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LONG MEMORY AND PERSISTENCE IN AGGREGATE OUTPUT*

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We examine persistence in U.S. aggregate output by estimating fractionally integrated ARIMA models. These models provide better low-frequency approximations to the Wold representation than previous stochastic specifications, and earlier results on the importance of a permanent component emerge as special cases. We find evidence of long memory, which induces persistence, though this long memory need not be associated with a unit root. Our point estimates indicate that macroeconomic shocks, while persistent, are distinctly less persistent than many earlier studies suggest; however, confidence intervals associated with the long-run response are quite wide.

1. Introduction

In the last five years, the permanent nature of macroeconomic fluctuations has become the subject of intense debate. Starting with Nelson and Plosser (1982), some have taken issue with the traditional view that macroeconomic time series are well described as transitory deviations from a deterministic trend. Instead, it has been suggested that aggregate output contains a substantial permanent component; that is, a given movement in aggregate output will persist and will not necessarily be reversed in the future through reversion to trend. Campbell and Mankiw (1987a), for instance, fit autoregressive integrated moving-average (ARIMA) models to post-war real gross national product (GNP) and conclude that a 1 percent innovation to current GNP should change long-run forecasts of GNP by *more* than 1 percent.

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We contribute to this debate by examining the low-frequency components in real output movements in greater detail than has been done previously. The results have implications for the nature and existence of business cycles, the persistence of macroeconomic shocks, and the specification of statistical models of economic fluctuations. We explore an approximation to the Wold representation that is more general than, and includes as special cases, the ARIMA and unobserved-components (UC) representations that have been used by others. We use long-memory, fractionally integrated representations that allow for increased flexibility in modeling low-frequency dynamics. The results provide a unification of existing disparate persistence estimates as well as, given the wide confidence intervals obtained, a cautionary note against focusing on any point estimate of the permanent component on the basis of a limited span of macroeconomic data.

In the next section, we discuss the importance of understanding the nature of persistence in aggregate output and describe our measure of persistence. In section 3, the fractionally integrated ARIMA model is introduced and economic motivation is provided. Section 4 describes the estimation procedure employed. Empirical results are contained in section 5, and section 6 concludes.

2. Measuring the permanent component

A measurement of the permanent component in aggregate output is crucial to both the theory and practice of macroeconomics. The presence of a large permanent component would imply that a substantial portion of a given macroeconomic shock to the economy would persist through time. This conflicts with traditional formulations of both Keynesian and Classical macroeconomic theories, where output fluctuations, from a variety of causes, are temporary deviations from a slowly growing natural or equilibrium level of output. A large permanent component implies instead that almost all fluctuations in output represent permanent movements. Such fluctuations require description within an intrinsically stochastic economic theory, since deterministic models cannot be regarded as even approximately true when reversion to deterministic equilibrium paths is absent.[1] In addition, all of the standard econometric tasks – estimation, hypothesis testing, prediction, and control – are sensitive to the presence of a permanent component.

For policymakers, the implications of macroeconomic persistence are just as unsettling. Strong persistence would call into question, at a fundamental level, the appropriateness of countercyclical policy. If the cyclical component is insignificant in aggregate output and there is no steady trend to which to

[1] Much recent work on stochastic growth models, such as King, Plosser, and Rebello (1988), is at least partially motivated by this fact.

return the economy, attempts at countercyclical policy are at best misguided. In addition, when each movement in output is largely permanent, the costs and benefits of policy actions are far different than when movements are transitory; the price of higher or lower output over the whole future path of the economy must be weighed in the policy calculus.

Much of the empirical literature relevant to the persistence debate examines the *existence* of a permanent component, which provides, as we shall discuss below, a first step to the more interesting question of the *importance* of the permanent component. The existence of a permanent component is commonly examined by testing for unit roots in autoregressive lag-operator polynomials. If a unit root is found in an ARMA representation, then, as shown by Beveridge and Nelson (1981), the series may be decomposed into the sum of a random walk component and a stationary component. The permanence of the random-walk movements implies a permanent component in the original series.

To formalize matters, consider the ARMA model

$$\Phi(L)Y_t = \Theta(L)\varepsilon_t, \qquad \varepsilon_t \sim (0, \sigma_\varepsilon^2), \tag{1}$$

where $\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\Theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$, all roots of $\Phi(L)$ are on or outside the unit circle, and all roots of $\Theta(L)$ are outside the unit circle. If $\Phi(L)$ has a unit root, it is assumed to be positive and real, in which case we can difference the series and write $\Phi(L) = (1 - L)\Phi'(L)$, where $\Phi'(L)$ is of order $p - 1$; thus, the process is ARIMA$(p - 1, 1, q)$.

The Dickey–Fuller tests [e.g., Fuller (1976)] and generalizations by Phillips (1987) and others have been widely applied in economics to test the unit-root hypothesis. Most such studies conclude that real output and many other economic time series are well described by low-order ARMA processes with a single unit root.[2] Broad surveys such as Nelson and Plosser (1982) and Schwert (1987), which make use of a variety of unit-root tests on scores of economic variables, find pervasive and robust evidence of unit roots. In addition, the growing literature documenting cointegration in various economic relationships implicitly extends the above list, because cointegrated variables have unit roots in their univariate representations. In this paper, we provide qualification to the wide-ranging evidence on the existence of unit roots, and more crucially their importance, by considering more general models that allow for rich low-frequency dynamics and include unit roots as a special case.

