

Better to Give than to Receive: Predictive Directional Measurement of Volatility Spillovers

Francis X. Diebold
University of Pennsylvania and NBER
fdiebold@sas.upenn.edu

Kamil Yilmaz
Koç University, Istanbul
kyilmaz@ku.edu.tr

First draft/print: November 2008
This draft/print: June 2009

Abstract: Using a generalized vector autoregressive framework in which forecast-error variance decompositions are invariant to variable ordering, we propose measures of both total and directional volatility spillovers. We use our methods to characterize daily volatility spillovers across U.S. stock, bond, foreign exchange and commodities markets, from January 1999 through October 2008. We show that despite significant volatility fluctuations in all markets during the sample, cross-market volatility spillovers were quite limited until the global financial crisis that began in 2007. As the crisis intensified, so too did volatility spillovers, with particularly important spillovers from the bond market to other markets.

JEL classification numbers: G1, F3

Keywords: Asset Market, Asset Return, Stock Market, Market Linkage, Financial Crisis, Contagion, Vector Autoregression, Variance Decomposition

Acknowledgements: For helpful comments we thank participants in the Cemapre / IIF International Workshop on the Predictability of Financial Markets, especially Nuno Crato, Antonio Espasa, Antonio Garcia-Ferrer, Raquel Gaspar, and Esther Ruiz. All errors, however, are entirely ours. We thank the National Science Foundation for financial support.

1. Introduction

Financial crises occur with notable regularity, and moreover, they display notable similarities (e.g., Reinhart and Rogoff, 2008). During crises, for example, financial market volatility generally increases sharply and spills over across markets. One would naturally like to be able to measure and monitor such spillovers, both to provide “early warning systems” for emergent crises, and to track the progress of extant crises.

Motivated by such considerations, Diebold and Yilmaz (DY, 2009) introduce a volatility spillover measure based on forecast error variance decompositions from vector autoregressions (VARs).¹ It can be used to measure spillovers in returns or return volatilities (or, for that matter, any return characteristic of interest) across individual assets, asset portfolios, asset markets, etc., both within and across countries, revealing spillover trends, cycles, bursts, etc. In addition, although it conveys useful information, it nevertheless sidesteps the contentious issues associated with definition and existence of episodes of “contagion” or “herd behavior”.²

However, the DY framework as presently developed and implemented has several limitations, both methodological and substantive. Consider the methodological side. First, DY relies on Cholesky-factor identification of VARs, so the resulting variance decompositions can be dependent on variable ordering. One would like a spillover measure invariant to ordering. Second, and crucially, DY addresses only the aggregate phenomenon

¹ VAR variance decompositions, introduced by Sims (1980), record how much of the H -step-ahead forecast error variance of some variable, i , is due to innovations in *another* variable, j .

² On contagion (or lack thereof) see, for example, Forbes and Rigobon (2002).

of *total* spillovers (from/to each market i , to/from all other markets, added across i). One would also like to examine *directional* spillovers (from or to a particular market).

Now consider the substantive side. DY considers only the measurement of spillovers across identical assets (equities) in different countries. But various other possibilities are also of interest, including individual-asset spillovers within countries (e.g., among the thirty Dow Jones Industrials in the U.S.), across asset classes (e.g., between stock and bond markets in the U.S.), and of course various blends. Spillovers across asset classes, in particular, are of key interest given the global financial crisis that began in 2007 (which appears to have started in credit markets but spilled over into equities), but they have not yet been investigated in the DY framework.

In this paper we fill these methodological and substantive holes. We use a generalized vector autoregressive framework in which forecast-error variance decompositions are invariant to variable ordering, and we explicitly include directional volatility spillovers. We then use our methods in a substantive empirical analysis of daily volatility spillovers across U.S. stock, bond, foreign exchange and commodities markets, including during the recent financial crisis.

We proceed as follows. In section 2 we discuss our methodological approach, emphasizing in particular our new use of generalized variance decompositions and directional spillovers. In section 3 we describe our data and present our substantive results. We conclude in section 4.

2. Methods: Generalized Spillover Definition and Measurement

Here we extend the DY spillover index, which follows directly from the familiar notion of a variance decomposition associated with an N -variable vector autoregression. Whereas DY focuses on *total* spillovers in a *simple* VAR framework (i.e., with potentially order-dependent results driven by Cholesky factor orthogonalization), we progress by measuring *directional* spillovers in a *generalized* VAR framework that eliminates the possible dependence of results on ordering.

