Central Bank Digital Currency: When Price and Bank Stability Collide

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Abstract

This paper shows the existence of a central bank trilemma. When a central bank is involved in financial intermediation, either directly through a central bank digital currency (CBDC) or indirectly through other policy instruments, it can only achieve two of three objectives: a socially efficient allocation, financial stability (i.e., absence of runs), and price stability. In particular, a commitment to price stability can cause a run on the central bank. Implementation of the socially optimal allocation requires a commitment to inflation. We illustrate this idea through a nominal version of the Diamond and Dybvig (1983) model. Our perspective may be particularly appropriate when CBDCs are introduced on a wide scale.

Keywords: CBDC, currency crises, monetary policy, bank runs, spending runs, financial intermediation, central bank digital currency, inflation targeting
JEL classifications: E58, G21.

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1 Introduction

Diamond and Dybvig (1983) (DD hereafter) taught us that implementing the social optimum via banks' financial intermediation comes at the cost of making banks prone to runs. This dilemma becomes a trilemma when a central bank with a price stability objective acts as the intermediary in the financial market by offering nominal savings accounts to households, e.g., a central bank digital currency (CBDC). A central bank concerned with price stability is exposed to the risk of spending runs and their associated inflations. Our main result is to show that a central bank involved in financial intermediation (directly or indirectly) that wishes to concurrently achieve a socially efficient allocation, financial stability (i.e., absence of runs), and price stability will see its desires foiled.[1] A central bank can only realize two of the three goals at a time. We call this phenomenon the central banking trilemma.

To make this point, we build a nominal version of DD with a central bank and strategic agents. The central bank issues money in $t = 0$ to purchase goods from agents and invest them in illiquid, real long-term projects. In $t = 1$, the central bank sees the fraction of agents wishing to purchase goods and liquidates a share of its projects to create supply. The agents draw on their nominal central bank accounts to purchase goods and prices clear markets.

In our environment, the deposit withdrawals in DD become spending decisions and a "bank run" a "spending run." Excessive spending (i.e., more spending than in the social optimum) is a run on the central bank in all but name. When prices adjust flexibly, we characterize run-deterring liquidation policies that prevent excessive spending ex-ante. These policies require a guaranteed positive real return on nominal deposits and a credible commitment to sufficiently low asset liquidation, irrespective of demand. Put differently, run-deterrence requires the central bank's credible threat to tolerate off-equilibrium price increases in $t = 1$ compared to the desired level (trilemma, part I), creating a time-consistency problem for a central bank that also cares about price stability. With a sufficiently strong price stability objective, a time-consistent policy avoids runs only at the expense of an inefficient no-run allocation (trilemma, part II)

[1] These three objectives are enshrined in legal instruments like the Federal Reserve's 1977 "dual mandate" in the U.S. or Article 127 of the *Treaty on the Functioning of the European Union* regulating the ECB.

or implements the efficient solution but faces the possibility of a run equilibrium, i.e., financial instability (trilemma, part III). The latter arises because keeping prices stable when a high fraction of agents spend in $t = 1$ means that the central bank will run out of goods in $t = 2$.

In sum, the challenges pointed out by DD do not disappear even in the extreme case where the central bank replaces run-prone banks and runs the entire financial system through a CBDC instead.[2] The central bank has the unenviable choice to either let prices move away from their desired level or liquidate long-term investments, risking a run. These trade-offs are particularly transparent in our benchmark economy with a consolidated central bank. Section 6 shows that these trade-offs also exist in decentralized economies with competitive firms and banks and households holding cash or nominal deposits at private banks. In such an environment, the central bank indirectly enforces a given price level or liquidation policy by granting loans to firms via banks and charging penalty rates whenever the firms or banks fail to meet loan repayments.

In relation to the literature, we follow Skeie (2008), Allen et al. (2014) (ACG hereafter), and Andolfatto et al. (2020) by building a nominal version of DD. Skeie (2008) is closest to our setup. He shows the impossibility of a DD-style run when banks offer nominal contracts and goods prices are flexible. However, he does not consider a central bank with a price stability or optimal risk-sharing objective. ACG study the implementation of optimal allocations under flexible prices where firms react to prices via their supply. However, in ACG, the liquidation of illiquid firm assets is ruled out, which deters inflation in equilibrium. Unlike ACG, we study how implementing optimal allocations hampers the central bank's price stability objective and vice versa in a framework where liquidating illiquid assets is possible. Also, we show how the design of interest rates on central bank loans can deter runs ex-ante and implement the optimum in dominant strategies. ACG study a representative firm whereas our firms are strategic with one another. In comparison to Andolfatto et al. (2020), we abstract from the role of money as a fundamental means of exchange. As in Green and Lin (2003), we demonstrate that the

[2] Fernández-Villaverde et al. (2021) show that a CBDC offered by the central bank may be such an attractive alternative to private bank deposits that the central bank becomes a deposit monopolist and the financial intermediator of the economy (in fact, that is the stated goal of some proponents of CBDCs).

efficient allocation can be implemented in dominant strategies when the bank can condition the allocations on the number of agents seeking to spend in $t = 1$, but we use nominal contracts. Like Ennis and Keister (2009), we study the depositors' incentives to spend and issues of efficiency once a run takes place, but we employ nominal instead of real demand-deposit contracts, giving the central bank an additional tool –the price level– to prevent runs.

Our paper contributes to the study of CBDCs; see the survey by Infante et al. (2022). We differ from this literature by paying attention to the central bank's trade-off between efficiency, financial stability, and price stability when CBDCs have eroded the deposit base at private banks. Barlevy et al. (2022) expand our analysis by showing that lending of last resort is possible without creating inflation.

Finally, our paper is related to the literature on self-fulfilling currency crises: a currency crisis is a form of a run on a central bank. As in Obstfeld (1984, 1996), multiple equilibria can arise due to self-fulfilling expectations of rationally behaving agents. In Obstfeld (1996), a government holds foreign reserves to defend an exchange rate peg or needs to give it up. Analogously, our central bank can respond to shocks by liquidating real investments or devaluing its currency. The latter can be seen as akin to repudiating a nominal government obligation as in Calvo (1988). Similar to Velasco (1996), the central bank can deter the run on currency by credibly committing to abandon the peg whenever output is threatened in the short run. The novelty of our analysis is its focus on the maturity-transforming role of the central bank. Price stabilization via liquidation is costly because premature liquidation increases output today at the expense of reducing output tomorrow. Due to this liquidation externality, short-term inflation can be socially optimal as an off-equilibrium threat to deter speculation against the real value of the currency.

2 The model

There are three periods $t = 0, 1, 2$, and no discounting. There is a $[0, 1]$-continuum of agents, each endowed with 1 unit of a consumption good in $t = 0$. Agents are symmetric at $t = 0$ but can be subject to a shock in $t = 1$, turning an agent impatient with probability $\lambda \in (0, 1)$ or staying patient. The agent's type is private information and random and independently drawn at the beginning of $t = 1$. By a law of large numbers, λ is also the deterministic share of impatient agents in the economy.

Let $x_t \geq 0$ represent goods consumed by an agent at time t. Preferences for each agent are $U(x_1, x_2) = u(x_1)$ if the agent is impatient and $U(x_1, x_2) = u(x_2)$ if she is patient. The function $u(\cdot) \in \mathbb{R}$ is strictly increasing, strictly concave, and continuously differentiable for all $x > 0$. Also, $-x \cdot u''(x)/u'(x) > 1$, for all $x > 1$.

There exists a long-term, illiquid production technology in the economy. For each unit of the good invested in $t = 0$, liquidation yields either 1 unit at $t = 1$ or $R > 1$ units at $t = 2$. Partial liquidation is possible. Additionally, there is a goods storage technology between $t = 1$ and $t = 2$, yielding 1 unit of the good in $t = 2$ for each unit invested in $t = 1$.[3]

Optimal risk sharing. Consider a social planner that collects and invests the agents' aggregate endowment in the long-term technology to maximize their ex-ante expected utility, $W = \lambda u(x_1) + (1 - \lambda)u(x_2)$, by choosing (x_1, x_2), subject to the feasibility constraint $\lambda x_1 \leq 1$ and the resource constraint $(1 - \lambda)x_2 \leq R(1 - \lambda x_1)$. We call W the **allocative welfare** to distinguish it from the broader objective $V(y; n, \overline{P})$, where additional price stability considerations are included. From DD, the optimal allocation (x_1^*, x_2^*) must satisfy the interior first-order condition $u'(x_1^*) = Ru'(x_2^*)$ and the resource constraint $R(1-\lambda x_1^*) = (1-\lambda)x_2^*$, yielding $x_1^* < x_2^*$, $x_1^* > 1$, and $x_2^* < R$.

DD show that a bank offering a real demand-deposit contract (i.e., a contract that promises to pay out goods in future periods) can implement the efficient allocation. Due to a maturity

[3] Our model is equivalent to DD's, where storage between $t = 1$ and $t = 2$ does not exist, but where patient agents can also consume in $t = 1$.

mismatch between real long-term investment and real deposit liabilities, the DD environment also features a bad bank-run equilibrium. In DD, the bad equilibrium can be deterred if a suspension of convertibility or real deposit insurance is in place.

A central message of our paper is that a central bank can always implement the efficient allocation above when using nominal instead of real demand deposits, even without suspension or insurance in place. The reason is that the central bank can set the price level, thereby controlling the wedge between real long-term investment and nominal deposit liabilities. However, this accomplishment comes at the cost of price-level stability. To develop these arguments, we must first introduce a central bank.

The central bank. In our benchmark model, we consider a consolidated central bank that aggregates different roles: it creates liquidity for depositors, finances real projects, and targets price stability. We abstract from private banks and firms because as in the classic papers by Calvo (1988), Obstfeld (1996), and Velasco (1996), it simplifies the analysis and makes the main economic mechanism more transparent.[4] More precisely, our central bank offers agents nominal, interest-bearing demand-deposit contracts. A straightforward interpretation of this deposit is as a CBDC, fully replacing bank deposits. Nonetheless, Section 6 shows that our mechanism works in a decentralized economy with private banks offering nominal deposit contracts and firms running the real economy, and Section 7 discusses the equivalence between nominal demand deposits at private banks vs. CBDC vs. cash.

To pin down the tools of the central bank, we define its policy as follows:

Definition 1. *A central bank policy is a triple* $(M, y(\cdot), i(\cdot))$*, where* M *is the money supply in* $t = 0$*,* $y : [0, 1] \to (0, 1]$ *is the central bank's liquidation policy and* $i : [0, 1] \to [-1, \infty)$ *is the nominal interest rate paid on deposits between* $t = 1$ *and* $t = 2$ *for all* $n \in [0, 1]$*.*

At $t = 0$, the central bank sets and commits to a policy $(M, y(\cdot), i(\cdot))$. The policy is common knowledge in $t = 0$. Then, the central bank creates a zero-balance account for each agent in the

[4] Also, the literature worries that financial disintermediation induced by a CBDC may be harmful because private banks are more skillful at investment than central banks. We show that a CBDC triggers a conflict between preventing runs and price stability, even if the central bank is as skilled as private banks.

economy. All agents sell their unit endowment of the good to the central bank in exchange for $P_0 > 0$ dollars, credited to that agent's deposit account. The nominal contract with the central bank promises P_0 nominal units if the agent decides to spend in $t = 1$ and offers $P_0(1 + i(n))$ units if the agent decides to spend in $t = 2$.[5] The agents cannot store or consume the good by themselves at $t = 0$. Thus, $M = \int_{[0,1]} P_0 \, di = P_0$. The central bank invests all goods in the long-term production technology.

At $t = 1$, before making the spending decision, all agents privately observe their type and simultaneously decide whether to spend their balances in $t = 1$ or roll them over to spend on goods in $t = 2$. Impatient types only care for consumption in $t = 1$, whereas patient types only care for late consumption but can spend nominal units on goods early in $t = 1$ and store the goods privately until $t = 2$. Let $n \in [0, 1]$ be the endogenous share of agents that spend money on goods in $t = 1$. To allow consumption, the central bank opens a centralized goods market to all agents, offering goods for sale by (partially) liquidating the long-term production technology. More concretely, the central bank observes the measure of spenders, n, liquidates a fraction $y = y(n)$ of the long-term technology at value one, and sells the resulting goods at the market-clearing unit price $P_1(n)$ to the agents against money. Because the agents' types are unobservable, the central bank cannot refuse to sell goods to a patient agent. We restrict attention to strictly positive liquidation policies $y(\cdot) > 0$ to rule out equilibria where impatient agents do not spend dollars early since there are no goods to purchase. While an agent does not know aggregate spending n when making her spending decision, the agent knows the provision of goods for every possible n. For simplicity, we assume that an agent spends all of her balances or none. Also, agents cannot hold negative deposit balances. Given n, the central bank sets the nominal interest rate $i = i(n)$ according to its announced policy in $t = 0$. Each dollar held at the end of $t = 1$ turns into $1 + i(n)$ dollars at the beginning of $t = 2$. Since agents cannot hold negative balances, $i(n) \geq -1$.

[5] We set the nominal interest rate between $t = 0$ and $t = 1$ to zero because its value does not change any results. Also, unlike a nominal deposit contract with a private bank, the central bank controls the money supply and can always deliver on these nominal units. Our mechanism is not steered via scarcity of money but through scarcity of the consumption good in the market.

In $t = 2$, the remaining investment matures, and the central bank supplies $R(1 - y(n))$ units of goods in exchange for the unspent money balances (we assume no free disposal). Each depositor who rolled over has $(1 + i(n))P_0$ dollars to spend on goods at a market-clearing price $P_2(n)$. The market-clearing conditions on (P_1, P_2) are $nP_0 = P_1 \cdot y(n)$ and $(1-n)(1+i(n))P_0 = P_2R(1 - y(n))$, which are just the quantity theory equations for each t ($MV = P_1y$, where velocity on unspent dollars is zero and velocity of spent dollars is one). A higher interest rate $i(n)$ induces a higher nominal monetary supply in t_2 and causes a higher price level P_2 when n and $y(n)$ remain unchanged, a "Fisherian" effect.

Implied real deposit contract. Patient agents have no consumption needs in $t = 1$. Because there is storage, a patient agent can strategically spend early or late. To make that decision, she compares the real allocation she can afford when spending her nominal balances early vs. late. The real value of the balances, x_t, in each t equals:

$$x_1 = \frac{P_0}{P_1(n)} \qquad \text{and} \qquad x_2 = \begin{cases} \frac{(1+i(n))P_0}{P_2(n)}, & P_2 < \infty \\ 0, & P_2 = \infty. \end{cases} \tag{1}$$

With the market-clearing conditions, we get the alternative formulae:

$$x_1(n) = \begin{cases} \frac{y(n)}{n}, & n > 0 \\ \infty, & n = 0 \end{cases} \qquad \text{and} \qquad x_2(n) = \begin{cases} \frac{1-y(n)}{1-n}R, & n < 1 \\ 0, & n = 1, y(n) = 1 \\ \infty, & n = 1, y(n) < 1. \end{cases} \tag{2}$$

That is, for a given n, the central bank sets the real value of the dollar in $t = 1, 2$ through its liquidation policy. Because all agents spending dollars in the same period have the same nominal expenses, the available goods are also allocated equally among all spending agents.[6] For now, the central bank is fully committed to carrying through with its policy $(M, y(\cdot), i(\cdot))$, regardless of the implications for (P_1, P_2).