While the presence of a unit root in real output provides evidence for the existence of persistence, the more relevant question for macroeconomic analysis involves quantifying the extent of that persistence. We are more interested

[2] See, for example, Stock and Watson (1986), Perron and Phillips (1987), and Campbell and Deaton (1987).

in the size or importance of the response of output to a unit innovation than in the mere existence of some nonzero response. One measure of persistence, used by Watson (1986), Campbell and Mankiw (1987a, b), and others, is the cumulative impulse-response function, i.e., the sum of the coefficients of the moving-average lag-operator polynomial of the first-differenced series. Specifically, consider

$$\Delta Y_t = A(L)\varepsilon_t = \left(1 + a_1 L + a_2 L^2 + \cdots\right)\varepsilon_t. \tag{2}$$

The impact of a unit shock in period t on the *growth rate* of Y at time $t+k$ is a_k, while the impact on the level of Y at time $t+k$ is $c_k \equiv 1 + a_1 + \cdots + a_k$. In the limit, we obtain c_∞, which is the effect of a unit shock today on the level of Y infinitely far in the future. For any stationary or trend-stationary series, $c_\infty = 0$, because the effect of any shock is transitory as reversion to mean or trend eventually dominates. For a random walk, $c_\infty = 1$; that is, the effect of any shock is exactly permanent. In general, unit roots lead to a nonzero long-run response; however, the particular value of c_∞ depends on the specific parameterization of the process.

Any reasonable persistence measure must be related to the form of the Wold-representation lag-operator polynomial $A(L)$, which completely characterizes the mapping from inputs $\{\varepsilon_t\}$ to outputs $\{Y_t\}$. This is true of all persistence measures that have appeared in the literature, including the height of the spectral density of ΔY at frequency zero, the proportion of variation in ΔY due to movements in the underlying random-walk component, or the limiting value of Cochrane's (1988) variance ratio.[3] In this paper, we part with the tradition of using the infinite cumulative response, c_∞, to measure persistence and instead examine the *entire* sequence of cumulative impulse responses, $C = \{1, c_1, c_2, c_3, \ldots, c_\infty\}$. We use C to study persistence, because it directly answers the question of interest: 'How does a shock today affect the level of output in the short, medium, long, and very long run?' For example, with quarterly data, c_{40} is the impact of a unit shock on the level of output ten years hence. This approach is more informative than concentrating on c_∞, because the economic horizons of interest are typically much shorter than infinity. In the long-memory models that we consider below, the cumulative impulse response, even at quite long horizons, can differ substantially from c_∞.

3. Modeling the permanent component

The modeling of persistent processes is an issue closely related to the measurement of persistence; in particular, construction of the cumulative impulse-response sequence requires estimates of the parameters of the

[3]A more detailed discussion of alternative persistence measures and their interrelationships is contained in Diebold and Nerlove (1989).

moving-average representation. A number of authors have recently addressed the issue of modeling persistence in real output, notably Campbell and Mankiw (1987a), who use unrestricted ARIMA representations. Fitting an ARIMA(2, 1, 2) model to the logarithm of post-war quarterly real GNP, they obtain a c_∞ persistence estimate of 1.52, so that a unit innovation leads to a long-run response substantially *larger* than the initial innovation. This striking result of very strong shock persistence (and, in fact, shock magnification) runs counter to the findings of Watson (1986) and Clark (1987), who obtain smaller estimates of persistence ($c_\infty = 0.6$) with estimated unobserved components (UC) models.[4] They argue that the long-run behavioral implications of the unrestricted ARIMA models are misleading because such models concentrate on representations of the short-run dynamics. Both Watson and Clark note, however, that the UC model can be viewed as a special case of the more general ARIMA model: UC models are simply (nonlinearly) restricted ARIMA models.[5] Campbell and Mankiw (1987b) argue that the restrictions implied by the UC specification are unsupported.

We believe that there is some truth in both of these arguments. On the one hand, long-run properties of data series are likely to be difficult to determine in the context of an unrestricted ARIMA model. Mean reversion in economic time series depends crucially on correlations at long lags, which easily can be misspecified in simple ARIMA representations [see, e.g., Gagnon (1988)]. We shall provide implicit support for this thesis below. We assert, however, that what is required is not a specialization of the ARIMA model but a generalization, one that can capture a variety of long-run, low-frequency responses. Very high-order autoregressive models could capture such responses if degrees of freedom were plentiful; instead, we adopt a parsimonious model that achieves the same goal. Furthermore, our specification nests both the ARIMA and UC models.

Specifically, consider a generalization of the ARIMA(p, d, q) model to allow *fractional* integration,

$$\Phi(L)(1-L)^d Y_t = \Theta(L)\varepsilon_t, \qquad \varepsilon_t \sim \left(0, \sigma_\varepsilon^2\right), \tag{3}$$

where $\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\Theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$, all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle, and d is allowed to assume values in the real, as opposed to the integer, set of numbers.[6] Econometricians

[4]Like Campbell and Mankiw, of course, Watson obtains high persistence estimates ($c_\infty = 1.5$) from unrestricted ARIMA models.

[5]In particular, the restrictions implied by common UC representations *force* c_∞ to be less than unity.

[6]Stationarity and invertibility require $|d| < \frac{1}{2}$, which can always be achieved by taking a suitable number of differences. In what follows, we achieve a local generalization of unit root behavior by considering $\frac{1}{2} < d < \frac{3}{2}$; note that the first-differenced series then has an integration order less than $\frac{1}{2}$ in absolute value, so that stationarity and invertibility are achieved.

typically have considered only integer values of d; the leading special cases are the discrete options $d = 0$ (stationarity) and $d = 1$ (unit-root nonstationarity). Formally, however, an integer-d restriction in eq. (3) is arbitrary. We shall demonstrate that noninteger d values, i.e., fractional integration, provide for parsimonious yet flexible modeling of low-frequency variation; we denote such models as ARFIMA (AutoRegressive Fractionally Integrated Moving Average) models.

