

We now explore the empirical link between r^* and monetary policy. We start by assuming that the central bank's reaction function can be expressed as

$$i_t = \sum_{k=1}^{K} \rho_k i_{t-k} + \left(1 - \sum_{k=1}^{K} \rho_k\right) \Phi\left(\pi_t, \mathbb{E}_t\left[\pi_{t+1}\right], ..., \mathbb{E}_t\left[\pi_{t+N}\right], \mathbb{E}_t\left[\tilde{y}_{t+1}\right], ..., \mathbb{E}_t\left[\tilde{y}_{t+N}\right]\right),$$

where $\sum_{k=1}^{K} \rho_k i_{t-k}$ is an autoregressive term that accounts for the potential smoothing in nominal rates and $\Phi(\cdot)$ is a nonlinear function that maps inflation, inflation expectations, the output gap $\tilde{y}$, and gap expectations to nominal interest rates. This is a general formulation that provides a flexible mapping to the reaction function of central banks in reality.

By taking a first-order Taylor expansion around the inflation target $\overline{\pi}$ and an output gap of zero, we express the monetary policy rule as:

$$i_t \approx \sum_{k=1}^{K} \rho_k i_{t-k} + \Phi_0 + \sum_{n=0}^{N} \left(\frac{\partial \Phi}{\partial \mathbb{E}_t\left[\pi_{t+n}\right]}\right) \left(\mathbb{E}_t\left[\pi_{t+n}\right] - \overline{\pi}\right) + \sum_{n=0}^{N} \left(\frac{\partial \Phi}{\partial \mathbb{E}_t\left[\tilde{y}_{t+n}\right]}\right) \left(\mathbb{E}_t\left[\tilde{y}_{t+n}\right]\right).$$

In a steady state, we have that $i_{t-k} = i_{ss}$ for all k, $\mathbb{E}_t\left[\pi_{t+n}\right] = \pi_{ss}$ and $\mathbb{E}_t\left[\tilde{y}_{t+n}\right] = 0$ for all n. Thus, we can express the rule as $i_{ss} \approx \overline{r} + \overline{\pi} + \phi_\pi\left(\pi_{ss} - \overline{\pi}\right)$, where $\phi_\pi = \sum_{n=0}^{N} \left(\frac{\partial \Phi}{\partial \mathbb{E}_t[\pi_{t+n}]}\right)$ and the intercept $\overline{r} = \Phi_0 - \overline{\pi}$. If we substitute the steady-state nominal interest rate using the Fisher equation, we obtain equation (4) outside of the ZLB:

$$r^* + \pi_{ss} \approx \overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}).$$

We can then also obtain equation (5) in this case:

$$\pi_{ss} \approx \overline{\pi} + \frac{r^* - \overline{r}}{\phi_\pi - 1},$$

which relates the deviation of long-term inflation from the inflation target $\pi_{ss} - \overline{\pi}$ with the *policy gap* $r^* - \overline{r}$ between the natural rate and the intercept in the central bank's reaction function.

If the steady-state objects inherit the time-varying nature of fiscal shocks, we can think of them as random variables instead of constant parameters. More precisely, if the intercept $\overline{r}$ perfectly tracks the natural rate r^*, then the policy gap is always zero, and steady-state inflation remains constant at the central bank's target $\pi_{ss} = \overline{\pi}$. Alternatively, if the Taylor rule remains stable in the sense that its intercept does not co-move with changes in long-term inflation ($cov(\overline{r}, \pi_{ss}) = 0$), then we can compute the variance of inflation as

$$var(\pi_{ss}) \approx \frac{cov(r^*, \pi_{ss}) - cov(\overline{r}, \pi_{ss})}{\phi_\pi - 1} = \frac{cov(r^*, \pi_{ss})}{\phi_\pi - 1},$$

where we have assumed that ϕ_π is constant. In this case, the policy gap can be computed as

$$r^* - \overline{r} = \frac{cov(r^*, \pi_{ss})}{var(\pi_{ss})}(\pi_{ss} - \overline{\pi}). \tag{7}$$

Equation (7) provides a measure of the policy gap that can be estimated using market data.

