Central Bank Digital Currency: When Price and Bank Stability Collide

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What is a CBDC?

Definition (According to Gandalf)

A wizard is never late! Nor is he early; he arrives precisely when he means to.

- Term 'CBDC' is underdetermined:

1. **A digital payment system**.
2. **A new digital currency**: unit of account, store of value, medium of exchange.
3. **In this paper:** Electronic, 24x7, national-currency-denominated and interest-bearing access to the central bank balance sheet via accounts held directly at the central bank or dedicated depositories (Barrdear and Kumhof, 2016; Bordo and Levin, 2017).

Motivation

- Traditional central bank objectives: Price stability.
- Central bank objectives accompanying the introduction of a CBDC:
  - Financial intermediation (Optimal risk-sharing).
  - Maturity transformation (No proneness to runs).

⇒ **Conflict of interest among three competing objectives.**

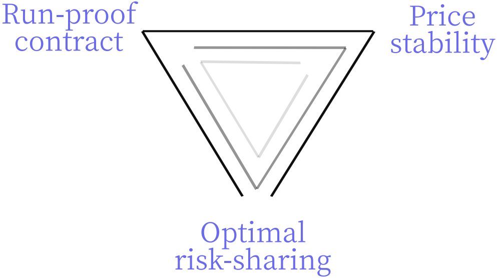
Key result

Impossibility (CBDC Trilemma)

- Impossible to attain all three goals simultaneously.
- Implementing optimal risk-sharing + stability against runs requires a commitment to high inflation (off-equilibrium threat).

Key Mechanism

- Central bank can always deliver on its nominal obligations (‘print money’).
- But central bank runs can happen in form of **run on the price level**.



Other contributions

- Central bank strategically plays against depositors.
- Nominal Jacklin extension: Trilemma can be resolved under trade in equity shares.

Literature

- Financial intermediation and bank runs:
  - Diamond and Dybvig (1983): real banking theory, deposit insurance, and bank regulation.
  - Allen and Gale (1998): system-wide run may alter price level.
  - Brunnermeier and Niepelt (2019): Equivalence of private and public money.
  - Diamond and Rajan (2006): nominal deposits serve as a hedge for real liquidity shocks and disregard panic equilibrium.
  - Skeie (2008): nominal deposits, unique no-run equilibrium by price flexibility, allows self-fulfilling runs.
  - Allen-Carletti-Gale (2014): nominal deposits, the social optimum can be implemented as competitive equilibrium, disregards strategic early withdrawal (runs are exogenous).
- Policy:
  - Barrdear and Kumhof (2016): The macroeconomics of CBDCs.
  - Bordo and Levin (2017): CBDC and the future of monetary policy.
  - Adrian and Mancini-Griffoli (2019): The rise of digital money.

The model: The Diamond and Dybvig block

- Time $t = 0, 1, 2$.
- Continuum $[0, 1]$ of agents:
  - In $t = 0$: symmetric, endowed with one unit of a real good.
  - In $t = 1$: types reveal: "impatient" λ, "patient" $1 - \lambda$.
  - $u(\cdot)$ strictly increasing, concave, and RRA greater than one, $-x \cdot u''(x)/u'(x) > 1$.
- Real technology, available to all:
  - Long term: $1 \to 1 \to R$.
  - Storage.
- **Optimal solution**: $u'(x_1^*) = Ru'(x_2^*)$.
- Classical result: $x_1^* > 1$.

The model: Nominal banking via CBDC

CBDC Contract $(M, i(\cdot))$:

- $t = 0$: Agent opens CBDC account, promising M units of 'CBDC balance' in $t = 1$ for each unit of good delivered now.
- $t = 1$: Learns type. Share n of agents spends M.
- $t = 2$: If "not spent" in $t = 1$: spends $M(1 + i(n))$.

Given policy $(M, y(\cdot), i(\cdot))$, the central bank:

- $t = 0$: Invests all collected real goods in long-term technology.
- $t = 1$:
  - Observes aggregate spending n in $t = 1$.
  - Liquidates fraction $y = y(n) \in [0, 1]$ of investment.
  - Sells goods y to spending agents at price P_1.
- $t = 2$:
  - Return $R(1 - y)$ on long term investment.
  - Sells these goods to agents at market price P_2.

Meaning of a central bank run

Definition (Monetary Distrust)

A **run on the central bank** occurs if $n > \lambda$ (i.e., patient agents also spend).