Definition 2. *An* **equilibrium** *consists of a central bank policy* $(M, y(\cdot), i(\cdot))$*, aggregate spend-*

[6]These equations remain intuitive even if $y(n) = 0$ or $y(n) = 1$. Thus, we assume that they continue to hold despite one of the price levels being potentially ill-defined or infinite.

ing behavior $n \in [0,1]$, and price levels (P_1, P_2) such that:

(i) The spending decision of each agent is optimal given aggregate spending decisions n, the announced policy $(M, y(\cdot), i(\cdot))$, and the price levels (P_1, P_2).

(ii) Given aggregate spending n, the central bank provides $y(n)$ goods and sets the nominal interest rate $i(n)$; given $(n, y(n), M)$, the price level P_1 clears the market in $t = 1$; and given $(n, y(n), i(n), M)$, P_2 clears the market in $t = 2$.

This equilibrium concept allows the price levels (P_1, P_2) to flexibly adjust to the aggregate spending realization and the announced central bank policy:

$$P_1(n) = \frac{nP_0}{y(n)} \qquad \text{and} \qquad P_2(n) = \begin{cases} \frac{(1-n)(1+i(n))P_0}{R(1-y(n))}, & y(n) < 1 \\ \infty, & y(n) = 1,\, n < 1 \\ \in [0, \infty], & y(n) = 1,\, n = 1. \end{cases} \tag{3}$$

When $y(n) = 1$, $n < 1$, the supply of goods in $t = 2$ is zero while demand for goods exists. When $y(n) = 1$, $n = 1$, the supply and the demand for goods in $t = 2$ are zero. Define inflation as $\pi_1(n) \equiv P_1(n)/P_0$ and $\pi_2(n) \equiv P_2(n)/P_1(n)$ whenever possible.

The price levels $(P_1(n), P_2(n))$ are intertwined via the central bank liquidation policy $y(n)$ (a private bank, in contrast, takes P_1, P_2 as given). Marginally higher liquidation in $t = 1$ lowers $P_1(n)$ at the expense of lower output and a higher price level in $t = 2$, assuming that n does not move much.

3 Central bank runs and optimal allocations

Agents only care for consumption and not money. Given n, it is optimal for a patient agent to spend her balances in $t = 1$ if she believes that the central bank's policy implies a higher real value of the dollar balances in $t = 1$ than in $t = 2$, $x_1(n) \geq x_2(n)$, storing the purchased goods in private for consumption in $t = 2$. It is optimal to roll over if $x_1(n) \leq x_2(n)$. Since $x_1(n) > 0$ for all n, spending is always optimal for an impatient agent so that every equilibrium features

$n \geq \lambda$.[7]

Definition 3 (Central bank run). *A run on the central bank occurs if some patient agents spend in $t = 1$, i.e., $n > \lambda$.*

A nominal deposit does not rule out the possibility of a run on the central bank because a central bank run is not about the central bank running out of money; a central bank can produce as many additional dollars as it wants. Instead, a central bank run signals a lack of trust in the real value of money or the nominal deposit. In fact, a patient agent's optimal decision on whether to spend depends on the central bank's policy choices only through the real liquidation policy $y(\cdot)$ and not via the nominal policy tools M and $i(n)$; see below. In equilibrium, the aggregate spending behavior n has to be consistent with optimal individual choices. These considerations imply:

Lemma 3.1. *Given the central bank policy $(M, y(\cdot), i(\cdot))$,*
(i) "No run," $n = \lambda$, is an equilibrium if and only if $x_1(\lambda) \leq x_2(\lambda)$. "No run" is the unique equilibrium if and only if $x_1(n) < x_2(n)$ for all $n \in [\lambda, 1]$, implying $\pi_2(n) < 1 + i(n)$.
(ii) A central bank run, $n = 1$, is an equilibrium if and only if $x_1(1) \geq x_2(1)$.
(iii) A partial run $n \in (\lambda, 1)$ is an equilibrium iff patient agents are indifferent, $x_1(n) = x_2(n)$.

All the (non-trivial) proofs are in Online Appendix A. The socially optimal allocation is determined by equation (2) as $(x_1^*, x_2^*) = \left(\frac{y^*}{\lambda}, \frac{R(1-y^*)}{1-\lambda}\right)$ with the socially optimal liquidation level $y^*(\lambda) = x_1^*\lambda \in (\lambda, 1]$ and implied optimal price levels $P_1^*(\lambda) = \frac{\lambda P_0}{y^*}$ and $P_2^*(\lambda) = \frac{(1-\lambda)(1+i(\lambda))P_0}{(1-y^*)R}$ and inflations $\pi_1^*(\lambda) = \frac{P_1^*(\lambda)}{P_0} = \frac{\lambda}{y^*} = \frac{1}{x^*}$, and $\pi_2^*(\lambda) = \frac{P_2^*(\lambda)}{P_1^*(\lambda)}$.

Given the characterization in Lemma 3.1, "no run" $n = \lambda$ is the unique equilibrium of the coordination game if the central bank implements "spending late" as the dominant equilibrium strategy for patient agents. The central bank can deter runs by fine-tuning the supply of goods via its liquidation policy to the observed aggregate spending.

[7] We restrict attention to pure strategy Nash equilibria in the depositors' coordination game. If $x_1(n) = x_2(n)$ and $\lambda < n < 1$, $n - \lambda$ of patient agents spend their dollars in $t = 1$, and the remaining $1 - n$ do not.

Definition 4. *We call a central bank's liquidation policy $y(\cdot)$ run-deterring if it satisfies $y(n) < y^d(n)$ for all $n \in (\lambda, 1]$, with the* **run-deterrence boundary** $y^d(n) = \frac{nR}{1+n(R-1)}$, *for all $n \in (\lambda, 1]$.*

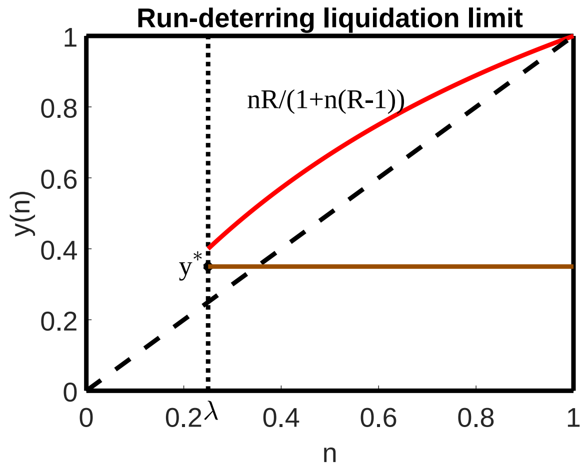


Figure 2: The red run-deterrence bound is an upper bound on liquidations as a function of n. For $n = \lambda$, the social optimum, y^*, is below the upper bound (here $\lambda = 0.25$).

The run-deterrence bound in Definition 4 captures the classic incentive-compatibility constraint in the bank run literature: by committing to liquidate sufficiently little in case of a run, the central bank threatens to make early spending sub-optimal ex-post for all patient types, i.e., $x_1(n) < x_2(n)$ for every $n \in (\lambda, 1]$. Via this threat, the central bank steers the incentives of the patient agents toward spending late at $t = 2$. Since the depositors' and the central bank's expectations are rational and the central bank policy is announced in $t = 0$ with full commitment, the depositors correctly anticipate the real value of their balances that would follow every n. Thus, the announcement of a run-deterring policy deters patient agents from spending ex-ante, and a central bank run never occurs, $n^* = \lambda$. That is, a run-deterring liquidation policy is an off-equilibrium threat that is never implemented in the unique equilibrium. Without this threat, central bank runs reoccur.

Implementing a run-deterring policy is possible because the contracts between the central bank and the agents are nominal, investment is real, and the central bank controls the price level. In contrast, in the DD case, the real claims of the agents pin down the liquidation policy one-for-one for all possible spending, and, in the case of high spending, rationing must occur. Similarly, in the case of nominal contracts between a private bank and depositors, the private bank has to take the price level as given, which then again pins down the liquidation policy. Here, instead, the central bank determines the liquidation of investments in the long-term technology independently of nominal withdrawals because it does not need to take the price level as given. The central bank can, however, only control one variable. By setting the liquidation, the central bank determines the supply of goods and, for a given n, the price levels and, with them, a spending-contingent real rate of return on the demand deposits. Thus, we get the first leg of our trilemma.

Given the optimal allocation $(x_1^*, x_2^*) = \left(\frac{y^*}{\lambda}, \frac{R(1-y^*)}{1-\lambda}\right)$, we have:

Corollary 5 (Trilemma part I: No price stability)**.** *Every central bank policy* $(M, y(\cdot), i(\cdot))$, $n \in [0, 1]$ *with* $y(\lambda) = y^*$ *and* $y(n) < y^d(n)$, *for all* $n \in (\lambda, 1]$, *deters central bank runs and implements the social optimum in dominant strategies. Such an "optimal run-deterring policy" requires the price bounds:*

$$P_1(n) > b(n) \equiv \frac{P_0}{R}(1 + n(R-1)), \quad P_2(n) < b(n)(1 + i(n)), \text{ for all } n \in (\lambda, 1], \tag{4}$$

implying inflation bounds $\pi_1(n) > \frac{b(n)}{P_0}$ *and* $\pi_2(n) < (1 + i(n))$ *for all* $n \in (\lambda, 1]$.

When a central bank follows a run-deterring policy, then the dominant strategy for all agents is to spend early if and only if the agent is impatient, regardless of how many other agents spend early. Thus, runs do not occur and the social optimum is achieved. Runs are prevented since the central bank commits to limiting supply, should a run occur. But this commitment also entails a commitment to sacrifice price stability in a run. By condition (4), the more agents spend, the higher P_1 and the higher $t = 1$-inflation π_1, as a high money supply chases a low supply of goods.[8]

The requirement of a lower bound on the interim price level and thus inflation π_1 for implementing the optimal allocation in dominant strategies is novel to the literature. ACG show

[8] It is impossible to avoid inflation by introducing a nominal interest rate between $t = 0$ and $t = 1$ unless the interest rate is spending-contingent and, thus, random in $t = 0$. See Section 5.

that the optimal allocation can be implemented through profit-maximizing firms and that equilibrium prices must follow deflation, $P_1 \geq P_2$, implying that prices can be stable between $t = 1$ and $t = 2$, $P_1 = P_2$ and that $t = 2$-inflation equals $\pi_2 = 1$. In their setting, the liquidation of illiquid assets, however, is not possible at a positive value. Here, though, we follow the DD framework where long assets can be liquidated at a cost, allowing for a spending-contingent transfer of resources from $t = 2$ to $t = 1$. In contrast to ACG, optimality in our setting requires the additional constraint that the price level in $t = 1$ is large enough to deter runs. If that is the case, prices can again satisfy $P_1 = P_2$ if the nominal interest rate is positive, $i > 0$. More generally, $t = 2$-deflation is not an equilibrium requirement: the optimum can be implemented under inflation $P_1 \leq P_2$ if $i(\cdot) > 0$, causing $[b(n), b(n)(1 + i(n))]$ to be non-empty. Section 6 shows that our results remain true in an economy closer to ACG, featuring firms that run the real economy and private banks that take deposits and make loans. There, in contrast to ACG, revenue-maximizing firms do not generically implement optimal allocations in response to market prices unless the central bank imposes penalty interest rates for non-repaid loans and for deviations of aggregate liquidation from the central bank's announced policy. Skeie (2008) also considers a nominal DD model, like ours, assuming that illiquid bank assets can be liquidated at a cost. He shows that flexible prices deter runs on nominal deposits altogether in the unique equilibrium. However, Skeie (2008) does not consider the implementation of optimal allocations.

Multiple monetary policies implement the optimal allocation since the pair $(M, i(\cdot))$ is not uniquely pinned down. While the pair $(M, i(\cdot))$ does not affect depositors' incentives, it has an impact on prices through equation (3) and market clearing $M = P_0$.

We learned in DD that offering the optimal amount of risk sharing via demand-deposit contracts requires private banks to be prone to runs. Thus, a bad bank run equilibrium also exists. Our result takes this dilemma to the next level. A central bank equipped with the power to set price levels and control the real supply of real goods can implement optimal risk sharing in dominant strategies such that a bank run never occurs but only at the expense of price stability. More pointedly, $y^* < y^d(\lambda)$ holds, and the run-deterrence boundary $y^d(n)$ is increasing in n; see

Figure 2.[9] As a special case, the constant liquidation policy $y(n) \equiv y^*$, for all $n \in [0, 1]$ implements optimal risk sharing in dominant strategies. Besides its simplicity, a constant liquidation policy is interesting since it is equivalent to the run-proof dividend policy in Jacklin (1987), which implements the social allocation with interim trade in equity shares. In other words, Jacklin (1987) features a special case of a run-deterring policy. The policy also implements the same allocation as the suspension-of-convertibility option that excludes bank runs in DD. There is a key difference, though, between suspension and our liquidation policy. Suspension of convertibility requires the bank to stop paying customers who arrive after a fraction λ of agents have withdrawn their deposits. In our environment, there is no restriction on agents to spend their dollars in $t = 1$. Instead, the restriction of the supply of goods offered for trade against those dollars and the resulting change in the price level generate the incentives for patient agents to wait. This reasoning also implies that neither nominal deposit insurance nor a rise in the nominal interest rate will deter a run on the central bank. Only a commitment to a run-deterring policy guarantees a positive real return on demand deposits between $t = 1$ and $t = 2$.

4 The classic policy goal: Price-level targeting

In practice, the policy selection $(M, y(\cdot), i(\cdot))$ of a central bank is heavily influenced by a price stability legal mandate, such as those ruling the Federal Reserve System or the ECB. We now analyze how this mandate interacts with the role of the central bank in implementing the socially optimal allocation we characterized above. To the best of our knowledge, such an analysis is novel to the literature.

Full price stability. We start by imposing a strong form of the price stability objective.

Definition 6. *We call a central bank policy:*

[9] Our result resembles Theorem 4 in Allen and Gale (1998) and has a similar intuition. In Allen and Gale (1998), a central bank lends to a representative bank an interest-free line of credit to dilute the claims of the early consumers so that they bear a share of the low returns to the risky asset. In their environment, private bank runs are required to achieve the optimal risk allocation.

(i) P_1***-stable at target level*** $\overline{P}$*, if* $P_1(n) \equiv \overline{P}$ *for all* $n \in [\lambda, 1]$*, implying a fixed* $t = 1$*-inflation target* $\pi_1(n) = \bar{P}/P_0$*.*

(ii) ***Price-stable at target level*** $\overline{P}$*, if both prices are stable at a target* $\bar{P}$*, achieving* $P_2(n) \equiv \overline{P} = P_1(n)$ *for all* $n \in [\lambda, 1)$*, implying inflation targets* $\pi_1(n) = \bar{P}/P_0$ *and* $\pi_2(n) = 1$*.*

The second price stability criterion is stronger, implying P_1-stability at $\overline{P}$. Our definition treats price stability as a commitment to the target $\overline{P}$ even for off-equilibrium realizations of n. We emphasize stability in $t = 1$ but not so much in $t = 2$, or inflation targeting in $t = 2$, because the former is harder to achieve. A stable price level P_1 in $t = 1$ requires a particular liquidation policy. In contrast, the central bank can use the nominal interest rate $i(n)$ to attain price stability in $t = 2$.[10] The same holds for inflation targeting between $t = 1$ and $t = 2$. For a *price-stable* policy, we exclude the possibility of a total run $n = 1$ by the absence of a demand for goods in $t = 2$; see Definition 3.

