

Measuring low fertility: Rethinking demographic methods ⁽¹⁾

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Introduction

Low fertility is a pervasive phenomenon. All European countries currently experience below-replacement fertility levels, and the proportion of the world's population living in a low fertility context continues to increase. The purpose of this contribution is to rethink demographic methods for the analysis of fertility from the perspective of recent research on low fertility and to assess data requirements for such analysis. In doing so, we will map many of the areas where the contributions of the late Gerard Calot were particularly relevant.

In a low fertility context, there are some demographic tools that become central. This is the case with tempo effects, parity specific analysis, and the introduction of

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fertility life table measures such as period parity-progression rates. While many of these methods have been available at least since the 1950s⁽²⁾, what is new is the possibility of combining these different elements of analysis. In particular, the logic and method of tempo adjustment can be extended to any fertility measure that is calculated from tempo adjusted age- and parity-specific fertility rates. This idea, while simple, has not yet caught on in research. It is still common to find some confusion about what particular methods can or cannot do. Fertility life table measures can eliminate compositional effects, that is, the role of the period distribution of women by parity, but they do not provide a measure that is free from tempo effects.⁽³⁾ Fortunately, the influence of parity distribution can be separated not only conceptually but also analytically from tempo distortions caused by changes in the timing of fertility, and we can gain more insight into fertility trends from such a separation.

The concept of *tempo effects* is therefore the first crucial point that is addressed in this article. The logic and mathematics of tempo effects are connected with the idea of *demographic translation* introduced by Ryder (1964, 1980) and developed further by Foster (1990), Calot (1993) and Keilman (1994, 2001).⁽⁴⁾ Despite the importance of tempo effects in these analyses, their goals are different. Demographic translation is concerned with the transformation from cohort to period measures of fertility and vice versa. In contrast, tempo adjustment does not concern itself with any cohort-period transformation. It was introduced by Bongaarts and Feeney (1998) (henceforth B – F) in order to obtain “the total fertility rate that would have been observed in a given year had there been no change in the timing of births during that year” (B – F: 275).⁽⁵⁾ It is therefore based on a counterfactual: assuming no postponement, what would the *TFR* be? This requires an analysis of the effects of postponement and the development of measures that compensate for those effects (Ortega and Kohler, 2002). While the procedure is used by B – F only for the adjustment of the *TFR*, the formula works at the age-specific level (B – F, 277). It can therefore be employed to adjust each age-specific fertility rate independently. In this sense, the B – F formula is a special case of tempo adjustment where the adjustment ratio is the same for all the rates.

(2) Hobcraft (1996) argues that the timely use of such measures would have allowed demographers to detect the temporary character of both the baby boom and baby bust, thus avoiding the sometimes ridiculous performance of fertility forecasting and analysis during this period.

(3) The relationship between life table measures of fertility and tempo adjustment is a recurrent topic in much of the recent debate on the subject (Ní Bhrolcháin, 1992; Rallu and Toulemon, 1993b; van Imhoff and Keilman, 2000; Bongaarts and Feeney, 2000; van Imhoff, 2001). The same can be said of measures based on duration specific fertility rates which control duration but are also affected by tempo changes. It is possible to extend the logic of tempo adjustment to that context as does Brass (1990), defining postponement for duration-specific rates.

(4) With further contributions by Pressat (1969), Leguina (1976), Deville (1977), Chávez (1979), Feichtinger (1979) and Keyfitz (1985). Personal perspectives and reviews of method are found in Hobcraft, Menken and Preston (1982), Keilman and van Imhoff (1995), van Imhoff (2001). Hobcraft, Menken and Preston refer to demographic translation as cohort-inversion.

(5) Lack of understanding of the difference between demographic translation and tempo adjustment is what lies behind the recent debate on the B-F procedure (B-F; Keilman and van Imhoff, 2000; Kim and Schoen, 2000; Bongaarts and Feeney, 2000). As Keilman and van Imhoff point out, the reason might be the discussion by B-F of the similarity between cohort fertility and a moving average of tempo adjusted TFRs.

The difference of approach between tempo adjustment and the translation approach is quite subtle, since the latter also provides a decomposition of period fertility in a tempo and a quantum index (Ryder, 1980; Hobcraft, 1996). However, the meaning attached to tempo and quantum is different. Within the translation approach, the idea is to relate the moments of the distribution of period fertility (period *TFR*, mean age, variance, asymmetry, ...) to those of cohort fertility. When only the first two moments are considered (cohort *TFR* and cohort mean age), the component associated with the cohort *TFR* is the quantum component, whereas the one associated with the mean age (more precisely, with the derivative of the mean age) is the tempo component. This quantum component cannot be interpreted as “the total fertility rate that would have been observed in a given year had there been no change in the timing of births during that year”, as in the B-F approach; that is not its purpose. The timing index for a particular year is calculated as the sum across cohorts of the proportions of completed cohort fertility that took place during the year in question.⁽⁶⁾ This index will be larger (or smaller) than one if fertility is being anticipated (or postponed). As a result, if any event takes place after the year of reference (for instance, a war), and some cohorts see their fertility permanently reduced, this procedure would lead to an ex-post interpretation of this as an anticipation of fertility. This is because the proportion of fertility cumulated before the war was larger for those cohorts than could have been expected at the time. Ward and Butz (1978) and Butz and Ward (1979) saw this problem and referred to it as an *Ex Post Timing Index*. They proposed an *Ex Ante Timing Index* which is similar to the timing index produced by Ryder but which only uses past information in the cohort completion process. They use economic models to forecast future fertility with the caveat that these models also require forecasting of the economic series themselves. This concept of tempo effects remains different from that of B – F in any case, since the idea is still connected to a translation concept irrelevant for the B – F counterfactual.

The practical relevance of tempo adjusted fertility measures stems from the extensive changes in childbearing age which occur in low fertility countries. In most countries, childbearing is being postponed (Council of Europe, 2000; Frejka and Calot, 2001 b; Kohler, Billari and Ortega, 2001). This is not a new phenomenon. Hooker (1898) and Yule (1906 a) studied the consequences of postponement at the turn of the century, and they developed the main ideas. They proposed the use of the mean age to measure postponement⁽⁷⁾ and showed that it was the speed of change of postponement that affected the marriage or birth rates. This analysis was connected with empirical evidence on the effects of abnormal circumstances,

(6) As a matter of fact Ryder (1980) estimates the tempo index in two alternative ways that lead to similar results. One is based in the translation formulas using the change in the mean age, while the other uses the sum of proportions of completed cohort fertility.

(7) Their analysis of postponement is basically connected to age at marriage. Since they did not have data on age at marriage they used the proportion of minors at marriage instead. They probably did not know that such data was already available in Ogle (1890). Hooker and Yule entered a very interesting debate about the possible causes of marriage postponement, Hooker favoring the spread of education and changes in the desired standard of comfort and Yule economic factors such as price changes (Yule, 1906 a, 1906 b; Hooker, 1906).

such as the Prussian war, on marriage and fertility rates. Rates fall below normal levels during those years, but there is recuperation after the war. Today we know that this pattern of *rebound* effects has been characteristic of short-term fluctuations in vital rates both in historical and in present times (Lee, 1997; Reher and Ortega, 2000). The effects of postponement and recuperation were again at play during World War II and the subsequent baby boom. Hajnal (1947) made a deep qualitative analysis of postponement in this context. He also pointed out that the widespread use of family limitation was making fertility postponement more relevant. He defined postponement in a general way very similar to the idea of tempo effects: "It is not even necessary to suppose that at the time the 'postponement' takes place [...] people have the idea clearly in their minds that they will later have the children they are 'postponing'" (p. 151). From Hajnal's time until today, tempo effects have played an important role in explaining fertility trends. First, there was a process of fertility anticipation which was essential for an understanding of the baby-boom process. This is the motivation for the work of Ryder (1964, 1980), Pressat (1969) and Deville (1977). Subsequently, there has been a surge of interest in studying fertility postponement, which is related to the fact that a delay in childbearing has become a pervasive characteristics of fertility patterns in low and lowest-low fertility countries.

Another crucial concept in the analysis of low fertility is the study of parity-specific fertility. There is broad agreement on this both with regard to cohort and period fertility (Lutz, 1989; Ní Bhrolcháin, 1992; Rallu and Toulemon, 1993 a; Keilman, 1993; Hobcraft, 1996): the reasons and contexts for having a first child are generally different from those for having a second or a third.⁽⁸⁾ Since the proportion of births of higher orders is becoming very small, these three transitions at least should be studied individually. There is a further reason to take parity into account in the context of demographic transition or tempo adjustment. As Hobcraft (1996) and B – F eloquently argue, tempo adjustments should be inferred from trends in parity-specific mean ages at childbearing. Otherwise a reduction in quantum might be taken as a tempo effect since the overall mean age generally declines when the proportion of higher order birth does. One example of this is shown by Lotka and Spiegelman (1940). Ryder (1980) was aware of this and, even within a quantum-tempo decomposition of general fertility, he devised an ad-hoc procedure to correct the timing index for the effect of a quantum change on the mean age at childbearing. Basically his procedure requires an estimate of the mean age at birth of the first child, and an estimate of the average inter-birth interval. This may be an interesting approach when no other information is available, but when it is possible to work with parity-specific births separately this is preferable, no matter whether we are studying cohort or period fertility. All the recently proposed tempo-adjustment procedures use parity-specific data. In the context of cohort fertility this is not always the case. We believe that the attempt should be made of basing any inferences from postponement, recuperation, etc., on parity-specific analysis.

(8) See, for instance, Namboodiri (1972), Seiver (1978), Louchart and Sagot (1984), de Cooman *et al.* (1987) or Heckman and Walker (1990).

In what follows, we will use the insights described above both to discuss and rethink the measurement of fertility with specific relevance to low fertility contexts. This discussion will be structured as follows: in a first section we will present the basic tools for the measurement of parity specific fertility: the childbearing rates. These can be classified as either incidence rates or intensities depending on whether exposure is measured explicitly (intensities), or only along the age dimension, irrespective of parity (incidence rates). This distinction proves particularly relevant in the context of tempo adjustment, since in a second section we will show that inference about tempo should be based on intensities rather than on incidence rates. Otherwise compositional effects will bias the measurement. An example is given of how to adjust childbearing intensities based on the K – O framework. These can also be converted to adjusted incidence rates.

Tempo adjusted childbearing rates can then be used to describe fertility behavior. Fertility life table measures are particularly common in such descriptions. In a third section we will discuss fertility tables for a parity-specific analysis and the different summary measures available.

Both period and cohort perspectives are relevant in specific contexts. A fifth section will concentrate on the description of period fertility. It is possible to summarize the effects of tempo change and the composition of the population by age and parity on the number of births as a set of ratios. This way of describing fertility trends is particularly valuable, since the different demographic influences on fertility can be separated. In searching for an explanation of fertility trends, one should concentrate on explaining the appropriate summary fertility measures that are free from tempo and compositional effects. A last section will examine the cohort approach to fertility analysis. There are many topics in demography that require a cohort dimension. Its strength is the ability to track dynamically groups of individuals. This is essential for some demographic topics such as family dynamics, kinship, Easterlin effects, etc. For cohorts with completed fertility, the cohort perspective is not problematical. It simply requires the use of fertility rates along the diagonals of the lexis diagram. However, problems arise in cohort completion. We will show that the analysis of tempo and compositional effects is particularly useful in this context, as it allows the decomposition of hypothesis about future fertility in two dimensions: quantum and tempo. The adjusted childbearing intensities can be used as a basis for the future evolution of quantum. The future evolution of tempo can be projected based on the mean ages at childbearing leading to different postponement scenarios. For each of these it is possible to complete fertility for the cohorts which presently are of childbearing age, in a way which is both demographically coherent and takes into account all the information available.