Consider a covariance stationary N -variable VAR(p), $x_t = \sum_{i=1}^p \Phi_i x_{t-i} + \varepsilon_t$, where $\varepsilon \sim (0, \Sigma)$. The moving average representation is $x_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i}$, where the $N \times N$ coefficient matrices A_i obey the recursion $A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + \dots + \Phi_p A_{i-p}$, with A_0 an $N \times N$ identity matrix and $A_i = 0$ for $i < 0$. The moving average coefficients (or transformations such as impulse-response functions or variance decompositions) are the key to understanding dynamics. We rely on variance decompositions, which allow us to parse the forecast error variances of each variable into parts attributable to the various system shocks. Variance decompositions allow us to assess the fraction of the H -step-ahead error variance in forecasting x_i that is due to shocks to x_j , $\forall j \neq i$, for each i .

Calculation of variance decompositions requires orthogonal innovations, whereas our VAR innovations are generally correlated. Identification schemes such as that based on Cholesky factorization achieve orthogonality, but the variance decompositions then depend on ordering of the variables. We circumvent this problem by exploiting the generalized VAR

framework of Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998), hereafter KPPS, which produces variance decompositions invariant to ordering.³

Variance Shares

Let us define *own variance shares* to be the fractions of the H -step-ahead error variances in forecasting x_i due to shocks to x_i , for $i=1, 2, \dots, N$, and *cross variance shares*, or *spillovers*, to be the fractions of the H -step-ahead error variances in forecasting x_i due to shocks to x_j , for $i, j = 1, 2, \dots, N$, such that $i \neq j$.

Denoting the KPSS H -step-ahead forecast error variance decompositions by $\theta_{ij}^g(H)$, for $H = 1, 2, \dots$, we have

$$\theta_{ij}^g(H) = \frac{\sigma_{ii}^{-1} \sum_{h=0}^{H-1} (e_i' A_h \sum e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \sum A_h' e_i)}.$$

Note that they do not have to sum to one, and in general they do not: $\sum_{j=1}^N \theta_{ij}^g(H) \neq 1$. Finally,

we normalize as:

$$\tilde{\theta}_{ij}^g(H) = \frac{\theta_{ij}^g(H)}{\sum_{j=1}^N \theta_{ij}^g(H)}.$$

Note that, by construction, $\sum_{j=1}^N \tilde{\theta}_{ij}^g(H) = 1$ and $\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(H) = N$.

³ KPPS focuses on “generalized impulse response functions,” but one can just as easily consider “generalized variance decompositions,” as we do. We refer to the overall framework as a “generalized VAR.”

Total Spillovers

Using the volatility contributions from the KPPS variance decomposition, we can construct a total volatility spillover index:

$$S^g(H) = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^N \tilde{\theta}_{ij}^g(H)}{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(H)} \cdot 100 = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^N \tilde{\theta}_{ij}^g(H)}{N} \cdot 100.$$

This is the KPPS analog of the Cholesky factor based measure used in Diebold and Yilmaz (2009).

Directional Spillovers

We now consider directional spillovers in addition to total spillovers. We measure directional volatility spillovers received by market i from all other markets j as:

$$S_{i \cdot}^g(H) = \frac{\sum_{\substack{j=1 \\ j \neq i}}^N \tilde{\theta}_{ij}^g(H)}{\sum_{j=1}^N \tilde{\theta}_{ij}^g(H)} \cdot 100.$$

In similar fashion we measure directional volatility spillovers transmitted by market i to all other markets j as:

$$S_{\cdot i}^g(H) = \frac{\sum_{\substack{j=1 \\ j \neq i}}^N \tilde{\theta}_{ji}^g(H)}{\sum_{j=1}^N \tilde{\theta}_{ji}^g(H)} \cdot 100.$$

One can think of the set of directional spillovers as providing a decomposition of total spillovers into those coming from (or to) a particular source.

Net Spillovers

Finally, we obtain the net volatility spillovers transmitted from market i to all other markets j as

$$S_i^g(H) = S_i^g(H) - S_i^g(H).$$

Net spillovers are simply the difference between gross volatility shocks transmitted to and gross volatility shocks received from all other markets.

3. Empirics: Estimates of Volatility Spillovers Across U.S. Asset Markets

Here we use our framework to measure volatility spillovers among four key U.S. asset classes: stocks, bonds, foreign exchange and commodities. This is of particular interest because spillovers across asset classes may be an important aspect of the global financial crisis that began in 2007 (which started in credit markets but spilled over into equities).