To do so, we need to observe variables that approximate the (potentially time-varying) steady-state values of real interest rates and inflation. One possibility is to employ market data on long-term interest rates and inflation compensation for the United States. We do not use the natural rate estimates of Lubik and Matthes (2015) because we also need a proxy for long-term inflation. Using market data provides us with a consistent set of variables.

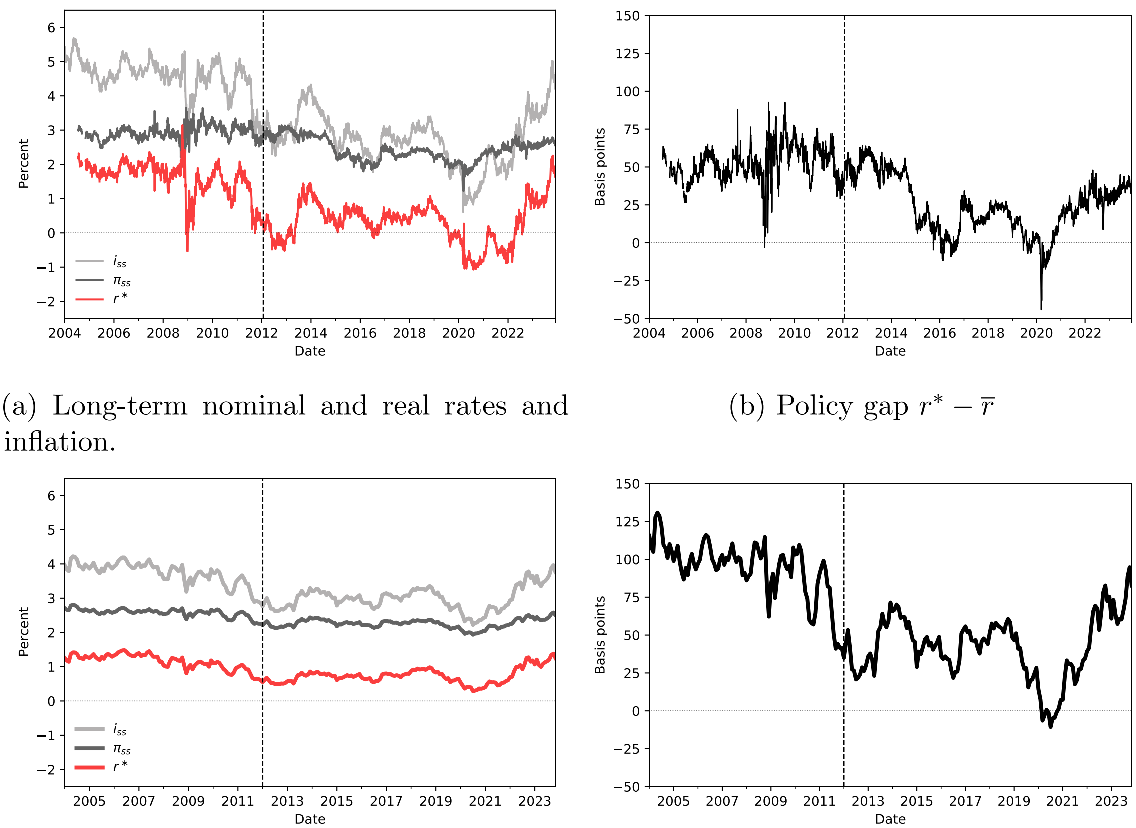
We collect daily data on the 5-year 5-year (5y5y) forward nominal yield. This is a measure of the 5-year yield expected five years ahead, which is commonly used as a proxy for long-term nominal interest rates. For long-term inflation, we employ the 5y5y inflation-linked swaps (ILS). These are swap contracts that transfer inflation risk from one party to another through an exchange of fixed cash flows. The real interest rate is computed as the difference between the 5y5y nominal rate and the 5y5y ILS.[18]

The value of the inflation target $\overline{\pi}$ is set to 2%. The Federal Reserve officially adopted this value in January 2012, but it was considered the implicit target long before that date, in line with other major central banks such as the ECB and the Bank of England. Nonetheless, to be on the safe side, we compute $\frac{cov(r^*, \pi_{ss})}{var(\pi_{ss})}$ only for the period starting in January 2012. The resulting value for this period is 0.56. This implies a value for the Taylor coefficient of $\phi_\pi = 1.56$, which is close to the standard values used in the literature.