The basic components: childbearing rates

We will start our analysis with the most basic components of common fertility measures: the childbearing rates. Depending on the sophistication of the analysis, rates can be made specific for a number of dimensions, the most common being the mother's age or the mother's birth cohort, parity, marital status, and duration

from marriage or last birth.⁽⁹⁾ These measures are obtained by dividing the number of births occurring to mothers in a specific category by a measure of exposure, that is, person-years lived by a certain group of women. Depending on the exposure measure in the denominator, we distinguish fertility intensities and incidence rates. If we divide by a measure of exposure in a specific category, we speak of *occurrence-exposure rates* or, following Hoem and Hoem (1989), *childbearing intensities*. These rates are also referred to as *rates of the first kind* or simply *rates* (Calot, 2001 c). When the denominator is a measure of exposure of all women in the age category, we call them *incidence rates* following Finnäs (1980) and Borgan and Ramlau-Hansen (1985). Lotka and Spiegelman (1940) and van Imhoff (2001) use the term *frequencies* for these rates. They are also called *rates of the second kind*, *reduced events*, and, again, just *rates*. There is a straight relation between them. If we call $m_c(a)$ and $f_c(a)$ the childbearing intensity and incidence rate for the women of class C and age a , $E(a)$ and $E_c(a)$ exposure to all women of age a , and to those in class C , and $B(a)$ and $B_c(a)$ births to all women age a , and to women in class C , we have the following relationship:

$$f_c(a) = \frac{B_c(a)}{E(a)} = \frac{E_c(a)}{E(a)} \cdot \frac{B_c(a)}{E_c(a)} = \frac{E_c(a)}{E(a)} \cdot m_c(a) \quad [1]$$

That is: in order to transform intensities into incidence rates we simply have to multiply the former by the proportion of exposure contributed by women of class C . As we have argued, we are especially interested in the analysis of fertility by birth order. In this particular case, the class C refers to births of a given order i , and exposure is limited to those women who can potentially have a birth of order i , that is, women of parity $i - 1$.

There are two kinds of considerations which determine the choice between rates and intensities: some connected to their intrinsic properties, some to measurement issues. We will start with the former.⁽¹⁰⁾

Intensities are generally advocated on theoretical grounds because, when they include all the relevant dimensions of fertility, they can reflect the instantaneous probability that a woman in that specific category gives birth (Hoem, 1976). However, this is only guaranteed when the subgroups of women are homogeneous with respect to their fertility behavior. If a group is not homogenous, the fertility intensity is a weighted average of the intensities for the different women where the weights are proportional to the respective intensities.

A second and important intrinsic advantage of intensities in the context of parity-specific analysis is their independence with respect to earlier childbearing behavior. Since past births are precisely the events that lead to transitions between parities, past fertility levels determine the proportion of exposure by women of parity $j - 1$, which according to [1] is the conversion factor from intensities to incidence rates.

(9) Other possibilities include the father's characteristics, education level, race or ethnicity, place of residence, etc. (see Lutz, 1989).

(10) See also Wunsch (2001 a, 2001 b), Vallin and Caselli (2001), Van Imhoff (2001), Toulemon (2001b) for discussions of these issues.

This means that trends in fertility incidence rates result not only from changes in fertility but also from changes in the population composition by parity (Whelpton, 1946). In this sense, the interpretation of trends in fertility intensities is easier since it is free from these compositional effects. This property is particularly important in the estimation of tempo effects as discussed extensively in the following section.

On the other hand, incidence rates have the advantage of their additivity: the age-specific fertility rate is the sum of the incidence rates for the different classes. This is not the case with intensities, since the sum of intensities for different parities does not make sense. One would need to convert them to incidence rates, using [1] or a life table distribution and add them up. This additive property also extends to the calculation of the *TFR* as we will discuss in Section devoted to live table measures of fertility.

When both intensities and incidence rates are available, one can choose between them according to the purpose at hand. The problem is that intensities require more data: class-specific exposure is required and this is not always readily available. This measurement issue very often explains why fertility incidence rates are used. Of the two factors needed in the calculation, births and exposure, births are the most widely available.

Vital statistics often provide a decomposition of births according to age and a number of characteristics including birth order or education. A connected choice is what kind of rate to calculate within the Lexis diagram: age-period, age-cohort, or cohort-period. The most common combinations in fertility studies are age-period and cohort-period. Not all countries provide the triple age-cohort-period classification (the Lexis diagram triangles) in their vital statistics, so this is not always a matter of choice. Where available, cohort-period parallelograms may be preferable since a unique cohort can be followed making them more apt for projection purposes.⁽¹¹⁾ The cross-classification of births is less of a problem when the micro data regarding births is available. This is becoming more common. It allows the researcher to cross-classify births as required. The only limitation is that the number of subclasses grows exponentially with the number of dimensions, which is the so-called “dimensionality curse”.

Regarding exposure, $E(a)$ is not generally known but it can be estimated. The most common estimates are mid-year populations or a population half-count (Wunsch, 2001b). These data are generally available from the intercensal population reconstructions made by statistical agencies. Using cross-classified births and aggregate exposure, fertility incidence rates can be calculated for many populations.⁽¹²⁾ The calculation of fertility intensities requires the additional knowledge of exposure specific to the different subgroups. This is usually problematic since vital statistics generally do not provide all the necessary data. In particular, death and migration statistics are not generally cross-tabulated according to the same

(11) On these issues and how to convert between the different rates see Calot (1984 a). Note also the possibility of using fertility probabilities instead of intensities. Here the denominator would be the population at the beginning of the period.

(12) A common limitation is that in some countries only birth-order within marriage is known. This limits the parity-specific analyses that can be done (Keilman, 1993).

criteria as births, in particular parity. In order to reconstruct the population it is usually assumed that mortality and migration are independent from parity.⁽¹³⁾ It is then possible to reconstruct the flows since, besides migration and mortality, the inflows into the parity j and age a category are the women of age $a - 1$ in the previous year who were of parity j and the women who had a birth of order j during the period. The outflows are the women of age a that had a birth of order $j + 1$ (Calot, 2001 c). This can be more reliable when combined with census information in the reconstruction of the intercensal population.⁽¹⁴⁾ The reconstruction of population is only the first step in the estimation of exposure. Exposure is usually measured again through the mid-year population or the population half-count. In general, as we can see, the reconstruction of exposure for subclasses is more problematic from the perspective of data quality.

While traditional measurement is based on vital statistics and population reconstruction, there are other possibilities. The ideal system may be registration as it is carried out in the Scandinavian countries. This covers the entire population, making it possible to estimate both births and exposure accurately according to the desired characteristics. A good alternative is the use of very large retrospective surveys, say over 100,000 women.⁽¹⁵⁾ Smaller surveys such as the FFS are not useful in this respect since the sample size is too small to estimate parity- and age-specific rates. Based on this micro data, it is possible to estimate directly either the intensities or the incidence rates, although it is generally the intensities or the probabilities that are obtained. The advantage of using individual data is that it allows the researcher to choose the relevant dimensions with sample size being the only limitation. An example of this is the estimation of age, parity and duration-specific rates as in Rallu and Toulemon (1993 a), where the number of categories grows to several thousand. In some of these categories, exposure is very limited, leading to rates with large variance. The alternative to the calculation of separate rates for each group is the use of a hazard regression model where the intensity is modeled as a function of a vector of characteristics (Hobcraft and Casterline, 1983; Hoem and Hoem, 1989; Lutz, 1989; Andersson, 1999, 2000). This eliminates the degrees of freedom problem at the cost of introducing a model that assumes some kind of additive effects of the covariates.

(13) See Hoem (1970), Finnäs (1980) and Borgan and Ramlau-Hansen (1985) for joint statistical modeling of fertility, migration and mortality and conversion of incidence rates or incidence rates to intensities.

(14) In this context it is important that censuses keep asking about children-ever-born. In some countries this question has been withdrawn on the assumption that fertility surveys are sufficient (INE, 2001). They are not, since they do not have enough sample size to provide reliable estimates of the very small proportions of women in some age and parity categories. A connected problem with intercensal reconstruction is that the results tend to be different depending on whether backward or forward reconstruction is used. This is connected with violations of the independence assumption.

(15) Examples of such surveys are the INSEE Enquête Famille in France (Rallu and Toulemon, 1993 b), the Encuesta Sociodemográfica in Spain (Requena, 1997), China's one per thousand fertility survey (Feeney and Yu, 1987) or Russia's 5% micro census (Scherbov and van Vianen, 2001).

Tempo effects on incidence rates and intensities

Fertility trends are the result of childbearing at the individual level. Here, two dimensions are relevant: how many children are given birth to, and at what age. At the aggregate level these two dimensions are intertwined: when there are shifts in the age at which childbearing takes place, the date at which childbirth occurs shifts as well. This means that the number of births taking place in a given period when there are age shifts in the fertility schedule is different from the number of births that would have occurred in the absence of an age shift. This is the basic idea behind *tempo effects*. Tempo effects are defined as the proportion by which fertility changes in the presence of age shifts. We are interested in tempo effects because, on the basis of aggregate data, they allow us to separate the two dimensions that at the individual level are evidently different: how many and when.

The history of tempo effects has been relatively short: Bongaarts and Feeney introduced the concept in 1998. They estimated these effects from the change in the mean ages at childbearing for different parities in successive years. They proposed the use of Ryder's translation formula for a linear case, which applies a factor $1/(1 - r)$ to the period *TFR*. In B – F reinterpretation, r is the pace at which the period parity-specific mean age at childbearing is shifted.⁽¹⁶⁾ While a useful approach, and a useful formula as a first approximation, some of its problems have led to reformulations.

A first problem lies in the assumption that age shifts are equal for all ages. This means that the B – F formula is valid only for parallel shifts in the fertility schedule. This is not generally the case. While the practical consequences of deviations from this pattern with regard to *TFR* adjustment may not be very important in most cases (Yi and Land, 2000) there is no guarantee for this. More importantly, when age shifts are not parallel the errors at different ages are partially cancelled out in the overall *TFR*, but the procedure is inadequate for adjusting each of the age-specific fertility rates separately. It is therefore important to develop adjustment formulas that allow for more general shifts in the fertility schedule. Kohler and Philipov (2001, henceforth K – P) have done this for a fairly general family of shifts, and have developed a procedure for adjusting rates in the case of variance changes. The resulting formula can be seen as a generalization of the B – F procedure, where each one of the age- and parity-specific fertility incidence rates, $f_j(a)$, is adjusted using an age- and parity-specific tempo effect, $r_j(a)$. The adjusted fertility rates are thus given by:

$$f_j(a) = f_j(a) / [1 - r_j(a)] \quad [2]$$

Their article provides the formula linking the tempo effects, $r_j(a)$, to the change in the mean age at childbearing, γ , and the proportional change in the standard deviation, δ . They also propose an iterative procedure to estimate the overall adjusted *TFR* taking into account variance effects.

