Better to Give than to Receive: Predictive Directional Measurement of Volatility Spillovers

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Abstract: Using a generalized vector autoregressive framework in which forecast-error variance decompositions are invariant to variable ordering, we propose measures of both total and directional volatility spillovers. We use our methods to characterize daily volatility spillovers across U.S. stock, bond, foreign exchange and commodities markets, from January 1999 through September 2009. We show that despite significant volatility fluctuations in all four markets during the sample, cross-market volatility spillovers were quite limited until the global financial crisis that began in 2007. As the crisis intensified so too did the volatility spillovers, with particularly important spillovers from the bond market to other markets taking place after the collapse of Lehman Brothers in September 2008.

JEL classification numbers: G1, F3

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1. Introduction

Financial crises occur with notable regularity, and moreover, they display notable similarities (e.g., Reinhart and Rogoff, 2008). During crises, for example, financial market volatility generally increases sharply and spills over across markets. One would naturally like to be able to measure and monitor such spillovers, both to provide "early warning systems" for emergent crises, and to track the progress of extant crises.

Motivated by such considerations, Diebold and Yilmaz (DY, 2009) introduce a volatility spillover measure based on forecast error variance decompositions from vector autoregressions (VARs).[1] It can be used to measure spillovers in returns or return volatilities (or, for that matter, any return characteristic of interest) across individual assets, asset portfolios, asset markets, etc., both within and across countries, revealing spillover trends, cycles, bursts, etc. In addition, although it conveys useful information, it nevertheless sidesteps the contentious issues associated with definition and existence of episodes of "contagion" or "herd behavior".[2]

However, the DY framework as presently developed and implemented has several limitations, both methodological and substantive. Consider the methodological side. First, DY relies on Cholesky-factor identification of VARs, so the resulting variance decompositions can be dependent on variable ordering. One would prefer a spillover measure invariant to ordering. Second, and crucially, DY addresses only the aggregate phenomenon of *total* spillovers (from/to each market *i*, to/from all other markets, added

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[1] VAR variance decompositions, introduced by Sims (1980), record how much of the *H*-step-ahead forecast error variance of some variable, *i*, is due to innovations in *another* variable, *j*.

[2] On contagion (or lack thereof) see, for example, Forbes and Rigobon (2002).

across *i*). One would also like to examine *directional* spillovers (from/to a particular market, or from a particular market to another market).

Now consider the substantive side. DY considers only the measurement of spillovers across identical assets (equities) in different countries. But various other possibilities are also of interest, including individual-asset spillovers within countries (e.g., among the thirty Dow Jones Industrials in the U.S.), across asset classes (e.g., between stock and bond markets in the U.S.), and of course various blends. Spillovers across asset classes, in particular, are of key interest given the global financial crisis that began in 2007 (which appears to have started in credit markets but spilled over into equities), but they have not yet been investigated in the DY framework.

In this paper we fill these methodological and substantive gaps. We use a generalized vector autoregressive framework in which forecast-error variance decompositions are invariant to variable ordering, and we explicitly include directional volatility spillovers. We then use our methods in a substantive empirical analysis of daily volatility spillovers across U.S. stock, bond, foreign exchange and commodities markets over a ten year period, including the recent global financial crisis.

We proceed as follows. In section 2 we discuss our methodological approach, emphasizing in particular our new use of generalized variance decompositions and directional spillovers. In section 3 we describe our data and present our substantive results. We conclude in section 4.

2. Methods: Generalized Spillover Definition and Measurement

Here we extend the DY spillover index, which follows directly from the familiar notion of a variance decomposition associated with an *N*-variable vector autoregression. Whereas DY focuses on *total* spillovers in a *simple* VAR framework (i.e., with potentially order-dependent results driven by Cholesky factor orthogonalization), we progress by measuring *directional* spillovers in a *generalized* VAR framework that eliminates the possible dependence of results on ordering.

Consider a covariance stationary *N*-variable VAR(*p*), $x_t = \sum_{i=1}^{p} \Phi_i x_{t-i} + \varepsilon_t$, where $\varepsilon \sim (0, \Sigma)$ is the vector of independently and identically distributed disturbances. The moving average representation is $x_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i}$, where the *N*x*N* coefficient matrices A_i obey the recursion $A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + ... + \Phi_p A_{i-p}$, with A_0 an *N*x*N* identity matrix and $A_i = 0$ for $i<0$. The moving average coefficients (or transformations such as impulse-response functions or variance decompositions) are the key to understanding the dynamics of the system. We rely on variance decompositions, which allow us to parse the forecast error variances of each variable into parts attributable to the various system shocks. Variance decompositions allow us to assess the fraction of the *H*-step-ahead error variance in forecasting x_i that is due to shocks to x_j, $\forall\ j \neq i$, for each *i*.

Calculation of variance decompositions requires orthogonal innovations, whereas our VAR innovations are generally contemporaneously correlated. Identification schemes such as that based on Cholesky factorization achieve orthogonality, but the variance decompositions then depend on ordering of the variables. We circumvent this problem by exploiting the generalized VAR framework of Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998), hereafter KPPS, which produces variance decompositions invariant to ordering. Instead of shocking all variables of the system at once and orthogonalizing these shocks through Cholesky decomposition or through the structural VAR approach, the generalized VAR approach shocks only one variable at a time and integrates out the effects of other shocks using the historically observed distribution of the errors. As the shocks to each variable are not orthogonalized, the sum of contributions to the variance of forecast error (that is, the row sum of the elements of the variance decomposition table) is not necessarily equal to one.

Variance Shares

Let us define *own variance shares* to be the fractions of the *H*-step-ahead error variances in forecasting x_i due to shocks to x_i, for i=1, 2,..,N, and *cross variance shares*, or *spillovers*, to be the fractions of the *H*-step-ahead error variances in forecasting x_i due to shocks to x_j, for i, j = 1, 2,.., N, such that $i \neq j$.

Denoting the KPPS *H*-step-ahead forecast error variance decompositions by $\theta_{ij}^{g}(H)$, for $H = 1, 2, ...$, we have

$$\theta_{ij}^{g}(H) = \frac{\sigma_{ii}^{-1} \sum_{h=0}^{H-1} (e_i' A_h \sum e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \sum A_h' e_i)} \tag{1}$$

where $\sum$ is the variance matrix for the error vector ε, σ_{ii} is the standard deviation of the error term for the *i*th equation and e_i is the selection vector with one as the *i*th element and zeros otherwise. As explained above, the sum of the elements of each row of the variance decomposition table is not equal to 1: $\sum_{j=1}^{N} \theta_{ij}^{g}(H) \neq 1$. In order to use the information available in the variance decomposition matrix in the calculation of the spillover index, we normalize each entry of the variance decomposition matrix by the row sum as[3]:

$$\tilde{\theta}_{ij}^{g}(H) = \frac{\theta_{ij}^{g}(H)}{\sum_{j=1}^{N} \theta_{ij}^{g}(H)} \tag{2}$$

Note that, by construction, $\sum_{j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = 1$ and $\sum_{i,j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = N$.