CBDC forfeits its purpose as 'store of value':

- Patient agents purchase goods instantaneously even though they do not need to consume them.
- Enable future consumption by storing apples in a barrel rather than storing value in the form of CBDC.
- Cause: An anticipated scarcity of goods/anticipated lack of CBDC purchasing power (expected future inflation).

$\Rightarrow$ **Expected future inflation causes inflation today.**

Toilet paper panic (t=2)

Figure: SkyNews, March 2020

Toilet paper panic (t=1). Actually: A run on cash

Figure: TheStreet, March 2020

Market clearing, I

$$
\begin{aligned}
nM &= P_1 y(n) \\
(1-n)(1+i(n))M &= P_2 R(1-y(n)),
\end{aligned}
$$

$\Rightarrow$ $(n, y(n), i(n))$ pin down the price level (P_1, P_2):

$$
\begin{aligned}
P_1(n) &= \frac{nM}{y(n)} \\
P_2(n) &= \frac{(1-n)(1+i(n))M}{R(1-y(n))}
\end{aligned}
$$

Market clearing, II

Via market clearing: $(n, y(n))$ determine the real goods allocation [**real CBDC backing**].

- For an agent, spending in $t = 1$:

$$x_1 = \frac{M}{P_1} = \frac{y(n)}{n}$$

- In $t = 2$:

$$x_2 = \frac{(1 + i(n))M}{P_2} = \frac{1 - y(n)}{1 - n}R$$

Equilibrium and runs

Definition

A **commitment equilibrium** consists of spending behavior $n \in [0,1]$, an initial money supply M, a liquidity policy $y : [0,1] \to [0,1]$, a nominal interest rate policy $i : [0,1] \to [-1,\infty)$, and price levels (P_1, P_2):

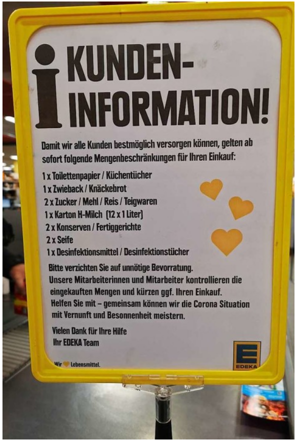
1. The individual spending decisions are optimal, given aggregate spending n, the central bank's policy $(M, y(\cdot), i(\cdot))$, the price level sequence (P_1, P_2).
2. Given the aggregate spending realization n, the central bank liquidates $y(n)$ and sets the nominal interest rate $i(n)$.
3. Given the realization $(n, y(n), i(n))$ and M, the price levels (P_1, P_2) clear the goods market in each period.

Important

(i) The central bank fully commits to its policy (M, y, i) in $t = 0$.

(ii) The price levels **flexibly adjusts** to (n, M, y, i) (vs. rationing or stockouts).

Rationing at Edeka



Badische Zeitung, 03/2020

Sueddeutsche Zeitung, 03/2020

Equilibria given central bank policy

Lemma

Given the central bank policy $(M, y(\cdot), i(\cdot))$,

1. $n = \lambda$ *is an equilibrium only if* $x_1(\lambda) \leq x_2(\lambda)$.
2. *A central bank run* $n = 1$ *is an equilibrium if and only if* $x_1(1) \geq x_2(1)$.

$$
\begin{aligned}
x_1(n) &= \frac{y(n)}{n} \\
x_2(n) &= \frac{1 - y(n)}{1 - n} R
\end{aligned}
$$

Implementing the social optimum, I

Proposition

The central bank policy $(M, y(\cdot), i(\cdot))$ implements the social optimum (x_1^*, x_2^*) in dominant strategies if:

i) for any $n = \lambda$, it sets $y(\lambda) = y^*$, where $x_1^*(\lambda) = y^*/\lambda$.