Proposition 7 (Policy under full price stability)**.** *A central bank policy is:*

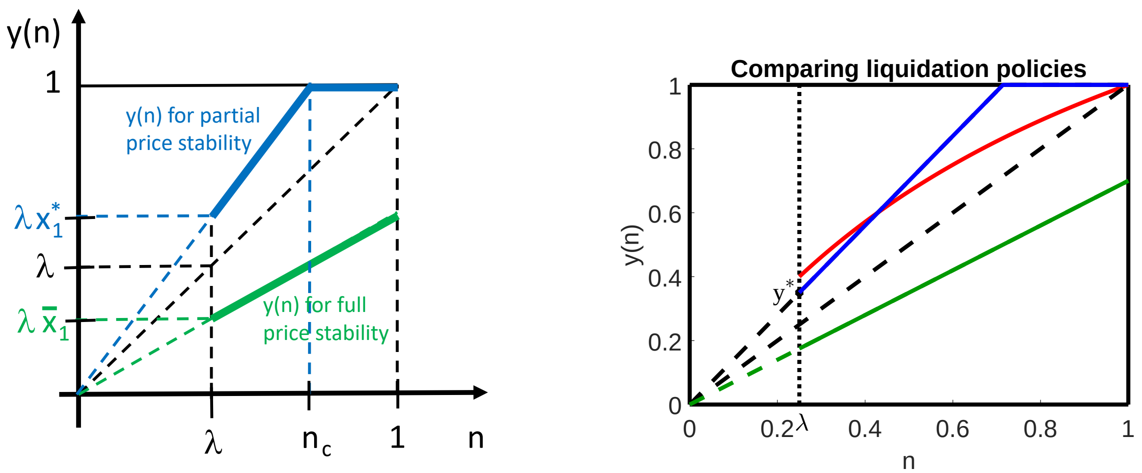
(i) P_1*-stable at level* $\overline{P}$*, if and only if its liquidation policy satisfies* $y(n) = \frac{P_0}{\overline{P}}n$*, for all* $n \in [0, 1]$*; implying a constant interim allocation* $x_1(n) \equiv \overline{x}_1 = \frac{P_0}{\overline{P}} \leq 1$*,* $t = 1$*-inflation* $\pi_1(n) = \bar{P}/P_0 \geq 1$*, and* $P_2(n) = \frac{(1-n)(1+i(n))P_0}{R(1-n\frac{P_0}{\overline{P}})}$*.*

(ii) price-stable at level $\overline{P}$*, iff its liquidation policy satisfies* $y(n) = \frac{P_0}{\overline{P}}n$*, for all* $n \in [0, 1]$*, and* $i(n) = \frac{\frac{\overline{P}}{P_0} - n}{1-n}R - 1$*, for* $n < 1$*. Then,* $x_1(n) = \frac{P_0}{\overline{P}}$*, and* $x_2(n) = (1 + i(n))\frac{P_0}{\overline{P}}$*.*

A price-stable liquidation policy requires investment liquidation in constant proportion to aggregate spending for all $n \in [0, 1]$; see the green line in Figure 3a. Hence, the interim real value of the balances x_1 is constant in n but undercuts 1: the central bank cannot liquidate more than the entire investment. By the resource constraint $y \in [0, 1]$, for a given P_0, only price levels $\overline{P} \geq P_0$ can be P_1-stable or price-stable. The slope of the liquidation policy is, thus, equal to or below 1. In other words, the rationing problem shows up indirectly through an upper bound on liquidation and a low provision of goods per realized spending level. The case $\overline{P} = P_0$ is the only P_1-stable price-level target at which the run equilibrium occurs since spending by

[10] Recall that the interest rate policy achieves stabilizing the price level in $t = 2$ but is ineffective in moving allocations or the price level in $t = 1$.

all agents implies a total investment liquidation $y(1) = 1 = y^d(1)$. If the central bank commits to a price-stable policy, the nominal interest rate increases in n and is non-negative $i(n) \geq 0$ for all $n \in [\lambda, 1]$.



(a) Partial vs. full price-stable liq. policies (b) Price-stable vs. run-deterring policy

Figure 3: Fully price-stable policies are run-deterring but do not reach the social optimum y^*. Partially price-stable policies are not run-deterring but can reach the social optimum.

This previous argument provides the second part of our trilemma:

Corollary 8 (Trilemma part II: No optimal risk sharing)**.** *If the central bank commits to a P_1-stable policy, then the optimal risk-sharing allocation (x_1^*, x_2^*) is never implemented. If $\overline{P} > P_0$, the no-run equilibrium is implemented in dominant strategies with $n^* = \lambda$, and there are no central-bank-run equilibria.*

In short, a strong price stability mandate deters runs but is incompatible with implementing the optimal allocation. No runs occur under a P_1-stable policy since the implied real allocation in $t = 1$ is below one, the asset's liquidation value. For the same reason, a fully price-stable policy can never implement $x_1^* > 1$. One can interpret full price stability as arising from a strong form of price stickiness at $\bar{P}$ that holds irrespective of the level of spending, forcing the central bank to supply goods at that price: when prices are "stuck at the wrong level," optimal allocations cannot be implemented, but runs may be deterred.

Partial price stability. While full price stability and the absence of central bank runs are desirable, the impossibility of implementing optimal risk-sharing allocations is not. Since optimal risk sharing at $x_1^* > 1$ triggers potential bank runs in models of the DD variety, the proposition above is not a surprise. Demanding price stability for all possible spending realizations of n is too stringent. For attaining the social optimum, we examine a lesser goal: a central bank may still wish to ensure price stability but deviate from that goal in times of crisis. We capture this idea with the following definition.

Definition 9. *A central bank policy is:*

(i) ***partially*** P_1***-stable at level*** $\overline{P}$*, if the policy attains the target* $P_1(n) = \overline{P}$ *for all* $n \in [\lambda, \overline{P}/P_0]$ *but may deviate from the target for* $n \in (\overline{P}/P_0, 1]$*. In the latter case, we require full liquidation,* $y(n) = 1$*.*

(ii) ***partially price-stable at level*** $\overline{P}$*, the policy attains the target* $P_1(n) = P_2(n) = \overline{P}$ *for all* $n \in [\lambda, \overline{P}/P_0]$ *but may deviate from* $\overline{P}$ *for* $n \in (\overline{P}/P_0, 1]$ *in which case* $y(n) = 1$*.*

The central bank tries to attain the target price level whenever possible, that is, for small runs, by appropriate liquidation. However, when n is too high and the central bank runs out of assets to liquidate, the price target is abandoned. See the blue line in Figure 3a for a graphical illustration. Obviously, P_1-stable central bank policies are also partially P_1-stable, and price-stable central bank policies are also partially price-stable.

Partial price stability restricts central bank policies as follows:

Proposition 10 (Policy under partial price stability)**.** *Suppose that* $P_0 > \overline{P} \geq \lambda P_0$*.*

(i) A central bank policy is partially P_1*-stable at level* $\overline{P}$*, if and only if its liquidation policy satisfies* $y(n) = \min\left\{\frac{P_0}{\overline{P}}n, 1\right\}$*. In that case, there exists a critical aggregate spending level* $n_c \equiv \frac{\overline{P}}{P_0} \in (0, 1)$ *such that:*

1. *For all* $n \leq n_c$*, the price level is stable at* $P_1(n) = \overline{P}$ *and the real allocations to the agents equal* $x_1(n) = \overline{x}_1 = \frac{P_0}{\overline{P}} > 1$*,* $x_2(n) = \frac{R(1-\overline{x}_1 n)}{(1-n)}$*, and* $P_2(n) = \frac{(1-n)(1+i(n))P_0}{R(1-n\frac{P_0}{\overline{P}})}$*.*

2. *For all* $n \in (n_c, 1]$*, the price level* $P_1(n)$ *is unstable, increasing proportionally with total spending:* $P_1(n) = P_0 n$*. The allocations equal* $x_1(n) = \frac{1}{n}$*,* $x_2(n) = 0$*, and* $P_2 = \infty$*.*

(ii) A central bank policy is partially price-stable at $\overline{P}$ if and only if $y(n) = \min\left\{\frac{P_0}{\overline{P}} n, 1\right\}$ and $i(n) = \frac{\frac{\overline{P}}{P_0} - n}{1-n} R - 1$ for all $n \leq n_c$, i.e., it declines monotonically in n. For $n > n_c$, the supply of goods is zero in $t = 2$. Thus, $P_2 = \infty$ and $i(n)$ are irrelevant. Given a partially price-stable policy, there exists a spending level $n_0 = \frac{Rn_c - 1}{R-1} \in [0, n_c)$, such that $i(n)$ turns negative for all $n \in (n_0, n_c)$. For $R \in (1, \frac{1}{n_c})$, $i(n)$ is negative for all $n \in [0, n_c)$.

To understand these restrictions, recall that only lower price targets $\bar{P} < P_0$ can attain optimality since the latter requires $1 < x_1^* = P_0/\overline{P}$. Further, price stabilization at target $\bar{P}$ for all $n \in [\lambda, \frac{\bar{P}}{P_0}]$ requires the central bank to liquidate less than the entire investment, $y(n) = \frac{P_0}{\overline{P}} n \in [0, 1]$, implying the feasibility constraint $\lambda \frac{P_0}{\overline{P}} \leq 1$, and thus a lower bound on all possible partially stable price levels, $\overline{P} \geq \lambda P_0$.

Proposition 10 reflects the central bank's capacity to keep x_1 and the price level stable for spending behaviors below the critical level n_c. A partially price-stable policy may arise from the central bank's commitment to offering the optimal allocation x_1^* to all n agents shopping in $t = 1$.[11] The liquidation policy is then $y(n) = \min\{1, nx_1^*\}$. Stabilizing the price level requires the liquidation of real investment proportionally to aggregate spending by a factor $P_0/\overline{P}$. At n_c, the central bank runs out of assets to liquidate, and price-level stabilization becomes impossible for all $n > n_c$. Rationing of goods occurs through a decline in the real allocation $x_1(n)$ and an increase in aggregate spending in the price level in $t = 1$.[12] Since the supply of goods in $t = 2$ is zero, the price level in $t = 2$ explodes.[13]

At the spending level n_0 the real allocations equalize $x_1(n_0) = x_2(n_0) = \bar{x}_1$, indicating that a partial run equilibrium exists; see the spending level at which the red and the blue line in Figure 3b cross. Notice that $x_2(n)$ declines in n for $n \in [0, n_c]$. Thus, if fewer than a measure n_0 of agents spend early, rolling over is optimal for patient agents. But for all $n > n_0$, the real interest rate on the deposits becomes negative, $x_2(n) < x_1(n)$, and spending early (run)

[11] A motive for that can be that the central bank does not know who among the n shoppers is impatient.

[12] This is in the spirit of DD but without the sequential service constraint. There, as the bank runs out of assets, some depositors try to withdraw but get zero since they are late in the queue. Here, all supplied goods are evenly divided among the shopping agents that try to spend, and x_1 declines.

[13] The price level in $t = 2$ can be artificially maintained by setting $i(n) = -1$, such that zero deposit balances meet zero goods in the market. But the results are the same.

becomes optimal for all patient agents. Hence, self-fulfilling runs reappear. As a corollary to Proposition 10, we obtain the third part of our trilemma:

Corollary 11 (Trilemma part III: Runs on the central bank (fragility))**.** *For every partially P_1-stable central bank policy with $P_0 > \overline{P} \geq \lambda P_0$, there is a multiplicity of equilibria:*
(i) There exists a good equilibrium in which a run is absent, $n^* = \lambda$, and both the social optimum (x_1^*, x_2^*) and the price-level target $P_1 = \overline{P}$ are attained.
(ii) There also exists a bad equilibrium in which a central bank run occurs, $n^* = 1$, the social optimum is not attained, and the price-level target is missed.

In short, under a partial price stability mandate, implementing the socially optimal allocation is possible but not certain because central bank runs may arise. Proposition 10 is in marked contrast to Proposition 7. When banking creates value, i.e., $x_1^* > 1$, the goal of price stability creates the possibility of runs on the central bank, the necessity for negative nominal interest rates, and the abolishment of price stability if a run occurs.[14]

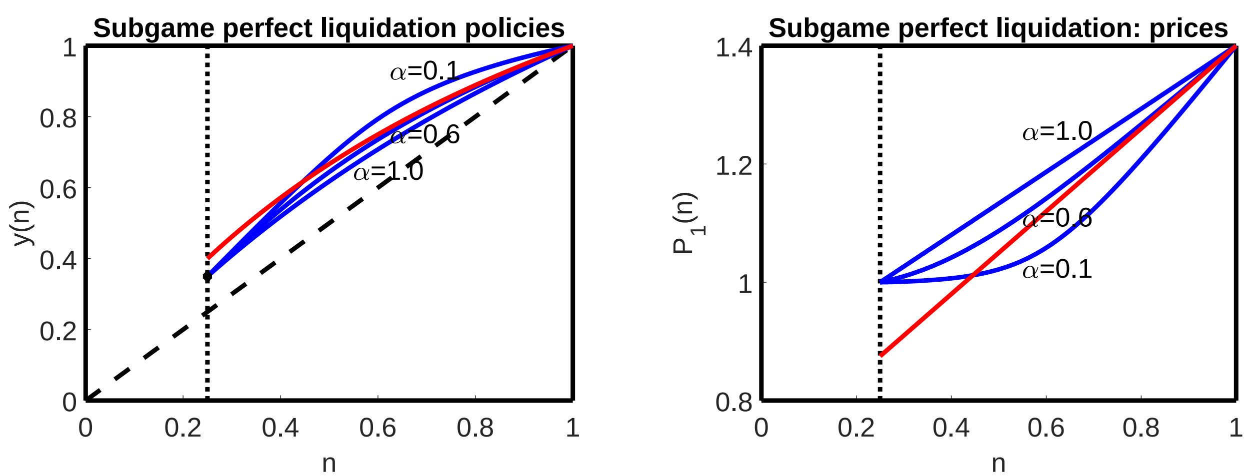
Time consistency. It is hard to believe that a central bank would commit to bad outcomes in terms of allocations or prices should central bank runs occur. Each time we have an off-equilibrium threat, we should worry about the possibility of time inconsistency. So far, we have assumed that the central bank fully commits such that the threat is credible. But what if the central bank is concerned with price stability and refuses to induce a high price level? We next analyze the subgame of the central bank liquidating y after observing n. Given n, allocative welfare resulting from liquidating y is:

$$W(y; n) = nu\left(\frac{y}{n}\right) + (1-n)u\left(\frac{R(1-y)}{1-n}\right). \tag{5}$$

[14] Ennis and Keister (2009) have already pointed out that too lenient but potentially ex-post efficient regulatory policies may give rise to bank runs ex-ante. Our analysis differs from theirs along two dimensions. First, they consider a real banking model (withdrawals cause liquidation one-for-one), while, in our nominal model, liquidation follows spending in proportion only if the central bank wants to stabilize prices. This proportion varies with the price-level target. Second, Ennis and Keister (2009) assume the bank follows a sequential service constraint, while we assume the central bank observes n and grants each spending agent the symmetric allocation $x_1(n) = y/n$. That is, our mechanism works via the goods market by constraining the total supply y, and not by constraining the spending (withdrawal) behavior of the agents.

where $x_1 = \frac{y}{n}$ respectively $x_2 = \frac{R(1-y)}{(1-n)}$ are the goods obtained by each spending agent in $t = 1$ respectively $t = 2$. Allocative welfare (5) should be viewed as part of a larger macroeconomic environment where price stability is desirable. Thus, following common practice, we expand this objective function with a concern for price stability, expressed by a quadratic loss of the resulting price $P_1(n) = nP_0/y$ deviating from a target $\overline{P}$, where $\alpha \in [0,1]$ is the weight of the allocative objective relative to the price stability objective $V(y; n, \overline{P}) = \alpha W(y; n) - (1-\alpha)\left(P_1(n) - \overline{P}\right)^2$.