A problem common to B – F and K – P is the use of fertility incidence rates for the estimation of tempo effects. Kohler and Ortega (2002 a, henceforth K – O)

(16) Hobcraft (1996) carried out very similar calculations in applying Ryder's method to England.

address this very issue by using fertility intensities instead of incidence rates. In order to understand their approach, it is helpful to first consider the different concepts of mean and variance that can be calculated from fertility incidence rates and intensities. It then becomes possible to see why inference based on incidence rates is wrong, and this in turn enables us to see how to estimate tempo effects which are free of compositional biases. The basic idea is to measure tempo effects only from the fertility behavior for the parity of interest.

What is usually referred to as the mean age at birth and variance for a particular birth-order is based on the fertility incidence rate schedule. If we are using cohort-period incidence rates and age attained during the year the expressions for the mean, μ_C^F , and variance, Var_C^F , are respectively⁽¹⁷⁾:

$$\begin{aligned} \mu_C^F &= \frac{\sum_a a \cdot f_c(a)}{\sum_a f_c(a)} \\ \text{Var}_C^F &= \frac{\sum_a (a - \mu_C^F)^2 \cdot f_c(a)}{\sum_a f_c(a)} \end{aligned} \quad [3]$$

We have already shown that fertility incidence rates at a given parity are the result of the combination of present fertility (as measured by intensities) and past fertility (as present in the parity composition of the population). Because of this, we cannot base our estimate of changing fertility behavior on the incidence rates. Two examples might clarify the kind of compositional effects that are present in mean ages calculated from incidence rates. First, let us consider that in previous years there has been a delay in first births combined with a reduction in quantum. This is a common scenario in many countries. Let us assume that from a base year onwards no further changes occur. The result is that since many women had their first births when the rates were higher and births took place earlier, the proportion of women at parity zero at older ages is out of equilibrium with relatively few women in that category. As we move towards the future, even if there is no further change in the birth intensities, the proportions of older women at parity zero will increase and there will be a shift in the mean age at birth for parity one and above. This would seem to indicate a tempo effect where there is none. Take a second example: let us assume that the quantum of first-birth falls dramatically from one year to the next, while higher parities continue unaltered. This will lead to fewer young women entering parity one. Over the following years, the mean age at second birth will increase, but this is not due to a change in behavior. Again this would be taken within B – F adjustment as a tempo effect in second births. Similar arguments could be drawn up for the calculation of variances.

A second commonly used mean age (and variance) comes from the equilibrium distribution of women by parity. We will call this the mean age at birth of the stable distribution, μ^S . This may be seen as a special application of the previous

(17) When using age-period or age-cohort incidence rates we have to add 0.5 to the mean age. When using age at the beginning of the year we have to add 1.

formula where the parity composition of the population instead of the current one is assumed to be the equilibrium one. Still, the dependence of the equilibrium distribution on the intensities at all parities is undesirable. Take for instance a postponement of first birth with no change in quantum. The next year the equilibrium population distribution will have relatively fewer young women at high parities. This means that the mean age at second and higher order births in the stationary distribution will increase even if there has been no change in behavior at those parities. A possible solution would be the use, for the purpose of estimating tempo effects, of the stationary distribution corresponding to the first year (or the second), to estimate the mean ages at birth in both years. While this would solve the problem of a changing parity distribution, it is still not entirely appropriate, since the tempo effects we want to remove are present in the intensities. Therefore the stable distribution associated with the unadjusted intensities is different from the stable distribution associated with the tempo adjusted rates. While a possible iterative system could be worked out to solve this conundrum of jointly adjusting the rates and estimating the mean ages, a simpler solution is proposed by K – O: to use the mean age at birth calculated from the intensity schedule. The corresponding formulas are the same as [3] where the incidence rates, $f_c(a)$, are replaced by the intensities, $m_c(a)$. We call these expressions the mean age at birth and the variance of the intensity schedule (μ^i and Var^i). They correspond to the mean age and variance that would be observed in a population where the distribution of women according to parity is perfectly uniform. This is obviously not realistic, but it serves the purpose of estimating the tempo effects. A surprising result of the application of this mean age is that the usual sequence in mean ages according to birth-order is generally absent: it is possible, for instance, for the mean age at third birth to be lower than the mean age at second birth. This indicates that the profile of birth probabilities might be as young for third births as for second ones. While this choice is natural given the use of fertility intensities, as an alternative it is possible to use a different fixed distribution to calculate mean ages and variances.

A second aspect in the estimation of tempo effects is the presence of variance effects. Again, if the shifts of the fertility schedule are parallel, one could use the shifts in the mean age as the estimate of tempo effects. When the shifts are not parallel and variance changes as in the K – P model, the mean age and the variance computed from the observed schedule are affected by tempo distortions. An iterative procedure is needed to derive the mean age and variance that would have been observed in the absence of tempo effects. The procedure is essentially the same as K – P with the exception that K – O apply it to the intensity schedule. Once tempo and variance changes are estimated, the adjustment formulas are given by:

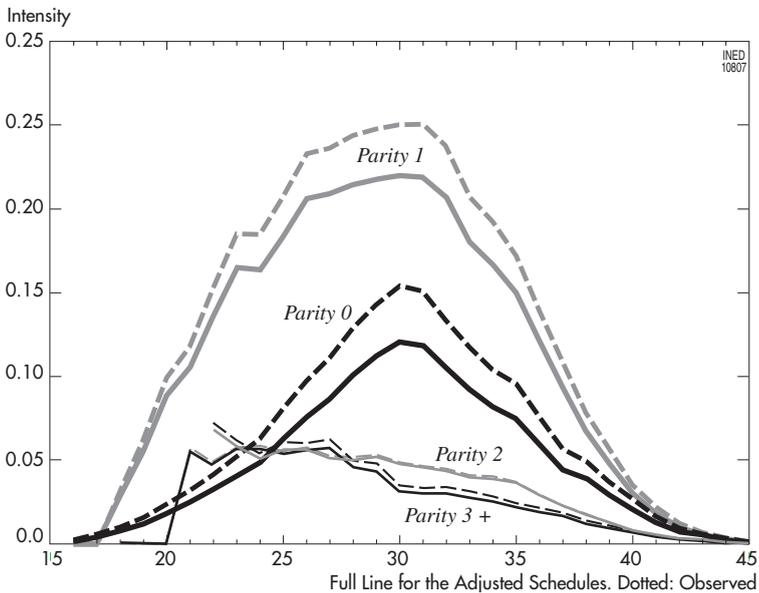
$$\begin{aligned}
 m_j'(a) &= \frac{m_j(a)}{1 - r_j(a)} \\
 r_j(a) &= \gamma_j + \delta_j(a - \bar{a})
 \end{aligned}
 \tag{4}$$

Where γ_j is the overall tempo change (the increase in the mean age of the adjusted intensity schedule), δ_j is the increase in the log of the standard deviation,

and \bar{a}_j is the mean age of the adjusted schedule. As we can see, older ages are more strongly adjusted when the variance is increasing and vice versa. The iterative procedure designed by K – O consists of using the adjustment formulas [4] based on the observed intensity schedule using a smoothing procedure to estimate γ and δ from time series of mean ages and standard deviations. From the adjusted schedules, new time series of γ , δ and \bar{a}_j are obtained. The process continues until convergence is reached.

As an example of the adjustment formulas we can take the particular case of Sweden in 1998.⁽¹⁸⁾ Table 1 shows the observed fertility intensities for births of order 1, 2, 3 and the incidence rates for births of order 4 and above. The analysis in K – O provides the estimates for gamma and delta as given in the last rows of the table. The \bar{a}_j , also from K – O, are the mean ages of the adjusted intensity schedule. The application of the adjustment formulas [4] leads to the particular age- and parity-specific tempo effects in Table 1 from which we can then obtain the adjusted intensity schedules in the last columns of Table 1. The table also provides the sums (the cumulated intensity) and the mean ages of the intensity schedules. Figure 1 plots the schedules. The cumulated intensity corresponds to the quantum index used by K – O. We see that the adjustment is particularly intense for parity zero and especially parity one. Variance effects are stronger for parities one and three and above. For those parities the range of tempo effects is therefore wider. Note that the tempo effects can be interpreted as the percentage by which each observed intensity rate must be adjusted to remove tempo distortions.

Figure 1 – Adjusted and Unadjusted Intensity Schedules Sweden 1998



(18) The original data come from Andersson (2001). They have been transformed from period-cohort form to age-period by means of smoothing splines.

Table 1 – Observed and Adjusted Intensities. Sweden 1998. Age-Period data

Age\Parity	Intensities × 1000				r(a,t) × 100				Adjusted Intensities × 1000			
	0	1	2	3+	0	1	2	3+	0	1	2	3+
16	1.6				21.6				2.1			
17	3.9				21.6				5.0			
18	7.5	29.6			21.5	9.7			9.6	32.8		
19	11.9	55.4			21.5	9.9			15.2	61.5		
20	18.4	89.0			21.5	10.1			23.5	99.0		
21	25.0	106.0	57.0		21.5	10.4	-3.2		31.8	118.2	55.2	
22	32.4	136.8	48.9	71.9	21.5	10.6	-3.2	-6.2	41.2	153.0	47.4	67.8
23	40.8	164.8	58.0	61.9	21.5	10.8	-3.1	-6.7	52.0	184.8	56.3	58.0
24	49.0	164.2	58.4	54.4	21.5	11.0	-3.0	-7.3	62.4	184.5	56.7	50.7
25	63.3	183.6	55.6	60.7	21.4	11.2	-3.0	-7.8	80.5	206.8	54.0	56.3
26	76.1	206.7	57.6	60.6	21.4	11.4	-2.9	-8.4	96.9	233.3	56.0	55.9
27	87.1	209.0	52.7	62.2	21.4	11.7	-2.8	-9.0	110.8	236.6	51.2	57.1
28	101.6	214.8	51.5	49.9	21.4	11.9	-2.8	-9.5	129.3	243.8	50.1	45.6
29	112.4	217.7	52.9	48.2	21.4	12.1	-2.7	-10.1	143.0	247.6	51.5	43.8
30	121.0	219.6	49.0	34.5	21.4	12.3	-2.6	-10.6	153.9	250.4	47.7	31.2
31	118.5	219.1	46.4	33.1	21.4	12.5	-2.6	-11.2	150.6	250.5	45.3	29.8
32	104.8	207.1	44.6	33.9	21.3	12.7	-2.5	-11.7	133.2	237.3	43.5	30.4
33	92.5	180.1	40.7	31.4	21.3	12.9	-2.5	-12.3	117.6	206.9	39.7	27.9
34	81.6	166.7	39.9	28.1	21.3	13.2	-2.4	-12.9	103.7	192.0	39.0	24.9
35	75.2	149.1	36.6	23.9	21.3	13.4	-2.3	-13.4	95.5	172.1	35.8	21.1
36	59.3	120.3	29.8	21.7	21.3	13.6	-2.3	-14.0	75.3	139.2	29.1	19.1
37	44.4	92.9	22.7	18.7	21.3	13.8	-2.2	-14.5	56.4	107.7	22.2	16.3
38	38.8	67.1	17.4	13.7	21.3	14.0	-2.1	-15.1	49.2	78.1	17.1	11.9
39	29.3	48.0	12.3	10.8	21.2	14.2	-2.1	-15.6	37.1	56.0	12.1	9.3
40	21.0	29.9	8.7	7.8	21.2	14.5	-2.0	-16.2	26.7	35.0	8.5	6.7
41	12.7	18.2	4.5	4.9	21.2	14.7	-1.9	-16.7	16.2	21.3	4.4	4.2
42	7.1	9.8	2.7	3.3	21.2	14.9	-1.9	-17.3	9.0	11.6	2.7	2.8
43	4.9	5.2	1.7	2.8	21.2	15.1	-1.8	-17.9	6.2	6.1	1.6	2.3
44	2.6	2.8	0.4	1.4	21.2	15.3	-1.7	-18.4	3.3	3.3	0.4	1.2
45	0.8	1.2	0.5	1.3	21.2	15.5	-1.7	-19.0	1.0	1.4	0.5	1.1
Cum. Intensities	1.450	3.310	0.850	0.740					1.840	3.770	0.830	0.680
Mean Ages	30.74	29.48	29.22	29.05					30.74	29.54	29.23	28.92
Gamma x 100	21.37	11.98	-2.76	-9.46								
Delta x 10000	-1.44	21.57	6.44	-55.63								

Source: Andersson (2001) for the cohort-period observed intensities, Kohler and Ortega (2001) for the gamma and delta parameters. Own calculations.