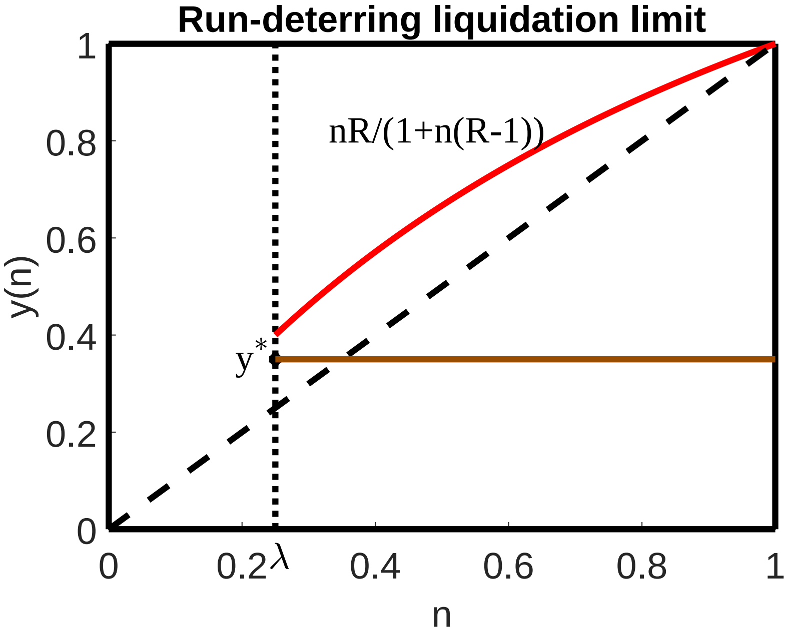
ii) for all $n > \lambda$: it sets a liquidation policy that implies $x_1(n) < x_2(n)$.

Definition

We call a liquidation policy $y(\cdot)$ *"run-deterring"* if it satisfies:

$$y^d(n) < \frac{nR}{1 + n(R-1)}, \qquad \text{for all } n \in (\lambda, 1]$$

Such a liquidation policy implies that "roll over" is *ex-post* optimal $x_1(n) < x_2(n)$, even though patient agents are withdrawing $n \in (\lambda, 1]$.


Run-deterring liquidation limit
nR/(1+n(R-1))
y*
λ
y(n)
n
0
0.2
0.4
0.6
0.8
1

Implementing the social optimum, II

Corollary (Trilemma I)

Every policy choice $(M, y(\cdot), i(\cdot))$, $n \in [0,1]$ *with* $y(\lambda) = y^*$ *and:*

$$y^d(n) < \frac{nR}{1 + n(R-1)}, \qquad \textit{for all } n \in (\lambda, 1],$$

deters central bank runs and implements the social optimum in dominant strategies.
Flipside: Such a deterring policy choice requires the interim price level $P_1(n)$ *to exceed the withdrawal dependent bound:*

$$P_1(n) > \frac{M}{R}(1 + n(R-1)), \qquad \textit{for all } n \in (\lambda, 1].$$

Remark. This looks like a version of "suspension of convertibility," but not quite. The central bank does not stop customers from spending their CBDCs. Instead, the supply of goods traded against these CBDCs is restricted.

A brief pause

So far

- Nominal banking model for a central bank and its CBDC.
  $\Rightarrow$ Central bank can always deliver on its nominal obligations.
- To deter runs, the central bank threatens with a high price level (or "inflation") for $t = 1$, making running *ex-post* suboptimal.

2 Issues to discuss

- Central banks usually wish to keep prices stable (for reasons outside this model)!
  $\Rightarrow$ Time inconsistency?
- If the central bank is constrained by price stability objective: $\Rightarrow$ Can runs reoccur?

Time consistency, I

Consider the subgame $n > \lambda$: Central bank realizes that a run is occurring.

- Depositor utility in the subgame is:

$$W(y, n) = n\, u\left(\frac{y}{n}\right) + (1-n)u\left(\frac{R(1-y)}{1-n}\right)$$

- Additional asset liquidation y (beyond intended level) has a price-stabilizing effect: Price level $P_1(n) = \frac{nM}{y}$.
- Impose concern for price stability at level $(1-\alpha) \in (0,1)$.
- Allocative welfare: Central bank reoptimizes via liquidation policy y:

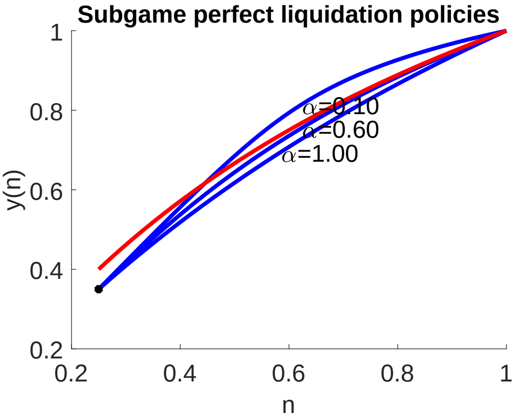
$$V(y, n, \overline{P}) = \alpha W(y, n) - (1-\alpha)\left(\overline{P} - P_1(n)\right)^2$$