The solution to the time-consistent equilibrium or subgame perfect equilibrium is computed by maximizing this central bank objective function via y given n and $\overline{P}$. When $\alpha = 1$, the first-order condition (FOC) is $u'\left(\frac{y}{n}\right) = Ru'\left(\frac{R(1-y)}{1-n}\right)$. If $u(c)$ is CRRA, $u(c) = c^{1-\eta}/(1-\eta)$, the FOC becomes $y(n) = \frac{n}{n+R^{(1/\eta)-1}(1-n)}$, which is neither constant nor proportional to n. The implied price level is $P_1(n) = \frac{Mn}{y(n)} = (n + R^{(1/\eta)-1}(1-n))$, and thus affine-linear in n. The subgame perfect solution is run-deterring for every $n < 1$, since patient agents always receive more if they wait until $t = 2$ (at $n = 1$, full liquidation $y(n) = 1$ takes place, and $x_2 = 0 < x_1$). This follows directly from the FOC and the strict concavity of $u(\cdot)$, since $R > 1$ and x_1 and x_2 are the arguments of the derivative $u'(\cdot)$.



Liquidation Policies

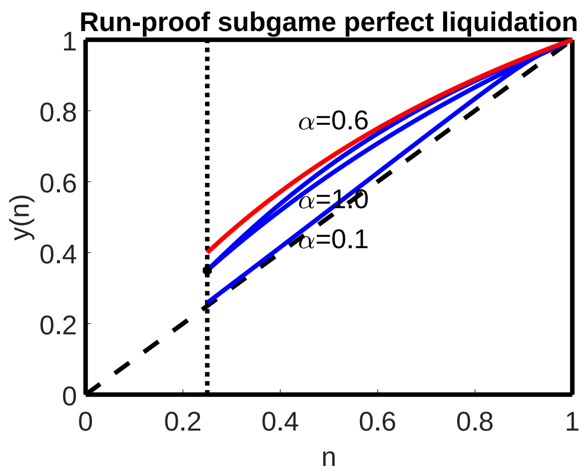
Prices

Figure 4: Subgame perfect liquidation policies and their pricing implication.

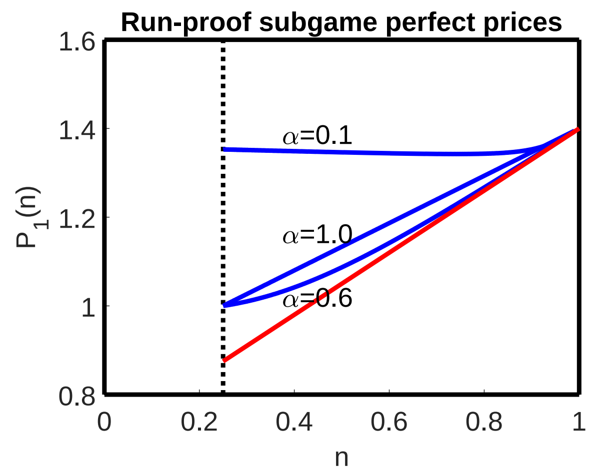
The situation changes when a concern for price stability is included, i.e., when $\alpha < 1$. In this case, the solution can only be obtained numerically. We do so in Figure 4 for the case with $R = 2$, $\lambda = 0.25$, and $\eta = 3.25$ for the utility function $u(c) = c^{1-\eta}/(1-\eta)$, so that $x_1^* = 1.4$.

The quantity of money $M = P_0 = 1.4$ implies $P_1^* = 1$ if $n = \lambda$.

The left panel in Figure 4 shows the subgame perfect liquidation policies $y_\alpha(n)$ for the three weights $\alpha = \{0.1, 0.6, 1\}$ and $\overline{P} = P_1^*$. They are compared to the run-deterrence boundary $y^d(n)$, plotted in red. All subgame perfect liquidation policies go through the allocative optimal solution y^* at $n = \lambda$ since the price level coincides with the target $\overline{P} = P_1^*$ at that point.[15] For $\alpha = 1$, the subgame perfect liquidation policy is below the red line and run-proof. However, as α decreases and the weight on the price stability objective increases, the liquidation policy eventually cuts through and exceeds the run-deterrence boundary at values below $n = 1$. This is more clearly visible in the right panel for $t = 1$ prices implied by these liquidation policies. For $\alpha = 0.1$, the central bank puts a large weight on stabilizing prices, which drop below the price boundary (the red line) necessary to deter runs. While $\alpha = 0.6$ still yields a run-proof liquidation strategy, this is no longer true for $\alpha = 0.1$.



Liquidation Policies



Prices $P_1(n)$

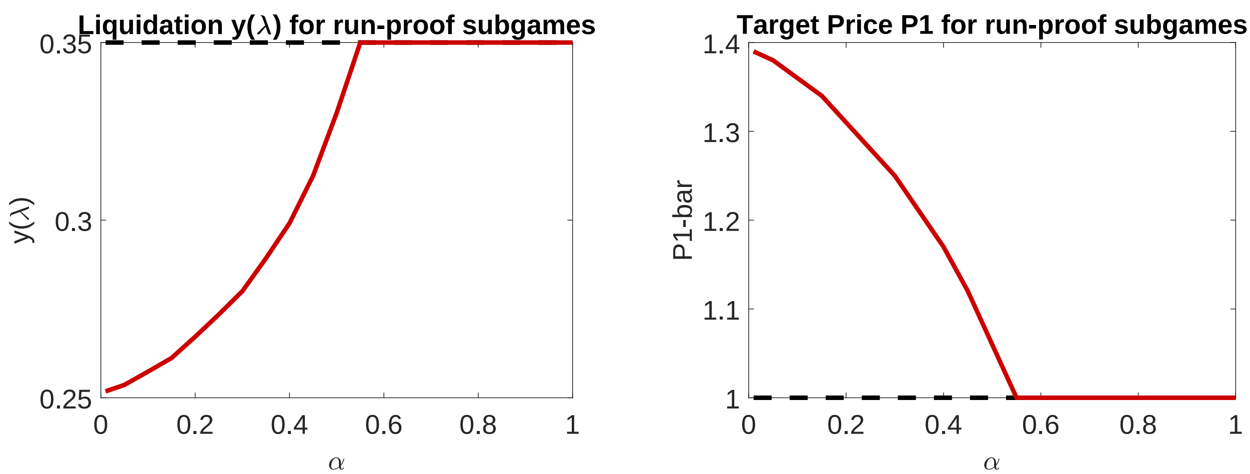
Figure 5: Subgame perfect liquidation policies and their pricing implication when $\overline{P}$ is set minimally so that the liquidation is run-proof for $n < 1$.

A central bank may thus be concerned in $t = 0$ about setting a price target $\overline{P}$ for $t = 1$ that might escalate to runs. The solution is to set $\overline{P}$ sufficiently high in $t = 0$ to deter runs.[16]

[15]This is akin to "divine coincidence" of New Keynesian models: a zero output gap coincides with achieving the inflation target.

[16]This may seem inconsistent with a central bank concerned about price stability. However, this price target is already known in $t = 0$. Thus, if the price stability objective arises from costs for adjusting prices between the unmodelled market in $t = 0$ and $t = 1$, prices in $t = 0$ need to be set high enough. Alternatively, the central

Figure 5 plots, for each α, the minimal $\overline{P}(\alpha) \geq P_1^*$ compatible with a subgame perfect run-proof liquidation policy. For $\alpha = 1$ and $\alpha = 0.6$, $\overline{P} = P_1^*$ delivers the desired result. However, for $\alpha = 0.1$, the price target must be raised to ensure that the run-deterrence boundary is no longer crossed. By design, the equilibrium prices now lie above the run-deterring price bound, plotted as a red line in the right panel. However, the liquidation policies $y(n;\alpha)$ no longer achieve the efficient outcome y^* for $n = \lambda$ when $\alpha = 0.1$. Also, the liquidations $y_\alpha(n)$ and prices $P_{1;\alpha}(n)$ are no longer monotone functions of α for intermediate values of n.



Liquidation Policies at $n = \lambda$ Price target $\overline{P}(\alpha)$

Figure 6: Adjustment of the price target $\overline{P}$ as a function of α required to achieve a run-deterring liquidation policy in the subgame perfect equilibrium, provided that $n < 1$. The dashed black lines show the ex-ante efficient liquidation level $y^* = \lambda x_1^*$ and P_1^*.

Figure 6 compares these run-proof liquidation policies at $n = \lambda$ and the minimal price targets $\overline{P}(\alpha)$ as a function of the weight α on the allocative objective (5). The liquidation increases, and the price target declines until they eventually hit the levels y^* and P^* compatible with the allocative efficient solution.

The limit $\alpha \to 0$ is particularly clean. In that case, the liquidation policies become linear until they hit full liquidation. This corresponds to the partially P_1-stable central bank policies analyzed above. Furthermore, the precise functional form of incorporating the price stability objective is unimportant as long as the same limit is reached.

bank can adjust the money supply to make $\overline{P}$ compatible with some given price level: it is only P in relationship to M that matters.

5 CBDCs and resolving the trilemma

A natural interpretation of the nominal deposits in our model is as a CBDC. Our consolidated central bank formulation is particularly appropriate when CBDCs are introduced widely. Fernández-Villaverde et al. (2021) show that a CBDC offered by the central bank may be such an attractive alternative to private banks that the central bank becomes a deposit monopolist and the only financial intermediator.[17] Thinking about nominal deposits as CBDCs opens several important discussions. First, the trilemma can be resolved when the central bank controls the agents' money balances, such as in the case of a CBDC.

State-contingent money balance adjustment. As in our baseline model, suppose the central bank learns the fraction n of agents planning to go shopping at $t = 1$ and then sets $y(n)$ and $i(n)$. Additionally, the central bank now seeks to control the resulting $P_1(n)$ by altering the total money supply away from $M = P_0$, to some $M_1(n)$. For simplicity, assume the desired liquidation policy is not state-contingent, $y(n) \equiv y^*$ (but can be generalized to other liquidation policies), which is a run-deterring policy. To maintain price stability at $\overline{P}$ even off-equilibrium, $n > \lambda$, market clearing demands $nM_1(n) = \overline{P}y^*$ for all $n \in [0, 1]$. That is, the total money balances spent in $t = 1$ are required to stay constant in n, implying $n\, M_1(n) \equiv \lambda\, M_1(\lambda)$, for all $n \in [\lambda, 1]$. To achieve that, spending per agent and total money quantity $M_1(n)$ must change with n. That is, the central bank must commit to reducing the quantity of money in circulation in response to a random positive demand shock encapsulated in n: the more people go shopping, the lower the individual money balances required to stabilize the price. With policy $nM(n) = \overline{P}y^*$, $y(n) \equiv y^*$ and $i(n) \equiv i^*$ chosen such that $P_2 = \overline{P}$, the central bank can now achieve full price stability, efficiency, and financial stability. The trilemma appears to be resolved.[18]

[17] Many CBDC proposals limit the amount of a CBDC agents can hold. We are skeptical that these limits will be adhered to when financial crises heighten agents' desire to hold liquid assets with government guarantees. Our environment can be read as what will happen when these limits are ultimately lifted.

[18] This state-contingent mechanism cannot be applied to cash since personal cash holdings are out of the central bank's control. A physical dollar today is still a physical dollar tomorrow (unless some cumbersome stamping requirement is introduced, as in some monetary reforms in history).

This policy can be implemented in several ways. First, via state-contingent money balances: the balance of a CBDC deposit is adjusted after the central bank observes n but before payments for goods are processed. This adjustment is technically trivial with a CBDC (e.g., instantaneous token-burning or state-contingent nominal taxes on CBDC holdings). Second, via a state-contingent nominal return paid on CBDC accounts between $t = 0$ and $t = 1$. Only in $t = 1$, and depending on n, agents learn the nominal value of their savings. This transforms the deposit contract into an equity contract.[19] Third, we can think about a state-contingent M_1 as a classic monetary injection in the form of state-contingent lump-sum payments ("helicopter drops") $M_1(n) - \overline{M}$ (or taxes, if negative), compared to a baseline $\overline{M}$. If one wishes to insist that $M_1(n) - \overline{M} \geq 0$, i.e., only allowing helicopter drops, then the central bank would choose $\overline{M} = P_0 \leq M(1)$ as payment for goods in $t = 0$ and distribute additional helicopter money in the "normal" case $n = \lambda$ in $t = 1$.

With a CBDC, there is yet another drastic policy tool at the central bank's disposal: a "digital corralito." The central bank can disallow agents to spend more than a certain amount of their account balance, ensuring that not more than the initially intended amount of money $\lambda M(\lambda)$ is spent in $t = 1$. This policy differs from the standard suspension of convertibility, as the central bank can still determine the liquidation amount of long-term investments as a separate tool. In terms of implementation, the central bank would observe all spending requests at once. If the total spending requests exceeded the overall threshold, it would restrict spending through a pro-rata spending limit or a first-come-first-served policy. Again, this unconventional policy might create havoc. The experience in Argentina at the end of 2001 provides ample proof.

State-contingent money balances cannot replace the central bank's liquidation policy as the active policy variable. A state-contingent money balance does not impact the agent's spending behavior and thus cannot target the deterrence of runs: the individual agents exclusively care for their allocation, $x_1 = y/n$ vs. $x_2 = R(1 - y)/(1 - n)$. These allocations are independent of

[19] In the DD literature, the depositors who roll over their deposits become equity investors in the bank. But here, even the depositors who spend (withdraw) in $t = 1$ face a random state-contingent balance.

nominal quantities $(M, P_1, P_2, i(n))$ and money is neutral. Given a realization of an individual real allocation y/n, the identity $\frac{y}{n} = \frac{M_1(n)}{P_1}$ pins down a relationship between the money supply and the price level.[20] Only by adjusting y per its liquidation policy can the central bank impact agents' behavior n.