In the remainder of this section we proceed as follows. We begin by describing our data in section 3a. Then we calculate average (i.e., total) spillovers in section 3b. We then quantify spillover dynamics, examining rolling-sample total spillovers, rolling-sample directional spillovers, and rolling-sample net directional spillovers in sections 3c, 3d, and 3e, respectively.

Stock, Bond, Exchange Rate, and Commodity Market Volatility Data

We examine daily volatilities of returns on U.S. stock, bond, foreign exchange, and commodity markets. In particular, we examine the S&P 500 index, the 10-year Treasury bond yield, the New York Board of Trade U.S. dollar index futures, and the Dow-Jones / AIG commodities index. The data span January 25, 1999 through Oct 31, 2008, for a total of 2460 daily observations.

In the tradition of a large literature dating at least to Parkinson (1980), we estimate daily variance using daily high and low prices.⁴ For market i on day t we have

$$\tilde{\sigma}_{it}^2 = 0.361 \left[\ln(P_{it}^{\max}) - \ln(P_{it}^{\min}) \right]^2,$$

where P_{it}^{\max} is the maximum (high) price in market i on day t , and P_{it}^{\min} is the daily minimum (low) price. Because $\tilde{\sigma}_{it}^2$ is an estimator of the daily variance, the corresponding estimate of the annualized daily percent standard deviation (volatility) is $\hat{\sigma}_{it} = 100\sqrt{365 \cdot \tilde{\sigma}_{it}^2}$. We plot the four markets' volatilities in Figure 1 and we provide summary statistics in Table 1. Several interesting facts emerge, including: (1) The bond and stock markets have been the most volatile (roughly equally so), with commodity and FX markets comparatively less volatile, (2) volatility dynamics appear highly persistent, in keeping with a large literature summarized for example in Andersen, Bollerslev, Christoffersen and Diebold (2006), and (3) all volatilities are high during the recent crisis, with stock and bond market volatility, in particular, displaying huge jumps.

In 1999, daily stock market volatility was mostly below 25 percent, but it increased significantly to fluctuate above 25 percent until mid-2003. In mid-2003, it declined to less than 25 percent and stayed there until August 2007. Since August 2007, stock market volatility reflects well the dynamics of the sub-prime crisis.

In the first and last few months of 2001, interest rate volatility measured by the annualized standard deviation increased and fluctuated between 25-50 percent. Bond market volatility remained high until mid-2005, and fell only briefly in 2006 and early 2007. Since August 2007, volatility in bond markets has also increased significantly.

⁴ For background, see Alizadeh, Brandt and Diebold (2002) and the references therein.

Commodity market volatility used to be very low compared to stock and bond markets, but it increased slightly over time and especially in 2005-2006 and recently in 2008. FX market volatility has been the lowest among the four markets. It increased in 2008 and moved to a 25-50 percent band following the collapse of Lehman Brothers in mid-September.

Unconditional Patterns: The Full-Sample Volatility Spillover Table

We call Table 2 a volatility spillover table. Its ij^{th} entry is the estimated contribution to the forecast error variance of country i coming from innovations to country j .⁵ Hence the off-diagonal column sums (labeled contributions to others) or row sums (labeled contributions from others), are the “to” and “from” directional spillovers, and the “from minus to” differences are the net volatility spillovers. In addition, the total volatility spillover index appears in the lower right corner of the spillover table. It is approximately the grand off-diagonal column sum (or row sum) relative to the grand column sum including diagonals (or row sum including diagonals), expressed as a percent.⁶ The volatility spillover table provides an approximate “input-output” decomposition of the total volatility spillover index.

Consider first what we learn from the table about directional spillovers (gross and net). From the “directional to others” row, we see that gross directional volatility spillovers to others from the stock and bond markets are relatively large, at 21.75 percent and 21.60 percent, respectively. We also see from the “directional from others” column that gross directional volatility spillovers from others to FX is relatively large, at 19.82 percent. As for

⁵ All results are based on vector autoregressions of order 2 and generalized variance decompositions of 10-day-ahead volatility forecast errors.

⁶ The approximate nature of the claim stems from the properties of the generalized variance decomposition. With Cholesky factor identification the claim is exact rather than approximate; see Diebold and Yilmaz (2009).

net directional volatility spillovers, the largest are to others from the bond market and from others to the FX market.