Total Spillovers

Using the volatility contributions from the KPPS variance decomposition, we can construct a total volatility spillover index:

$$S^{g}(H) = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \tilde{\theta}_{ij}^{g}(H)}{\sum_{i,j=1}^{N} \tilde{\theta}_{ij}^{g}(H)} \bullet 100 = \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \tilde{\theta}_{ij}^{g}(H)}{N} \bullet 100. \tag{3}$$

This is the KPPS analog of the Cholesky factor based measure used in Diebold and Yilmaz (2009). The total spillover index measures the contribution of spillovers of volatility shocks across four asset classes to the total forecast error variance.

Directional Spillovers

Although it is sufficient to study the total volatility spillover index to understand how much of shocks to volatility spill over across major asset classes, the generalized VAR

[3] Alternatively, we can normalize the elements of the variance decomposition matrix with the column sum of these elements and compare the resulting total spillover index with the one obtained from the normalization with the row sum.

approach enables us to learn about the direction of volatility spillovers across major asset classes. As the generalized impulse responses and variance decompositions are invariant to the ordering of variables, we calculate the directional spillovers using the normalized elements of the generalized variance decomposition matrix. We measure directional volatility spillovers received by market *i* from all other markets *j* as:

$$S_{i\bullet}^{g}(H) = \frac{\sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{\theta}_{ij}^{g}(H)}{\sum_{j=1}^{N} \tilde{\theta}_{ij}^{g}(H)} \bullet 100 \tag{4}$$

In similar fashion we measure directional volatility spillovers transmitted by market *i* to all other markets *j* as:

$$S_{\bullet i}^{g}(H) = \frac{\sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{\theta}_{ji}^{g}(H)}{\sum_{j=1}^{N} \tilde{\theta}_{ji}^{g}(H)} \bullet 100 \tag{5}$$

One can think of the set of directional spillovers as providing a decomposition of total spillovers into those coming from (or to) a particular source.

<u>Net Spillovers</u>

Finally, we obtain the net volatility spillover from market *i* to all other markets *j* as

$$S_{i}^{g}(H) = S_{\bullet i}^{g}(H) - S_{i\bullet}^{g}(H) \tag{6}$$

The net volatility spillover is simply the difference between gross volatility shocks transmitted to and gross volatility shocks received from all other markets.

<u>Net Pairwise Spillovers</u>

The net volatility spillover (6) provides summary information about how much in net terms each market contributes to volatility in other markets. It is also of interest to examine net pairwise volatility spillovers, which we define as:

$$S_{ij}^{g}(H) = \left(\frac{\tilde{\theta}_{ij}^{g}(H)}{\sum_{k=1}^{N} \tilde{\theta}_{ik}^{g}(H)} - \frac{\tilde{\theta}_{ji}^{g}(H)}{\sum_{k=1}^{N} \tilde{\theta}_{jk}^{g}(H)} \right) \bullet 100 \qquad (7)$$

The net pairwise volatility spillover between markets *i* and *j* is simply the difference between gross volatility shocks transmitted from market *i* to *j* and gross volatility shocks transmitted from *j* to *i*.

3. Empirics: Estimates of Volatility Spillovers across U.S. Asset Markets

Here we use our framework to measure volatility spillovers among four key U.S. asset classes: stocks, bonds, foreign exchange and commodities. This is of particular interest because spillovers across asset classes may be an important aspect of the global financial crisis that began in 2007 (which started in credit markets but spilled over into equities).

In the remainder of this section we proceed as follows. We begin by describing our data in section 3a. Then we calculate average (i.e., total) spillovers in section 3b. We then quantify spillover dynamics, examining rolling-sample total spillovers, rolling-sample directional spillovers, rolling-sample net directional spillovers and rolling-sample net pairwise spillovers below.

<u>Stock, Bond, Exchange Rate, and Commodity Market Volatility Data</u>

We examine daily volatilities of returns on U.S. stock, bond, foreign exchange, and commodity markets. In particular, we examine the S&P 500 index, the 10-year Treasury bond yield, the New York Board of Trade U.S. dollar index futures, and the Dow-Jones/UBS commodity index.[4] The data span January 25, 1999 through September 30, 2009, for a total of 2688 daily observations.

In the tradition of a large literature dating at least to Parkinson (1980), we estimate daily variance using daily high and low prices.[5] For market *i* on day *t* we have

$$\tilde{\sigma}_{it}^2 = 0.361\left[\ln(P_{it}^{\max}) - \ln(P_{it}^{\min})\right]^2,$$

where $P_{it}^{\max}$ is the maximum (high) price in market *i* on day *t*, and $P_{it}^{\min}$ is the daily minimum (low) price. Because $\tilde{\sigma}_{it}^2$ is an estimator of the daily variance, the corresponding estimate of the annualized daily percent standard deviation (volatility) is $\hat{\sigma}_{it} = 100\sqrt{365 \bullet \tilde{\sigma}_{it}^2}$. We plot the four markets' volatilities in Figure 1 and we provide summary statistics in Table 1. Several interesting facts emerge, including: (1) The bond and stock markets have been the most volatile (roughly equally so), with commodity and FX markets comparatively less volatile, (2) volatility dynamics appear highly persistent, in keeping with a large literature summarized for example in Andersen, Bollerslev, Christoffersen and Diebold (2006), and (3) all volatilities are high during the recent crisis, with stock and bond market volatility, in particular, displaying huge jumps.

Throughout the sample, stock market went through two periods of major volatility. In 1999, daily stock market volatility was close to 25 percent, but it increased significantly to fluctuate above 25 percent until mid-2003, moving occasionally above 50 percent. After

[4] The DJ/AIG commodity index was re-branded as the DJ/UBS commodity index following the acquisition of AIG Financial Products Corp. by UBS Securities LLC on May 6, 2009.
[5] For background, see Alizadeh, Brandt and Diebold (2002) and the references therein.

mid-2003, it declined to less than 25 percent and stayed there until August 2007. Since August 2007, stock market volatility reflects well the dynamics of the sub-prime crisis.

In the first half of our sample, the interest rate volatility measured by the annualized standard deviation was comparable to the stock market volatility. While it was lower than 25 percent mark for most of 2000, in the first and last few months of 2001, it increased and fluctuated between 25-50 percent. Bond market volatility remained high until mid-2005, and fell below 25 percent from late 2005 through the first half of 2007. Since August 2007, volatility in bond markets has also increased significantly.