Time consistency, II

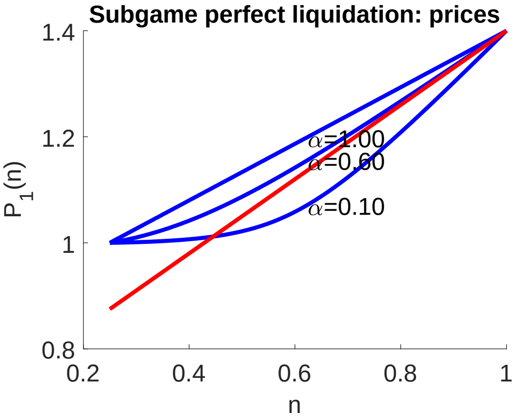
A numerical example:

- Set $R = 2$, $\lambda = 0.25$, $u(c) = c^{1-\eta}/(1-\eta)$, $\eta = 3.25$.
- Then $x_1^* = 1.4$ (the DD optimum for $\alpha = 1$ and $n = \lambda$).
- For $M = 1.4$, one obtains $P_1^* = M/x_1^* = 1$. Set price target $\overline{P} = P_1^*$.
- For $\alpha = \{0.1, 0.6, 1\}$: Calculate the subgame-optimal liquidation policy $y_\alpha(n)$ that maximizes V and the implied sub-game optimal price level $P_{1,\alpha}(n)$.

Time consistency, III



Liquidation Policies



Prices

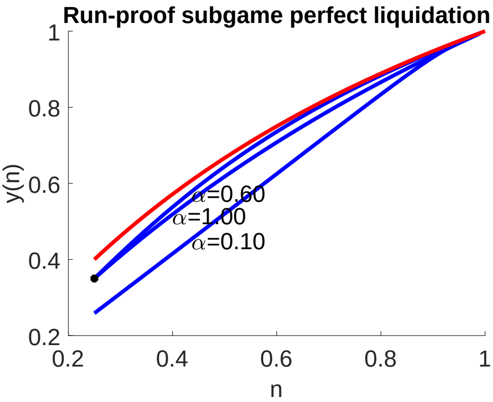
Time consistency, IV

- At $n = \lambda$: All levels α reach y^* (because $P_1(\alpha) = \overline{P}$).
- For $\alpha = 1$ (no price stability concern): At every run $n > \lambda$ the subgame-perfect liquidation policy is run-deterring (time-consistent).
- **Issue** for α small: subgame-perfect liquidation policies give rise to runs. Thus, the depositors' anticipation of a central bank deviation **rationalizes** runs *ex-ante*.

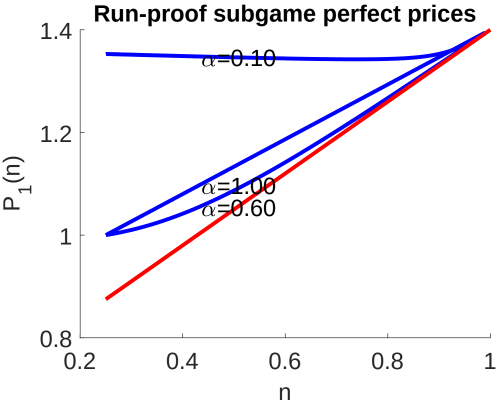
To prevent runs for sure: Raise price stability target.

- Given α: Compute the smallest $\overline{P}(\alpha) \geq P_1^*$ so that the subgame-perfect liquidation policy is run-deterring following every subgame $n > \lambda$.
- Problem: the resulting sub-game perfect run-deterring liquidation policies no longer attain the optimum x_1^* at $n = \lambda$.

Time consistency, V



Liquidation Policies



Prices $P_1(n)$

Taking stock

When incorporating a concern for price stability $\alpha < 1$:

- The *ex-ante* optimum x_1^* can be attained for all $\alpha \in (0, 1)$ when setting $\overline{P} = P_1^*$, but the central bank's reoptimization following some sub-games give rise to runs.

- When raising the price level target to fit α, runs can be deterred for sure (in all possible subgames), but the *ex-ante* optimum x_1^* is never attained.

From numerical analysis $\Rightarrow$ theory:

What happens under the predominant price stability objective?