Now consider the total (non-directional) volatility spillover, which is effectively a distillation of the various directional volatility spillovers into a single index. The total volatility spillover appears in the lower right corner of Table 2, which indicates that on average, across our entire sample, 15.20 percent of volatility forecast error variance in all four markets comes from spillovers.

Conditioning and Dynamics I: The Rolling-Sample Total Volatility Spillover Plot

Clearly, many changes took place during the years in our sample, January 1999-October 2008. Some are well-described as more-or-less continuous evolution, such as increased linkages among global financial markets and increased mobility of capital, due to globalization, the move to electronic trading, and the rise of hedge funds. Others, however, may be better described as bursts that subsequently subside.

Given this background of financial market evolution and turbulence, it seems unlikely that any single fixed-parameter model would apply over the entire sample. Hence the full-sample spillover table and spillover index obtained earlier, although providing a useful summary of “average” volatility spillover behavior, likely miss potentially important secular and cyclical movements in spillovers. To address this issue, we now estimate volatility spillovers using 10-day rolling samples, and we assess the extent and nature of spillover variation over time via the corresponding time series of spillover indexes, which we examine graphically in the so-called total spillover plot of Figure 2.

Starting from below ten percent in 1999, the total volatility spillover plot usually fluctuates between ten and twenty percent, occasionally falling below ten percent. However, there are important exceptions: the last quarter of 2000 and the first quarter of 2001, the aftermath of 9/11 terrorist attacks, the third quarter of 2002, and most importantly by far, the global financial crisis of 2007-2009. One can see four volatility waves during the recent crisis: July-August 2007, January 2008, June 2008, and September-October 2008. During these episodes the spillover index surges above twenty percent. Indeed, following the collapse of Lehman Brothers in mid-September, and consistent with the unprecedented evaporation of liquidity world-wide, the volatility spillover plot jumped to 56 percent on September 30, 2008, before declining somewhat.

Conditioning and Dynamics II: Rolling-Sample Gross Directional Volatility Spillover Plots

Thus far we have discussed the *total* spillover plot, which is of interest but discards directional information. That information is contained in the “Contribution to” row (the sum of which is given by $S_i^g(H)$) and the “Contribution from” column (the sum of which is given by $S_i^g(H)$).

We now estimate that row and column dynamically, in a fashion precisely parallel to the earlier-discussed total spillover plot. We call these *directional* spillover plots. In Figure 3 we present the directional volatility spillovers *from* our four asset classes. They vary greatly over time. During tranquil times, spillovers from each market are below five percent, but during volatile times, stock and bond directional spillovers increase to around twenty-five percent, and Commodity and FX volatilities increase to around fifteen percent.

In Figure 4 we present the directional volatility spillovers *to* our four asset classes. As with the directional spillovers *from* others, the spillovers *to* others vary noticeably over

time. The relative variation pattern, however, is reversed, with directional volatility spillovers *to* commodities and FX increasing relatively more in turbulent times.

Conditioning and Dynamics III: Rolling-Sample Net Directional Volatility Spillover Plots

We also calculate difference between the “Contribution from” column sum and the “Contribution to” row sum (given by $S_i^g(H)$), which we call the net directional spillover plot, as shown in Figure 5. First note that, overall, there has been very little net volatility transmission from the commodity and FX markets. Only in late 2004 and early 2005 do we observe net volatility spillovers from commodity markets to others reach almost five percent. Similarly, volatility in FX markets also had very little net impact on volatility in other markets, perhaps with the exception of the first half of 2006 and January 2008.

Instead, the clear channels of net directional volatility spillovers are from the stock and bond markets. Net volatility spillovers from the stock market appear the most consistently positive and large. After the terrorist attacks on September 11, 2001, net spillovers from the stock market affected mostly the commodity markets. During the increased U.S. stock market gyrations in June through October 2002, net spillovers from the stock market affected mostly the FX market. Finally, since August 2007, net spillovers from the stock market to other markets have increased dramatically.

Similarly, and interestingly, during most of the 2007-2009 financial crisis – and especially at the very end of our sample in September-October 2008 – the bond market was an important net transmitter of volatility. Indeed, following the collapse of Lehman Brothers from mid-September to mid-October, volatility spillovers originated mostly from the bond market, followed by the stock market, with the other two markets being net spillover

recipients, and during the second half of October the stock market also became a net recipient, leaving the bond market as the only net transmitter of volatility.