Commodity market volatility used to be very low compared to stock and bond markets, but it increased slightly over time and especially in 2005-2006 and recently in 2008. FX market volatility has been the lowest among the four markets. It increased in 2008 and moved to a 25-50 percent band following the collapse of Lehman Brothers in September 2008. Since then, FX market volatility declined, but it is still above its average for the last decade.

<u>Unconditional Patterns: The Full-Sample Volatility Spillover Table</u>

We call Table 2 a volatility spillover table. Its ij^{th} entry is the estimated contribution *to* the forecast error variance of market i coming *from* innovations to market j.[6] Hence the off-diagonal column sums (labeled contributions to others) or row sums (labeled contributions from others), are the "to" and "from" directional spillovers, and the "from minus to" differences are the net volatility spillovers. In addition, the total volatility spillover

[6] All results are based on vector autoregressions of order 4 and generalized variance decompositions of 10-day-ahead volatility forecast errors. To check for the sensitivity of the results to the choice of the order of VAR we calculate the spillover index for orders 2 through 6, and plot the minimum, the maximum and the median values obtained in Figure A1 of the Appendix. Similarly, we calculated the spillover index for forecast horizons varying from 4 days to 10 days. Both Figure A1 and Figure A2 of the Appendix show that the total spillover plot is not sensitive to the choice of the order of VAR or to the choice of the forecast horizon.

index appears in the lower right corner of the spillover table. It is approximately the grand off-diagonal column sum (or row sum) relative to the grand column sum including diagonals (or row sum including diagonals), expressed as a percent.[7] The volatility spillover table provides an approximate "input-output" decomposition of the total volatility spillover index.

Consider first what we learn from the table about directional spillovers (gross and net). From the "directional to others" row, we see that gross directional volatility spillovers to others from each of the four markets are not very different. We also see from the "directional from others" column that gross directional volatility spillovers from others to FX is relatively large, at 26.53 percent, followed by the bond market with the spillovers from others explaining 17.21 percent of the forecast error variance. As for net directional volatility spillovers, the largest are from the stock market to others (6.92 percent) and from others to the FX market (10.27 percent).

Now consider the total (non-directional) volatility spillover, which is effectively a distillation of the various directional volatility spillovers into a single index. The total volatility spillover appears in the lower right corner of Table 2, which indicates that on average, across our entire sample, 16.55 percent of volatility forecast error variance in all four markets comes from spillovers.

<u>Conditioning and Dynamics I: The Rolling-Sample Total Volatility Spillover Plot</u>

Clearly, many changes took place during the years in our sample, January 1999-September 2009. Some are well-described as more-or-less continuous evolution, such as increased linkages among global financial markets and increased mobility of capital, due to

[7] As we have already discussed in Section 2 in detail, the approximate nature of the claim stems from the properties of the generalized variance decomposition. With Cholesky factor identification the claim is exact rather than approximate; see also Diebold and Yilmaz (2009).

globalization, the move to electronic trading, and the rise of hedge funds. Others, however, may be better described as bursts that subsequently subside.

Given this background of financial market evolution and turbulence, it seems unlikely that any single fixed-parameter model would apply over the entire sample. Hence the full-sample spillover table and spillover index constructed earlier, although providing a useful summary of "average" volatility spillover behavior, likely miss potentially important secular and cyclical movements in spillovers. To address this issue, we now estimate volatility spillovers using 100-day rolling samples, and we assess the extent and the nature of spillover variation over time via the corresponding time series of spillover indices, which we examine graphically in the so-called total spillover plot of Figure 2.

Starting around twenty percent in 1999, the total volatility spillover plot usually fluctuates between ten and thirty percent. However, there are important exceptions: The spillovers exceed the thirty percent mark in the second half of 2000 and the first quarter of 2001, immediately after the 9/11 terrorist attacks, in the third quarter of 2002, in June 2006 and most importantly by far, during the global financial crisis of 2007-2009. One can see five volatility waves during the recent crisis: July-August 2007 (credit crunch), January 2008 (panic in stock and foreign exchange markets followed by an unscheduled rate cut of three-quarters of a percentage points by Federal Reserve), June 2008 (when the worries about the burden of the crisis on government budgets started to appear, the volatility increased in the bond market and then spread to the other markets), following the collapse of Lehman Brothers (September-October 2008) and in the first half of 2009 as the crisis started to have its real effects on the world economy. During these episodes, the spillover index surges well above thirty percent. Indeed, following the collapse of Lehman Brothers in mid-September, and consistent with the unprecedented evaporation of liquidity world-wide, the volatility spillover plot jumped to 64 percent on September 30, 2008, before declining slightly.

Conditioning and Dynamics II: Rolling-Sample Gross Directional Volatility Spillover Plots

Thus far we have discussed the *total* spillover plot, which is of interest but discards directional information. That information is contained in the "Contribution to" row (the sum of which is given by $S_{i\cdot}^{g}(H)$ in equation 4) and the "Contribution from" column (the sum of which is given by $S_{\cdot i}^{g}(H)$ in equation 5).

We now estimate that row and column dynamically, in a fashion precisely parallel to the earlier-discussed total spillover plot. We call these *directional* spillover plots. In Figure 3, we present the directional volatility spillovers *from* our four asset classes. They vary greatly over time. During tranquil times, spillovers from each market are below five percent, but during volatile times, directional spillovers increase to above 10 percent. Among the four markets, gross volatility spillovers from the commodity markets are in generally smaller than the spillovers from the other three markets.

In Figure 4, we present directional volatility spillovers *to* our four asset classes. As with the directional spillovers *from* others, the spillovers *to* others vary noticeably over time. The relative variation pattern, however, is reversed, with directional volatility spillovers *to* commodities and FX increasing relatively more in turbulent times.

Conditioning and Dynamics III: Rolling-Sample Net Directional Volatility Spillover Plots

Above we briefly discussed the gross spillover plots, because our main focus point is the net directional spillover plot, which is given in Figure 5. Each point in Figure 5a through 5d corresponds $S_{i}^{g}(H)$ (equation 6) and is the difference between the "Contribution from" column sum and the "Contribution to" row sum. In addition, as we described briefly at the end of section 2, we also calculate net pairwise spillovers between two markets (equation 7) and present these plots in Figure 6.

First note that, overall, there has been relatively little net volatility transmission from the commodity and FX markets. Only in the first five months of 2003 (before and immediately after the invasion of Iraq in March 2003), in late 2004 and early 2005 and during the commodity price boom in the first half of 2008 do we observe that net volatility spillovers from commodity markets to other markets reach five percent. Similarly, volatility in FX markets also had very little net impact on volatility in other markets, perhaps with the exception of the sizable spillovers at the end of 2001, throughout 2006 and in January 2008.