The ARFIMA model can be put in the moving-average form (2). First, write eq. (3) as

$$(1 - L)^d Y_t = B(L)\varepsilon_t, \tag{4}$$

where $B(L) = \Phi^{-1}(L)\Theta(L)$. Extracting the factor $(1 - L)$ gives

$$(1 - L)^{d-1}(1 - L)Y_t = B(L)\varepsilon_t \tag{5}$$

or

$$(1 - L)Y_t = A(L)\varepsilon_t, \tag{6}$$

where $A(L) = (1 - L)^{1-d}B(L)$.[7] Operationally, a binomial expansion of the operator $(1 - L)^d$ is used,

$$(1 - L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j - d)L^j}{\Gamma(-d)\Gamma(j + 1)} \tag{7}$$

$$= 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \cdots, \tag{8}$$

where $\Gamma(\cdot)$ denotes the gamma, or generalized factorial, function. Thus, the filter $(1 - L)^d$ provides an infinite-order lag-operator polynomial with slowly and monotonically declining weights.

The ARFIMA model (3) belongs to the class of long-memory processes, so named for their ability to display significant dependence between observations widely separated in time.[8] Standard ARMA processes are often labeled 'short-memory' processes because the dependence between observations τ periods apart decays rapidly as τ increases; indeed, it is well known that for

[7]More generally, we can allow for drift, $(1 - L)Y_t = \mu + A(L)\varepsilon_t$, as is done in the estimation reported below.

[8]Long-memory processes have their genesis in physics and in early hydrological work, such as Hurst (1951). Mandelbrot (1972) formalized many of the empirical insights about these processes and proposed R/S analysis, an early technique to characterize the extent of persistence. [For an application and generalization of R/S analysis, see Lo (1988).] Granger and Joyeux (1980) and Hosking (1981) independently proposed the use of fractionally integrated ARIMA procedures as long-memory models.

Table 1

Autocorrelation functions for AR(1) and ARFIMA(0, d, 0).

		Lag (τ)								
		1	2	3	4	5	10	25	50	100
$(1-0.5L)Y_t=\varepsilon_t$,	$\rho(\tau)=$	0.50	0.25	0.13	0.06	0.03	0.00	0.00	0.00	0.00
$(1-L)^{0.3}Y_t=\varepsilon_t$,	$\rho(\tau)=$	0.50	0.40	0.35	0.32	0.30	0.24	0.18	0.14	0.11

large τ ARMA autocorrelations decay approximately geometrically,

$$\rho(\tau) \sim r^{\tau},$$

where r is a constant such that $|r|<1$. ARFIMA processes, however, have a slower hyperbolic autocorrelation decay; for large τ we have the approximation

$$\rho_Y(\tau) \sim \tau^{2d-1}, \qquad d<\tfrac{1}{2}, \qquad d \neq 0.$$

To see how the autocorrelations vary with fractional d, it is instructive to consider pure fractional noise, denoted ARFIMA(0, d, 0), given by

$$(1-L)^d Y_t = \varepsilon_t. \tag{9}$$

Table 1 provides a comparison of the τth-order autocorrelations of fractional noise with those of a first-order autoregression [AR(1)]. The two models are parameterized to provide the same first-order autocorrelations, but as the interval between observations increases, the autocorrelations diverge. At lag 25, the AR(1) correlation is approximately 0.0, while the fractionally integrated series has a correlation of 0.18.

The intuition of long memory and the limitation of the integer-d restriction emerge clearly in the frequency domain. The series $\{Y_t\}$ displays long memory if its spectral density, f_Y, increases without limit as angular frequency tends to zero,

$$\lim_{\lambda \to 0} f_Y(\lambda) = \infty. \tag{10}$$

In fact, for an ARFIMA series, $f_Y(\lambda)$ behaves like λ^{-2d} as $\lambda \to 0$, so d parameterizes the low-frequency behavior. This is in contrast to the usual ARIMA model with $d=1$, where the spectral density is forced to behave like λ^{-2} as $\lambda \to 0$. Thus, a rich range of spectral behavior near the origin becomes possible when the integer-d restriction is relaxed. The ARFIMA model, by allowing a variety of spectral shapes near the origin (corresponding to the continuum of possible d values), can provide superior approximations to the Wold representations of economic time series. Indeed, the 'typical spectral shape' of economic variables [Nerlove (1964) and Granger (1966)], which has power that monotonically declines as frequency increases (except at seasonals), is well captured by the fractionally integrated process with d between zero and one. The fact that many economic time series in level form have spectra that appear to be infinite at the origin might suggest that a first difference is appropriate; however, after differencing, these time series often have no power at the origin, suggesting that a first difference is 'too much'. Such behavior is characteristic of a fractionally integrated process with d between zero and one [see Granger and Joyeux (1980)].

The potential macroeconomic relevance of the ARFIMA representation is also established by Granger (1980, 1988), who describes how fractional integration can be induced by aggregation. Specifically, if the underlying components of an aggregate series (e.g., individual firms' productions) follow AR(1) processes with parameters ρ_i and the ρ_i's are beta-distributed in the cross-section, then aggregation yields a fractionally integrated macroeconomic series. An example of a theoretical macroeconomic model producing fractionally integrated output is provided by Haubrich and Lo (1988), who exploit Granger's aggregation result in a real business cycle model with beta-distributed intrasectoral input–output coefficients to obtain fractionally integrated aggregate output.

4. Estimation of fractionally integrated models

The long-memory aspects of fractionally integrated ARIMA models make their estimation more difficult than the usual ARIMA model with integer d. We use a two-step estimation procedure suggested by Geweke and Porter-Hudak (GPH) (1983). We first obtain a consistent and asymptotically normal estimate of d and transform the series by the expansion of $(1-L)^d$. We then fit an ARMA model to the transformed series to obtain consistent estimates of the remaining model parameters Φ, Θ, and σ_ε^2.[9] Finally, these estimates are used to construct consistent estimates of the sequence of cumulative impulse responses.

The first-stage estimate of d is based on the order of the spectral density function near $\lambda = 0$. We start with the first difference of the relevant series, denoted $X_t = (1-L)Y_t$; thus, we wish to estimate $\tilde{d}$ in the model

$$(1-L)^{\tilde{d}} X_t = \Phi^{-1}(L)\Theta(L)\varepsilon_t \equiv u_t. \tag{11}$$

[9] The possibility of maximum-likelihood estimation (MLE) has also received some attention, as in Brockwell and Davis (1987) and Sowell (1987). Both the two-step procedure and MLE have associated costs and benefits; however, the former has certain advantages that make it our method of choice for the present application, as discussed below.