In summary, state-contingent money balances are an uncommon monetary policy tool. In the real world, central banks tend to accommodate an increase in demand with a rise rather than a decline in the money supply. A central bank that reacts to an increase in demand by making money scarce may undermine trust in the monetary system. Hence, this particular escape route from the trilemma must be treated cautiously. Finally, recall that changes in the nominal interest rate do not fix the trilemma. Online Appendix C demonstrates that open market operations cannot fix the trilemma either.

6 Decentralization with firms and private banks

Our framework above is an abstraction, describing a scenario where a central bank issues a CBDC that has crowded out deposits at private banks. We show next how the central bank can implement its desired liquidation policy in a decentralized economy with private banks and firms and where households hold nominal deposits at the private banks. Our decentralized setting builds on the framework in ACG, extended for a strategic central bank, costly asset liquidation, and strategic firms. Unlike in ACG, the central bank supplies money strategically, steering the liquidation of firms jointly with the announced liquidation policy and interest rates. Appendix B contains all the relevant proofs and additional material.

At $t = 0$, a continuum of competitive firms $[0, 1]$ have access to the long-run production technology but have no resources. There is a competitive sector of banks and a continuum of households $[0, 1]$. Households initially own one unit of the good, but have no money. Both households and firms require banking services and pick the banks that offer the best contracts.

[20] The central bank can implement all pairs (M_1, P_1) that satisfy this relationship. And as soon as P_1 is pinned down, contingent on the realization $\frac{y}{n}$, the money supply that solves $\frac{y}{n} = \frac{M_1(n)}{P_1}$ is unique. But in this case, the classic dichotomy holds: the choice of (M_1, P_1) cannot alter the incentives to run.

Without loss of generality, we assume that all banks offer the same conditions and make zero profits, and that each firm is associated with a "house bank" that passes funds through between the firm and the central bank. We assume households treat banks symmetrically, implying equally sized banks and symmetric deposit withdrawals across banks. Within $t = 0$, and across periods $t = 1$ and $t = 2$, the money supply created by the central bank circulates from banks to households and firms, and back to banks and the central bank. As before, choices by all agents are observable but the agents' types are private information.

Model and timing. At $t = 0$, the central bank sets and publicly announces its policy characterized by a positive money supply $M_0, M_1 = M_0, M_2(n, \hat{y}) = M_1(1+i(n, \hat{y}))$, a liquidation policy $y(n) \in [0, 1]$, and interest rate functions $i(n, \hat{y}), r_1^*(n, \widehat{y}), \tilde{r}_1(n, \widehat{y})$ for every $n \in [\lambda, 1]$ and every aggregate liquidation $\widehat{y} = \widehat{y}(n) = \int_{[0,1]} y_j \, dj$ across all firms. The aggregate liquidation $\widehat{y}(n)$ may potentially deviate from $y(n)$.

These policy choices imply the following central bank actions. At the beginning of $t = 0$, the central bank provides banks with a zero-interest intraperiod loan of M_0 per household served by that bank.[21] At the beginning of $t = 1$, and given the endogenous withdrawal demands $n \in [0, 1]$, the central bank provides banks with liquidity nM_1 per banking household in the form of an interperiod loan. At the end of $t = 1$, the central bank demands a payment of $\widehat{P}_1 y(n)$ per banking household, where $\widehat{P}_1$ is the market-clearing price.[22] $\widehat{P}_1 y(n)$ may differ from nM_1. If the bank cannot pay in full, the central bank provides a loan for the difference to be repaid in $t = 2$ at the interest rate $\tilde{r}(n, \widehat{y})$, unless it sets $\tilde{r}(n, \widehat{y}) = \infty$. No loans are provided in that case. If the bank repays more than required, excess funds can be held as reserves at the central bank, paying the interest rate $r^*(n, \widehat{y})$ in $t = 2$. At the beginning of $t = 2$, the central bank provides banks with liquidity $(1 - n)M_2(n, \widehat{y})$ per banking household in the form of an intraperiod loan. Let $\widehat{P}_2$ be the market-clearing price in $t = 2$. At the end of $t = 2$ and in addition to the repayment of the interperiod $t = 1$ loan with interest (should there be

[21] At the beginning of $t = 1$, the central bank first announces its policy. Then, banks announce their contracts. Households and firms pick banks. Finally, the central bank provides liquidity to the banks.

[22] If $\widehat{y} > y$, then $nM_1 > \widehat{P}_1 y$, i.e., the central bank may leave liquidity in the banking system between $t = 1$ and $t = 2$. If $\widehat{y} < y$, the central bank will demand back more liquidity than the average bank has available.

one) or the repayment of reserves held (if any), including interest, the central bank requires the repayment $\widehat{P}_2(1-y(n))R$ from the banks, if $n<1$.[23]

Firms. At $t=0$, firms require a loan from banks to purchase the goods endowment from the households and borrow $L_0=M_0$ from their house banks to do so. The following contract obligations follow from the monetary policy choices and bank competition. Firms agree to repay the amounts $\widehat{P}_1 y(n)$ in $t=1$ and $\widehat{P}_2(1-y(n))R$ in $t=2$, where $\widehat{P_1}$ and $\widehat{P}_2$ result per market clearing from the actual aggregate liquidation $\widehat{y}$, whereas $y(n)$ is the desired liquidation policy.[24] If the firm falls short of its payment scheduled in $t=1$, it agrees to repay the outstanding difference with the penalty interest rate $\tilde{r}(n,\widehat{y})$ in $t=2$. If the firm repays more than the required amount in $t=1$, it can invest the excess funds between $t=1$ and $t=2$ at its bank at the reserve rate $r^*(n,\widehat{y})$. In addition to the repayment of the interperiod $t=1$ loan with interest (should there be one), the central bank requires the repayment $\widehat{P}_2(1-y(n))R$ by the end of $t=2$ from the banks.

Households. In $t=0$, the firms use the loaned funds to purchase the goods from the households at the market-clearing price $P_0=\int_{[0,1]}P_0 di=M_0$, investing the goods in the production technology. In turn, the households invest the proceeds P_0 from the goods sales in a nominal demand-deposit contract with banks, allowing them to withdraw D_1 in $t=1$ or $D_2(n,\hat{y})$ in $t=2$. The banks use the deposited funds P_0 to repay their intraperiod loan to the central bank by the end of $t=0$. At the beginning of $t=1$, an endogenous share $n\in[0,1]$ of households seeks to withdraw their nominal deposit D_1 to purchase goods. To serve these withdrawals, banks use the liquidity provided by the central bank. Given the central bank policy, the liquidity-constrained bank must, thus, set the deposit coupons in $t=0$ equal to the central bank's announced money supply rule $D_1=M_1$ and $D_2(n,\widehat{y})=M_2(n,\widehat{y})$. Since

[23]No payment is due if $n=1$, since then $\widehat{P}_2=0$ or $\widehat{y}=1$. Suppose that $\widehat{y}>y(n)$. Then, $nM_1>\widehat{P}_1 y(n)$, i.e., the central bank provides banks on average with more funds at the beginning of $t=1$ than it asks back at the end of $t=1$. Likewise, the central bank is asking back on average more at the end of $t=2$ than the liquidity provided at the beginning of $t=2$, $(1-n)M_2<\widehat{P}_2 R(1-y(n))$. One can interpret this as an intertemporal loan of the amount $\widehat{y}-y$ at the rate $1+\check{r}_1=\widehat{P}_2 R/\widehat{P}_1$ between $t=1$ and $t=2$, provided the firm-bank pair liquidates exactly the amount asked for, $y_j=y$, with the rates becoming less favorable upon deviating.

[24]Mixing the desired liquidation policy y with the price level $\widehat{P}_1$ resulting from a potential deviation $\widehat{y}$ deters aggregate deviations; see below.

$M_2(n, \widehat{y}) = M_1(1 + i(n, \widehat{y}))$, the nominal interest rate on deposits between $t = 1$ and $t = 2$ equals the nominal interest rate chosen by the central bank, $D_2(n, \widehat{y})/D_1 = 1 + i(n, \widehat{y})$.[25]

The firms operate the production technology and, akin to ACG, take goods market prices in $t = 1$ and $t = 2$ and the interest rates on loans as given when maximizing profits via liquidation decisions $y_j \in [0, 1]$ of the technology, offering those goods for sale. Goods markets are centralized and market clearing implies that $\widehat{P}_1$ adjusts to $\widehat{y}(n)$, satisfying $\widehat{P}_1(n)\widehat{y}(n) = nM_1$. One can interpret n as the average velocity of money, in line with quantity theory.

In $t = 1$, firm j chooses to liquidate the share $y_j(n) \in [0, 1]$ of the long asset at value 1, sells the goods $y_j(n)$ at the market-clearing price $\widehat{P}_1(n)$, and uses the proceeds to pay part of its $t = 1$ contractual obligations $\widehat{P}_1 y(n)$ to its bank.[26] The firm would never liquidate and store the goods until $t = 2$ because staying invested in the technology yields a higher real return than storage $R > 1$.[27] The banks repay as much as possible of the $t = 1$ intraperiod central bank loan (we analyze the incentives to do so below). If all firms follow the central bank's announced policy $y_j(n) \equiv \widehat{y}(n) = y(n)$, all firms exactly pay their contractual obligations, and all banks exit the period with zero balances vis-a-vis the central bank.[28] If a firm liquidates less than the announced policy $y_j(n) < y(n)$, it only partially pays its contractual obligations, $\widehat{P}_1(n)y_j(n) < \widehat{P}_1(n)y(n)$, irrespective of what other firms do. Thus, the firm's bank cannot fully meet the payment to the central bank and requires an additional interperiod loan from the central bank at the penalty rate $\tilde{r}(n, \widehat{y})$.[29] The bank forwards that penalty rate to the firm. If the firm liquidates more than the announced policy $y_j(n) > y(n)$, it can pay more than its contractual obligations, $\widehat{P}_1(n)y_j(n) > \widehat{P}_1(n)y(n)$. Via the firm, the bank has excess liquidity,

[25]The central bank must dictate the deposit contract to the bank via the money supply and not the other way around, implying that the money supply jointly with a liquidation policy $y(n)$ yields particular price levels P_1, P_2 via the market-clearing condition. Introducing a nominal interest rate on deposits between $t = 0$ and $t = 1$ does not change the result.

[26]Note the mismatch between the outstanding loan amount $nM_1 = \widehat{P}_1(n)\widehat{y}(n) = P_1 y(n)$ the firm-bank pair owes the central bank and the required repayment $\widehat{P}_1 y_j$.

[27]This is an important distinction between our paper and ACG's, where liquidation of the long asset is not possible. Instead, firms can store proceeds from a short asset maturing in $t = 1$ until $t = 2$. In our setting, the latter is also possible but dominated by not liquidating the long asset.

[28]We analyze the scenario $\widehat{y} \neq y$ in the proof of Proposition 6.1 in Appendix B.

[29]We preclude interbank loans. Since interbank loans often need to be collateralized in the real world, the absence of interbank loans amounts to assuming that firm loans are not easily collateralizable.

which it deposits at the central bank at an interest rate $r^*(n, \widehat{y})$, and that interest accrues to the firm due to bank competition.

Suppose that $n \in [\lambda, 1)$, $0 < \widehat{y} < 1$, $\widehat{P}_1, \widehat{P}_2 \in (0, \infty)$. In the proof for Proposition 6.1 below, we show that the central bank can set interest rate functions $i(n, \widehat{y}), r^*(n, \widehat{y}), \tilde{r}(n, \widehat{y})$ so that

$$1 < 1 + r^*(n, \widehat{y}) < \frac{\widehat{P}_2 R}{\widehat{P}_1} < 1 + \tilde{r}(n, \widehat{y}) < \infty \tag{6}$$

Note how $\frac{\widehat{P}_2(n)R}{\widehat{P}_1(n)}$ is the endogenous nominal return on investment of the production technology.[30] Unlike in ACG, the central bank cannot generically set $r^* = \tilde{r} = 0$ for implementing its desired liquidation policy or the optimal allocation because these rates are required to incentivize the firms.

At $t = 2$, the remaining households liquidate their deposits, financed by a central bank loan of the amount $(1 - n)M_2(n, \hat{y})$ to banks. The assets of firms mature, yielding a goods quantity $R(1 - y_j(n))$ for firm j. Firms sell the quantities in the centralized goods market at the market-clearing price $\widehat{P}_2$. With their revenues $\widehat{P}_2 R(1 - y_j)$, they pay the remaining contractual obligations. Market clearing implies $\widehat{P}_2(n)R(1 - \hat{y}(n)) = (1 - n)M_2$. Banks then repay the intraperiod central bank loan. Because of competition, banks and firms make zero profit. We rule out the possibility that the firm-bank pair can invest in other banks' deposits at a nominal interest rate i, but it can either store via central bank reserves at interest rate r^*, explained above, or via vault cash. In the special cases where markets are absent in $t = 1$ via $\widehat{y} = 0$ or $t = 2$ through $\widehat{y} = 1$ or $n = 1$, we set the required loan repayment to the central bank to zero, since neither $\widehat{P}_1$ nor $\widehat{P}_2$ is defined.

Proposition 6.1 (Decentralized implementation)**.** *Fix $M_0 = M_1 > 0$. For every central bank liquidation policy with $0 < y(n) \leq 1$ for all $n \in [\lambda, 1]$, there exist state-contingent interest rate functions $r^*(n, \widehat{y}) < \tilde{r}(n, \widehat{y}) \leq \infty$ on reserves and loans, and a nominal interest rate on deposits $i(n, \widehat{y})$ pinning down $M_2(n, \widehat{y})$ such that given these policy choices it holds that $y_j(n) = y(n)$ for all $n \in [0, 1]$ is the unique Nash equilibrium of the firm's liquidation game, as long as cash is*

[30]If $r_1 * > 0$, keeping excess reserves at the central bank dominates cash storage if cash was also available.

absent. This statement remains true if cash available as an alternative to reserves, if $y(1) = 1$.

We disregard the case $y = 0$ since it is inefficient per $\lambda > 0$. If nominal interest rates are fixed at zero in $t = 0$ for exogenous reasons, Proposition 12 shows that the result of Proposition 6.1 holds in the absence of cash, and holds with cash provided that $y(n) \geq n$.

7 Nominal deposits vs. CBDC vs. cash

We conclude the paper by comparing nominal deposits with CBDCs and cash, using the extended framework of Section 6. The presence of nominal deposits slightly restricts the implementable liquidation policies compared to the CBDC-only case.