Instead, the clear channels of net directional volatility spillovers are from the stock and bond markets. Net volatility spillovers from the stock market for the most part appear to be positive and large especially in the earlier parts of the sample period (from 2000 to 2002, in particular). As the technology bubble burst in March 2000, net volatility spillovers from the stock market reached close to 5 percent (Figure 5a). At the time, the bulk of the volatility spillovers from the stock market went to the bond market (Figure 6a). However, the stock market volatility and the spillovers from the stock market continued to run high towards the end of 2000 and in 2001, as the problems in technology stocks spread to other stocks.

Volatility spillovers from the stock market ran as high as 3-4 percent from the end of 2000 until the 9/11 terrorist attacks in September (Figure 5a). During this period, shocks to stock market volatility were transmitted to the commodities and the FX markets. After the terrorist attacks on September 11, 2001, volatility spillovers from the stock market took place only in the direction of the commodity market. During the increased U.S. stock market gyrations in June through October 2002, net spillovers from the stock market reached close to 10 percent, but affected only the FX and bond markets. Finally, from August 2007 to June 2008, net spillovers from the stock market to bond and commodity markets reached close to 5 percent mark. While volatility spillovers from the stock market affected the FX market after the collapse of Lehman Brothers, during the panic in January 2008 it was the stock market that received volatility spillovers from the FX market.

Similarly, and interestingly, during the global financial crisis and especially since the summer of 2008, the bond market was the most important transmitter of volatility. Indeed, in June 2008 when the worries about the heavy burden of the crisis on government finances intensified, and following the collapse of Lehman Brothers in mid-September, net volatility spillovers from the bond market reached as high as 10 percent (Figure 5b). The bulk of the volatility spillovers from the bond market during this time period were transmitted to the stock market (see Figure 6a). Over the same period, the commodity and FX markets were also net receivers of volatility spillovers from the bond market (Figure 6d and 6e).

Volatility spillovers from commodity markets increased in 2003 just before the invasion of Iraq by U.S. forces, at the end of 2004 and early 2005, when the surge in Chinese demand for oil and metals surprised investors sending commodity prices higher (shocks mostly transmitted to the bond and FX markets), and especially during the commodity price boom in the first half of 2008 (shocks mostly transmitted to the bond market). The volatility shocks in the commodity market before and during the initial phases of the Iraqi invasion spilled over mostly to the stock market, but also to the bond market. During the late 2004-early 2005 and the first half of 2008, the volatility shocks in the commodity market mostly spilled over to the bond market, but also to the FX market.

Finally, volatility spillovers from the FX market increased at the end of 2001 and early 2002. It also increased in May 2006, following the FED's decision to increase the Federal Funds target rate further to 5 percent on May 10, 2006, and in January 2008, following the FED's decision to cut the Federal Funds target rate by 75 basis points on January 22, 2008. In all three episodes, the volatility shocks in the FX market spilled over to the stock market. In only two out of these three episodes (May 2006 and January 2008) shocks to FX return volatility spilled over to the bond market, albeit at a lower scale.

Our results show that when there is a significant shock to volatility in one of the four major asset markets, this volatility shock is likely to spill over to other asset markets. Before the global financial crisis of 2008-2009 mostly this was the case; shocks did not occur simultaneously. When one of the markets was hit by a volatility shock, the shock was likely to spill over to other markets if it was significant enough. As a case in point we can have a closer look at volatility spillovers in 2006. When the Federal Reserve decided to increase the FED Funds target rate from 4.75 percent to 5.00 percent in May 2006 and signaled it might increase it further in June, it had a rather unexpected effect on FX markets. The dollar started to gain momentum as the investors decided to move part of the portfolio they invested in many emerging markets back to the US safe haven. During this period the volatility in FX markets spilled over to other markets, but most importantly to the commodity market.

We know that during the global financial crisis volatility shocks take place simultaneously in some or all asset markets. In such a case it is not possible to argue that the volatility spillovers will always take place from a particular market towards the others. For this we can focus on January 2008 episode. In an unscheduled emergency meeting on January 21, 2008, which was an official holiday in the US, the Federal Reserve's FOMC decided to lower Federal Funds target rate by three-quarters of a percent. Before that day volatility was high in all asset markets. After this meeting volatility increased further because it was taken as an acknowledgement by the Federal Reserve of how grave the situation was. While the volatility in the stock and bond markets increased by 30 and 123 percent, respectively, on the next business day (January 22), volatility in the commodity and FX markets increased by 186 and 174 percent. Furthermore, in the following weeks investors moved away from other assets towards commodities as they were deemed less vulnerable during the financial crisis. In addition investors also reduced their US dollar holdings while increasing that of Euro and Yen. As a result following the January 21, 2008 decision the volatility shocks were transmitted from the FX market to the stock and bond markets, whereas shocks in the commodity markets were transmitted to the bond market.

4. Concluding Remarks

We have provided both gross and net *directional* spillover measures that are independent of the ordering used for volatility forecast error variance decompositions. When applied to U.S. financial markets, our measures shed new light on the nature of cross-market volatility transmission, pinpointing the importance during the recent crisis of volatility spillovers from the bond market to other markets.

We are of course not the first to consider issues related to volatility spillovers (e.g., Engle et al. 1990; King et al., 1994; Edwards and Susmel, 2001), but our approach is very different. It produces continuously-varying indexes (unlike, for example, the “high state / low state” indicator of Edwards and Susmel), and it is econometrically tractable even for very large numbers of assets. Although it is beyond the scope of this paper, it will be interesting in future work to understand better the relationship of our spillover measure to a variety of others based on measures ranging from traditional (albeit time-varying) correlations (e.g., Engle, 2002, 2009) to the recently-introduced CoVaR of Adrian and Brunnermeier (2008).

References

Adrian, T. and Brunnermeier, M. (2008), "CoVaR," Staff Report 348, Federal Reserve Bank of New York.

Alizadeh, S., Brandt, M.W. and Diebold, F.X. (2002), "Range-Based Estimation of Stochastic Volatility Models," *Journal of Finance*, 57, 1047-1092.

Andersen, T.G., Bollerslev, T., Christoffersen, P.F. and Diebold, F.X. (2006), "Practical Volatility and Correlation Modeling for Financial Market Risk Management," in M. Carey and R. Stulz (eds.), *Risks of Financial Institutions*, University of Chicago Press for NBER, 513-548.

Diebold, F.X. and Yilmaz, K. (2009), "Measuring Financial Asset Return and Volatility Spillovers, With Application to Global Equity Markets," *Economic Journal*, 119, 158-171.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH Models," *Journal of Business and Economic Statistics*, 20, 339-350.

Engle, R.F. (2009), *Anticipating Correlations*. Princeton: Princeton University Press.

Engle, R.F., Ito, T. and Lin, W.-L. (1990), "Meteor Showers or Heat Waves: Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market," *Econometrica*, 58, 525-542.