Panel (a) of Figure 13 plots the three series starting in the year they are first available (2004). Two patterns are immediately apparent. First, market expectations of long-term nominal and real rates and inflation are neither constant nor evolve exclusively according to low-frequency secular trends, but display a significant level of high- and medium-term volatility. Second, both nominal and real rates display a larger volatility than inflation.

Panel (b) of Figure 13 plots the estimated policy gap. This gap was significantly different from zero before 2014. From 2015 to 2020, the gap largely closed, but it reopened again after the large fiscal expansion that followed the COVID-19 pandemic. These results provide

[18] For robustness, we have also employed 5y5y TIPS as a proxy for long-term real rates and computed the break-even inflation rate as the difference between nominal and real rates. The results are quite similar.



(a) Long-term nominal and real rates and inflation.

(b) Policy gap $r^* - \overline{r}$

(c) Data adjusted for term premia

(d) Policy gap $r^* - \overline{r}$ (adj. data)

Figure 13: Policy gaps

Note: Panel (a): i_{ss} is the 5y5y forward nominal rate obtained from the zero-coupon U.S. yield curve. π_{ss} is the 5y5y ILS. r^* is computed as the difference $i_{ss} - \pi_{ss}$. Panel (b) policy gap is based on Panel (a) data. In Panel (c), the estimated term premia are removed from Panel (a) series using the methodology described by Hördahl and Tristani (2014). The policy gap in Panel (d) is based on Panel (c) data. The dashed vertical line marks the date when the 2% inflation target was announced (January 24, 2012).

evidence supporting the idea that market participants perceive that the Federal Reserve's reaction function has not always tracked the natural rate perfectly, which explains the dynamics in long-term inflation.

However, the previous series do not reflect genuine market participants' expectations, as they are contaminated by term premia that reflect the compensation that risk-averse investors demand for bearing interest and inflation risk. As a robustness analysis, we remove term premia from the interest rate and inflation data using the methodology described by Hördahl and Tristani (2014). The adjusted series are displayed in Panel (c). Then, in Panel (d), we redo the analysis and display the policy gap estimated using these data. The new series is remarkably similar to the previous one, especially in the post-2012 period. The main difference is due to the higher slope of the Taylor coefficient, $\phi_\pi = 2.63$.

8 Conclusions

This paper analyzes a novel type of monetary-fiscal interaction in a HANK model with a fiscal block. In our economy, the stock of public debt affects the natural interest rate, forcing the central bank to adapt its monetary policy rule to the fiscal stance in order to guarantee price stability. We show that there is a threshold of minimal debt, below which the steady-state inflation deviates from its target as the ZLB constraint binds.

We also analyze the response to a debt-financed fiscal expansion and quantify the impact of different timings in the adaptation of the monetary policy rule, as well as the performance of alternative monetary policy rules à la Orphanides and Williams (2002) that do not require an assessment of the natural rates.

The data validate the key properties of our model: the reaction of natural rates to fiscal shocks and the subsequent response of central banks to them.

An important question, which we leave unaddressed, is how these new fiscal-monetary interactions would affect the optimal conduct of monetary policy. We leave this for future research.

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Appendix

A Derivation of the nonlinear wage Phillips curve with trend inflation

Each household i supplies a continuum of labor services $n_{i,k,t}$ that are imperfect substitutes. There is a union for each type of labor k. Each union k aggregates the efficient units of work of its members into a union-specific task:

$$N_{k,t} = \int z_{i,t} n_{i,k,t} di.$$

Unions set a common wage per efficiency unit $W_{k,t}$, and their members have to supply labor demanded at that wage. A competitive labor packer then packages these tasks into aggregate employment services using the constant-elasticity-of-substitution technology:

$$N_t = \left(\int N_{k,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dk \right)^{\frac{\epsilon_w}{\epsilon_w - 1}},$$

and sells these services to final goods firms at a price W_t. In this setup, all unions choose to set the same wage $W_{k,t} = W_t$ at time t and all households work the same number of hours, equal to $n_{i,t} = N_t$, so efficiency-weighted hours worked $\int z_{i,t} n_{i,t} di$ are also equal to aggregate labor demand N_t.