Life table measures of fertility

Life table measures are probably *the* central tool of demographic analysis. Most of the commonly used measures in demography such as life expectancy, total fertility rate, parity progression ratios, net reproduction ratios, can be interpreted as life table measures. Period life table measures are synthetic measures, since they do not refer to a real cohort but to a synthetic one that experienced the period rates over the lifetimes (Vallin and Caselli, 2001). Life table measures thus provide a unifying framework for the study of both period and cohort demographic indicators. In this section, we will briefly review the different life table methods available for the study of fertility through parity and age.⁽¹⁹⁾ We will call these tables fertility tables.

Like Feeney and Yu (1987), we can distinguish two kinds of fertility tables: additive and multiplicative. Additive tables are based on age-specific fertility incidence rates. They were first used by Böckh (1899) in the Berlin Statistical Yearbook.⁽²⁰⁾ They are called additive because the total fertility measure, the parity-specific *TFR*, is arrived at by adding up fertility rates for a given parity. However, this implies treating parity-specific births as if they were repeatable events, which they are not (Henry, 1972; Keilman 1994). Adding the parity-specific *TFRs*, the general *TFR* is obtained. This also applies when other dimensions besides parity are considered. In general the class-specific *TFR*, TFR_c is given by:

$$TRF_c = \sum_a f_c(a)$$

The overall *TFR* is then obtained either from the addition of the different class-specific *TFRs*, or from the addition of the overall fertility incidence rates:

$$TRF_c = \sum_c TRF_c(a) = \sum_a f(a)$$

(19) There are alternative fertility life tables which use other relevant dimensions such as birth interval and parity (Feeney, 1983; Feeney and Yu, 1987; Nì Bhrolcháin, 1987), birth interval, parity and age jointly (Rallu and Toulemon, 1993a), age and marital status (Farr, 1880, Ansell, 1874), parity only (Chiang and van der Berg, 1982; Lutz, 1989), age, parity and marital status (Whelpton, 1946; Oechsli, 1975). Also, we are dealing with pure fertility tables, while many early examples of fertility life tables were combined with mortality, leading to reproduction tables. Systematic work on these tables stems from Kuczynski (1928, 1932), although earlier examples can be found in Farr (1880), or Böckh (1886 and other years). See Stolnitz and Ryder (1949) and Lewes (1984) for a survey and a historical note.

(20) For a number of years Richard Böckh tabulated births according to parity and age but did not standardize the figures. In 1899 he standardized them using the age distribution of women thereby obtaining fertility incidence rates for the first time. Kuczynski, who was a student of Böckh at the Berlin statistical office, popularized the measures in England and the United States (Kuczynski, 1928, 1932; an example of the impact of his work is Glass and Blacker (1938), and we owe to him many of the terms used in fertility analysis like total fertility. However, he neglected the analysis of fertility by parity. On the other hand, when Charles (1937) consulted him during his exile in England regarding the study of fertility by birth order, he suggested the use of fertility incidence rates in what was to be the comeback of additive fertility tables which would be followed by Lotka and Spiegelman (1940). After WW2 Henry and Pressat took the distinction between rates of the first and the second kind.

This simplicity is both the strength and the weakness of additive life table measures. Since the *TFR* is merely a sum of rates, it can be regarded as a simple summary measure of fertility that takes into account fertility at all ages with equal weight. This simplicity makes period *TFR* time series quite volatile, which is a strong point of the measure.⁽²¹⁾ A second strong point is its relatively lax data requirements. Its first weak point is the lack of built-in demographic logic. As we have seen, incidence rates do not reflect the risk of giving birth for any particular women. They are influenced by the parity distribution of women at each age. When these tables are applied to a cohort, they acquire some demographic consistency, since they track the same group of women – or men – over time. Applied to period data, this consistency is lacking, and it is not unusual to find first-birth period specific *TFRs* of higher than one (Feeney and Yu, 1987; Hobcraft, Menken and Preston, 1982; Keilman, 1994).⁽²²⁾ This is not a problem if one interprets it as a sign of a very favorable – and inherently unstable – parity distribution together with high fertility for that order rather than just as an impossible fertility quantum. It is necessary to be aware of which application can legitimately be used for each measure. An example of an additive table is given in Table 2 where use is made of the tempo adjusted fertility incidence rates for Sweden, using the K – O procedure and the conversion formula [1]. We see that the tempo adjusted parity-specific *TFRs* are considerably larger than the observed ones for parities zero and one. This is particularly so for the first birth, where the observed rates would suggest a childlessness of almost 40% as compared with 20% from the adjusted rates. Allowing for the different parities, then, the overall adjusted *TFR* of 1.67 compares to the observed 1.43.⁽²³⁾ We can also see the mean ages for the incidence rate schedule. As we have commented in the previous section, these are higher with parity. There are no great differences in this case between the mean ages for the adjusted and observed schedule. It is also possible to estimate an implied parity distribution from the difference between the *TFRs* for different parities (Charles, 1937). This measure should be avoided when based on period data since tempo distortions and parity composition can lead to very erroneous estimates.

Multiplicative life tables treat births of a particular order as a non-repeatable event. Since this is precisely what they are, this brings some demographic consistency into the life table calculations. They are based on fertility intensities. Having been first used by Quensel (1939) and Whelpton (1946), they were revived in their parity and age form by Park (1976), Lutz (1989), Rallu and Toulemon (1993a), Giorgi (1993) and de Simoni (1995). In their classic form, they assume that the

(21) The volatility of the period *TFR* is desirable in that it makes it, at least potentially, easier to find the determinants of change. See Ní Bhrolcháin (1992) for a similar position. Ryder (1980, 1986) instead, sees this as an inconvenient.

(22) This is also a possibility in cohort parity-specific life tables when there is parity selective mortality or migration. Let us assume for instance that, for some reason, there is a one-time very high emigration of childless women in a particular country. Cohort second-order *TFR* might then well be higher than one since the population composition by parity has experienced a change with greater weight attached to parity-one women, who are the ones contributing to parity two births.

(23) This figure is somewhat below the published figure of 1.50 since certain categories of birth have been excluded, like those born to foreign women (see Andersson, 1999).

Table 2 – Observed and Adjusted Frequencies. Sweden 1998. Age-Period data

Age\Parity	Frequencies × 1000				Adjusted Freq. × 1000			
	0	1	2	3+	0	1	2	3+
16	1.62				2.07			
17	3.91				4.99			
18	7.46	0.25			9.51	0.28		
19	11.71	1.00			14.92	1.11		
20	17.73	2.90			22.59	3.22		
21	23.52	5.52	0.43		29.96	6.15	0.42	
22	29.20	10.76	0.82	0.09	37.19	12.03	0.80	0.09
23	35.09	17.34	1.99	0.11	44.68	19.44	1.93	0.11
24	39.66	21.49	3.09	0.35	50.49	24.15	3.00	0.33
25	46.77	29.32	5.07	0.58	59.54	33.03	4.93	0.54
26	50.93	37.70	7.45	1.23	64.82	42.56	7.24	1.13
27	51.16	42.82	9.41	1.72	65.10	48.46	9.15	1.58
28	51.52	47.24	11.86	2.19	65.55	53.60	11.54	2.00
29	48.90	49.19	14.88	2.78	62.19	55.95	14.49	2.53
30	44.39	48.95	16.03	2.83	56.45	55.82	15.61	2.55
31	37.38	46.80	17.23	3.35	47.53	53.50	16.80	3.02
32	29.07	41.33	17.77	4.21	36.95	47.36	17.33	3.77
33	22.68	33.89	17.07	4.62	28.83	38.93	16.67	4.12
34	18.11	29.32	17.29	4.75	23.01	33.76	16.89	4.21
35	15.58	24.32	16.11	4.50	19.80	28.07	15.75	3.97
36	11.72	18.89	13.17	4.42	14.88	21.86	12.88	3.88
37	8.20	13.93	10.13	4.07	10.41	16.16	9.91	3.55
38	6.76	9.78	7.78	3.20	8.58	11.38	7.62	2.78
39	4.88	7.19	5.47	2.57	6.19	8.38	5.36	2.23
40	3.36	4.44	3.86	1.92	4.26	5.20	3.78	1.66
41	2.04	2.70	2.00	1.20	2.59	3.16	1.96	1.03
42	1.11	1.51	1.22	0.81	1.41	1.78	1.20	0.69
43	0.75	0.80	0.76	0.66	0.96	0.94	0.74	0.56
44	0.39	0.43	0.19	0.34	0.50	0.51	0.19	0.29
45	0.11	0.18	0.25	0.30	0.15	0.22	0.25	0.25
TFRj	0.6257	0.5500	0.2013	0.0528	0.7961	0.6270	0.1964	0.0468
Mean Ages	28.08	30.38	32.69	34.42	28.07	30.43	32.71	34.33

Source: Andersson (2000) for the cohort-period observed frequencies. Own calculations as explained in text.

intensities are piecewise constant within the respective age intervals. However, it is also possible to disregard this assumption and work in continuous time. We refer the reader to K – O for such a context. The name of the tables stems from the multiplicative nature of the intensities. The basic measure we will use is the proportion of women of parity j at age a_0 experiencing at least n additional births between exact ages a_0 and a_1 : ${}_n P_j(a_0, a_1)$. These proportions are particularly simple to calculate for one additional birth:

$${}_1P_j(a_0, a_1) = 1 - \prod_{a=a_0}^{a_1-1} \exp[-m_j(a)] = 1 - \exp\left[-\sum_{a=a_0}^{a_1-1} m_j(a)\right] \quad [5]$$

where we see the multiplicative nature of the intensities. When the proportion is calculated in the remaining lifetime, we have Park's (1976) lifetime probability of n additional births, ${}_n P_j(a_0)$. The argument a_0 can also be dropped when the calculation is carried out through the whole reproductive age span. K – O very often use the measure ${}_1 P_j(a)$, which they call conditional parity progression probability at age a . As we have mentioned, it is generally the case that quantum measures based on multiplicative life tables are more stable than those based on additive measures. This is so because the additive measures only reflect the sum of the rates: when there are very high but transitory rates at several ages, they are simply added up. By contrast, the weight given to a particular age in a multiplicative quantum measure depends negatively on the quantum itself. For instance, the first derivative of the parity progression ratio for parity 0 is given by:

$$\frac{\partial {}_1 P_0}{\partial I_1(a)} = -\exp(-\sum m_1(a)) = 1 - {}_1 P_0$$