5. Concluding Remarks

This paper was entitled “Predictive Directional Measurement of Volatility Spillovers.” In particular, we have provided both gross and net *directional* spillover measures that are independent of the ordering used for volatility forecast error variance decompositions. When applied to U.S. financial markets, our measures shed new light on the nature of cross-market volatility transmission, pinpointing the importance during the recent crisis of volatility spillovers from the bond market to other markets.

We are of course not the first to consider issues related to volatility spillovers (e.g., Engle et al. 1990; King et al., 1994; Edwards and Susmel, 2001), but our approach is very different. It produces continuously-varying indexes (unlike, for example, the “high state / low state” indicator of Edwards and Susmel), and it is econometrically tractable even for very large numbers of assets. Although it is beyond the scope of this paper, it will be interesting in future work to understand better the relationship of our spillover measure to a variety of others based on measures ranging from traditional (albeit time-varying) correlations (e.g., Engle, 2002, 2009) to the recently-introduced CoVaR of Adrian and Brunnermeier (2008).

References

- Adrian, T. and Brunnermeier, M. (2008), "CoVaR," Staff Report 348, Federal Reserve Bank of New York.
- Alizadeh, S., Brandt, M.W. and Diebold, F.X. (2002), "Range-Based Estimation of Stochastic Volatility Models," *Journal of Finance*, 57, 1047-1092.
- Andersen, T.G., Bollerslev, T., Christoffersen, P.F. and Diebold, F.X. (2006), "Practical Volatility and Correlation Modeling for Financial Market Risk Management," in M. Carey and R. Stulz (eds.), *Risks of Financial Institutions*, University of Chicago Press for NBER, 513-548.
- Diebold, F.X. and Yilmaz, K. (2009), "Measuring Financial Asset Return and Volatility Spillovers, With Application to Global Equity Markets," *Economic Journal*, 119, 158-171.
- Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH Models," *Journal of Business and Economic Statistics*, 20, 339-350.
- Engle, R.F. (2009), *Anticipating Correlations*. Princeton: Princeton University Press.
- Engle, R.F., Ito, T. and Lin, W.-L. (1990), "Meteor Showers or Heat Waves: Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market," *Econometrica*, 58, 525-542.
- Forbes, K.J. and Rigobon, R. (2002), "No Contagion, Only Interdependence: Measuring Stock Market Comovements," *Journal of Finance*, 57, 2223-2261.
- King, M., Sentana, E. and Wadhvani, S. (1994), "Volatility and Links Between National Stock Markets," *Econometrica*, 62, 901-933.
- Koop, G., Pesaran, M.H., and Potter, S.M. (1996), "Impulse Response Analysis in Non-Linear Multivariate Models," *Journal of Econometrics*, 74, 119-147.

Parkinson, M. (1980), “The Extreme Value Method for Estimating the Variance of the Rate of Return,” *Journal of Business*, 53, 61-65.

Pesaran, M.H. and Shin, Y. (1998), “Generalized Impulse Response Analysis in Linear Multivariate Models,” *Economics Letters*, 58, 17-29.

Reinhart, C.M. and Rogoff, K.S. (2008), “Is the 2007 U.S. Subprime Crisis So Different? An International Historical Comparison,” *American Economic Review*, 98, 339–344.

Sims, C.A. (1980), “Macroeconomics and Reality,” *Econometrica*, 48, 1-48.