Forbes, K.J. and Rigobon, R. (2002), "No Contagion, Only Interdependence: Measuring Stock Market Comovements," *Journal of Finance*, 57, 2223-2261.

King, M., Sentana, E. and Wadhwani, S. (1994), "Volatility and Links Between National Stock Markets," *Econometrica*, 62, 901-933.

Koop, G., Pesaran, M.H., and Potter, S.M. (1996), "Impulse Response Analysis in Non-Linear Multivariate Models," *Journal of Econometrics*, 74, 119–147.

Parkinson, M. (1980), “The Extreme Value Method for Estimating the Variance of the Rate of Return,” *Journal of Business*, 53, 61-65.

Pesaran, M.H. and Shin, Y. (1998), “Generalized Impulse Response Analysis in Linear Multivariate Models,” *Economics Letters*, 58, 17-29.

Reinhart, C.M. and Rogoff, K.S. (2008), “Is the 2007 U.S. Subprime Crisis So Different? An International Historical Comparison,” *American Economic Review*, 98, 339–344.

Sims, C.A. (1980), “Macroeconomics and Reality,” *Econometrica*, 48, 1-48.

Figure 1. Daily U.S. Financial Market Volatilities

(Annualized Standard Deviation, Percent)

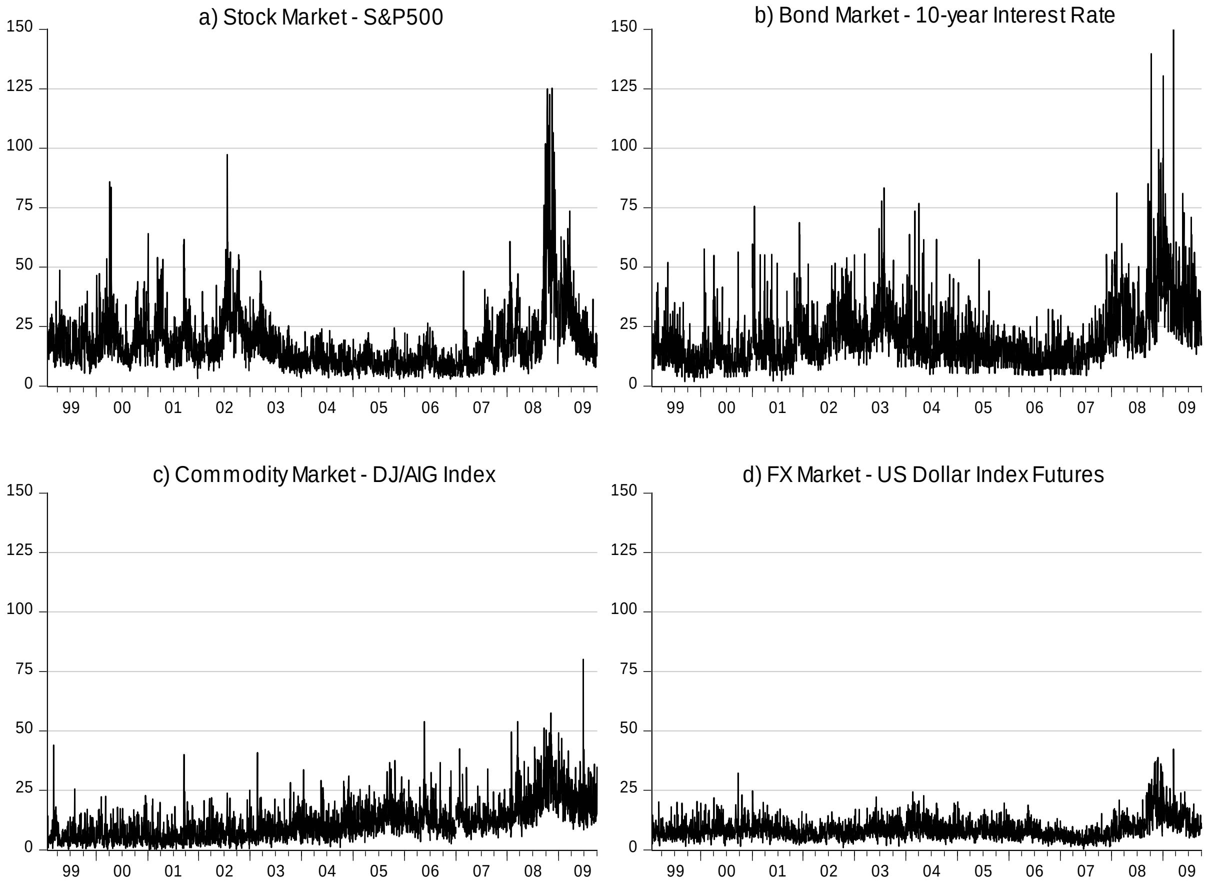


Table 1: Volatility Summary Statistics, Four Asset Classes

	Stocks	**Bonds**	**Commodities**	**FX**
Mean	18.05	20.34	11.57	8.74
Median	14.71	16.88	9.78	7.79
Maximum	125.17	230.77	80.11	42.34
Minimum	2.75	1.94	0.20	0.42
Std. Deviation	12.93	13.88	8.21	4.52
Skewness	2.98	3.11	1.66	1.82
Kurtosis	17.71	28.60	7.66	9.23

Table 2: Volatility Spillover Table, Four Asset Classes

	Stocks	**Bonds**	**Commodities**	**FX**	**Directional *FROM* Others**
Stocks	90.10	4.92	3.26	1.73	9.90
Bonds	2.95	82.79	4.20	10.06	17.21
Commodities	5.43	2.64	87.45	4.48	12.55
FX	8.44	11.34	6.75	73.47	26.53
Directional *TO* Others	16.82	18.90	14.21	16.26	
Directional Including Own	106.9	101.7	101.7	89.7	**Total Spillover Index (**66.19/400): **16.6%**