Proposition 7.1. *The optimal allocation (x_1^*, x_2^*) can be implemented as the unique Nash equilibrium in the decentralized economy via the optimal run-deterring central bank liquidation policy $y(n) = y^*$ for all $n \in [\lambda, 1]$ as long as cash is absent. With cash, the households' coordination game has two pure equilibria. In the "no run" equilibrium, only impatient households spend early, in which case there exist central bank interest rates on firm loans $r_1^*(\lambda, \hat{y}) < \tilde{r}_1(\lambda, \hat{y})$ such that firms liquidate optimal quantities y^*. In the bad equilibrium, all households spend early, $n = 1$, in which case firms deviate, liquidating everything $\widehat{y} = 1$, so the optimal allocation is not implemented.*

By Corollaries 5 and 11 the trilemma reoccurs. If cash is absent, the optimal run-deterring liquidation policy $y(n) = y^*$ for all $n \in [\lambda, 1]$ implies off-equilibrium price threats; see equation (4). If cash exists, partial price-stability holds at level P_1^*, but runs can happen. Only if runs are absent, is the optimal allocation implemented and the price target P_1^* reached. ACG's analysis differs from ours since we allow for asymmetric firm behavior, analyzing possibly profitable, strategic liquidation deviations that may result in shifts in the price levels. Ultimately, we find the Nash equilibria of the firm's liquidation coordination game. Without cash, the equilibrium is unique. Firms do not deviate from the announced policy not to liquidate everything, $y^* < 1$, even though the run $n = 1$ causes zero demand in $t = 2$. The uniqueness of a Nash equilibrium may require negative interest rates on reserves, which firms/banks can circumvent if cash coexists as a store of value. With cash, the Nash equilibrium is not unique, and the run equilibrium reemerges. That is, the extent to which the central bank can interfere with the economy's amount of maturity transformation is impaired when households invest in nominal deposits and if cash exists compared to the setting with a CBDC. We also derive an additional result:

Proposition 7.2. *The central bank can implement the fully price-stable policy $\bar{P} = P_0 = P_1(n) = P_2(n)$ as the unique Nash equilibrium of the decentralized economy via the liquidation policy $y(n) = n$ for all $n \in [\lambda, 1]$, even when cash coexists with central bank reserves.*

Recall that the real allocation to households satisfied $x_1(n) = y(n)/n = 1 < x_1^*$ for all $n \in [\lambda, 1]$. Thus, the optimal allocation is not implemented following policy $y(n) = n$, and the trilemma from Corollary 8 reappers.

Cash vs. CBDC. In the CBDC setting of the benchmark model, as long as cash and CBDCs are equivalent in terms of spending, there is no difference in terms of attaining optimal allocations or deterring runs because our mechanism works via the goods market. However, cash can usually be "hidden" by the agents from any policy that augments or reduces the balance of the deposit or the CBDC. Therefore, the central bank can neither pay an interest rate $i(n)$ on cash holdings nor could the central bank adjust the individual cash balances or suspend spending in a spending-contingent way. Thus, the central bank can neither attain a fully price-stable policy that requires fine-tuning $i(n)$ (see Proposition 7); (ii) nor can it "fix" the trilemma when cash is the only medium of exchange.

Cash and nominal deposits. In the decentralized economy, the presence of cash next to nominal deposits makes a large difference. If cash is not present, the central bank can force the firm-bank pair to pay negative interest rates on central bank reserves if the firm's liquidation is more than the desired policy. This allows the implementation of a larger range of liquidation policies as the unique Nash liquidation equilibrium of the firms in contrast to the case where cash is absent (see Proposition 6.1). Cash constrains the central bank's (indirect) involvement in maturity transformation even more in the decentralized intermediated setting than in our benchmark setting with CBDCs.

Decentralized CBDC. Another possibility is a decentralized economy with private banks, firms, and a decentralized CBDC. Since, in this case, the central bank commits to redirect CBDC funds to banks, this system is equivalent to the decentralized system with deposits at private banks; see Online Appendix B.2.

To summarize, inherent trade-offs between price stability, financial stability, and social optima exist in all settings: with a CBDC or nominal private bank deposits and with and without cash.

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Online Appendix

A Proofs

Proof. [Proposition 7]

Proof (i): Via the market-clearing condition (3), setting $P_1(n) \equiv \overline{P}$ for all n requires $y(n) = \frac{P_0}{\overline{P}} n$, for all $n \in [0,1]$. Thus, via equation (2), $x_1(n) = y(n)/n = \frac{P_0}{\overline{P}}$ is constant for all n. Last, since the central bank cannot liquidate more than the entire investment in the real technology, $y(n) \in [0,1]$ for all n, together with x_1 constant requires, in particular, $\frac{P_0}{\overline{P}} = x_1 = x_1(1) = y(1) \leq 1$. Thus, $P_0 \leq \overline{P}$.

Proof (ii): When additionally requiring price stability, $P_1(n) = P_2(n) \equiv \overline{P}$, the market-clearing condition (3) together with requirement $y(n) = \frac{P_0}{\overline{P}} n$, for all $n \in [0,1]$ yields:

$$i(n) = \frac{\frac{\overline{P}}{P_0} - n}{1-n} R - 1, \quad \text{for } n < 1.$$

$\square$

Proof. [Corollary 8]

Price stability demands $x_1 \leq 1$, but the social optimum satisfies $x_1^* > 1$. Since $\overline{x}_1 \leq 1$, $x_2(n) = \frac{1-y(n)}{1-n} R = \frac{1-n\overline{x}_1}{1-n} R \geq R > 1 \geq \overline{x}$. Also, since the real value of the allocation at $t = 2$ always exceeds the real value of the allocation at $t = 1$, patient agents never spend at $t = 1$. Thus, there are no runs. The fact that $\frac{\overline{P}}{P_0} \geq 1$ implies $i(n) = \frac{\frac{\overline{P}}{P_0} - n}{1-n} R - 1 \geq R - 1 > 0$ for all $n \in [\lambda, 1]$ by $R > 1$. Further, $\frac{\overline{P}}{P_0} \geq 1$ implies that $i(n)$ increases in n. $\square$

Proof. [Proposition 10]

Proof (i): The liquidation equation $y(n) = \min\left\{\frac{P_0}{\overline{P}} n, 1\right\}$ follows immediately from equation (3) and the constraint $y(n) \leq 1$. In $n = n_c$, we have $\frac{P_0}{\overline{P}} n = 1$. Hence, $n_c > 0$. By assumption $\overline{P} < P_0$, $n_c < 1$ with $n_c \in (0,1)$. Equation $y(n) = \min\left\{\frac{P_0}{\overline{P}} n, 1\right\}$ implies that $x_1(n) = y(n)/n$ is constant at the level $\overline{x} = P_0/\overline{P}$, as long as $y(n) < 1$; this is the case for $n < n_c$. For $n \geq n_c$,

$y(n) \equiv 1$. All goods are liquidated, so $x_1(n) = 1/n$. Equation $P_1(n) = P_0 n$ follows from equation (3).

Proof (ii): Equation $i(n) = \frac{\frac{\overline{P}}{P_0} - n}{1-n} R - 1$, for all $n \leq n_c$ follows from (3) combined with $y(n) = \min\left\{\frac{P_0}{\overline{P}} n, 1\right\}$. The remainder follows from plugging $y(n) = \min\left\{\frac{P_0}{\overline{P}} n, 1\right\}$ into $P_2(n)$ and observing that n_0 is positive only for $R > P_0/\overline{P}$. □

B Proofs: Decentralization with firms and private banks

Here, we provide the proof and more details on the decentralized economy in Section 6 of the main text.

Proof. Proof Proposition 6.1

Fix $M_0 = M_1 > 0$ and the central bank liquidation policy with $0 < y(n) \leq 1$ for all $n \in [\lambda, 1]$. We need to construct the policy choices $i(n, \widehat{y}), r^*(n, \widehat{y}), \tilde{r}(n, \widehat{y})$ for every $n \in [\lambda, 1]$ and every aggregate liquidation $\widehat{y}$ such that $y_j(n) = y(n)$ for all $n \in [0, 1]$ is the unique Nash equilibrium of the firm's liquidation game.

1) Determining incentive-compatible interest rates

We first pin down the interest rates and show below that these choices indeed render the central bank's announced liquidation policy the unique Nash equilibrium of the firms' liquidation game.

A. We start with the interior case $\widehat{y} \in (0, 1)$.

A1. For $\widehat{y} \in (0, 1)$, $n \in [\lambda, 1)$, and $y(n) \in (0, 1)$ for this considered n, pick $-1 < i(n, \widehat{y}) < \infty$ large enough such that $\frac{\hat{y}}{1-\hat{y}} \frac{1-n}{n}(1 + i(n, \hat{y})) > 1$ holds. Then pick $r^*(n) > 0$ and $\tilde{r}(n, \hat{y}) < \infty$ so that

$$1 < 1 + r^*(n) < \frac{\widehat{y}(n)}{(1 - \widehat{y}(n))} \frac{(1 - n)}{n}(1 + i(n, \widehat{y})) < 1 + \tilde{r}(n, \widehat{y}) < \infty \tag{7}$$

A2 For $\widehat{y} \in (0, 1)$, and the central bank demands full liquidation $y(n) = 1$ for some $n \in [\lambda, 1]$, pick $\tilde{r}(n, \widehat{y}) = \infty$ and set any finite $i(n, \widehat{y}) > -1$ and any $r^*(n, \widehat{y})$ with $-1 < r^*(n, \widehat{y}) < 0 < \tilde{r}(n, \widehat{y}) = \infty$. These choices are required for $n = 1$ and $y(1) = 1$.

A3. For $\widehat{y} \in (0,1)$, $n = 1$ and when $y(1) < 1$, set $r^*(1, \widehat{y}) < -1$ and some finite $i(1, \widehat{y}) > -1$ and $\tilde{r}(1, \widehat{y})$ such that $r^*(1, \widehat{y}) < -1 < 0 < \tilde{r}(1, \widehat{y}) < \infty$. This is the crucial case for which the absence of cash is required. For details, see the proof of Lemma B.1 right below.

B The corner cases $\hat{y} \in \{0, 1\}$ are special because the price levels $\hat{P}_1$ or $\hat{P}_2$ are undefined by the absence of supply either in $t = 1$ or $t = 2$.

B1) For all $n \in [\lambda, 1]$ and $\widehat{y} = 0 < y(n)$, pick some finite $i(n, 0) > -1$ and pick finite $-1 < r^*(n, 0) < 0 < \tilde{r}(n, 0) < \infty$.

B2) For all $n \in [\lambda, 1]$ and $\widehat{y} = 1$ and when $y(n) < 1$, set $r^*(n, 1) < -1$ and otherwise choose any finite $i(n, 1) > -1$ and $\tilde{r}(n, 1)$ such that $r^*(n, 1) < -1 < 0 < \tilde{r}(n, 1) < \infty$. Note, $r^*(n, 1) < -1$ requires the absence of cash to have a bite.

B3) For $n \in [\lambda, 1]$, $\widehat{y} = 1$ and when $y(n) = 1$, set $\tilde{r}(n, 1) = \infty$ and otherwise choose any finite $i(n, 1) > -1$ and $r^*(n, 1)$ such that $-1 < r^*(n, 1) < 0 < \tilde{r}(n, 1) = \infty$.

2) Uniqueness of Nash equilibrium

For all the cases above, we will now show that if cash is absent, the firms will either choose $y_j = y(n)$ or optimal choices cannot aggregate to $\widehat{y}$, implying that the aggregate liquidation $\hat{y}$ is not consistent with all firms playing a Nash equilibrium. Then we show, for the case where cash is available as an alternative to reserves, the same arguments go through, except for the case $n = 1$ and $y(1) < 1$.

A We start with the case $\widehat{y} \in (0, 1)$, and show that for every $y(n) \in (0, 1]$ that possibly deviates from $\hat{y}$, following the announcement $y(n)$ is optimal for each firm $j \in [0, 1]$ for every $n \in [\lambda, 1]$. As a consequence, the strategy $y_j = y(n)$ for a realized $n \in [\lambda, 1]$ is the unique Nash equilibrium of the firm's liquidation game. Recall that the case $y(n) = 0$ is inefficient due to $\lambda > 0$ and thus excluded from the analysis.

A1. Consider $\widehat{y} \in (0, 1)$, $n \in [\lambda, 1)$, and $y(n) \in (0, 1)$ for this considered n. For this case the market-clearing prices $\widehat{P}_1$, $\widehat{P}_2$ are nonzero and finite, satisfying $\widehat{y}\, nP_0 = \widehat{P}_1 \widehat{y}$ and $(1 - n)P_0(1 +$

$i(n, \widehat{y})) = \widehat{P}_2(1 - \widehat{y})R$. Thus,

$$\frac{\widehat{P}_2 R}{\widehat{P}_1} = \frac{\widehat{y}}{(1 - \widehat{y})} \frac{(1 - n)}{n}(1 + i(n, \hat{y})) \tag{8}$$

and (7) is equivalent to equation (6), restated here:

$$1 < 1 + r^*(n, \widehat{y}) < \frac{\widehat{P}_2 R}{\widehat{P}_1} < 1 + \tilde{r}(n, \widehat{y}) < \infty. \tag{9}$$

Regarding the choice $y_j(n)$, we need to examine three cases:

Case 1: The single firm j follows the announcement $y_j(n) = y(n)$. In that case, the firm and thus its house bank can exactly make the required payment to the central bank $\widehat{P}_1 y(n)$ in $t = 1$. Firm profits in $t = 2$ equal:

$$\text{Profits}(y_j(n)) = \widehat{P}_2 R(1 - y_j(n)) - \widehat{P}_2 R(1 - y(n)) = 0 \tag{10}$$

Case 2: The firm deviates by liquidating less than the announcement, $y_j(n) < y(n)$. In that case, irrespective of aggregate behavior $\widehat{y}$, the firm-house bank pair can only partially repay the loan to the central bank in $t = 1$ and pays the penalty interest rate $\tilde{r}(n)$ on the necessary interperiod loan $\widehat{P}_1(y(n) - y_j(n))$. With equation (6), firm profits in $t = 2$ satisfy

$$\begin{aligned} \text{Profits}(y_j(n)) &= \widehat{P}_2 R(1 - y_j(n)) - \widehat{P}_2 R(1 - y(n)) - (1 + \tilde{r}(n))\widehat{P}_1(y(n) - y_j(n)) \\ &< \widehat{P}_2 R(y(n) - y_j(n)) - \widehat{P}_2 R(y(n) - y_j(n)) = 0 \end{aligned} \tag{11}$$

Case 3: The firm deviates by liquidating more than the announcement, $y_j(n) > y(n)$. In that case, the firm-house bank pair must decide what to do with the excess liquidity. Since $r^* > 0$, the pair will deposit the excess liquidity at the central bank. With equation (6), firm profits in

$t = 2$ satisfy:

$$\begin{aligned} \text{Profits}(y_j(n)) &= \widehat{P}_2 R(1 - y_j(n)) - \widehat{P}_2 R(1 - y(n)) + (1 + r_1^*(n))\widehat{P}_1(y_j(n) - y(n)) \\ &< \widehat{P}_2 R(y(n) - y_j(n)) + \widehat{P}_2 R(y_j(n) - y(n)) = 0 \end{aligned} \tag{12}$$

In sum, we have established that if $\widehat{y} \in (0,1)$ and $n \in [\lambda, 1)$, the choice $y_j = y(n)$ is the only profit-maximizing choice for the firm.