**Figure 1. Daily U.S. Financial Market Volatilities
(Annualized Standard Deviation, Percent)**

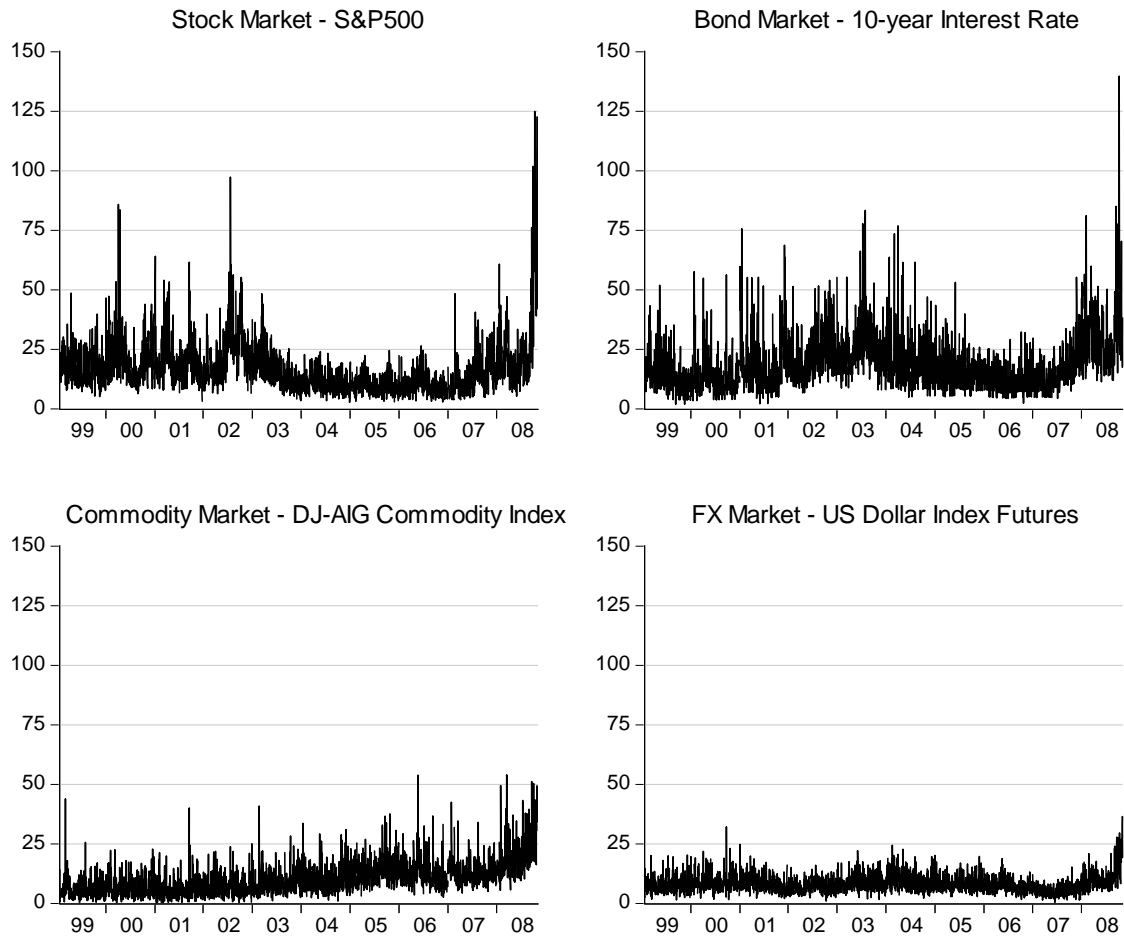


Table 1: Volatility Summary Statistics, Four Asset Classes

	Stocks	Bonds	Commodities	FX
Mean	16.93	18.44	10.55	8.21
Median	14.12	15.92	9.11	7.48
Maximum	124.93	139.70	53.93	36.42
Minimum	2.75	1.94	0.20	0.42
Std. Deviation	11.69	11.16	7.28	3.88
Skewness	3.23	2.12	1.65	1.42
Kurtosis	21.55	12.49	7.37	6.89

Table 2: Volatility Spillover Table, Four Asset Classes

	Stocks	Bonds	Commodities	FX	Directional <i>FROM</i> Others
Stocks	83.12	11.89	2.80	2.18	16.87
Bonds	5.36	89.68	1.57	3.39	10.32
Commodities	6.09	3.48	86.50	3.93	13.5
FX	9.30	6.23	4.29	80.18	19.82
Directional <i>TO</i> Others	20.75	21.60	8.66	9.50	65.12
Directional Including Own	103.9	111.3	95.2	89.7	Total Spillover Index: 15.2%