As d of the level series equals $1 + \tilde{d}$, a value of $\tilde{d}$ equal to zero corresponds to a unit root in Y_t.

The spectral density of X_t is given by

$$f_X(\lambda) = |1 - \exp(-i\lambda)|^{-2\tilde{d}} f_u(\lambda) = [2\sin(\lambda/2)]^{-2\tilde{d}} f_u(\lambda), \tag{12}$$

where $f_u(\lambda)$ is the spectral density of the stationary process u_t. Suppose that a sample of size T is available $\{X_t, t = 1, \ldots, T\}$; let $\lambda_j = 2\pi j/T$ $(j = 0, \ldots, T-1)$ denote the harmonic ordinates of the sample. Taking logarithms of eq. (12), adding and subtracting $\ln\{f_u(0)\}$, and evaluating at the harmonic ordinates, we obtain

$$\ln\{f_X(\lambda_j)\} = \ln\{f_u(0)\} - \tilde{d}\ln\{4\sin^2(\lambda_j/2)\} + \ln\{f_u(\lambda_j)/f_u(0)\}. \tag{13}$$

If we restrict consideration to the low-frequency ordinates near zero, say, λ_j, $j \leq K \ll T$, the last term in (13) can be dropped as negligible. Let $I(\lambda_j)$ denote the periodogram at ordinate j, then add $\ln\{I(\lambda_j)\}$ to both sides of (13) and rearrange to obtain

$$\ln\{I(\lambda_j)\} = \ln\{f_u(0)\} - \tilde{d}\ln\{4\sin^2(\lambda_j/2)\} + \ln\{I(\lambda_j)/f_X(\lambda_j)\}. \tag{14}$$

The particular utility of this formation is its formal similarity to a simple linear regression equation,

$$\ln\{I(\lambda_j)\} = \beta_0 + \beta_1 \ln\{4\sin^2(\lambda_j/2)\} + \eta_j, \qquad j = 1, \ldots, K, \tag{15}$$

where β_0 is the constant $\ln\{f_u(0)\}$, and the η_j, equal to $\ln\{I(\lambda_j)/f_X(\lambda_j)\}$, are independently and identically distributed across the harmonic frequencies.

Now let the number of low-frequency ordinates used in the above spectral regression be a function of the sample size, i.e., $K = g(T)$. Then, under regularity conditions on $g(\cdot)$, the negative of the OLS estimate of the slope coefficient provides a consistent and asymptotically normal estimate of $\tilde{d}$.[10] This is true regardless of the orders and parameterizations of the Φ and Θ polynomials underlying the stationary process u_t. Furthermore, the variance of the estimate of b_1 is given by the usual OLS estimator, and the theoretical

[10] Based upon theoretical considerations and Monte Carlo simulations, Geweke and Porter-Hudak (1983), Brockwell and Davis (1987), and Shea (1989) recommend using $g(T) = T^{\alpha}$, and obtain good results with $\alpha = 0.5$. Thus, for example, in a sample of size 144, the first twelve periodogram ordinates would be used.

asymptotic variance of the regression error η_j is known to be equal to $\pi^2/6$, which can be imposed to increase efficiency. A formal statement of the Geweke and Porter-Hudak theorem appears in the appendix.

Given an estimate of $\tilde{d}$, we transform the series X_t by the long-memory filter (8), truncated at each point to the available sample. The transformed series is then modeled as an ARMA(p, q) process. Because the estimate of the order of fractional integration from the periodogram regression is consistent, the second-stage estimates of Φ and Θ are also consistent. Finally, consistent estimates of the sequence of cumulative impulse responses, C, are constructed.

It is worth noting at this point the benefits of the semiparametric first-stage estimator of $\tilde{d}$ (and hence d); its asymptotic distribution does not depend on the infinite-dimensional nuisance parameter $\Phi^{-1}(L)\Theta(L)$. This is a desirable property in the present application, because the estimate of C turns out to depend largely on the estimated value of d, not on the estimates of the parameters in Φ and Θ. Thus, it is valuable to have an estimator of d whose properties do not depend on correct specification of Φ and Θ, the orders of which are typically unknown *a priori*. Alternative procedures such as simultaneous maximum-likelihood estimation of d, Φ, and Θ, which may have certain desirable properties under correct model specification, may be inconsistent under misspecification of Φ and Θ.

5. Empirical results

In this section, we examine evidence for fractional integration in ten different measures of U.S. macroeconomic activity. These include post-war quarterly real GNP, which was used in Campbell and Mankiw (1987a, b), as well as post-war quarterly real GNP per capita. Real GNP is the most comprehensive measure of the macroeconomy; however, we will focus much of our attention on per capita GNP because movements induced in aggregate output by a varying population will be naturally persistent and may obscure the persistence intrinsic to the market economy, which is our main interest.

We also examine the Federal Reserve Board's index of industrial production on a quarterly basis. This provides a more specialized measure of real output, including just the manufacturing, mining, and utilities sectors (which account for roughly one-fourth to one-third of GNP), but over a substantially longer time range (1919–1987). In addition, we are able to control for seasonality and the effects of seasonal adjustment filters by examining both seasonally adjusted and nonseasonally adjusted (NSA) industrial production data.[11] Another specialized series examined is the quarterly average unemployment rate, which has cyclical movements closely related to those of aggregate output. The

[11]Ghysels (1987) argues that the smoothing effects of seasonal adjustment filters might lead to spurious unit-root-like behavior in seasonally adjusted series.

persistence of unemployment has been the focus of recent work on employment hysteresis, such as Blanchard and Summers (1986).

