Challenges Solving Economic Models

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Motivation

- Many interesting questions in macroeconomics require:

  1. Nonlinear techniques. Examples: How do financial crises arise? Why do countries or firms default? When do firms invest in large, lumpy projects? Why do individuals decide to migrate?

  2. Heterogeneous agents. Examples: What mechanisms account for changes in income and wealth inequality? Is there a trade-off between inequality and economic growth? How does inequality affect monetary and fiscal policy? What are the consequences of entry-exit in models of industry dynamics?

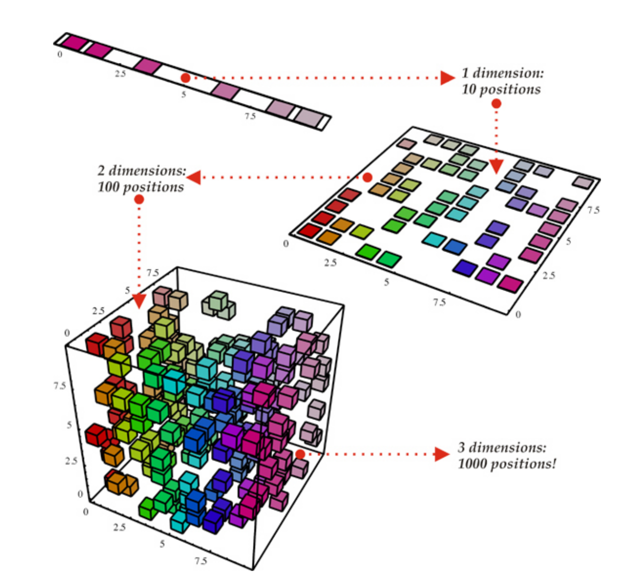
  3. Many state variables. Examples: Discrete node models, corporate finance models, rich life-cycle models, models where parameters are quasi-states.

- Often, all three elements come together. Example: heterogeneous agents models with nominal frictions and many assets.

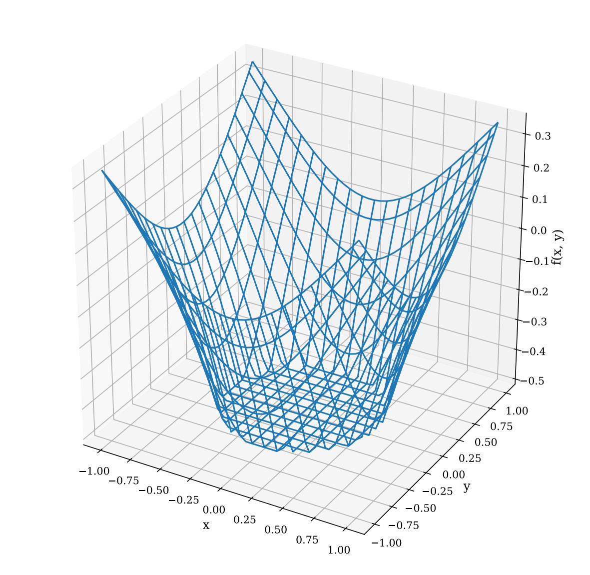
The challenge

- Modeling this class of problems rarely leads to analytic solutions. Thus, we must resort to numerical techniques.
- We want accurate and fast solution methods that can handle these models.
- Fast includes both coding and running time.
- While standard dynamic programming techniques (value function iteration, policy function iteration, or a combination of both) can tackle, in theory, most environments, we would need to struggle with the "curse of dimensionality" (three aspects of it).
- Similar concern for projection methods.
- Challenges for perturbation approaches.