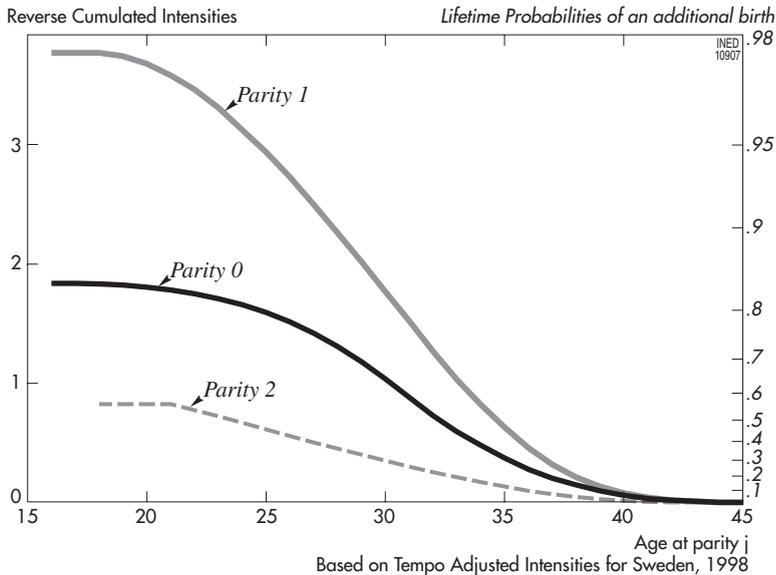
That is: the derivative is equal to the proportion of childless women in the table. We see then how the effect of a single intensity is muted according to how close the quantum measure is to the maximum. This means, for instance, that whenever fertility is very high at young ages, the weight given to older ages diminishes. This is a strong point when using cohort data, but it may be a disadvantage when we are only interested in a pure period measure. In such a case it is possibly more interesting to use cumulated intensities which are additive and can be converted in probabilities through a direct transformation. Figure 2 is a useful graphical representation of the reversed cumulated intensities. As suggested by Toulemon (2001 b), a double-scale graph of the cumulated intensities in the second scale shows the lifetime probability measure, ${}_1 P_j(a)$.

Fertility tables are useful for the organization of the fertility intensities and the computation of summary measures. They are increment-decrement tables where the state variable is parity, and access to each parity requires previous transition through the lower states. In order to build the tables, a first step is to calculate the transition probabilities from the intensities. This is a relatively tricky issue. On the one hand, there is the possibility of using the formulas based on Markov chain theory as in Hoem and Jensen (1982), alternatively, standard simplifications such as those described by Schoen (1988) or Palloni (2001) may be used. The problem is that these formulas do not easily capture the particular context of the birth sequence. The simplest approach is probably the direct estimation of birth probabilities instead of intensities as in Rallu and Toulemon (1993b), or the use of the simple exponential formula which is then compatible with [5].⁽²⁴⁾ In such a case we would get the age- and parity- specific probability of birth, $q_j(a)$ as:

$$q_j(a) = 1 - \exp[-m_j(a)] \quad [6]$$

(24) An interesting alternative is to apply the exponential formula to six month age intervals obtained by duplication of the rates. This would allow for two transitions within a year, but no more, in correspondence with the biological limitations.

Figure 2 – Reverse Cumulated Intensities and Probabilities of an Additional Birth



We can then use the birth probabilities to work out the remaining life table measures. For the general fertility table, the remaining columns are the number of women of parity j and exact age a , $D_j(a)$, and the number of births occurring to those women at age a , $b_j(a)$. These quantities are calculated iteratively over age from the formulas:

$$b_j(a) = D_j(a) q_j(a) \tag{7}$$

$$D_j(a + 1) = D_j(a) + b_{j-1}(a) \tag{8}$$

It is common that the last parity category, J , includes parities J and above, the rates being fertility incidence rates for this group. The formulas are then slightly different for this last parity:

$$b_j(a) = D_j(a) f_j(a)$$

$$D_j(a + 1) = D_j(a) + b_{j-1}(a)$$

Note also the initial conditions:

$$D_0(a) = N; D_j(a) = 0, j > 0$$

where N is the radix of the table which is equal to the size of the synthetic cohort. Table 3 shows the general fertility table based on the tempo adjusted intensities for Sweden in 1998.

Table 3 – Fertility Table. Tempo Adjusted Intensities, Sweden 1998

Age/ Parity	Women				Probabilities × 1000				Births			
	0	1	2	3+	0	1	2	3+	0	1	2	3+
16	1000.00	0.00	0.00	0.00	2.07	0.00	0.00	0.00	2.07	0.00	0.00	0.00
17	997.93	2.07	0.00	0.00	5.00	0.00	0.00	0.00	4.99	0.00	0.00	0.00
18	992.95	7.05	0.00	0.00	9.56	32.28	0.00	0.00	9.49	0.23	0.00	0.00
19	983.45	16.32	0.23	0.00	15.08	59.62	0.00	0.00	14.84	0.97	0.00	0.00
20	968.62	30.18	1.20	0.00	23.17	94.25	0.00	0.00	22.45	2.84	0.00	0.00
21	946.17	49.78	4.05	0.00	31.34	111.48	53.74	0.00	29.66	5.55	0.22	0.00
22	916.52	73.89	9.38	0.22	40.36	141.89	46.29	67.76	36.99	10.48	0.43	0.01
23	879.53	100.40	19.43	0.65	50.66	168.71	54.71	58.03	44.56	16.94	1.06	0.04
24	834.97	128.02	35.30	1.71	60.48	168.47	55.13	50.71	50.50	21.57	1.95	0.09
25	784.47	156.94	54.92	3.66	77.36	186.85	52.54	56.30	60.69	29.32	2.89	0.21
26	723.79	188.31	81.36	6.55	92.33	208.11	54.48	55.89	66.83	39.19	4.43	0.37
27	656.96	215.94	116.12	10.98	104.89	210.68	49.91	57.09	68.91	45.50	5.80	0.63
28	588.05	239.36	155.82	16.77	121.29	216.32	48.85	45.60	71.32	51.78	7.61	0.76
29	516.73	258.90	199.98	24.39	133.25	219.32	50.16	43.78	68.86	56.78	10.03	1.07
30	447.87	270.98	246.73	34.42	142.62	221.53	46.61	31.22	63.87	60.03	11.50	1.07
31	384.00	274.82	295.26	45.92	139.84	221.57	44.25	29.77	53.70	60.89	13.07	1.37
32	330.30	267.63	343.09	58.99	124.73	211.20	42.61	30.38	41.20	56.52	14.62	1.79
33	289.10	252.30	384.99	73.60	110.93	186.86	38.94	27.93	32.07	47.15	14.99	2.06
34	257.03	237.23	417.15	88.60	98.53	174.69	38.21	24.92	25.32	41.44	15.94	2.21
35	231.71	221.11	442.65	104.54	91.09	158.10	35.11	21.05	21.11	34.96	15.54	2.20
36	210.60	207.26	462.06	120.08	72.52	129.98	28.68	19.05	15.27	26.94	13.25	2.29
37	195.33	195.59	475.75	133.33	54.88	102.12	21.92	16.34	10.72	19.97	10.43	2.18
38	184.61	186.34	485.29	143.76	48.01	75.10	16.92	11.88	8.86	13.99	8.21	1.71
39	175.74	181.21	491.07	151.98	36.46	54.41	12.02	9.30	6.41	9.86	5.90	1.41
40	169.34	177.75	495.03	157.88	26.31	34.37	8.48	6.70	4.46	6.11	4.20	1.06
41	164.88	176.10	496.94	162.07	16.03	21.05	4.38	4.20	2.64	3.71	2.18	0.68
42	162.24	175.04	498.47	164.25	8.91	11.49	2.68	2.83	1.45	2.01	1.34	0.47
43	160.79	174.47	499.15	165.59	6.18	6.07	1.64	2.34	0.99	1.06	0.82	0.39
44	159.80	174.41	499.39	166.41	3.25	3.24	0.42	1.21	0.52	0.57	0.21	0.20
45	159.28	174.36	499.74	166.62	1.00	1.37	0.54	1.07	0.16	0.24	0.27	0.18
46	159.12	174.28	499.71	166.88								
Sum per woman									0.841	0.667	0.167	0.024
Mean Age									28.22	30.93	33.5	35.11
Source: Own calculation based on intensities in Table 1.												

One of the advantages of the fertility table is that many summary measures can be constructed directly from the table births. For instance, mean numbers of births for women in the synthetic cohort can be defined by rectangular sums of births in the table (de Simoni, 1995) as:

$$b_{j_1, j_2}(a_0, a_1) = \sum_{a=a_0}^{a_1} \sum_{j=j_1}^{j_2} b_j(a) \quad [9]$$

In particular, the completed fertility of the fertility table corresponds to $b_{0,j}(\alpha, \omega) / N = b_{0,j}(\alpha) / N$. In our case, the value is 1.7. This is the index called *PATFR* by Rallu and Toulemon (1993a), meaning that it is the index of total fertility obtained from a parity and age fertility table. This index is free from compositional effects (because it is based on the table distribution of women) and from tempo distortions (because it has been computed from tempo adjusted intensities). The cumulated sums $CF(a) = b_{0,j}(\alpha, a - 1) / N$ are also a useful measure since they provide the cumulated fertility before age a . They are commonly used especially in cohort studies, where they are referred to as incomplete cohort fertility. From the sum of columns corresponding to each parity, the parity-specific total fertility indexes can be obtained, $PATFR_j$. This can be interpreted as the proportion of women in the synthetic cohort that had at least $j + 1$ children. They are shown as the last row of the birth column in Table 4. They also correspond to Park's lifetime probabilities of $j + 1$ births. In this example it appears that both the proportion of women having an additional birth at parity 0 and 1 are very high, with less than 16% childlessness and more than two thirds of the women having at least 2 children. In contrast, the parity 2 index is very small, below 17%. The differences between the $PATFR_j$ and the TFR_j obtained in the additive table, Table 2, are the result of the parity distribution of women in 1998 being different from that of the stable distribution. Since the values based on the multiplicative table are higher than those for the additive table for low parities (0 and 1) and lower for high parities (2, 3 and more), this shows that the proportion of women in the lower parities is surprisingly low. This is the result of the roller coaster fertility phenomenon that took place in Sweden during the early 1990s, when the rates for the first and second births were high for a few years (e.g., see Hoem and Hoem, 1996). Fertility beyond the third birth is almost negligible.

Table 4 – Specific Fertility Table for a woman age 35 and parity 1

Age\ Parity	Women			Probabilities × 1000			Births		
	1	2	3+	1	2	3+	1	2	3+
35	1.0000	0.0000	0.0000	158.097	35.115	21.047	0.1581	0.0000	0.0000
36	0.8419	0.1581	0.0000	129.980	28.680	19.050	0.1094	0.0045	0.0000
37	0.7325	0.2630	0.0045	102.121	21.925	16.341	0.0748	0.0058	0.0001
38	0.6577	0.3320	0.0103	75.103	16.921	11.880	0.0494	0.0056	0.0001
39	0.6083	0.3758	0.0159	54.412	12.019	9.301	0.0331	0.0045	0.0001
40	0.5752	0.4044	0.0204	34.366	8.476	6.700	0.0198	0.0034	0.0001
41	0.5554	0.4207	0.0239	21.046	4.385	4.198	0.0117	0.0018	0.0001
42	0.5437	0.4306	0.0257	11.491	2.681	2.833	0.0062	0.0012	0.0001
43	0.5375	0.4357	0.0269	6.071	1.639	2.338	0.0033	0.0007	0.0001
44	0.5342	0.4382	0.0276	3.245	0.421	1.212	0.0017	0.0002	0.0000
45	0.5325	0.4398	0.0278	1.366	0.535	1.071	0.0007	0.0002	0.0000
46	0.5318	0.4403	0.0280						
Sum							0.4682	0.0280	0.0008
Mean Age							37.211	38.965	40.630

Source: Own calculation based on intensities in Table 1.