A2. Consider the case $\widehat{y} \in (0,1)$, where the central bank requires full liquidation $y(n) = 1$ for some particular $n \in [\lambda, 1]$. The single firm cannot deviate by liquidating more than desired. Instead, the central bank only needs to deter a too-low liquidation via the interest rates. If the firm only partially liquidates by following the strategy $y_j = \widehat{y} < 1$, the firm-bank pair can only partially repay the required amount $\hat{P}_1 y$ to the central bank. Following that strategy, thus, requires the firm-bank pair to take out an interperiod loan in $t = 1$. However, the central bank charges a penalty rate of $\tilde{r}(n, \widehat{y}) = \infty$ on that loan, which results in an infinite loss. In contrast, following the central bank policy $y_j = y(n) = 1$ results in zero profits as in (10). The infinite penalty rate can be interpreted as the central bank disallowing borrowing in $t = 1$.

A3. For $\widehat{y} \in (0,1)$, $n = 1$ and when $y(1) < 1$, the argument is similar to B2 and crucial, because run-deterring policies require $y(1) < 1$. Absent cash, Lemma B.1 shows that the central bank can find interest rates $r^*(n, \widehat{y}) < -1$ and penalty rates on loans $\tilde{r}(n, \widehat{y}) > 0$ to ensure that the firms follow the announcement, $y_j(1) = y(1) < 1$ and $n = 1$ and that partial liquidation is a Nash equilibrium even though there is no demand and no goods sales in $t = 2$.

B. We now examine the corner cases $\hat{y} \in \{0, 1\}$, in the order as stated above. We show in each case that rational and optimal firm behavior cannot lead to an aggregate liquidation $\hat{y}$. As a consequence, an aggregate liquidation $\hat{y} \in \{0, 1\}$ as discussed in this section **B** is never observed following any Nash equilibrium of the firm's liquidation game.

B1. Consider $n \in [\lambda, 1)$ and $\widehat{y} = 0 < y(n)$. Then the goods supply in $t = 1$ is zero, meeting a positive demand nP_0. This causes the price level to explode, $\widehat{P}_1 = \infty$, making a single deviation from the aggregate action $y_j > \widehat{y} = 0$ infinitely profitable since the given interest rates are finite. That is, setting $y_j = \widehat{y} = 0$ for all $j \in [0, 1]$ is not a Nash equilibrium, implying that for $n \in [\lambda, 1)$ the individually optimal firm choices y_j never aggregate to $\widehat{y} = 0$.

For $n = 1$ and $\widehat{y} = 0 < y(n)$, the supply of goods in $t = 1$ is zero, and the demand for goods in $t = 2$ is zero. By the same argument as above, setting $y_j = \widehat{y} = 0$ for all $j \in [0, 1]$ is not a Nash equilibrium: $y_j = \widehat{y} = 0$ generates zero sales proceeds in $t = 1$ and $t = 2$ and a loss in $t = 2$, whereas following the announcement $y_j = y(n) > 0$ results in zero profits as in (10). Hence, zero supply in $t = 1$, $\widehat{y} = 0$, cannot be the aggregation of optimal choices y_j.

B2. Consider $n \in [\lambda, 1]$, $\widehat{y} = 1$ and $y(n) < 1$. The supply in $t = 2$ is zero. For $n < 1$, that supply meets a positive demand $(1 - n)$ and a deviation $y_j < \widehat{y} = 1$ is infinitely profitable, implying that playing $y_j = \widehat{y}$ for all $j \in [0, 1]$ is not a Nash equilibrium so that the aggregate liquidation $\hat{y} = 1$ cannot occur as a result of rational, individually optimal liquidation.

For $n = 1$, $\widehat{y} = 1$ and $y(1) < 1$, the analysis is provided in greater detail in Lemma B.1. This step is crucial in the proof: because there is no demand in $t = 2$, the central bank needs to set $r^*(n, 1) < -1$ and requires the absence of cash to keep firms from following $\hat{y}$, thus, liquidating too much. In particular, this part of the proof shows that the central bank can find interest rates that implement run-deterring liquidation policies in the decentralized economy. Recall that run-deterring liquidation policies require $y(1) < 1$ at a run $n = 1$ to render "spend early" ex-post suboptimal for patient types. Lemma B.1 shows why the presence of cash invalidates this argument.

B3. For $n \in [\lambda, 1]$, $\widehat{y} = 1$ and when $y(n) = 1$: we need to show that following $y(n) = 1 = \hat{y}$ is optimal for a firm j. The single firm cannot deviate upward from $y = 1$ by liquidating more than everything. The central bank only needs to ensure that the firm does not liquidate less than the desired amount. Liquidating $y_j < y$ implies that the payment of the firm-bank pair falls short of the required amount in $t = 1$. Since the central bank sets the penalty interest rate on the short amount infinitely high, $\tilde{r}(n, 1) = \infty$, they are forced to borrow from the central bank and incur infinite losses. By contrast, liquidating everything as demanded by the central bank $y_j = y$ results in zero profits as in (10), a better choice. Because all firms, in response, liquidate everything in $t = 1$, the goods price equals $P_2 = \hat{P}_2 = \infty$ for $n < 1$ and $P_2 = \hat{P}_2 \in [0, \infty]$ for $n = 1$. $\square$

We did not impose symmetry of equilibria: the other firms with $\widehat{y} = \int_{i \in [0,1], i \neq j} y_i \, di$ may set asymmetric liquidations. In a nutshell, because the nominal interest rate $i(n, \widehat{y})$ is state-contingent, as long as markets exist in both periods, we can always find positive interest rates $0 < r^*(n, \widehat{y}) < \tilde{r}(n, \widehat{y})$ and a unique Nash equilibrium exists even when cash is present. Yet, when markets are absent ($n = 1$ or $y \in \{0, 1\}$) negative interest rates $r^* < 0$ may be required. When cash is absent, firms cannot circumvent the negative interest rates, and the central bank's announcement is implemented as the unique Nash equilibrium. If cash exists, negative interest rates $r^*(\widehat{y}) < 0$ have no bite. Hence, policies with $y(1) < 1$ at $n = 1$ can only be implemented as the unique Nash equilibrium absent cash.

Note that every run-deterring liquidation policy requires $y(1) < 1$ at $n = 1$. The following lemma is important for implementing run-deterring and optimal liquidation policies:

Lemma B.1. *Consider a liquidation policy $y(n) \in [0, 1]$ that requires $y(1) < 1$ at $n = 1$. Given the realization $n = 1$, such a liquidation policy is implementable as the unique Nash equilibrium in the decentralized economy only if cash is absent. In that case, interest rates on reserves require $r^*(1, \widehat{y}) < -1$ and penalty rates on loans $\tilde{r}(1, \widehat{y}) > 0$.*

Intuitively, as the firms observe the full run, $n = 1$, they understand that the goods demand in $t = 2$ is zero. A strategy to not liquidate everything $y_j < 1$ in $t = 1$ can only maximize profits if the central bank's penalty rate on reserves is large. If cash exists, the negative interest rate on reserves has no effect, and the central bank can no longer deter the single and aggregate deviations $y_j = 1$, respectively $\widehat{y} = 1$.

Proof. [Lemma B.1] Assume the central bank desires a liquidation $y(n) \in [0, 1]$ with $y = y(1) < 1$ at $n = 1$. Given a full run realizes, $n = 1$, the resulting goods demand in $t = 2$ is zero.

Case A. Assume firm j deviates by liquidating more than required, $y_j > y(1)$, repaying more than its central bank loan. If $\widehat{y} < 1$, then $\widehat{P}_2 = 0$. Thus, the value of the required repayment to the central bank is zero in $t = 2$. If the firm-bank pair invests the excess proceeds from sales $(y_j - y)\widehat{P}_1$ at the central bank, profits to firm j in $t = 2$ equal:

$$\text{Profits}(y_j) = 0 - 0 + (1 + r^*(1, \widehat{y}))(y_j - y)\widehat{P}_1,$$

If the central bank sets $r^*(1, \widehat{y}) < -1$, then this deviation is not profitable, $\text{Profits}(y_j) < 0$. And indeed, when cash is absent, the firm-bank pair has to invest in reserves at interest rate $r^*(1, \widehat{y})$.

However, if cash exists, the firm-bank pair can circumvent the negative interest rate $r^*(1, \widehat{y})$ on central bank reserves by storing the sales proceeds from $t = 1$ as vault cash, resulting in

$$\text{Profits}(y_j) = 0 - 0 + (y_j - y)\widehat{P}_1 > 0.$$

Therefore, firm profits in $t = 2$ are positive. Thus, with cash, if the central bank demands $y(1) < 1$ at $n = 1$, a profitable deviation exists. All firms will play $y_j^* = 1$, resulting in $\widehat{y}(1) = 1$. Fixing $n = 1$ and $\widehat{y} = 1$, the goods demand and the supply in $t = 2$ are zero, so that, without a market in $t = 2$, the price $\widehat{P}_2$ is undefined, and we set the required repayment to the central bank in $t = 2$ to zero as in the case $\widehat{y}(1) < 1$.

Case B. The deterrence of deviations in the other direction does not pose an issue. Assume first that the aggregate of firms liquidate $\widehat{y}(1) < 1$, then $\widehat{P}_2 = 0$. Assume firm j deviates by liquidating less than required, $y_j(1) < y(1)$, repaying less than the required amount $\widehat{P}_1 y(1)$ to the central bank. The firm-bank pair thus needs to take out an additional interperiod loan with the central bank. Any choice of a penalty rate $\tilde{r}(1, \widehat{y}) > 0$ results in negative firm profits in $t = 2$:

$$\text{Profits}(y_j) = 0 - 0 - (1 + \tilde{r})(y - y_j)\widehat{P}_1 < 0.$$

Thus, any penalty rate $\tilde{r}(1, \widehat{y}) > 0$ can deter a liquidation deviation $y_j < y(1) = 1$. The argument is analogous for $\widehat{y}(1) = 1$. □

To highlight the difference between the cash and no-cash situation, it is instructive to consider the case where the nominal interest rate must be fixed for some exogenous reason, $i(n, \hat{y}) \equiv i$, regardless of n and $\hat{y}$. This could arise due to some alternative timing where the decision by the central bank regarding the interest rate i has to be made before n and $\hat{y}$ is known. This could also arise due to external considerations of the effects of nominal interest rates outside the model. We do so in the next proposition, fixing that nominal interest rate at zero. The proposition imposes restrictions on the liquidation policy in the presence of cash, while no such restrictions are necessary absent cash.

Proposition 12. *Fix $M_0 = M_1 > 0$. Fix $i = 0$. Absent cash, the conclusion in Proposition 6.1 still holds. With cash, the conclusion in Proposition 6.1 holds if $y(n) \geq n$.*

Proof. [Proposition 12] We only need to consider the cases $\hat{y} \in (0, 1)$, $n \in [\lambda, 1)$ and $y(n) \in (0, 1)$ since the exclusion of the corner cases works by the same logic as above. Reexamine the case A1 of the proof above: For $1 > \hat{y} > n$ it holds that $1 < \frac{\hat{y}}{n}\frac{1-n}{1-\hat{y}}$ for all $n \in [\lambda, 1)$. For $i = 0$, with equations (8) and (7) it follows $\frac{\hat{P}_2 R}{\hat{P}_1} = \frac{\hat{y}}{n}\frac{1-n}{1-\hat{y}} > 1$, or $\widehat{P}_1 - \widehat{P}_2 R < 0$. This implies via equation (6) that a choice $r_1^* > 0$ is possible even at $i = 0$. As a consequence, following the central bank liquidation policy $y_j = y(n)$ remains the optimal firm choice given that $1 > \hat{y} > n$, $i = 0$, and $0 < r_1^*(n, \hat{y}) < \tilde{r}(n, \hat{y})$ chosen to satisfy equation (7).

By the same logic, equations (6) and (7) also show that for $i = 0$ the central bank will need to choose $r^*(n, \widehat{y}) < 0$ whenever $\widehat{y} < n$, implying $\widehat{P}_1 - \widehat{P}_2 R > 0$. This plays no role when cash is absent because the firm-bank pair has to store excess sales proceeds via reserves. That is, $y_j = y(n)$ remains optimal as long as $r_1^*(n, \hat{y}) < 0 < \tilde{r}(n, \hat{y})$ are chosen to satisfy (7). However, when cash is available and $r^*(n, \widehat{y}) < 0$, cash is a better store of value than reserves at the central bank. The firm-bank pair chooses to hold any excess liquidity as vault cash to avoid negative interest on reserves $r_1^*(n, \hat{y}) < 0$. Firm profits in $t = 2$ then satisfy:

$$\begin{aligned}
\text{Profits}(y_j(n)) &= \widehat{P}_2 R(1 - y_j(n)) - \widehat{P}_2 R(1 - y(n)) + \widehat{P}_1(y_j(n) - y(n)) \\
&= \widehat{P}_2 R(y(n) - y_j(n)) + \widehat{P}_1(y_j(n) - y(n)) \\
&= (\widehat{P}_1 - \widehat{P}_2 R)(y_j(n) - y(n)) > 0.
\end{aligned}$$

That is, given the realization $\widehat{y} < n$ and thus $\widehat{P}_1 - \widehat{P}_2 R > 0$, the firm makes a profit when deviating from the announced policy by liquidating more than desired by the central bank $y_j(n) > y(n)$. This profitable individual deviation implies an aggregate liquidation $\hat{y} > y(n)$ whenever $\widehat{y} < n$, which is a contradiction for $y(n) > n$. For $y(n) < n$, the profitable deviation remains in the case of cash. $\square$

B.1 Run-deterrence, optimality, and price stability

Proof. [Proof Proposition 7.1] Assume $y(n) = y^*$ for all $n \in [\lambda, 1]$, and thus $y(n) \in (0,1)$. We know from the main text that this liquidation policy implements the optimal allocation in dominant strategies and deters runs if the firms implement it as the unique Nash equilibrium. Further, by Proposition 6.1, we know that this liquidation policy is the unique Nash equilibrium of the firm's liquidation game absent cash. Technically, the proof finishes here, but we give the details for greater clarity.

To see that $\widehat{y} = y$ is a Nash equilibrium, consider $n = \lambda$. It holds that $\frac{P_2 R}{P_1} = \frac{y^*}{1-y^*}\frac{1-\lambda}{\lambda}(1 + i(\lambda)) = \frac{x_1^* - x_1^*\lambda}{1-x_1^*\lambda}(1 + i(\lambda))$, where we have plugged in $y^* = x^*\lambda$. See that $\frac{x_1^* - x_1^*\lambda}{1-x_1^*\lambda} > 1$ by $x_1^* > 1$ so that for any choice $i(n, y^*) \geq 0$ it holds that $\frac{P_2 R}{P_1} > 1$. Thus, the central bank can find $\tilde{r}_1 > r_1^* > 0$ with $1 + r_1^* < \frac{P_2 R}{P_1} < 1 + \tilde{r}_1$. Now consider $n \in (\lambda, 1)$. Then $\frac{P_2 R}{P_1} = \frac{y^*}{1-y^*}\frac{1-n}{n}(1 + i(n)) > 1$ for all $n \in (\lambda, 1)$ if $i(n)$ grows sufficiently fast in n. Therefore, likewise, positive interest rates can be found with $1 < 1 + r_1^* < \frac{P_2 R}{P_1} < 1 + \tilde{r}_1$.