We assume that there are quadratic utility costs for adjusting the nominal wage $W_{k,t}$ set by union k:

$$\frac{\psi}{2} \log \left(\frac{W_{k,t+\tau}}{W_{k,t+\tau-1} (1 + \overline{\pi})} \right)^2.$$

The problem solved by the union at each date t is:

$$\max_{\{W_{k,t+\tau}\}} \sum_{\tau \geq 0} \beta^{t+\tau} \left[\int \left[u(c_{i,t}) - v(n_{i,t}) \right] di - \frac{\psi}{2} \log \left(\frac{W_{k,t+\tau}}{W_{k,t+\tau-1} (1 + \overline{\pi})} \right)^2 \right],$$

subject to the demand curve $N_{k,t} = \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon_w} N_t$, where $W_t^{1-\epsilon_w} = \int W_{k,t}^{1-\epsilon_w} dk$. The solution is a wage Phillips curve of the form:

$$\log \left(\frac{1 + \pi_t^w}{1 + \overline{\pi}} \right) = \kappa_w \left[-\frac{(\epsilon_w - 1)}{\epsilon_w} (1 - \tau) \frac{W_t}{P_t} \int u'(c_{it}) z_{it} di + v'(N_t) \right] N_t + \beta \log \left(\frac{1 + \pi_{t+1}^w}{1 + \overline{\pi}} \right),$$

with slope $\kappa_w \equiv \frac{\epsilon_w}{\psi}$.

B Aggregate saving behavior in a steady state

The consumer problem of our model in a steady state can be shown to be isomorphic to the consumer problem in a typical Bewley-Imhoroğlu-Aiyagari model.

First, notice that the union makes the labor choice on behalf of the households, which take their labor input as given. Moreover, we have assumed a proportional allocation of labor, which implies $n_{it} = N_t = Y_t/\Theta$, where the second equality follows from the functional form of the production function. Profit maximization by firms implies $\frac{W_t}{P_t} = \Theta$.

Taken together, these relations imply that the problem solved by households can be rewritten as

$$V(a_{i,t}, z_{i,t}) = \max_{c_{i,t}, a_{i,t+1}} u(c_{i,t}) - v\left(\frac{Y_t}{\Theta}\right) + \beta \mathbb{E}_t[V(a_{i,t+1}, z_{i,t+1})]$$
$$\text{s.t.} \;\; c_{i,t} + a_{i,t+1} = (1 + r_t)a_{i,t} + (1 - \tau)Y_t z_{i,t},$$
$$a_{i,t+1} \geq 0.$$

In a steady state, output is constant at the value Y_{ss} and the consumer problem for $Y_t = Y_{ss}$ can be written as

$$V(a_{i,t}, z_{i,t}) = \max_{c_{i,t}, a_{i,t+1}} \tilde{u}(c_{it}) + \beta \mathbb{E}_t[V(a_{i,t+1}, z_{i,t+1})]$$
$$\text{s.t.} \;\; c_{i,t} + a_{i,t+1} = (1 + r_t)a_{i,t} + \tilde{w} z_{i,t},$$
$$a_{i,t+1} \geq 0,$$

with $\tilde{u}(c_{it}) \equiv u(c_{i,t}) - v\left(\frac{Y_{ss}}{\Theta}\right)$ and $\tilde{w} \equiv (1-\tau)Y_{ss}$, which is a constant. Written in this form, the optimization problem is identical to the one that appears in equations (1a) and (1b) in Aiyagari (1993) and Aiyagari (1994). Thus, under regularity conditions described in Aiyagari (1993), there exists a unique invariant distribution over assets associated with a steady state, and this invariant distribution behaves continuously with respect to the parameters r^* and $\tilde{w}$ (Aiyagari, 1993, Proposition 5). Moreover, aggregate savings A_{ss} satisfy $\lim_{r* \to \frac{1-\beta}{\beta}} A_{ss}(r^*) = +\infty$ and $\lim_{r* \to -\infty} A_{ss}(r^*) = 0$.

C Existence and uniqueness

Here, we show the existence and uniqueness of a DSS of our model.