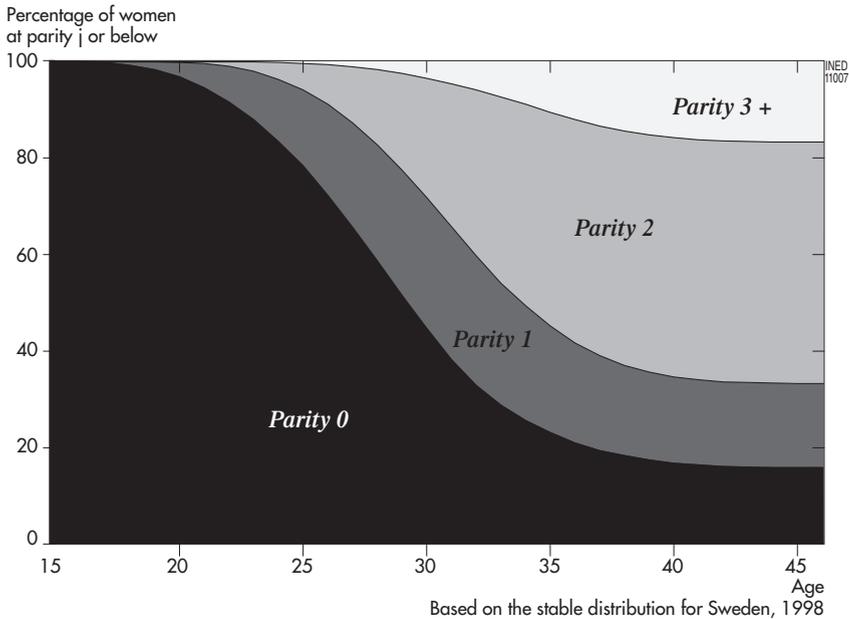
From the $PATFR_j$ the Parity Progression Ratios ($PPRs$), π_j , (Henry, 1953; Ryder, 1986; Ní Bhrolcháin, 1987; Feeney and Yu, 1987) can be defined. They represent the proportion of women that were ever at parity j who moved on to parity $j + 1$. They can be calculated as

$$\pi_j = PATFR_{j+1} / PATFR_j$$

In our case the parity progression rates for parity 1 and 2 are 79.3% and 25.0% respectively.

The distribution of women by parity at the different ages is immediately available in the fertility table as $D_j(a)$. If the fertility intensities remained constant long enough, these proportions would be the ones observed in the population. It is in this sense that these proportions provide the stable age distribution of women by parity. The final distribution is particularly evident in the last row. In this specific example the high $PPRs$ for parities 0 and 1 in connection with the low PPR for parity 2 lead to approximately half the women in the stable distribution having two children. Figure 3 shows the cumulated proportions of women at the different parities by age.

Figure 3 – Cumulated Proportions of Women at Different Parties



The calculation of the additional birth proportions ${}_n P_j(a,b)$ is more complicated for the general case. The fertility table can be interpreted as the exercise of tracking the fertility of a woman initially parity 0 from age α until age ω . The additional birth proportions require similar calculations, but for a woman initially parity j and age a and followed only until age b and the birth of order $j + n$. They can be constructed by forming a specific fertility table with a radix of one chosen for the desired initial category (age a and parity j) and where the absorbent parity

is $j + n$. K – O call this the synthetic cohort age a and parity j . The additional birth proportions can then be read either as the proportion of women in the absorbent state at age $b+1$, or as the cumulated sum $b_{j+n-1}(a,b)$. Table 4 shows an example of such a calculation for women at parity 1 at thirty-five years of age. As can be seen, all the associated measures, such as number of births or mean age, can also be applied here. In this case there is a 46.82% chance that the woman in question will have an additional birth, but at 2.8%, the probability of having more than two children is very low.

It is also possible to obtain birth-interval measures, since the mean birth interval from parity j to parity $j + 1$ is equal to the difference between the mean age at birth at parity $j + 1$ minus the mean age at birth at parity j for the women who had additional children. This can be computed by splitting the number of births in the general fertility table into two columns: those by women that had additional children, ${}_+b_j(a)$, and those by women that remained at parity $j + 1$, ${}_1b_j(a)$. They are given respectively by:

$${}_+b_j(a) = b_j(a) \cdot \pi_{j+1}(a) \quad [10]$$

and

$${}_1b_j(a) = b_j(a) \cdot [1 - \pi_{j+1}(a)] \quad [11]$$

One can then get the mean ages at birth for those that progressed and those that did not, and by subtracting this from the mean age at the next birth, we get the mean birth interval for the transition for parity j to $j + 1$ (Feichtinger, 1987). In Table 5 we show an example of the calculations. We observe that those women who progress to second birth were much younger when having their first birth than those that did not (27.2 versus 33 years). The same applies for progression to third birth. Because of this, the difference between the overall mean ages at birth would be an underestimate of the birth interval. The mean birth intervals obtained are therefore 3.7 years from first to second birth and 4.7 from second to third.

It is also possible to obtain a mean birth order for different ages (de Simoni, 1995). For a particular age a , the mean birth order would be:

$$\bar{O}(a) = \frac{\sum_{j=0}^J (j+1) b_j(a)}{\sum_{j=0}^J b_j(a)}$$

We can base any of the standard life table functions for increment-decrement tables on fertility tables. In particular, one can also define conditional person-years lived or waiting times (Palloni, 2001). Waiting times at age 15 and parity 0 can be calculated easily from the fertility table by calculating a number of women-years lived table $L_j(a) = [D_j(a) + D_j(a + 1)]/2$ and adding up the number of years lived in each state. Table 6 shows such a calculation. In this particular case, we see how late childbearing implies that women pass most of their childbearing life at parity zero (15 years), and the proportion spent at parity one is very low given the late progression to parity one and the high progression rate to parity two. These waiting times can be generalized for other ages and parities. The basic idea is to

Table 5 – Mean Birth Intervals, First to Second and Second to Third Births

Age \ Class	Births in the life table			
	1 to 2	Remain at 1	2 to 3	Remain at 2
18	0.222	0.005	0.000	0.000
19	0.950	0.023	0.000	0.000
20	2.773	0.072	0.000	0.000
21	5.394	0.155	0.122	0.095
22	10.154	0.330	0.234	0.200
23	16.316	0.621	0.548	0.515
24	20.616	0.951	0.949	0.997
25	27.769	1.555	1.321	1.564
26	36.631	2.556	1.896	2.536
27	41.748	3.748	2.289	3.507
28	46.374	5.403	2.764	4.848
29	49.221	7.561	3.314	6.717
30	49.789	10.239	3.393	8.108
31	47.551	13.343	3.404	9.662
32	40.613	15.911	3.308	11.311
33	30.321	16.824	2.877	12.115
34	23.254	18.187	2.537	13.404
35	16.368	18.588	1.954	13.589
36	9.924	17.015	1.244	12.007
37	5.473	14.501	0.700	9.730
38	2.679	11.315	0.380	7.832
39	1.240	8.619	0.176	5.726
40	0.461	5.647	0.076	4.120
41	0.158	3.548	0.021	2.158
42	0.044	1.967	0.007	1.329
43	0.011	1.048	0.002	0.816
44	0.003	0.563	0.000	0.210
45	0.000	0.238	0.000	0.267
Mean Ages	29.617	34.457	30.925	34.144
Age next birth	33.498		35.110	
Birth Interval	3.881		4.185	

Source: Own calculation based on Table 3.

base the calculation on the table for the synthetic cohort age a and parity j . De Simoni (1995) also proposes a number of age-span measures based on person-years-lived at a given parity by women who progress to an additional birth and by women who do not, which are all based on the table of $L_j(a)$.

While we have concentrated on the measurement of fertility by parity and age, there are alternative schemes for the study of fertility (Lutz, 1989; see also footnote 18). Each of them leads to related summary measures. The most common

alternatives are parity and duration life tables. Here, it is possible to define mean birth intervals and parity progression rates (Henry, 1953; Feeney and Yu, 1987; Ní Bhrolcháin, 1987), and an index of total fertility, Rallu and Toulemon's (1993 a) *PDIFR*, Parity and Duration *TFR*. Using large surveys, it is possible to extend this to additional dimensions, as shown by Rallu and Toulemon's (1993 a, 1993 b) calculation of *PADIFR*: the parity, age and duration index of Total Fertility. Both the summary measures proposed for parity and age and parity and duration schemes can be extended to the parity, age and duration scheme. Regarding the choice

Table 6 – Waiting Times at the Different Parities

Age \ Parity	Women Years Lived			
	0	1	2	3+
16	0.999	0.001	0.000	0.000
17	0.995	0.005	0.000	0.000
18	0.988	0.012	0.000	0.000
19	0.976	0.023	0.001	0.000
20	0.957	0.040	0.003	0.000
21	0.931	0.062	0.007	0.000
22	0.898	0.087	0.014	0.000
23	0.857	0.114	0.027	0.001
24	0.810	0.142	0.045	0.003
25	0.754	0.173	0.068	0.005
26	0.690	0.202	0.099	0.009
27	0.623	0.228	0.136	0.014
28	0.552	0.249	0.178	0.021
29	0.482	0.265	0.223	0.029
30	0.416	0.273	0.271	0.040
31	0.357	0.271	0.319	0.052
32	0.310	0.260	0.364	0.066
33	0.273	0.245	0.401	0.081
34	0.244	0.229	0.430	0.097
35	0.221	0.214	0.452	0.112
36	0.203	0.201	0.469	0.127
37	0.190	0.191	0.481	0.139
38	0.180	0.184	0.488	0.148
39	0.173	0.179	0.493	0.155
40	0.167	0.177	0.496	0.160
41	0.164	0.176	0.498	0.163
42	0.162	0.175	0.499	0.165
43	0.160	0.174	0.499	0.166
44	0.160	0.174	0.500	0.167
45	0.159	0.174	0.500	0.167
Waiting Time	15.052	4.901	7.960	2.086

Source: Own calculation based on Table 3.

between parity and age versus parity and duration, there are good arguments for both. Tables with no duration loose sight of the low fertility period following birth and the conception-birth interval. Tables with no age loose sight of the fact that age conditions parity progression: it is not the same for a woman to reach parity one at thirty, thirty-five or forty years. Indeed, its postponement is one of the characteristic elements of present fertility, and one of its effects is what K – O call the *fertility aging effect*: the lower fertility achieved by women who delay their fertility. This, together with the technical possibility of removing tempo effects within the parity and age scheme, is the main reason for our choice here.

Layers in period fertility analysis: from births to fertility behavior

Fertility table modeling as described in the preceding section is valid both for a period and a cohort perspective. In the first case, rates are applied to a synthetic cohort; in the second, to a birth cohort. Period measures frequently are of interest, since they provide a measure of fertility at a given moment in time. This is very important, since time is one of the main dimensions of change in fertility (Ní Bhrolcháin, 1992). Understanding these trends is one of the main purposes of fertility analysis. Only by using period measures can we gain insight into the effects which current events have on fertility. This is important irrespective of whether we are looking at the effect of socioeconomic evolution on fertility, that of public policies, etc. Having a life table interpretation is then a good property for a period measure of fertility. The basic possibility opened up by the techniques described in the preceding sections is the removal of compositional and tempo distortions from period fertility measures. This is important because neither the composition of the population nor tempo distortions on quantum are directly connected to behavior. They should therefore be removed before trying to explain period fertility.