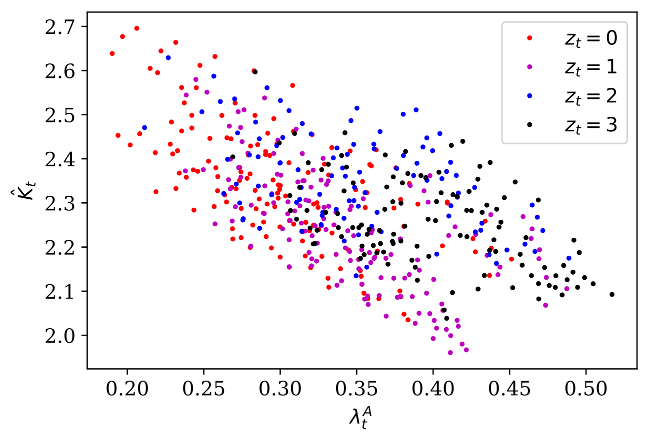
Too many dimensions

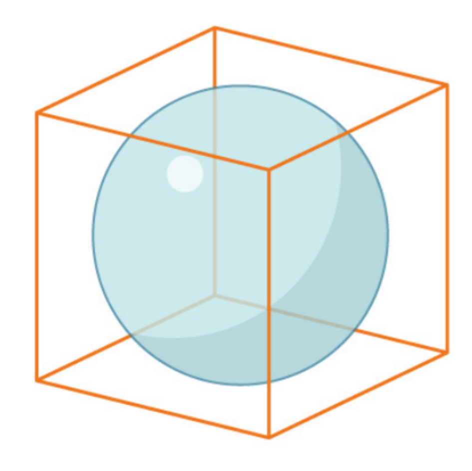


Too many kinks



Unknown and non-hypercubic domains of interest

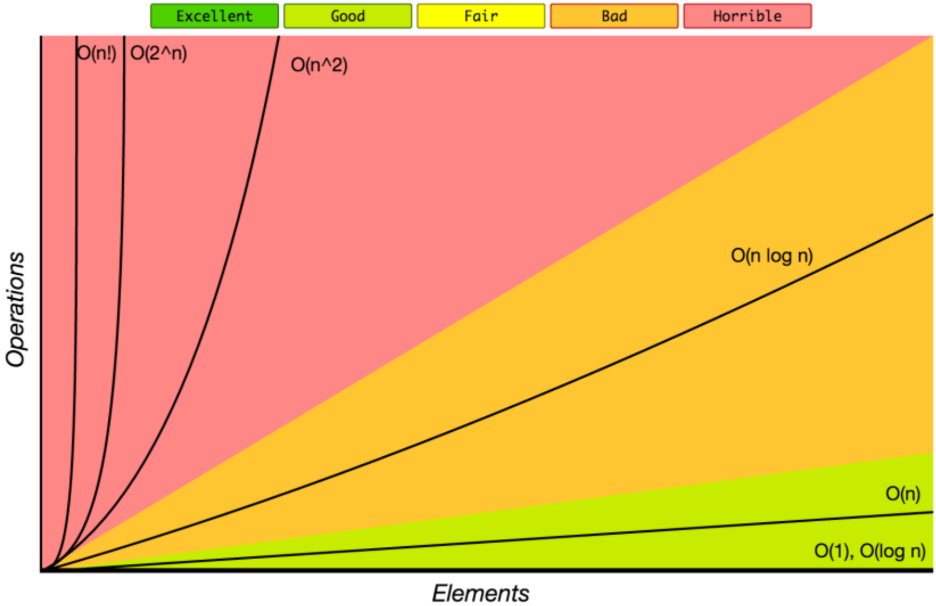




Our goal

- We want to find ways to keep the "curse of dimensionality" under control.
- In particular, we want to move to the "feasible" region of the Big-O complexity chart.
- This is relevant both for time and memory complexity.
- But key, as well, in terms of coding time. In practice, given modern computational resources, this is the real constraint for researchers.

Big-O Complexity Chart

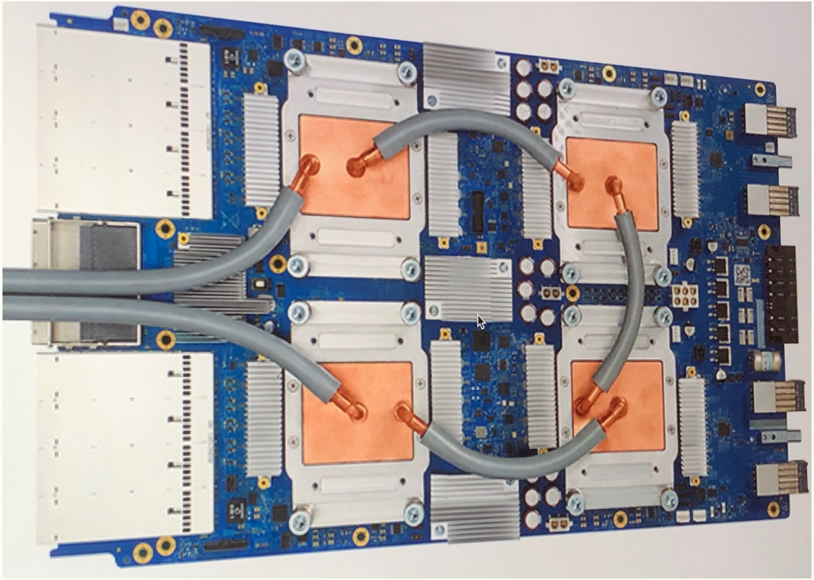


Taming the "curse of dimensionality"

- Three strategies:
  1. Better numerical algorithms (e.g., continuous-time methods, deep learning).
  2. Better software implementations (e.g., robust OS, modern programming languages, functional programming, flexible data structures, advances in massive parallelization).
  3. Better hardware designs (e.g., GPUs, TPUs and other AI accelerators, FPGAs).
- Some of these techniques are relatively new in economics or, at least, less familiar to many researchers.
- A complete treatment of the material would require, at the very least, a whole semester.
- In this course, we will focus on better numerical algorithms: deep learning.

GPUs

TPUs



Programming field-programmable gate arrays for economics

Why neural networks?

Approximation method	High-dimensional input	Can resolve local features accurately	Irregularly shaped domain	Large amount of data
Polynomials	✓	✗	✓	✓
Splines	✗	✓	✗	✓
Adaptive (sparse) grids	✓	✓	✗	✓
Gaussian processes	✓	✓	✓	✗
Deep learning	✓	✓	✓	✓