Central bank constraint: full price stability

Definition

i) A central bank policy is **P_1-stable at level $\overline{P}$** if it achieves $P_1(n) \equiv \overline{P}$ for the **price level target $\overline{P}$** at all spending fractions $n \in [\lambda, 1]$.

ii) A central bank policy is **price-stable at level $\overline{P}$** if it achieves $P_1(n) = P_2(n) \equiv \overline{P}$ for the **price level target $\overline{P}$** for all spending fractions $n \in [\lambda, 1]$.

Recall market clearing:

$$
\begin{aligned}
P_1(n) &= \frac{nM}{y(n)} \\
P_2(n) &= \frac{(1-n)(1+i(n))M}{R(1-y(n))}
\end{aligned}
$$

Thus, the liquidation and interest rate (y, i) adjust to $(n, \overline{P})$.

Characterizing P_1-stable central bank policies

Feasibility constraint: $y(1) \leq 1$ requires $\frac{M}{\overline{P}} \leq 1$.

Proposition (Characterization of (y, i) to attain P_1-stability)

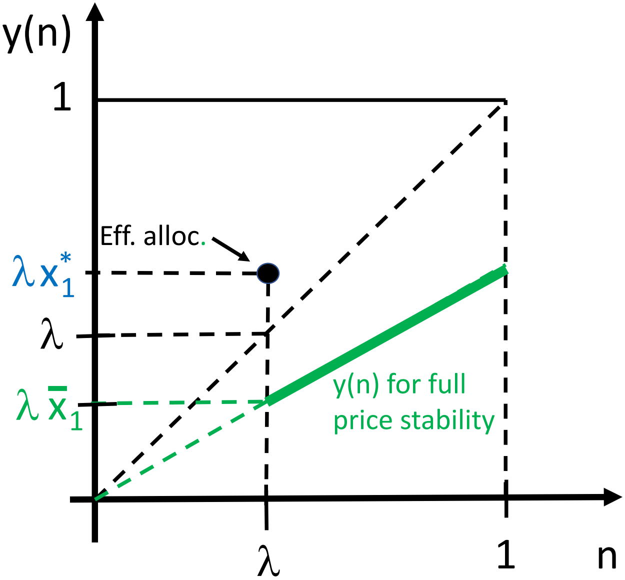
A central bank policy is:

i) P_1-stable at level $\overline{P}$ if and only if its liquidation policy satisfies:

$$y(n) = \frac{M}{\overline{P}} n, \text{ for all } n \in [0,1], \text{ and, thus, } x_1(n) \equiv \overline{x}_1 = \frac{M}{\overline{P}} \leq 1. \tag{1}$$

ii) A central bank policy is price-stable if and only if its liquidation policy satisfies equation (1) and its interest policy satisfies:

$$n = \frac{\frac{\overline{P}}{M} - n}{1 - n} R - 1 \text{ and } \overline{P} \geq M.$$


y(n)
1
Eff. alloc.
λ x*1
λ
λ x̄1
y(n) for full
price stability
λ
1
n

P_1-stable central bank policies are inefficient

Corollary (Trilemma II)

If the central bank commits to a P_1-stable policy, then:

i) *The socially optimal allocation is not implemented.*

ii) *There is a unique equilibrium where only impatient agents spend, $n^* = \lambda$, i.e., no central bank run equilibria.*

iii) *If the central bank commits to a price-stable central bank policy, then the nominal interest rate is non-negative $i(n) \geq 0$ for all $n \in [\lambda, 1]$. The interest rate $i(n)$ is increasing in n.*