In $n = 1$, because $y(1) = y^* < 1$, Lemma B.1 states that an interest rate $r_1^*(1, \hat{y}) < -1$ and $\tilde{r}_1(1, \hat{y}) > 0$ implement y as the unique Nash equilibrium for any deviation $\hat{y}$, given cash does not exist. Consider a deviation $\widehat{y} \in (0, 1)$ and $n \in [\lambda, 1)$. One can always find a nominal interest rate $i(n, \widehat{y})$ such that $\frac{\widehat{P}_2 R}{\widehat{P}_1} = \frac{\widehat{y}}{1-\widehat{y}}\frac{1-n}{n}(1 + i(n, \widehat{y})) > 1$. Thus, $\tilde{r}_1(n, \widehat{y}) > r_1^*(n, \widehat{y}) > 0$ exist with $1 + r_1^* < \frac{\widehat{P}_2 R}{\widehat{P}_1} < 1 + \tilde{r}_1$. Following the proof of Proposition 6.1 shows that $\widehat{y}$ cannot be Nash. Likewise, the cases $n = 1$ and $\widehat{y} = 1$ and $\widehat{y} = 0$ are covered there.

With cash: Then given a run, $n = 1$, the central bank cannot deter a deviation $\hat{y} = 1$ by the firms; see the reasoning in Lemma B.1. The households internalize the firms' deviation ex-ante. They know, given the run, that the firms are liquidating everything, implying that the goods supply in $t = 2$ equals zero. This makes running on the central bank optimal ex-post. Therefore, the run-equilibrium reemerges.

Given $n \in [\lambda, 1)$, the central bank can find interest rates to deter every deviation $\hat{y} \neq y^*$; see the proof to Proposition 6.1. The households anticipate this ex-ante. Therefore, given $n = \lambda$, the firms provide goods $\hat{y} = y^*$, making "spend late" ex-post optimal for patient households. $\square$

Proof. [Proof Proposition 7.2]

Let $y(n) = n$ for all $n \in [\lambda, 1]$, the liquidation policy desired by the central bank. Following Proposition 7, we know this liquidation policy can be implemented as fully price stable if the nominal interest rate $i(n)$ is fine-tuned. Further, for $n \in [\lambda, 1)$ we have $y = n < 1$, $\frac{P_2 R}{P_1} = (1 + i(n)) = \frac{\frac{\overline{P}}{P_0} - n}{1-n} R = R > 1$. Thus, there exist positive interest rates $0 < r_1^* < \tilde{r} \leq \infty$, such that following the liquidation policy desired by the central bank is the unique Nash equilibrium and thus optimal for all firms. Moreover, $y = 1$ in $n = 1$ so that by Proposition 6.1 interest rates exist such that profitable deviations are absent, even when cash is present.

Recall that the real allocation to households satisfied $x_1(n) = y(n)/n = 1 < x_1^*$ for all $n \in [\lambda, 1]$. Thus, the optimal allocation is not implemented, but runs are absent by $x_1(n) < x_2(n)$ for all n. $\square$

B.2 Decentralization with private banks, firms, and a CBDC

Section B assumed that households hold deposits at private banks, with the central bank providing interperiod loans to banks to meet withdrawal demands. To connect Section B with our benchmark model, we shall demonstrate that we can equally well assume that households hold a CBDC rather than deposits across periods. In contrast to the benchmark model in the main text, the central bank no longer runs projects directly but funds banks, which in turn fund firms running projects. Hence, in this version of the model, the disintermediation problem of banks losing deposit funding to the central bank in the form of a CBDC is resolved by having the central bank replenish that funding via intertemporal loans.

In $t = 0$ and as above, households are endowed with one unit of the good but have no

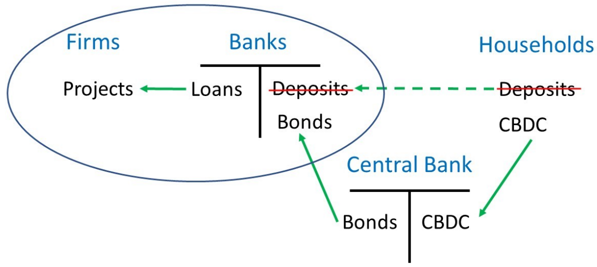

Figure 7: The financial system with a CBDC: Households, firms, banks, and the central bank.

access to the production technology. Firms have no funds of their own but have access to the technology. They require a loan from banks to purchase the goods from the households. The central bank provides banks with a loan M_0, which they lend out to firms to purchase goods from households. Assume now that households hold the money obtained from the goods sales in the form of a CBDC at the central bank across periods rather than redepositing it with the banks. Holding a CBDC allows paying a nominal interest.[31] Because the households do not redeposit the sales proceeds with the banks in $t = 0$, banks can repay loans to the central bank only when firms sell goods in $t = 1$ and $t = 2$, depositing their proceeds with their house bank to repay their bank loan. Hence, the central bank's loans to banks must now be intertemporal rather than intratemporal. The equivalence to the formulation above is best seen by using the same notation but giving it a different interpretation.

Let $(D_1, D_2) = (M_1, M_2)$ be the CBDC balances available to the household when spending in either $t = 1$ or $t = 2$. The case $M_0 = M_1 = M_2$ and $i(n) = 0$ covers the case of cash. Note, in this model version, D_1 and D_2 are set directly by the central bank, whereas in Section B, the bank would set the deposit contract as $D_1 = M_1$ and $D_2 = M_2$ following the central bank's announced money supply in $t = 0$. The central bank loan to a bank then requires the bank to repay nD_1 units of money in $t = 1$ and $(1-n)D_2$ units of money in $t = 2$, where n is the fraction of households spending their CBDC balances in $t = 1$, and via market clearing $nD_1 = P_1 y$ and $(1-n)D_2 = (1-y)RP_2$, the outstanding loan amounts equal the revenue of the "average firm," liquidating the aggregate and average quantity y.[32] Penalties are applied as before should the

[31] This holds because the mechanisms to control runs are implemented via the goods market and not in the form of deterring depositors from withdrawing money (which would be extremely hard in the case of cash).

[32] As before, one may wish to think of this as a one-period loan from $t = 0$ to $t = 1$, of which a fraction $1 - n$

bank deviate from these repayments. The contract between a bank and a firm is as before. It is clear then that the analysis above applies here and that one obtains the same allocations and prices.

C Fixing the trilemma

Open market operations. We argued before that changes in the interest rate do not fix the trilemma. We will show now that open market operations also fail at this task. Consider an open market operation by the central bank, given n and its other policy choices. In $t = 1$, the central bank sells one-period nominal bonds $B > 0$ to be repaid in $t = 2$ with interest i_B. If $B = M - M(n)$ and all agents buy these bonds, then shopping agents are left with the quantity $M(n)$ of money, and only $nM(n)$ gets spent.

This intervention does not fix the trilemma, regardless of B and i_B. When the central bank sells these bonds, agents' types and n have already been revealed. Impatient agents have no desire to buy these bonds because they pay off in $t = 2$ when they have no use for balances. For patient agents, consider first the case $i_B = i(n)$. Non-shopping patient agents are indifferent between holding deposit balances or bonds. If $i_B < i(n)$, non-shopping patient agents strictly prefer to hold their balances rather than purchase bonds, and no other agents buy the bonds. If $i_B > i(n)$, then all non-shopping patient agents will seek to purchase up to the amount of their deposit balances. If the bond supply is lower than that, the bonds are sold pro rata, or the buyers are chosen randomly to achieve bond market clearing. But in all three cases, patient agents will not change their shopping behavior because bond purchases do not alter real allocations, and the net result is only a higher price level in $t = 2$, leaving the price level in $t = 1$ unaffected.

can be rolled over without further penalty.

D Extensions

Token-based CBDCs. With a token-based CBDC, a central bank issues anonymous electronic tokens to agents in $t = 1$ rather than accounts. Whether this is done with or without a blockchain is irrelevant to our paper. Similarly, we do not need to specify which walls should exist between the CBDC and the central bank to guarantee the anonymity of tokens. These electronic tokens are more akin to traditional banknotes than to deposit accounts. Trading with tokens only requires trust in the token's authenticity rather than knowledge of the token holder's identity. Thus, token-based transactions can be made without the knowledge of the central bank.

With appropriate software, digital tokens can be designed in such a way that each unit of a token in $t = 1$ turns into a quantity $1 + i$ of tokens in $t = 2$, with i to be determined by the central bank at the beginning of $t = 2$: even a negative nominal interest rate is possible.[33]

With that, the analysis in the main paper still holds since nothing of essence depends on the identity of the spending agents other than the total CBDC tokens spent in the goods market. With a token-based CBDC, agents obtain M tokens in $t = 0$ and decide how much to spend in $t = 1$ and $t = 2$. Hence, the same allocations can be implemented except for those that require the suspension of spending, as discussed in Section 5.

For the latter, the degree of implementability depends on technical details outside the scope of this paper. Even with a token-based system, the transfer of tokens usually needs to be registered somewhere, e.g., on a blockchain. Limiting the total quantity of tokens that can be transferred on-chain in any given period is technically feasible. A pro-rata arrangement can be imposed by taking all of the pending transactions waiting to be encoded in the blockchain, taking the sum of all the spending requests, and dividing each token into a portion that can be transferred and a portion that cannot. Such an implementation is even easier when a centralized third party operates the token-based CBDC.

[33] Historically, we have examples of banknotes bearing positive interest (for instance, during the U.S. Civil War, the U.S. Treasury issued notes with coupons that could be clipped at regular intervals) and negative interest (demurrage-charged currency, such as the prosperity certificates in Alberta, Canada, during 1936). Thus, an interest-bearing electronic token is novel only in its incarnation but not in its essence.

Synthetic CBDC and retail banking. With a synthetic CBDC, agents do not hold the central bank's digital money directly. Rather, agents hold accounts at their retail bank, which in turn holds a CBDC not much different from current central bank reserves. This may be due to tight regulation by the monetary authority. In our analysis above, the retail banks undertake the real investments envisioned for the central bank.

The key difference from the current cash-and-deposit-banking system is that cash does not exist as a separate central bank currency or means of payment. That is, in a synthetic CBDC system, agents can transfer amounts from one account to another, but these transactions are always observable to the banking system and, thereby, the central bank. Likewise, agents (and banks) cannot circumvent negative nominal interest, while they could do so in a classic cash-and-deposit banking system by withdrawing and storing cash.

For our analysis, observability is key. Our analysis is relevant in the case of a systemic bank run, i.e., if the economy-wide fraction of spending agents exceeds the equilibrium outcome. Much then depends on the interplay between the central bank and the system of private banks. For example, if the liquidation of long-term real projects is up to the retail banks, and these retail banks decide to make the same quantity of goods available in each period, regardless of the nominal spending requests by their depositors, then the aggregate price level will have to adjust. The central bank may seek to prevent this by suspending spending at retail banks or forcing banks into higher liquidation of real projects: both would require considerable authority from the central bank.

E Bank runs vs. spending runs

Deposit insurance or lender-of-last resort policies have been proposed to address the bank run issues raised by DD. Conceptually, these policy discussions view a private bank as small relative to a deep-pocketed government, allowing for a partial equilibrium perspective. Such traditional policies do not restrict early consumption or behavior but provide additional consumption in $t = 2$ to ease rollover incentives.

By contrast, our analysis takes a general equilibrium approach. Providing insurance in case of a system-wide bank run needs to respect aggregate resource constraints. DD do so by proposing a real tax on withdrawals in $t = 1$ to finance deposit insurance. Their tax depends on the aggregate withdrawals, reduces real investment liquidation, and can be designed in such a way as to prevent a run.

In our framework, such a tax can be imposed as a real tax on goods purchased after the agents have gone shopping or as a nominal tax on dollar balances before agents can spend them. The first case is then a particular form of our liquidation policy, rewritten as selling a gross amount of goods to agents and reducing it with a real sales tax to the net amount delivered. The key insight of our analysis in the main text is that such a run-deterring policy is at odds with the price stability objectives. The second case of a nominal tax does not deter spending runs in our model. Nominal taxes are a version of the state-contingent money balances considered in Section 5. As we show there, state-contingent dollar balances are insufficient on their own. Spending runs can only be deterred if, in addition, the liquidation policy is run-deterring. The same logic applies to nominal bailouts and nominal deposit insurance at $t = 1$: whether a spending run can happen depends entirely on the real liquidation policy, not on nominal quantities.

Only real deposit insurance or real lender-of-last-resort policies could prevent runs. Because this paper takes a general equilibrium approach, the only way to guarantee high consumption in the future is by constraining liquidation during the interim period. This liquidation constraint can be interpreted as the central bank's early intervention to implement a (real) lender of last resort or insurance policy in $t = 2$.

The provision of real deposit insurance in $t = 1$ while adhering to an aggregate budget constraint requires the central bank to liquidate investment in proportion to withdrawals. These additional liquidations stabilize the price level in $t = 1$. A central bank's full price stability commitment can be understood as a commitment to providing real $t = 1$-deposit insurance in a nominal world, but is inefficient, as we have pointed out in Corollary 8. As we saw above, maintaining efficiency and providing real deposit insurance in $t = 1$ are bound to fail if withdrawals exceed the critical threshold n_c.

As an alternative way of providing real insurance, Keister (2016) proposes taxing depositor resources in $t = 0$ to finance bailouts. The tax there reduces the real claims by depositing households in $t = 1$. With sufficient reduction, the tax collected can then provide real insurance in case of a run. Such a mechanism per se would not necessarily deter spending runs in the context of our model. That holds because, with or without tax, our framework has no fixed real claims in $t = 1$. Instead, real goods obtained in $t = 1$ result from endogenous purchase decisions and market clearing, given the liquidation policy of the central bank.

Rather, the nominal claims remain unchanged in our model, but, as already explained above, even a spending-contingent change in nominal claims could not deter spending runs. Because real taxation in $t = 0$ does not necessarily translate into a real claim reduction in $t = 1$, such taxation in $t = 0$ is ineffective in preventing spending runs. Moreover, such taxation does not free up additional resources for allocation in the form of a bailout in $t = 1$. This holds because all resources available for bailing out or insuring the households in $t = 1$ are under the central bank's control due to its investments in $t = 0$. There are no additional resources in the economy up for grabs.

The discussion above highlights the difference between a more traditional perspective on bank runs on the one hand and the spending run on the central bank in our analysis on the other hand. In a traditional bank run, agents run away from deposits and into cash. If that bank is small relative to the aggregate economy, a central bank or lender of last resort can alleviate such a run by providing emergency lending. This is still true for a system-wide bank run, when deposit claims are nominal, and the conversion into cash can be satisfied by a central bank, providing the appropriate quantities of cash.

That kind of deposit-to-cash conversion during a classic bank run keeps the money aggregate $M_1 = D + C$, that is, the sum of cash and deposits in the economy, constant. If that deposit-to-cash conversion does not result in higher spending or liquidation by the central bank, aggregate real allocations and the price level remain unaffected. By contrast, our focus here is a spending run where households run away from M_1, such as currency, into goods on an aggregate scale.

This now requires the liquidation of long-term projects on an aggregate level. Aggregate resource constraints have to be obeyed, and the consequences for the aggregate price level have to be analyzed, and indeed, we do.