Figure 2. Total Volatility Spillovers, Four Asset Classes

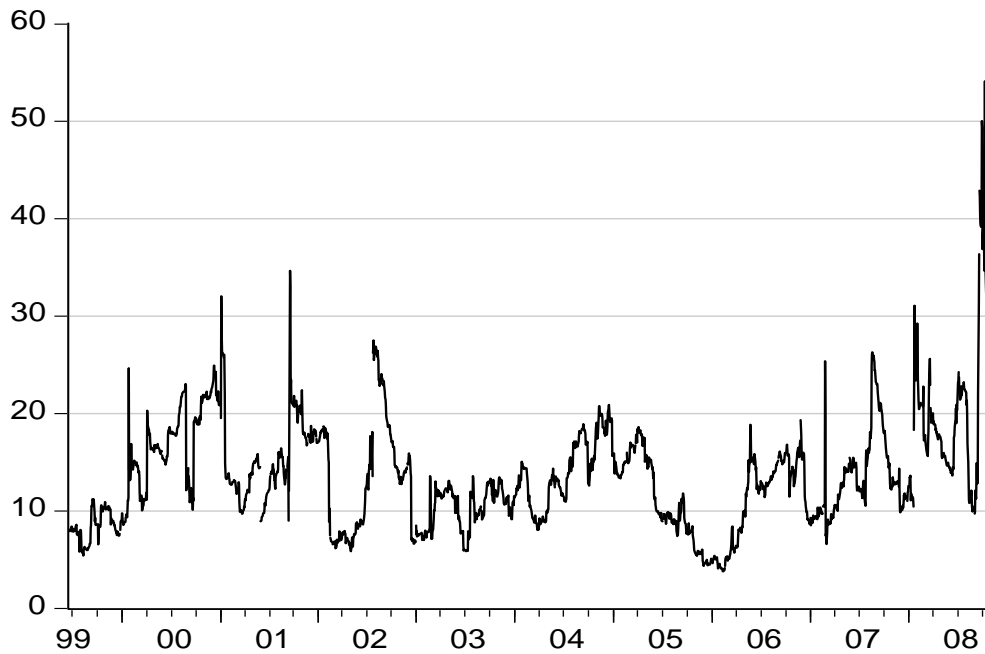


Figure 3. Directional Volatility Spillovers, FROM Four Asset Classes

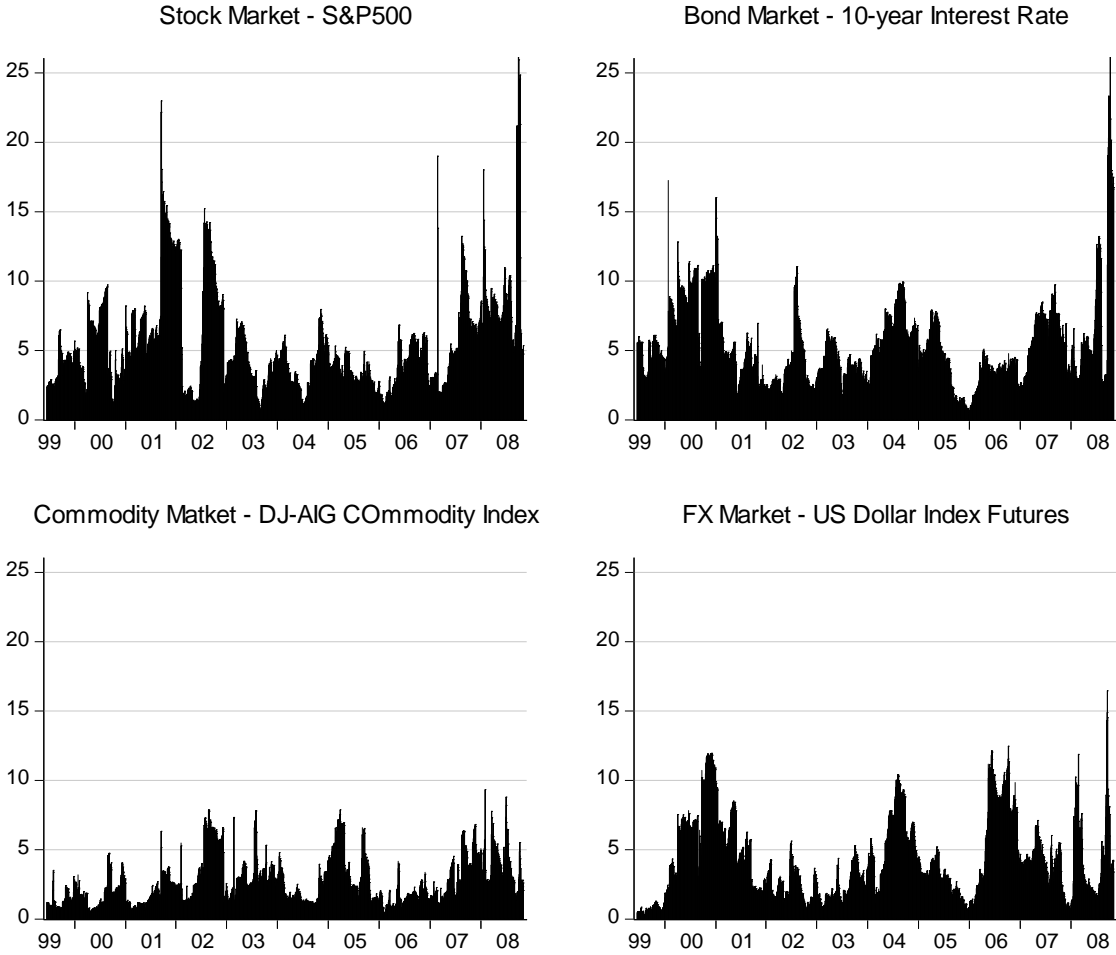