Existence. Start from the debt accumulation equation

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t.$$

Now substitute G_t with the fiscal rule. This leads to

$$\begin{aligned} B_t &= (1+r_t)B_{t-1} + \overline{G} - \phi_G(B_{t-1} - \overline{B}) - T_t \\ &= (1 + r_t - \phi_G)B_{t-1} + \overline{G} + \phi_G\overline{B} - T_t. \end{aligned}$$

In a DSS,

$$\begin{aligned} B_{ss} &= (1 + r^* - \phi_G)B_{ss} + \overline{G} + \phi_G\overline{B} - T_{ss} \\ &= (1 + r^* - \phi_G)B_{ss} + \overline{G} + \phi_G\overline{B} - (G_{ss} + r^* B_{ss}) \\ &= (1 - \phi_G)B_{ss} + \overline{G} + \phi_G\overline{B} - G_{ss}, \end{aligned}$$

or:

$$B_{ss} = \frac{(\overline{G} - G_{ss})}{\phi_G} + \overline{B}.$$

If $G_{ss} = \overline{G}$, then $B_{ss} = \overline{B}$. Hence, a DSS with $B_{ss} = \overline{B}$ and $G_{ss} = \overline{G}$ exists. These values are independent of the level of the interest rate and a DSS with $B_{ss} = \overline{B}$ and $G_{ss} = \overline{G}$ exists for any value that r^* may take.

Uniqueness. We show next how, as long as ϕ_G exceeds the rate of time preference, i.e., $(1-\beta)/\beta < \phi_G$ (as in our calibration of the model), the system of equations consisting of the fiscal rule together with the condition for no debt accumulation in the DSS can have at most one solution in the (B_{ss}, G_{ss}) space. In fact, it will have exactly one without further constraints, and at most one if an additional constraint, like $B_{ss} \geq 0$, is imposed.

The fiscal rule in the DSS and no-debt-accumulation evaluated at the DSS values imply:

$$G_{ss} = \overline{G} - \phi_G(B_{ss} - \overline{B}) = (\overline{G} + \phi_G\overline{B}) - \phi_G B_{ss} = T_{ss} - r^* B_{ss} = \tau Y_{ss} - r^* B_{ss}.$$

These two equations are linear relations between G_{ss} and B_{ss} with intercepts $(\overline{G} + \phi_G\overline{B})$ and τY_{ss} and slopes $-\phi_G$ and $-r^*$, respectively. For any $r^* \leq (1-\beta)/\beta < \phi_G$, they can cross at most once in the (B_{ss}, G_{ss}) plane. Taken by itself, this crossing point need not be the same for all r^*. However, combined with the prior result, there exists a DSS located at the point $(\overline{B}, \overline{G})$ regardless of the interest rate, and the uniqueness result implies that $(B_{ss}, G_{ss}) = (\overline{B}, \overline{G})$.

The intercept τY_{ss} is itself a function of the real interest rate. This does not change the conclusion at which we have arrived because, for any value of Y_{ss}, there can be at most one crossing point in the (B_{ss}, G_{ss}) plane. Finally, because the demand for bonds from households is upward-sloping, there is a unique r^* associated with $B_{ss} = \overline{B}$, i.e., for which $A_{ss}(r^*) = \overline{B}$.