On the other hand, whenever we are interested in the consequences of period fertility, it is usually the number of births that matter (Calot, 2001 a, 2001 b; Toulemon, 2001 a; van Imhoff, 2001): this determines the size of future generations with its impact on their labor market conditions, housing, pension systems, education, etc. (Ryder, 1965). The basic purpose of demographic analysis in this respect is the separation of the different contributing factors to the number of births. It then becomes relevant to explore how tempo effects and population composition combine with fertility behavior in determining the number of births. In this section, we will concentrate on providing such a decomposition of the different layers in period fertility from births to fertility behavior.

The idea of removing compositional effects from period fertility is central in fertility analysis. It goes back at least to Newsholme and Stevenson (1906), who stated that “the corrected rate measure is a force, the crude rate the result of the

operation of this force”.⁽²⁵⁾ Based on our discussion, there is a growing number of factors to remove from observed births until we get to fertility: age composition, parity composition, and tempo distortions. The first is the composition of the population by age, which is removed by using the Total Fertility Rate. The relationship between the *TFR* and the number of births has been studied by Ryder (1964, 1980) and Calot (1984, 1985). Calot uses the term *mean generation size* for the factor that translates the *TFR* into births. Ryder (1980) calls *age distribution factor* the number that converts the *TFR* into the birth-rate.⁽²⁶⁾ Obviously, meangeneration size is equal to the age distribution factor multiplied by the average population size (or mean exposure). Because of the shortcomings of the *TFR* as a measure of completed fertility, Calot (2001a; 2001b) advocates that the period *TFR* should not be interpreted as a measure of fertility quantum at all, but rather as a measure of period generation replacement: as the ratio of the newborn generation to the generation of mothers.

A second factor in the number of births are tempo effects. Ideally we can remove tempo distortions using fertility intensities. These adjusted intensities can be converted back into fertility incidence rates in order to obtain a *TFR* measure which is free from tempo effects. This procedure for adjusting the *TFR* leads to a result which is analogous to the B – F procedure with the exception that the inference on tempo effects rests on a sounder methodological basis. We can thus give a mean tempo effect measure similar to B – F’s *r* where:

$$r = 1 - \frac{TFR}{Adj.TFR} \quad [12]$$

The interpretation, when put in percentage terms, is the percentage of births “missing” because of tempo effects. It can be defined separately for the different parities.

Finally, the effect of parity composition can be removed using the adjusted *PATFR*, the total fertility estimate from the multiplicative fertility table of tempo adjusted intensities. Since the adjusted *PATFR* is a pure index of fertility in the sense that it is free from tempo and compositional distortions, it is called the *Period Fertility Index*, *PF* by Ortega and Kohler (2002). We can define a parity distribution effect similar to *r* which we call *d*, the parity distribution effect:

$$d = \frac{Adj.TFR}{.PF} - 1 \quad [13]$$

We have inverted the sign so that this index will be positive when the parity composition favors high fertility, and negative when it rather leads to low fertility.

(25) They try to correct for age and marital composition: “the corrected birth-rate must be a measure of fertility, which operating upon a population of given constitution as to age, sex and marriage, produces as its result the crude birth-rate” (Newsholme and Stevenson, 1906: 35).

(26) It is possibly better to use the mean generation size since introducing the age distribution leads to confounding factors: past fertility determines the size of the population at young ages. Therefore after a baby-boom the crude birth rate might fall merely because the proportion of women at child-bearing ages in the population falls (Whelpton, 1963).

Again, the parity distribution effect can be defined separately for the different parities. They could also be defined for a particular age.

This procedure provides a coherent partition of the *TFR* into its demographic components which can be fruitfully exploited for forecasting or analysis purposes. Putting together [12] and [13], we have:

$$TFE_i = (1 - r_i) \cdot (1 + d_i) \cdot PF_i \quad [14]$$

From the definition of the mean generation size, G , we can directly connect the number of births to the adjusted *PATFR*, our measure of quantum that is free from non-behavioural effects, as:

$$B_i = G_i \cdot (1 - r_i) \cdot (1 + d_i) \cdot PF \quad [15]$$

Other combinations of terms are also possible: for instance, one can define a mean generation size for the tempo adjusted *TFR* which would include G_t and r_t . These issues are further developed in Ortega and Kohler (2002).

A minor caveat for this procedure is the sensitivity of the partition to the sequence of operations: the decomposition is different if the effect of parity distribution is removed before the tempo effect, which is also possible. We prefer the decomposition given here because the interpretation of the tempo effect is simpler, it refers to the actual proportion of births being missed. It also provides a useful comparison to the adjustment procedures based on incidence rates such as B – F and K – P.

In Table 7 we show examples of such decompositions for Sweden in 1990 and 1998. The first was the year when fertility reached its highest levels in Sweden for many years after a very fast increase and before a very sudden collapse (Andersson, 1999). This collapse reached a trough in 1998 (Andersson, 2001). Looking at the period fertility index *PF*, we can see that the peak was real: for all parities the values are much higher in 1990 than in 1998. In 1990 tempo effects and parity distribution played in opposite directions: there were some tempo effects, on average 11% of the fertility level, but the parity composition partially offset this effect. Tempo effects were particularly strong for first birth and the adjusted *TFR* is actually higher than one. Of course, this can only happen when the parity distribution favors high fertility. The strong parity composition effect discounts that effect leading to a period fertility index of 0.89. To understand the large parity distribution effect we have to remember that the period fertility increase happened basically at all ages. If the rates had been sustained for some time, the proportion of women at higher parities would have been larger, and there would have been fewer births accordingly. This is what the ratio d picks up. The situation is different in 1998. Tempo effects are more important than before, especially for first and second birth. On the other hand, now the compositional effect is also leading to low fertility, as we can see from the negative values of d at parities 0 and 1. As we have shown in Table 3, this is precisely the result of the fertility peak of the 1990s. Since many women had progressed at the time to higher parities, there is a lower observed fertility at low parities and substantially more births at high parities (d is 17.7% and 91.7% for parities 2 and 3 respectively). Table 7 also provides the *PATFR* so that the alternative decomposition can be

Table 7 – Period Contributions of Tempo, Parity Composition and Generation Size to the Number of Births

	Year	Parities				
		0	1	2	3 +	Total
Births	1990	45.182	36.273	16.232	4.696	10.2383
	1998	30.085	26.645	9.627	2.462	6.8819
TFR	1990	0.895	0.742	0.345	0.101	2.083
	1998	0.626	0.550	0.201	0.053	1.430
Adjusted TFR	1990	1.021	0.828	0.384	0.107	2.340
	1998	0.796	0.627	0.196	0.047	1.666
Adjusted PATFR	1990	0.890	0.765	0.365	0.139	2.159
	1998	0.841	0.667	0.167	0.024	1.699
PATFR	1990	0.854	0.704	0.306	0.110	1.974
	1998	0.764	0.562	0.136	0.022	1.484
Mean Age Stable Distr.	1990	26.44	29.09	31.79	33.54	28.62
	1998	28.08	30.38	32.69	34.42	29.84
Adj. Mean Birth Interval	1990		3.54	4.45		
	1998		3.71	4.68		
Mean Generation Size	1990	50,487	48,889	47,020	46,321	49,141
	1998	48,074	48,451	47,805	46,772	48,133
Mean Tempo Effect (r) %	1990	12.4%	10.4%	10.1%	4.9%	11.0%
	1998	21.4%	12.3%	-2.5%	-12.4%	14.2%
Parity Composition Effect (d) %	1990	14.8%	8.2%	5.1%	-23.1%	8.4%
	1998	-5.3%	-5.9%	17.7%	91.7%	-1.9%
Mean Tempo Effect in PATFR %	1990	4.0%	8.1%	16.2%	20.6%	8.6%
	1998	9.1%	15.8%	18.2%	10.4%	12.6%
Mean Age Frequency Sched.	1990	26.38	29.12	31.86	34.30	28.65
	1998	28.22	30.93	33.50	35.11	29.90
Mean Birth Interval (Unadj.)	1990		3.64	4.40		
	1998		3.82	4.56		

Source: Data from Andersson (2001). Own Calculations.

worked out as well. The mean tempo effect in *PATFR* is obtained by comparing the adjusted *PATFR* and the unadjusted one.

It may also be of interest to look at mean ages and birth intervals. These are shown in the table as well. We see how substantial postponement has taken place irrespective of whether we look at the mean age of the stable distribution or that of the incidence rate schedule. The mean birth intervals can also be estimated from the life table of observed or tempo adjusted intensities. In the latter case, we obtain a tempo adjusted birth interval. We see that the results in this case are not very sensitive to tempo distortions. They also operate in different directions for the transition to second and third births. Comparing the figures for 1990 with those of 1998 we see that the birth intervals have widened slightly. This should be seen as a period effect: in the year of high fertility, 1990, not only was fertility higher but

the birth interval shorter. It is also possible to compute the rest of the fertility measures presented in Section 5.

It is important to bear in mind that these are all period measures, they therefore measure fertility in a given year and should not be interpreted as what will be observed in cohorts of women. That is the object of cohort analysis which we analyze in the next section.

Cohort fertility

Cohort fertility is a particularly interesting strand of fertility analysis. In its present form, it owes much to the work of Whelpton (1949), Ryder (1951) and Hajnal (1947).⁽²⁷⁾ The idea is to follow the fertility experience of a cohort of women (or men) through time. When the fertility rates are of the cohort-period or cohort-age type, cohort analysis follows from simply reading the rates along the diagonal of the Lexis diagram instead of the vertical lines. What lends cohort fertility its attraction is that the rates refer to roughly the same group of women. If parity-selected migration and mortality are not important, there will be a correspondence between fertility measures based on the cohort additive fertility tables and retrospective measures of fertility based on children-ever-born to women in a survey (Wunsch, 2001a). This therefore provides an interesting connection between fertility and family dynamics which is not so straightforward in period analysis.

The key characteristic of cohort fertility dynamics is that its variability from cohort to cohort is very small compared to the variability over time of period total fertility. There are two reasons for this. One is purely mathematical: since the variability of period fertility is very high, averaging over different periods, which is what cohort fertility does, must reduce the variance. The second is connected to the concept of a cohort: women may decide according to period circumstances when to have children but they might have an *ex-ante* idea about what ultimate level of fertility they want. If the variance of this *ex-ante* idea is not large across cohorts because of common socialization, the variance of cohort fertility should be smaller. Ryder (explained in Hobcraft *et al.*, 1982) actually showed that the variance of fertility is reduced to similar levels irrespective of whether we follow real cohorts or other combinations of period fertility which do not correspond to a cohort. This does not necessarily mean that cohorts have no effect. On the contrary, while there are reasons why the variance should be smaller, there are also factors leading to inter-cohort variability like cohort size, education (which is always more comparable for people who were educated at the same time), contraceptive knowledge, etc. (Ryder, 1965). These can lead to differences in fertility across cohorts. In recent times, interest in the study of cohort fertility has grown as exemplified by Lesthaeghe and Willems (1999), Lesthaeghe (2001), Frejka and Calot (2001 a; 2001 b), van Imhoff (2001), and K – O.

(27) Earlier studies of fertility using cohorts of women are Ansell (1874) and Maynard (1923). Sal-lume and Notestein (1932) focused on completed fertility of birth cohorts.