While real GNP is the most comprehensive macroeconomic indicator, on a quarterly basis it is only available from the *National Income and Product Accounts* (NIPA) for the last forty years. Annual GNP data, which have been constructed by a variety of researchers, provide a span of up to 120 years, and previous investigations using such long annual series have found less shock persistence than in the post-war quarterly data.[12] In light of this, we examine the long annual series of real net national product (NNP), as reported in Friedman and Schwartz (1982, table 4.8),[13] and real GNP, as reported in Romer (1989) and in Balke and Gordon (1989).[14] In addition, we examine two annual real output per capita series: real NNP per capita from Friedman and Schwartz (1982, table 4.8) and real GNP per capita from *Long-Term Economic Growth* (1973) spliced in 1929 to the annual NIPA record.

In summary, we examine a variety of real macroeconomic time series at quarterly and annual frequencies and in level and per capita terms. This range of combinations enables us to explore the robustness of our results. As a first step, we obtain estimates of the fractional-integration parameter d for each of the ten series. We then focus our attention on the leading case of post-war quarterly real GNP per capita and examine persistence through the sequence of cumulative impulse responses, C. The sensitivity of our results is then examined.

5.1. Estimation of d

Table 2 reports d estimates for all ten measures of aggregate economic activity along with asymptotic standard errors and the associated p values for the unit-root null hypothesis ($d = 1$). The asymptotic standard errors are constructed using the known theoretical GPH regression error variance of $\pi^2/6$ to increase efficiency.[15] The p values give the asymptotic probability, under the null hypothesis that $d = 1$, of obtaining the estimated d value; they are against the one-sided alternative $d < 1$. The number of low-frequency

[12]See, for example, Cochrane (1988). Recent annual data are also more accurate than the quarterly series, which interpolate a substantial portion of detail from annual surveys [see Carson (1987)]. However, important information contained in short-run, high-frequency fluctuations may be lost in the annual series.

[13]The Friedman and Schwartz series, which is net of a capital consumption allowance, is widely mislabeled as gross national product in the literature.

[14]As part of a debate that focuses primarily on cyclical volatility, Romer (1989) and Balke and Gordon (1989) have each constructed revised estimates of real GNP before 1929 based on reassessments of the sources and assumptions underlying previous estimates of early GNP.

[15]The Monte Carlo evidence presented by Geweke and Porter-Hudak (1983) and Diebold and Rudebusch (1989b) indicates that asymptotic normality is a good approximation for the sample sizes considered here.

Table 2

Estimates of d.[a]

Data series and source	α = 0.5	α = 0.525	α = 0.55
Annual series			
Real NNP	0.67	0.61	0.61
1869–1975	(0.29)	(0.26)	(0.24)
[Friedman and Schwartz (1982)]	$p = 0.19$	$p = 0.07$	$p = 0.05$
Real GNP	0.59	0.70	0.65
1869–1987	(0.27)	(0.26)	(0.23)
[Romer (1989)]	$p = 0.06$	$p = 0.13$	$p = 0.06$
Real GNP	0.50	0.71	0.64
1869–1987	(0.27)	(0.26)	(0.23)
[Balke and Gordon (1989)]	$p = 0.03$	$p = 0.13$	$p = 0.06$
Real NNP, per capita	0.52	0.48	0.49
1869–1975	(0.29)	(0.26)	(0.24)
[Friedman and Schwartz (1982)]	$p = 0.05$	$p = 0.02$	$p = 0.02$
Real GNP, per capita	0.65	0.57	0.55
1900–1986	(0.32)	(0.29)	(0.26)
[*Long-Term Economic Growth*]	$p = 0.14$	$p = 0.07$	$p = 0.04$
Quarterly series			
Real GNP	0.90	0.92	0.88
1947:Q1–1987:Q2	(0.24)	(0.23)	(0.21)
[NIPA]	$p = 0.34$	$p = 0.36$	$p = 0.28$
Real GNP, per capita	0.68	0.72	0.71
1947:Q1–1987:Q2	(0.24)	(0.23)	(0.21)
[NIPA]	$p = 0.09$	$p = 0.11$	$p = 0.08$
Industrial Production	0.85	0.86	0.81
1919:Q1–1987:Q2	(0.19)	(0.19)	(0.17)
[Federal Reserve Board]	$p = 0.21$	$p = 0.23$	$p = 0.13$
Industrial Production, NSA	0.84	0.86	0.80
1919:Q1–1987:Q2	(0.19)	(0.19)	(0.17)
[Federal Reserve Board]	$p = 0.20$	$p = 0.23$	$p = 0.12$
Civilian Unemployment Rate	0.72	0.65	0.71
1948:Q1–1987:Q2	(0.24)	(0.24)	(0.21)
[Bureau of Labor Statistics]	$p = 0.12$	$p = 0.06$	$p = 0.08$

[a]The sample size for the GPH spectral regression is T^{α}. All variables except the unemployment rate are in logarithms, and quarterly variables are seasonally adjusted except for Industrial Production, NSA. The standard errors given in parentheses are constructed imposing the known theoretical regression error variance of $\pi^2/6$. The p values for the unit-root null hypothesis ($d = 1$) are against the one-sided alternative $d < 1$.

periodogram ordinates included in the GPH regression introduces a judgemental aspect; improper inclusion of medium-frequency ordinates will contaminate the estimate of d, while too small of a regression sample will lead to imprecise estimates. Table 2 reports d estimates for each series for three different regression sample sizes. The sample sizes are equal to T^{α}, for $\alpha = 0.5$, 0.525, and 0.55. The estimates of the order of fractional integration are quite robust across this variation.

While the d estimate for quarterly per capita GNP is about 0.7, quarterly GNP yields a higher estimate of 0.9, suggesting, not surprisingly, more persistence.[16] The annual national output series, which span more time, have smaller d estimates; ranging from 0.5 to 0.65, depending on the particular series. Romer's GNP series has an estimated fractional-integration parameter very similar to those of the other annual series. The estimated d for quarterly industrial production is close to that for quarterly GNP, and seasonal adjustment makes virtually no difference. Finally, quarterly unemployment exhibits a fractional-integration parameter of about 0.70.