Central bank constraint: partial price stability

Definition

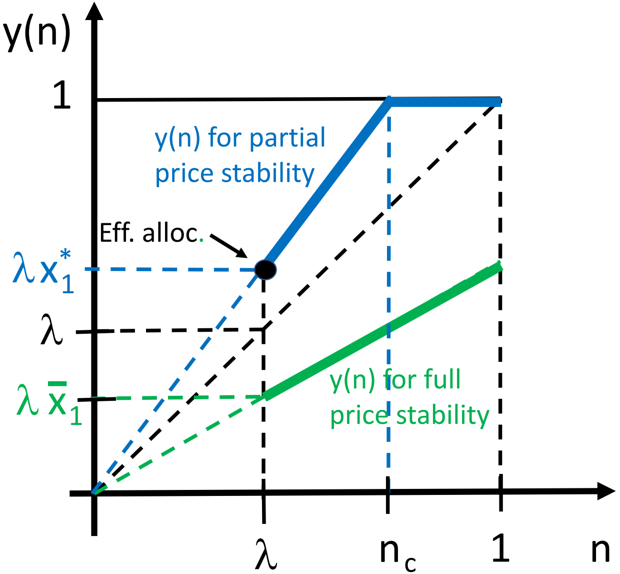
1. A central bank policy is **partially P_1-stable at level $\overline{P}$** if either it achieves $P_1(n) = \overline{P}$ for some **price level target** $\overline{P}$, or the central bank fully liquidates real investment $y(n) = 1$.
2. A central bank policy is **partially price-stable at level $\overline{P}$**, if either it achieves $P_1(n) = P_2(n) = \overline{P}$ for some **price level target** $\overline{P}$, or the central bank fully liquidates real investment $y(n) = 1$.

Proposition

Suppose that $M > \overline{P} \geq \lambda M$. A central bank policy is partially P_1-stable at level $\overline{P}$ if and only if its liquidation policy satisfies:

$$y(n) = \min\left\{\frac{M}{\overline{P}}n, 1\right\}$$

Full vs. partially price-stable liquidation policies



Characterizing partially P_1-stable central bank policies

Proposition

Suppose that $\overline{P} \in [\lambda M, M]$. Consider a partially P_1-stable central bank policy at level $\overline{P}$. Define the critical aggregate spending level:

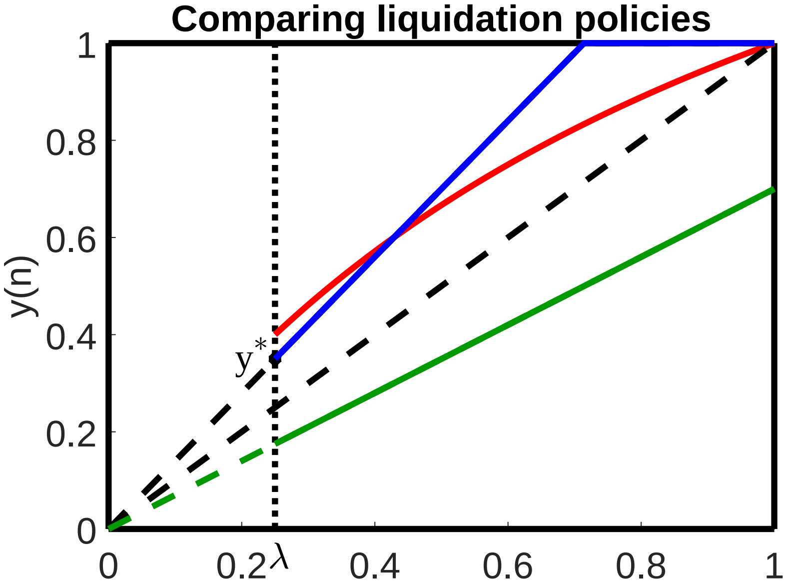
$$n_c \equiv \frac{\overline{P}}{M}$$

For all $n \leq n_c$,

- the price level is stable at $P_1(n) = \overline{P}$.

For all $n > n_c$ (full liquidation),

- price level **not stable**: $P_1(n)$ proportionally increasing with n: $P_1(n) = Mn$,
- real goods per agent: $x_1(n) = 1/n$, $x_2 = 0 \Rightarrow$ **runs occur in equilibrium** + negative real interest rate.


Comparing liquidation policies
y(n)
1
0.8
0.6
0.4
0.2
0
0
0.2
λ
0.4
0.6
0.8
1
y*

Characterizing partially P_1-stable central bank policies

Corollary (Trilemma III)

Suppose that CB policy is partially price-stable at $\overline{P} \in [\lambda M, M]$

1. *then runs on the central bank can occur (multiple equilibria)* $n^* \in \{\lambda, 1\}$.
2. *Given no run: the social optimum and the price goal are attained.*
3. *Given a run: the social optimum and the price goal are not attained.*

Price targeting via state-contingent money supply in $t = 1$?

- Assume state-contingent individual money balances $M(n)$ in $t = 1$.
- Suppose $y(n) \equiv y^*$. To maintain price stability at some $\overline{P}$:

$$n\, M(n) = \overline{P} y^* = \lambda M(\lambda)$$

- Implementations:
  1. Taxation of individual money holdings (helicopter grab).
  2. Suspension of spending (supermarket stockout).
  3. Rationing (only some of the money can be used).