D Additional tables and figures

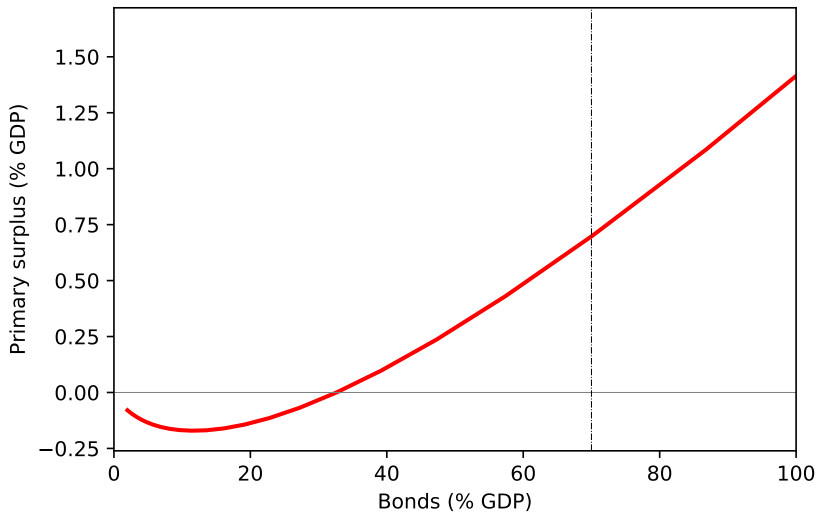

Figure 14: Equilibrium primary surplus

Note: The graphs use the baseline calibration but allow the level of bonds to vary.

	HANK		RANK	
	Expansionary	Contractionary	Expansionary	Contractionary
Bonds (% GDP)	10.000	-10.000	10.000	-10.000
Real interest rate	0.162	-0.193	0.000	0.000
Nominal interest rate	0.165	-0.197	0.000	0.000
Output	-0.097	0.096	-0.043	0.043
Consumption	0.158	-0.145	0.069	-0.069
Govt. consumption	-0.255	0.241	-0.112	0.112
Tax revenue	-0.027	0.027	-0.012	0.012
Primary surplus (% GDP)	0.229	-0.214	0.100	-0.100

Table 4: Asymmetric effects of expansionary and contractionary fiscal policy: DSS values

	HANK		RANK	
	Expansionary	Contractionary	Expansionary	Contractionary
Bonds (% GDP)	2.126	-2.108	2.210	-2.206
Real interest rate	1.795	-1.953	1.127	-1.072
Nominal interest rate	5.423	-5.853	3.583	-3.406
Inflation	4.191	-4.525	2.856	-2.720
Output	1.557	-1.581	1.606	-1.579
Consumption	-1.754	1.760	-1.774	1.795
Govt. consumption	3.311	-3.342	3.380	-3.375
Tax revenue	0.431	-0.438	0.445	-0.437
Primary surplus (% GDP)	-2.880	2.904	-2.935	2.937

Table 5: Asymmetric effects of expansionary and contractionary fiscal policy: $t = 2$