Figure 4. Directional Volatility Spillovers, *TO* Four Asset Classes

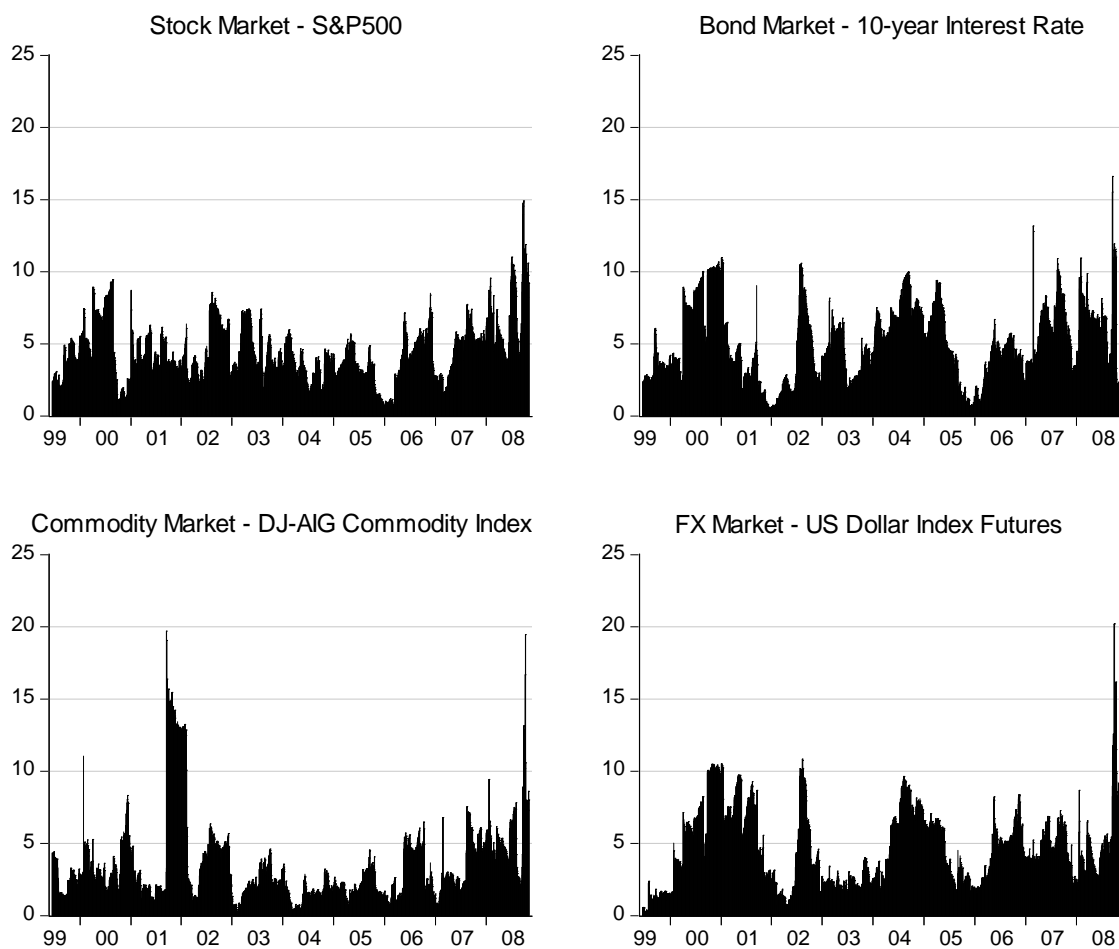


Figure 5. Net Directional Volatility Spillovers, Four Asset Classes