Figure 2. Total Volatility Spillovers, Four Asset Classes

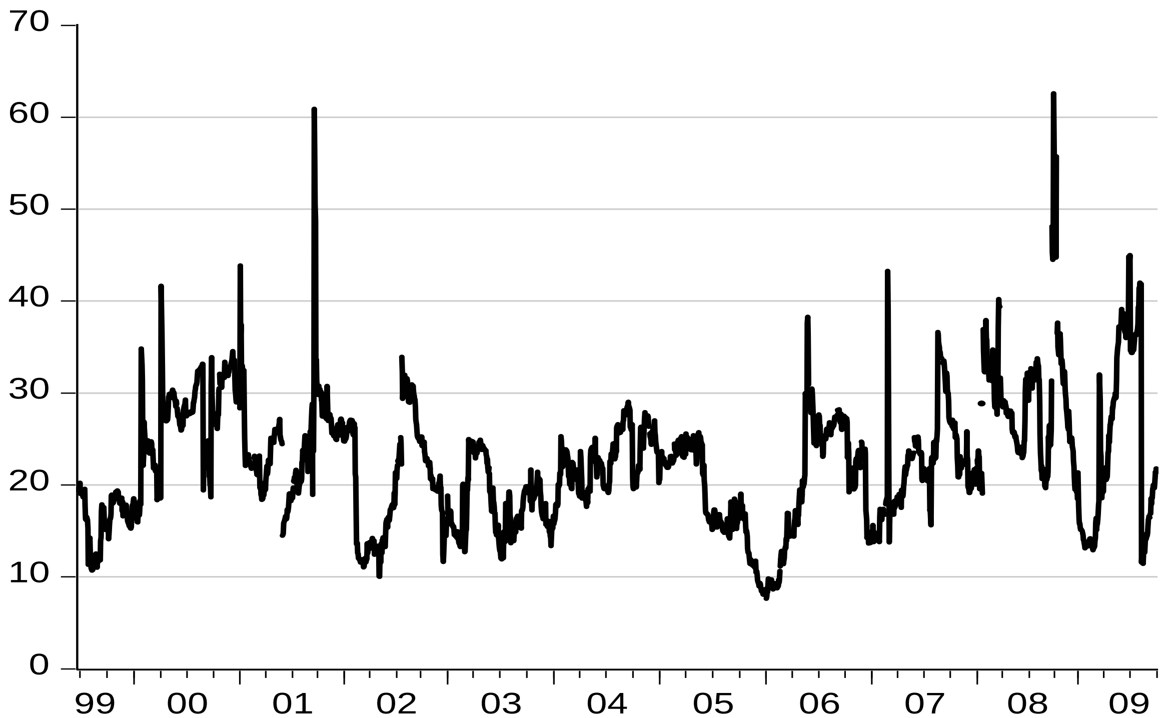


Figure 3. Directional Volatility Spillovers, *FROM* Four Asset Classes

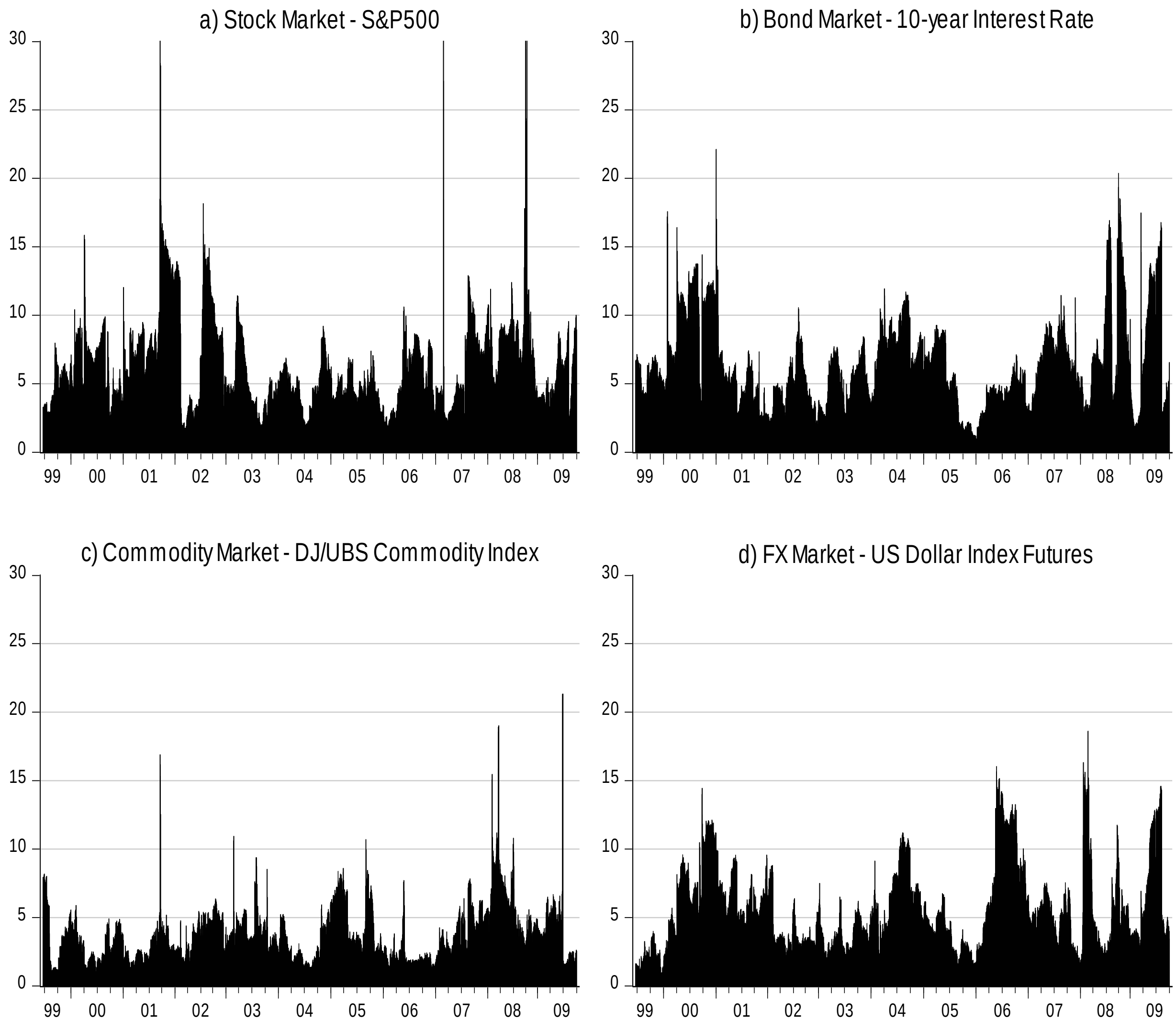


Figure 4. Directional Volatility Spillovers, *TO* Four Asset Classes