The evidence from all of these series should be considered, as each provides a different perspective on the persistence of shocks to aggregate economic activity. The point estimates of d are quite striking, as *all* are less than unity, and some are very much less than unity; however, as we shall discuss below, the standard errors for these estimates are quite large. The results call for a deeper exploration of the nature of low-frequency economic dynamics.[17] In particular, the knife-edged parameterizations '$d = 1$' and '$d = 0$', which arise in standard ARIMA modeling and are the implicit subject of the unit-root literature, may be overly restrictive. In what follows, we focus on the leading case of interest, quarterly real GNP per capita. We first estimate the remaining ARFIMA model parameters; then, we proceed to construct persistence estimates and investigate their robustness.

5.2. *Estimation of cumulative impulse responses*

Computation of the sequence of cumulative impulse responses requires estimation of all of the parameters of the ARFIMA(p, d, q) model. For quarterly per capita real GNP (denoted hereinafter as *QY82PC*), the GPH

[16]For post-war quarterly per capita real GNP, fig. 1 provides the complete range of estimates of d obtained for increasing numbers of periodogram ordinates included in the GPH regression. The number of periodogram ordinates corresponding to the square root of the sample size is indicated with a vertical dashed line.

[17]Moreover, one might conjecture that standard unit-root tests, described in section 2, may have low power against fractional alternatives. This appears to be the case; see Diebold and Rudebusch (1989b).

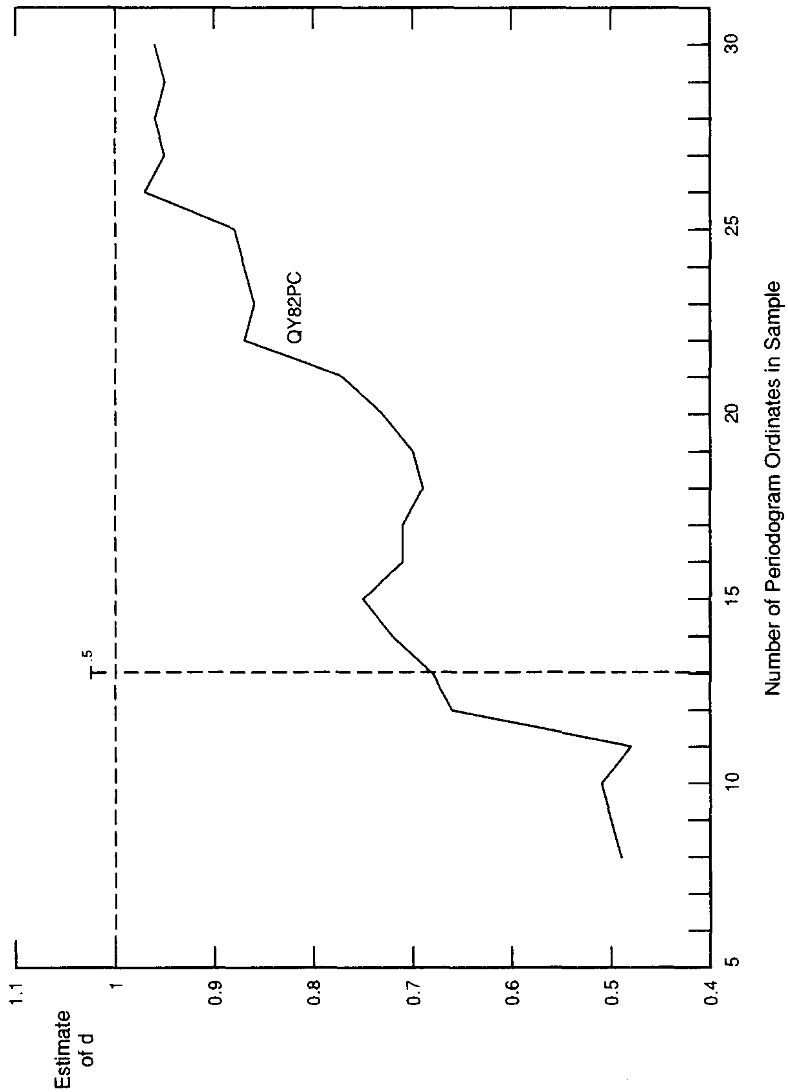

Fig. 1. Estimate of d for quarterly real GNP per capita from the GPH spectral regression as a function of sample size.

Table 3

Model selection criteria, log of quarterly real GNP per capita ($QY82PC$).[a]

ARFIMA(p, d, q) model: $(1-L)^{0.7}\Phi(L)QY82PC_t = \Theta(L)\varepsilon_t$

Number of AR parameters (p)	Number of MA parameters (q)			
	0	1	2	3
0	927.165 924.083	981.278 975.115	1008.250 999.007	1017.190 1004.870
1	1016.720 1010.560[b]	1015.440 1006.190	1018.520[c] 1006.190	1017.550 1002.140
2	1015.800 1006.550	1013.740 1001.410	1017.280 1001.870	1015.790 997.299
3	1018.090 1005.770	1016.230 1000.830	1018.180 999.689	1016.340 994.770

[a]For each model, we report the Akaike Information Criterion ($2\ln L - 2k$) and, below that, the Schwartz Information Criterion ($2\ln L - k\ln T$).

[b]Maximum SIC value.

[c]Maximum AIC value.

regression results suggest a fraction-integration parameter in the range of 0.7; thus, we transform the data by applying the filter $(1-L)^{-0.3}$ to the first-differenced series. To capture the remaining short-run dynamics, we consider ARMA models with up to three autoregressive parameters and three moving-average parameters. We distinguish these models through the Akaike and Schwarz information criteria (AIC and SIC, respectively), which are differentiated by their degrees-of-freedom adjustment of the maximized log-likelihood function. Table 3 reports the selection criteria for the sixteen models under consideration, ranging from white noise to ARMA(3, 3). The Akaike criterion identifies an ARMA(1, 2), while the Schwarz criterion identifies a more parsimonious ARMA(1, 0). Like Campbell and Mankiw (1987a), we are not interested in selecting one 'best' model of short-run fluctuations in GNP; rather, we seek robust evidence from a variety of models on the effects of economic shocks and use the information criteria for guidance.