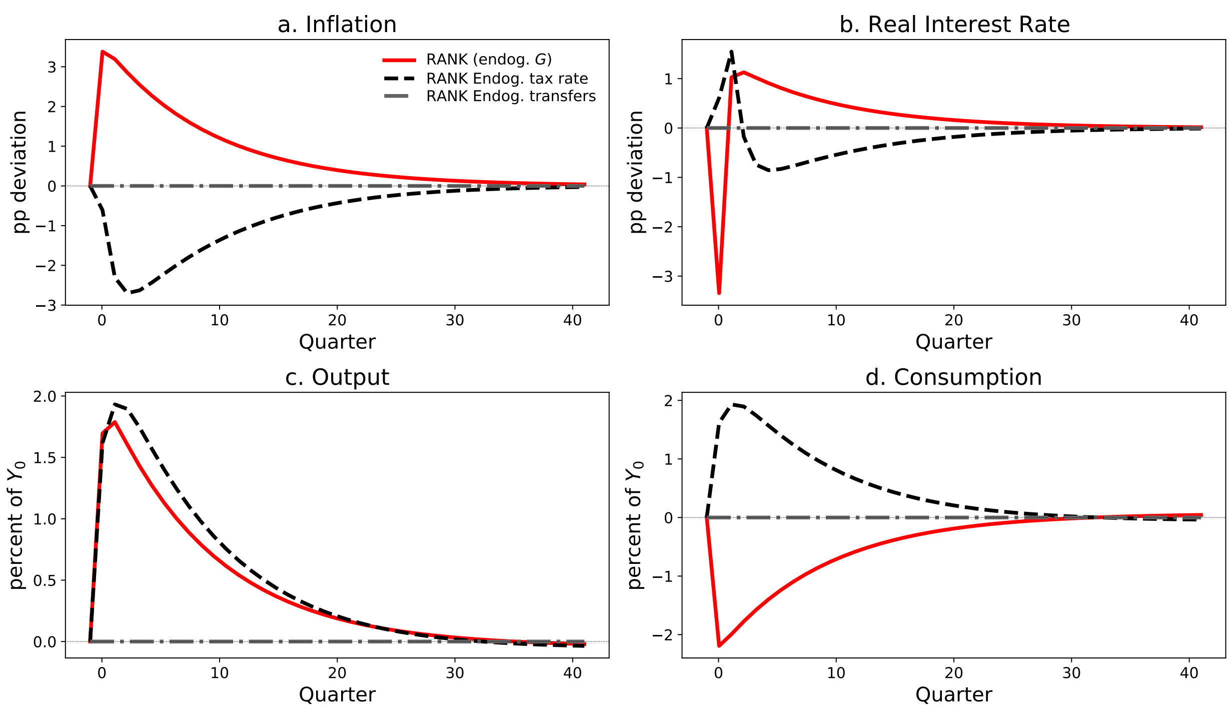

Figure 15: Dynamics after a surprise debt-financed fiscal expansion. Comparison of a RANK model with alternative fiscal rules

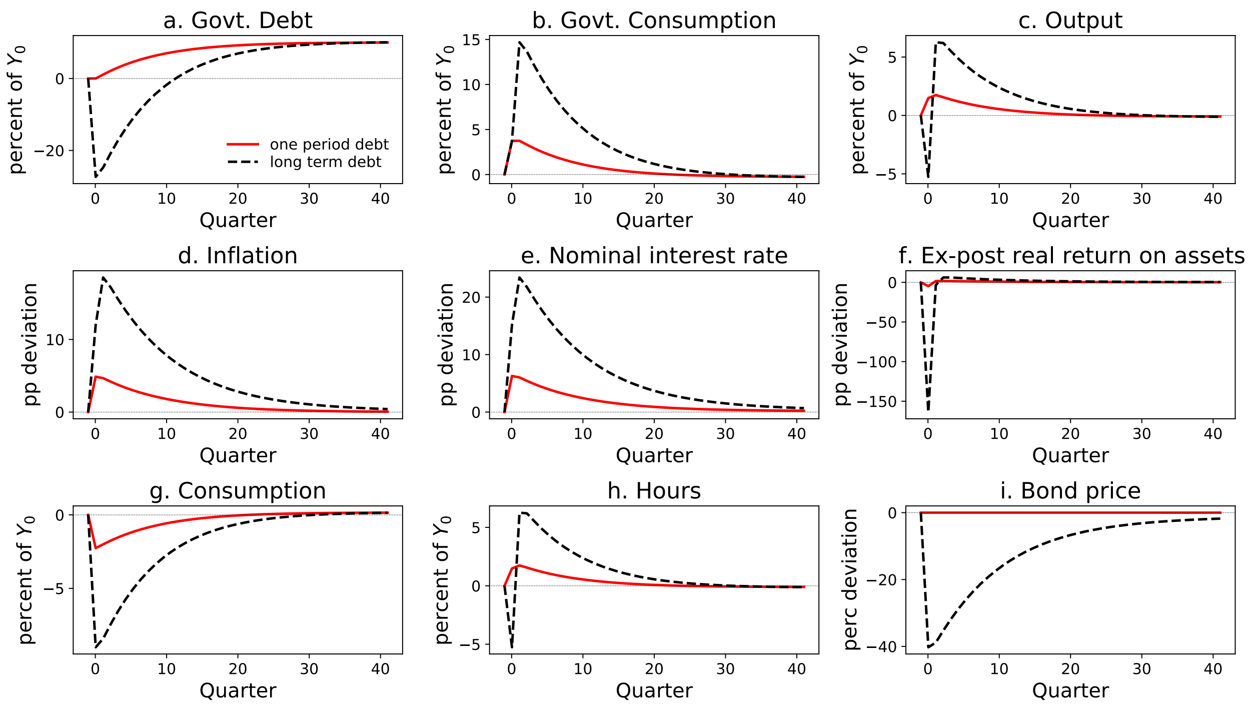

Figure 16: Dynamics of a debt-financed fiscal expansion with long-term debt

Note: Deviation with respect to the initial DSS after a shock that increases the value of the debt target $\overline{B}$ by 10% of initial GDP. Interest rates, inflation, and the ex-post real return on assets are annualized.