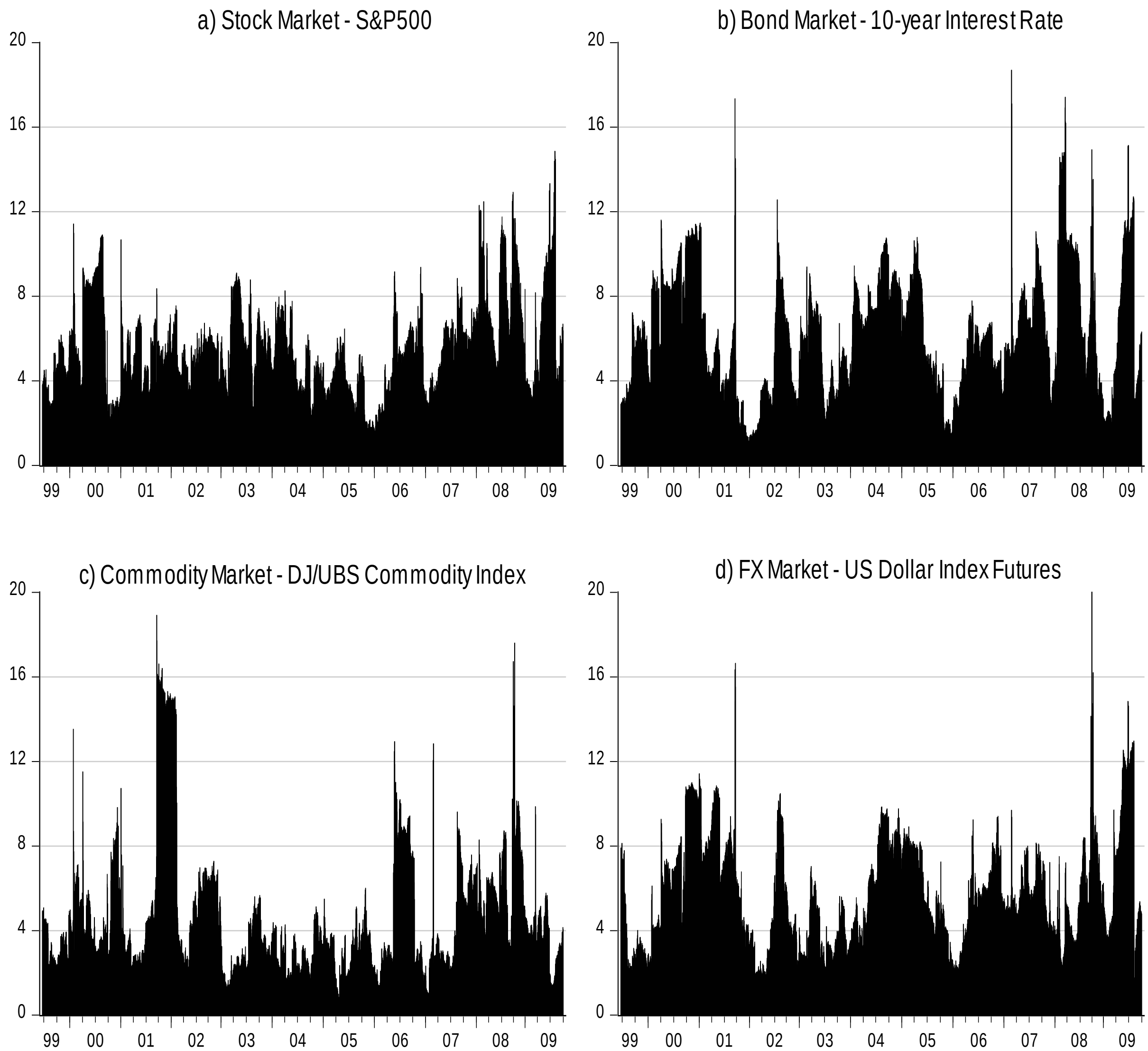


Figure 5. Net Volatility Spillovers, Four Asset Classes

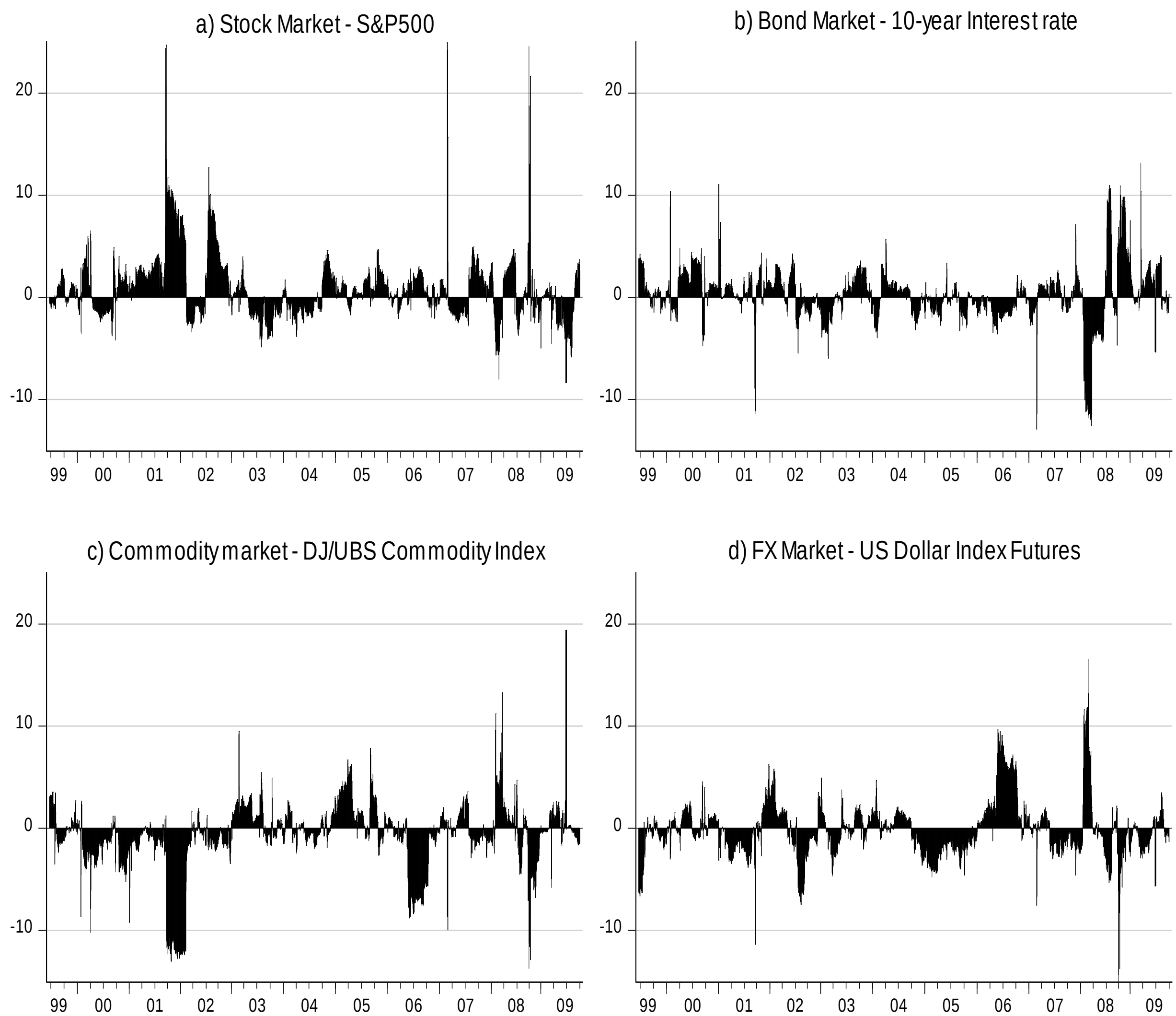
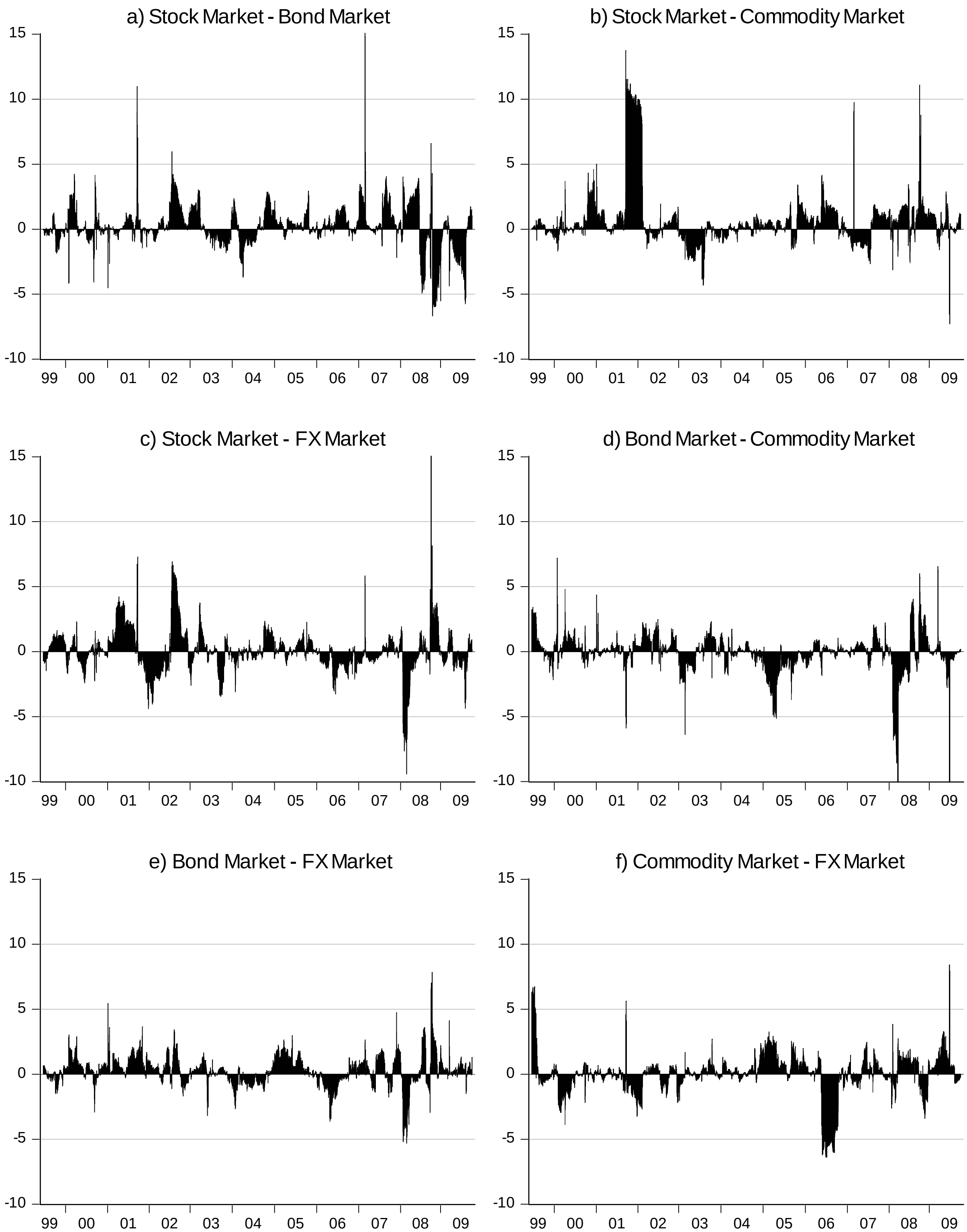


Figure 6. Net Pairwise Volatility Spillovers



APPENDIX