Table 4 provides the estimated cumulative impulse responses for all sixteen ARFIMA(p, 0.70, q) models of $QY82PC$. These responses demonstrate the effect of a unit growth rate innovation on the level of output k periods hence, with k ranging from 1 to 400 quarters. The cumulative impulse responses are hump-shaped, with initial shock magnification, followed by shock dissipation. The maximum c_k value occurs at less than 8 quarters; after 16 quarters the cumulative impulse response has fallen back to approximately unity. By fifty quarters out, the response has dropped to about 0.7, and after one hundred quarters, it is less than 0.6 and continues to decline. These results are robust to

Table 4

Cumulative impulse responses, ARFIMA(p, 0.7, q) model, $QY82PC$.

$(1-L)^{0.7}\Phi(L)QY82PC_t = \Theta(L)\varepsilon_t$

Model	Quarters							
p, q	1	2	4	8	16	50	100	400
0,1	1.175	0.927	0.750	0.609	0.494	0.351	0.285	0.188
0,2	1.260	1.372	1.024	0.815	0.657	0.464	0.377	0.248
0,3	1.262	1.506	1.314	1.014	0.809	0.569	0.461	0.304
1,0[a]	1.359	1.491	1.497	1.295	1.023	0.707	0.571	0.375
1,1	1.324	1.472	1.529	1.377	1.099	0.753	0.608	0.399
1,2[b]	1.304	1.591	1.563	1.280	1.003	0.697	0.564	0.371
1,3	1.306	1.602	1.544	1.188	0.938	0.657	0.532	0.350
2,0	1.306	1.468	1.544	1.411	1.133	0.774	0.624	0.410
2,1	1.356	1.477	1.439	1.120	0.654	0.068	−0.564	−0.014
2,2	1.307	1.605	1.538	1.149	0.912	0.640	0.518	0.341
2,3	1.308	1.603	1.504	1.113	0.888	0.623	0.505	0.333
3,0	1.319	1.591	1.556	1.209	0.943	0.660	0.535	0.352
3,1	1.316	1.589	1.559	1.171	0.911	0.641	0.520	0.342
3,2	1.316	1.600	1.543	1.280	0.990	0.687	0.556	0.365
3,3	1.309	1.600	1.547	1.332	1.027	0.710	0.574	0.378

[a]Selected by the SIC.
[b]Selected by the AIC.

the particular values of p and q chosen.[18] It is interesting to note that our estimates of the long-run cumulative impulse response are closer to those obtained in other studies using UC models (although we do not impose their implicit restrictions), than to the shock magnification results obtained with ARIMA models. The short-run hump-shaped cumulative impulse-response pattern is very similar across all three classes of models; they differ, however, in their implied medium- and long-run dynamics.

Thus far, we have shown that the point estimates of the cumulative impulse responses are typically smaller than estimates obtained from ARIMA representations and are highly robust to the form of the second-stage ARMA model fitted. Also of interest is the sensitivity of the point estimates of the cumulative impulse responses to the first-stage estimate of d, obtained from the GPH regression, upon which we condition. The results of such a sensitivity analysis are contained in table 5 for quarterly per capita real GNP. For each d value,

[18]The exception is the ARFIMA(2, 0.7, 1), where the AR and MA polynomials have an approximate common factor of unity, which forces an estimated long-run cumulative response of zero. Both information criteria detect the redundancy and favor ARFIMA(1, 0.7, 0) models. The unit moving-average root most likely reflects the 'pileup' problem [see Diebold and Nerlove (1989)].

Table 5

Shock persistence in estimated ARFIMA(p, d, q) models, $QY82PC$.[a]

Fractional integration parameter (d)	Stationary model ARMA(p, q); AIC, SIC	Persistence measures c_{16}	c_{50}	c_{400}
1.0	(2,2); 1018.28, 1002.87	1.50	1.50	1.50
	(1,0); 1014.96, 1008.80	1.57	1.57	1.57
0.90	(2,2); 1019.05, 1003.64	1.24	1.10	0.90
	(0,3); 1019.68, 1007.35	1.40	1.24	1.01
	(1,0); 1016.54, 1010.38	1.32	1.17	0.95
0.85	(2,2); 1019.22, 1003.82	1.13	0.95	0.69
	(0,3); 1018.42, 1003.01	1.23	1.03	0.76
	(1,0); 1017.12, 1010.96	1.21	1.02	0.74
0.80	(2,2); 1019.09, 1003.68	1.03	0.82	0.54
	(0,3); 1020.24, 1007.91	1.08	0.85	0.56
	(1,0); 1017.43, 1011.27	1.13	0.89	0.58
0.75	(2,2); 1018.49, 1003.09	0.96	0.72	0.43
	(0,3); 1019.38, 1007.05	0.94	0.70	0.41
	(1,0); 1017.34, 1011.18	1.06	0.79	0.46
0.70	(2,2); 1017.28, 1001.87	0.91	0.64	0.34
	(1,2); 1018.52, 1006.19	1.00	0.70	0.37
	(1,0); 1016.72, 1010.56	1.02	0.71	0.38
0.65	(2,2); 1015.37, 999.97	0.90	0.59	0.28
	(1,2); 1016.95, 1004.62	0.97	0.63	0.30
	(1,0); 1015.50, 1009.34	1.02	0.66	0.31