Stable prices! **Problem solved? Issues:**

- Trust: Individual CBDC accounts decrease with n (\$1 today not \$1 tomorrow).
- Money supply is not effective in preventing runs. Individual real allocation $y(n)/n$ is independent of money supply [neutrality] $\Rightarrow$ The important policy variable is $y(n)$.

Nominal Jacklin (1987): Equity shares in the central bank, I

- Agents invest in equity shares of the central bank.
- In $t = 0$: Central bank promises nominal dividends (D_1, D_2) to be paid in $t = 1$, $t = 2$.
- In $t = 1$: types reveal, agents can go shopping for goods, but before doing so, they trade in a market claims on nominal dividends.
- Assumption: nominal dividends expire and cannot be stored.
- **Central bank run:** $n > \lambda$ (patient types shop early and trade in equity shares collapses).

Nominal Jacklin (1987): Equity shares in the central bank, II

- **Market clearing**

$$D_1 = P_1(n)y(n)$$

$$D_2 = P_2(n)R(1 - y(n))$$

- Main difference to demand-deposit model: dividends are predetermined, pinning down the money supply in $t = 1, 2$.

- **Still:** liquidation is at the discretion of the central bank

Lemma (Price stability)

Consider the central bank policy $(D_1, D_2, y(\cdot))$ with $D_1, D_2 > 0$. Every constant (demand-insensitive) liquidation policy $y(n) \equiv y \in (0, 1)$ for all $n \in [0, 1]$ implies constant price levels in $t = 1$ and $t = 2$, $P_1(n) = \overline{P}_1$, $P_2(n) = \overline{P}_2$ for all $n \in [0, 1]$.

Nominal Jacklin (1987): Equity shares in the central bank, III

$$x_1 = \frac{D_1}{P_1 n} = \frac{y(n)}{n}$$

$$x_2 = \frac{D_2}{P_2(n)(1-n)} = \frac{R(1-y)}{1-n}$$

Remark (Run-deterring price-dividend pairs)

A price-dividend pair $(D_1, P_1(\cdot))$ deters runs on equity shares if

$$\frac{D_1}{P_1(n)} < \frac{nR}{1+n(R-1)}, \qquad \textit{for all } n \in (\lambda, 1]. \tag{2}$$

Define the constant liquidation policy

$$\hat{y} := \frac{\lambda R}{1+\lambda(R-1)} \in (0,1)$$

as the minimum of the right-hand side of (2).

Nominal Jacklin (1987): Equity shares in the central bank, IV

Proposition (No trilemma with nominal dividends)

Consider the central bank policy $(D_1, D_2, y(\cdot))$ *with* $D_1, D_2 > 0$:
(i) [run-deterrence and price-stability]: If the central bank sets a constant liquidation policy $y(n) = \tilde{y} \in (0, \hat{y}]$ *for all* $n \in [0,1]$, *it implements the stable price level* $P_1(n) \equiv \frac{D_1}{\tilde{y}} =: \overline{P}$ *in* $t = 1$ *for all* $n \in [0,1]$ *and simultaneously deters runs.*
(ii) [run-deterrence, price-stability, and social optimality]: If the central bank sets the constant liquidation policy $y(n) = y^*$ *for all* $n \in [0,1]$, *not only runs are deterred, but the social optimum is implemented in dominant strategies. In addition, the price target* $P_1 = \overline{P}$ *is attained in* $t = 1$. *The trilemma vanishes.*
(iii) If the late dividend payment D_2 *additionally satisfies*

$$D_2 = \overline{P} R (1 - \hat{y})$$

then the price target is also implemented in $t = 2$.

Conclusions

In a nominal banking model for a central bank and its CBDC.

- The central bank can always deliver on its nominal obligations.
- But: runs can still occur.
- We show the following CBDC TRILEMMA
  - Implementation of the social optimum $x_1^* > 1$ requires the threat of inflation to deter runs. (price stability lost).
  - Full price stability. requires giving up the social optimum, $x_1 \leq 1$. But runs do not occur.
  - Under partial price stability, runs can occur (multiple equilibria). But absent a run, the social optimum can be implemented.
- Ways around the trilemma? Predetermined nominal equity shares with expiring dividends or spending-contingent money supply.