Figure A1. Sensitivity of the index to VAR lag structure (Max, Min and Median values of the index for VAR order of 2 through 6)

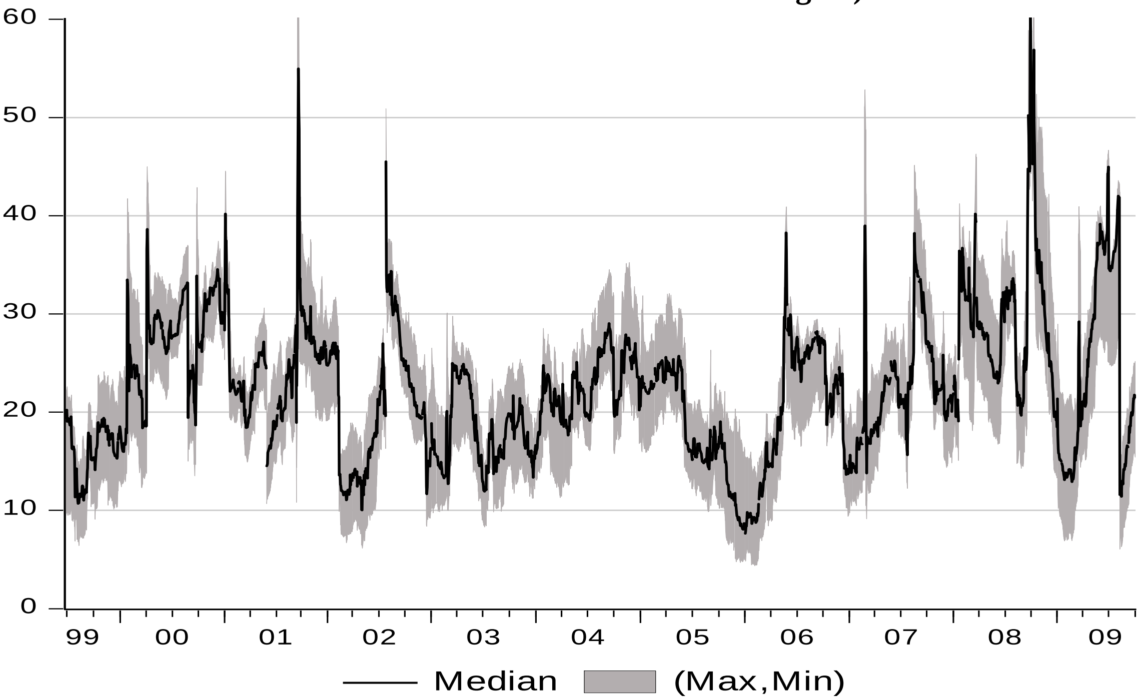


Figure A2. Sensitivity of the Index to Forecast Horizon (Min, Max and Median values over 5 to 10-day horizons)