[a]For each model, we report the AIC, $2\ln L - 2k$, and the SIC, $2\ln L - k\ln T$. The ARFIMA(p, d, q) model, $(1-L)^d Y_t = \text{ARMA}(p, q)$, is estimated conditional on d. Up to three ARMA stationary models are reported for each d: the ARMA(2,2) and those selected by the AIC and SIC. The measure of persistence, c_k, is the sum of the first k coefficients of the moving-average representation of the first-differenced series.

we report the estimates of c_{16}, c_{50}, and c_{400} for the ARFIMA(2, d, 2) model and, if not redundant, two other ARFIMA models corresponding to those selected by the AIC and SIC. Recall the preferred d value as estimated via the GPH spectral regression is 0.70, and the associated cumulative impulse responses in table 5 summarize the persistence estimates given in table 4. The estimated cumulative impulse responses are highly dependent upon d; in particular, forcing a unit root on the data (i.e., imposing $d = 1$) leads to large long-run persistence estimates, on the order of 1.5, which is what Campbell and Mankiw (1987a) report. This inflates the preferred estimate of c_{400}, corresponding to $d = 0.7$, by a factor of four, and it inflates the preferred estimate of c_{50} by a factor of two.

The information provided by the point estimates of C obtained from ARFIMA models is useful in that it represents a 'best guess' at the shape of

C, based upon a generalized approximation to the Wold representation. The results also indicate, however, that the *interval* estimates associated with the long-run response are wide. To approximate $k\%$ confidence intervals for the elements of C, we can vary d over its $k\%$ confidence range (obtained by exploiting the asymptotic normality of the first-stage GPH estimate) and condition upon estimated Φ and Θ values at each d value.[19] The sensitivity of the estimated cumulative impulse responses to the estimate of d, together with the standard errors of the d estimates reported in table 2, imply that interval estimates of the cumulative impulse responses will be quite wide. Clearly, varying the d estimate across just one standard error will encompass both long-run shock magnification as well as dissipation. The wide confidence intervals underscore a fundamental econometric reality: precise inference about low-frequency behavior is very difficult given the short time series available in macroeconomics. One hundred years of data can provide only one independent observation about the long-run response one hundred years hence.

6. Summary and conclusions

We examine persistence in U.S. real output using a generalized approximation to the Wold representation. Our application of long-memory models, associated with *fractional* integration via the operator $(1-L)^d$ and noninteger d, allows flexible modeling of low-frequency behavior, with important implications for the measurement of macroeconomic shock persistence. Evidence of long memory is found in all of the macroeconomic series studied, though it is not necessarily associated with a unit root, as estimated d values range from 0.5 to 0.90. Furthermore, the knife-edged cases of 'unit root' and 'no unit root', which correspond to $d=1$ and $d=0$, respectively, lose their exaggerated importance once d is allowed to vary on a continuum.

Post-war real GNP per capital is investigated in detail, and estimated long-run responses to a unit innovation are shown to depend crucially on d. Using d estimates obtained from a spectral regression procedure, an estimated 50-quarter cumulative impulse response of roughly 0.7 is obtained. For GNP at annual frequencies, the estimates of d suggest even less persistence. In short, the point estimates strongly suggest that aggregate shocks are partially dissipated, not magnified.

However, and most importantly, we argue that the confidence intervals associated with any univariate persistence estimates are likely to be quite

[19] Two implicit, and potentially offsetting, assumptions underlie this approximation. The first is that the elements of C are monotone in d, which is the case in table 5 but need not be true in general. Second, we ignore stochastic variation in the elements of C associated with variability of the second-stage Φ and Θ estimates.

wide.[20] Even when conducted on a common data sample as in Stock and Watson (1988), various modeling methodologies provide very different persistence estimates. The UC representations, which contain restrictions on low-frequency behavior, produce low estimates of macroeconomic persistence. When the UC restrictions are relaxed by using ARIMA models, larger persistence estimates are obtained. However, when the ARIMA restrictions are relaxed to consider fractional integration, persistence point estimates again fall. This is consistent with misspecification in the unit-root models (UC or ARIMA). However, the confidence we can place in *any* estimate of the long-run response is low because there are so few independent observations on long-run behavior available in the data.

Appendix: The Geweke and Porter-Hudak (1983) theorem

Suppose $\{Y_t\}$ is an ARFIMA(p, d, q) process, with $d < 0$. Let $I(\lambda_{j,T})$ denote the periodogram of $\{Y_t\}$ at the harmonic frequencies $\lambda_{j,T} = 2\pi j/T$ in a sample of size T. Let $b_{1,T}$ denote the OLS estimator of β_1 in the regression equation

$$\ln\left\{I(\lambda_{j,T})\right\} = \beta_0 + \beta_1 \ln\left\{4\sin^2(\lambda_{j,T}/2)\right\} + u_{j,T}, \qquad j = 1, \ldots, K.$$

Then there exists a function $g(T)$, which will have the properties $\lim_{T\to\infty} g(T) = \infty$, $\lim_{T\to\infty} g(T)/T = 0$, such that if $K = g(T)$, then $\operatorname{plim} b_1 = -d$. If $\lim_{T\to\infty}(\ln(T))^2/g(T) = 0$, then $(b_1 + d)/\{\hat{\mathrm{var}}(b_1)\}^{1/2} \xrightarrow{d} \mathrm{N}(0,1)$, where $\hat{\mathrm{var}}(b_1)$ is the usual OLS estimator, i.e., the (2,2) entry of $s^2(X'X)^{-1}$. Furthermore, under the stated conditions, $\operatorname{plim} s^2 = \pi^2/6$.

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[20] In related work [Diebold and Rudebusch (1989a)], we show that the confidence intervals associated with the predicted response of consumption to income, under the permanent-income hypothesis, are also quite wide, which casts doubt on the existence of the 'Deaton Paradox'. In future work, it may prove beneficial (in terms of efficiency gains) to work with multivariate models; common trends may then manifest themselves in terms of fractional cointegration, in a fashion analogous to King, Plosser, Stock, and Watson (1987), who allow for 'integer' cointegration.

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