From an analytic point of view the study of fertility does not present particular difficulties. The fertility measures presented in Section 4 are all applicable both to cohort and period analysis. The only difference is in the notation. The cohort equivalent of the *TFR* is generally called Completed Cohort Fertility, *CCF*. The reason for this is that it is possible to use census information to provide estimates of cumulated fertility and parity distribution at a particular moment in time (Sal-lume and Notestein, 1932). While demographic translation formulas can be used for translating period measures into cohort measures and vice versa, this should not in fact be done. Not only is it an impossible task (van Imhoff, 2001), but also a quite useless one: if we know the matrix of rates over time we should compute both cohort and period measures directly from the rates.

The main problem from the point of view of cohort analysis is cohort completion: what will the completed cohort fertility be of the generation still of childbearing age. This issue is very much connected to fertility forecasting, and we can see cohort-completion methods as forecasting methods and vice versa. Akers (1965) draws a distinction between the period method, the cohort method and the parity progression method in forecasting births or completing cohorts. This distinction is still valid today. The period method is based on the assumption that the period *TFR* remains constant or follows a specific trajectory. The cohort method is based on some hypothesis about the completed cohort fertility of different cohorts, the “remaining” births to these cohorts being spread through time. The parity progression method is based on the calculation of birth intensities that are specific to, at least, parity and age, and possibly marital status and birth interval. It is then assumed that these intensities will remain constant or will follow a particular trajectory over time.

The period method of cohort completion/fertility projection is possibly the first (Whelpton, 1928) and by far the most commonly used despite its methodological shortcomings (Lee, 1974, 1993; Tuljapurkar and Boe, 1999). It is based on the assumption of future paths for the total fertility rate and the application of a generally fixed age-specific fertility schedule. It is useful because of its simplicity and relatively lax data requirements. This makes the introduction of statistical forecasting methods easier. However, it is misleading in periods of fertility change, that is, precisely in those periods when forecasts are more relevant. Fertility peaks associated merely with fertility anticipation would be taken to be more permanent, instead of assuming that they will end when tempo stabilizes. Conversely, fertility observed in times of postponement might be interpreted as downward trends. The particular applications of Lee and Tuljapurkar and Boe get around this problem by introducing ad-hoc ultimate levels of fertility. Projections based on parity- and age-specific fertility incidence rates such as Yi and Land (2001) can also be considered as period projections. Their approach is based on assuming an ongoing change in the shape of the incidence rate schedule as given by its mean age and variance. Even if parity is considered, the use of incidence rates implies that the parity composition of the population is not.

The cohort method for cohort completion consists in setting up a “target” fertility level for the different cohorts and then using an appropriate method to distribute the births. An example of such a procedure has recently been advanced by Lesthaeghe (2001). It is based on taking a cohort as a reference and measuring

“postponement” and “recuperation” against it. Postponement is defined as the relative lagging of cumulated fertility in a given cohort born in year b before age 30 with respect to the reference cohort. Recuperation is given by the relative catching up of fertility after age 30. For a given cohort, completed cohort fertility is then given by two parameters: k_b , the intensity of postponement, and R_b , the intensity of recuperation:

$$CCF_b = CCF^* - d_n(30) \cdot k_b + r_n(50) \cdot R_b$$

$d_n(a)$ and $r_n(a)$ being some country-specific calendars of age-related relative postponement and recuperation defined over the intervals 15-30 and 30-50 respectively. This method solves some of the issues involved in cohort methods such as how to use the information from incomplete cohorts. It is here used to estimate the k_b parameters for all cohorts and the R_b parameter for those cohorts that are over 30. It is then possible to extrapolate based on time series of k_b and R_b . Analogously incomplete cohort fertility can be worked out as:

$$CF_b(a) = CF^*(a) - d_n(a) \cdot k_b, a \leq 30$$

$$CF_b(a) = CF^*(a) - d_n(30) \cdot k_b + r_n(a) \cdot R_b, a > 30$$

The simplicity of this method makes it a useful descriptive device where the two parameters have a meaningful interpretation, but in forecasting fertility it relies on many hidden or explicit assumptions. A first problem arises from the constant fertility trough at age 30 and the discontinuity in modeling before and after that age. The age at which this trough happens is a function of the decline in quantum and the difference between the mean ages of the fertility schedule for the reference cohort and cohort b . This will inevitably shift over time. Where both k and R are large, the discontinuity around age 30 may lead to spikes around age 30 that might not be very congruent. It would be interesting to analyze each birth order separately as in Bosveld (1996). The reason is that the constants k and R when defined for births of all orders must simultaneously play the role of both quantum and tempo. Take a case like that of Spain, where CCF has declined fast and there has been intense postponement of first births. The model will say there is no recuperation, which means that there are fewer births at older ages than before. This is not surprising if fertility at higher parities has fallen and it was very high in the cohort of reference. Recuperation could only make sense at a parity-specific level, meaning that the first births that did not happen at young ages are taking place at older ages. This is the reason why the method works for countries like the Netherlands, where quantum has not changed much in the last thirty years, but it does not work in countries with large quantum changes. A second aspect is that the method provides a comparison between cohorts. In order for the procedure to have predictive content, the comparison with the reference cohort would have to contain relevant information for the prediction of cohorts having children now. In countries that have experienced sharp socioeconomic changes in the interim, like those of Eastern European, Spain, or Portugal, it is unclear whether one can gain predictive power through comparison with a cohort which lived under very different conditions. This last problem is not exclusive to Lesthaeghe's method, but also applies to cohort methods in general.

The parity progression approach consists of using period intensities specific to several dimensions including parity. Here the idea is that the most relevant way of obtaining information on what current childbearing generations will do is to look at what women in the same circumstances were doing in the last periods for which data is available. Early examples are Akers' (1965) and Ryder's (1980, 1986) completion of cohort fertility which was based on the last period intensities specific for parity, marital status and birth interval.

K – O's method of cohort completion also belongs to the parity progression approach. This method is based on the projection of fertility intensities for alternative scenarios of quantum, tempo and variance. While it can be applied to any given set of postponement/quantum scenarios, they concentrate on two: the postponement stops and the postponement continues scenario. The postponement stops consists of using the current tempo adjusted parity- and age-specific fertility intensities for completing cohort fertility. While this is an interesting scenario to consider, it is probably not the most likely. Postponement trends seem to be very persistent (Kohler, Billari and Ortega, 2001), therefore it is more likely that postponement will continue in the future. The postponement continues scenario assumes that the tempo-adjusting parameters, γ and δ , will continue in future and that the quantum, given by the cumulated intensity for a given parity, will remain unaltered. It is similar to Yi and Land's (2001) scenario but defined for tempo-adjusted fertility intensities instead of tempo-distorted incidence rates. K – O provide formulas for obtaining completed cohort fertility and related measures directly from the intensities observed. Conceptually this is the same as introducing tempo and variance effects back into the future intensities by means of system [4], the difference being that now the adjusted rates come from the profile of adjusted fertility schedules in the reference year T . The mean age and variance of the adjusted schedule will change over time. In a given year t they will be given by:

$$\begin{aligned} \bar{a}_j(t) &= \bar{a}_j(T) + \gamma_j \cdot (t - T) \\ s_j^2(t) &= s_j^2(T) \exp((2\delta_j \cdot (t - T))) \end{aligned} \quad [16]$$

We can then transform the adjusted intensity schedule into one that has mean and variances given by [16]. Once this adjusted schedule has been obtained, we can get the observed intensities for the given year t from system [4]. While formula [16] is valid only for the postponement continues scenario with constant γ and δ parameters, it can be generalized to cover any future profile of tempo and variance parameters. This makes the procedure amenable to the incorporation of statistical forecasting methods for the future paths of ${}_t p_j$, γ_j and δ_j . K – O is an improvement on previous parity progression projections in the explicit consideration of tempo. This was a flaw of earlier attempts that were based on tempo-distorted measures of fertility and did not take into consideration the effect of tempo change on the intensities. This is the reason, for instance, why earlier attempts such as Akers' (1965) needed to combine, even in an ad-hoc manner, parity-progression with a cohort approach to ensure sensible demographic results. Kohler and Ortega (2002 b) apply the methods to three countries, Sweden, the Netherlands and Spain. A comparison of completed cohort fertility with the projected cohort fertility for

different base years, shows that the projection fares well when the assumptions about quantum, tempo and variance are close to the period evolution of fertility as in the Netherlands, while they fail in years of unusual developments like the Swedish roller coaster fertility of the 1990s. While this is an improvement on previous methods, more effort needs to be put into forecasting turning points both in the quantum and tempo of fertility.

The reason why procedures based on parity progression lead to results different from period methods basically lies in the feedback effects of progression at lower parities: since second births only occur to women at parity one, the parity progression rate to second birth declines when the first birth is postponed. Such feedback effects are called *fertility aging effects* by Ortega and Kohler (2002). These effects can be partially or totally compensated when births at higher parities are also being postponed. Empirical patterns show that fertility aging has a stronger effects in some countries like Spain or Sweden than in others like the Netherlands.

Discussion

In this article we have provided an overview of old as well as more recent methods for the analysis of fertility. We have tried to emphasize the flexibility of the methods and the broadness of interest of demographers and fertility researchers. It is a fortunate circumstance that we can adapt the methods to our interests, and that no matter what our interests are, there are a small number of key issues that we should address. It does not make sense to wage war over the question of whether the *TFR* is a good or a bad measure. It is good in some contexts, too simple in others, and indeed too complex in others. Accordingly, it is the specific context of our research which should guide us in the adoption of the appropriate measures. We have tried to present a wide range of techniques that are at the disposal of the researcher and we have placed particular emphasis on how to combine them.

In a context of low fertility a key issue is the separate analysis of fertility by birth order. Since most births are of an order lower than 3, the separate analysis of transition to first, second and third birth is sufficient. Given that in many low fertility countries a larger proportion of births occur outside marriage, it is advisable to study births irrespective of marital status. To make this possible, vital statistics must provide the appropriate tabulation of births, which is not always the case. We have also seen that it is important to use intensities, real exposure-occurrence rates, instead of incidence rates or rates of the second kind. The second ones are not suitable for the study of parity progression, one of the main predictable factors in fertility. They also lead to better estimation of tempo effects, which is a second important predictable component of fertility. The study of tempo is also especially important in a low fertility context since the timing of childbearing becomes more flexible. The adjustment of timing to the socioeconomic circumstances can potentially lead to large variations in period fertility. Trends in postponement may also be connected to social interactions. The use of tempo adjustment techniques makes it possible to elucidate the contribution of changing tempo to period fertility rates.

The application of life table techniques to tempo adjusted fertility intensities leads to the isolation of the behavioral component in fertility from parity composition and tempo effects. Only at this stage should we attempt to explain fertility trends, or make hypotheses about future developments. We provide a toolkit of ratios for the study of period fertility that isolate the contribution to the number of births of generation size, mean tempo effects and the parity distribution effect. This set of tools can be applied either to general fertility or to order-specific fertility.

The knowledge gained from the analysis of tempo-adjusted fertility intensities is also valuable for completing cohort fertility following the so-called *parity progression method*. This is based on the assumption that current patterns of childbearing according to age and parity and current postponement trends provide information about future developments. This procedure can be used either for cohort completion or fertility forecasting. It makes full use of those observable elements besides pure fertility quantum that have predictable implications for fertility, such as the parity composition of the population and the existence of tempo distortions in the presence of tempo change.

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