The Distribution of Stock Return Volatility*

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Abstract

We exploit direct model-free measures of daily equity return volatility and correlation obtained from high-frequency intraday transaction prices on individual stocks in the Dow Jones Industrial Average over a five-year period to confirm, solidify and extend existing characterizations of stock return volatility and correlation We find that the unconditional distributions of the variances and covariances for all thirty stocks are leptokurtic and highly skewed to the right, while the logarithmic standard deviations and correlations all appear approximately Gaussian. Moreover, the distributions returns scaled by the realized standard deviations are also Gaussian. Furthermore, the realized logarithmic standard deviations and correlations all show strong dependence and appear to be well described by long-memory processes, consistent with our documentation of remarkably precise scaling laws under temporal aggregation. Our results also show that positive returns have less impact on future variances and correlations than negative returns of the same absolute magnitude, although the economic importance of this asymmetry is minor. Finally, there is strong evidence that equity volatilities and correlations move together, thus diminishing the benefits to diversification when the market is most volatile. By explicitly incorporating each of these stylized facts, our findings set the stage for improved high-dimensional volatility modeling and out-of-sample forecasting, which in turn hold promise for the development of better decision making in practical situations of risk management, portfolio allocation, and asset pricing.

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1. Introduction

Financial market volatility is central to the theory and practice of asset pricing, asset allocation, and risk management. Although most textbook models assume volatilities and correlations to be constant, it is widely recognized among both finance academics and practitioners that they vary over time. This recognition has spurred an extensive and vibrant research program into the distributional and dynamic properties of stock market volatility.[1] Most of what we have learned from this burgeoning literature has been based on the estimation of parametric ARCH or stochastic volatility models for the underlying returns, or on the analysis of implied volatilities from options or other derivatives prices. However, the validity of such volatility measures generally depends upon specific distributional assumptions, and in the case of implied volatilities, further assumptions concerning the market price of volatility risk. As such, the existence of multiple competing models calls into question the robustness of previous findings. An alternative approach, based for example on squared returns over the relevant return horizon, provides model-free unbiased estimates of the ex-post realized volatility. Unfortunately, however, squared returns are also very noisy and hence do not allow for reliable inference regarding the true underlying latent volatility.

The limitations of the traditional approaches motivate a very different approach for measuring and analyzing the properties of stock market volatility, which we adopt in this paper. Using a five-year sample of continuously recorded transactions prices for all thirty stocks in the Dow Jones Industrial Average (DJIA), we construct estimates of the ex-post *realized* daily volatilities by summing the squared intraday high-frequency returns. Volatility estimates so constructed are model-free, and as the sampling frequency of the returns approaches infinity, they are also free from measurement error (Andersen, Bollerslev, Diebold and Labys, ABDL, 1999a).[2] Of course, market microstructure frictions may be operative, including price discreteness, infrequent trading, and bid-ask bounce, and in order to

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[1] For an early survey, see Bollerslev, Chou and Kroner (1992). A selective and incomplete list of studies since then includes Andersen (1996), Bekaert and Wu (1999), Bollerslev and Mikkelsen (1999), Braun, Nelson and Sunier (1995), Breidt, Crato and de Lima (1998), Campbell and Hentschel (1992), Campbell and Lettau (1999), Canina and Figlewski (1993), Cheung and Ng (1992), Christensen and Prabhala (1998), Day and Lewis (1992), Ding, Granger and Engle (1993), Duffee (1995), Engle and Ng (1993), Engle and Lee (1993), Gallant, Rossi and Tauchen (1992), Glosten, Jagannathan and Runkle (1993), Hentschel (1995), Jacquier, Polsen and Rossi (1994), Kim and Kon (1994), Kroner and Ng (1998), Kuwahara and Marsh (1992), Lamoureux and Lastrapes (1993), and Tauchen, Zhang and Liu (1996).

[2] Nelson (1990, 1992) and Nelson and Foster (1994) demonstrate that mis-specified ARCH models may work as consistent filters for the latent *instantaneous* volatility as the return horizon approaches zero. Similarly, Ledoit and Santa-Clara (1998) show that the Black-Scholes implied volatility for an at-the-money option provides a consistent estimate of the underlying latent instantaneous volatility as the time to maturity time approaches zero.

mitigate them we use a five-minute return horizon as the effective "continuous time record." Treating the resulting daily time series constructed by the summation of the cross-products of the intraday five-minute returns as the realizations of the variances and covariances allows us to characterize the distributional properties of the daily return volatilities for a large set of equities -- the DJIA stocks -- without attempting to fit multivariate ARCH or stochastic volatility models.

Our approach is directly in line with earlier work by French, Schwert and Stambaugh (1987), Schwert (1989, 1990a, 1990b), and Schwert and Seguin (1991), who rely primarily on daily return observations for the construction of monthly realized stock volatilities.[3] The earlier studies, however, do not provide a formal justification for such measures, and the diffusion theoretic underpinnings provided here explicitly hinge on the length of the return horizon approaching zero. Intuitively, following the work of Merton (1980) and Nelson (1992), for a continuous time diffusion process, the diffusion coefficient can be estimated arbitrarily well with sufficiently finely sampled observations, and by the theory of quadratic variation, this same idea carries over to estimates of the integrated volatility over fixed horizons.[4] Moreover, our focus centers on *daily*, as opposed to monthly, volatilities. This mirrors the focus of the extant ARCH and stochastic volatility literatures and more clearly highlights the important intertemporal volatility fluctuations.[5] Finally, because our methods are trivial to implement, even in the high-dimensional situations relevant in practice, we are able to study the distributional and dynamic properties of *correlations* in much greater depth than is possible with traditional multivariate ARCH or stochastic volatility models, which rapidly become intractable as the number of assets grows.

Turning to the results, we find it useful to segment them into unconditional and conditional aspects of the distributions of volatilities and correlations. As regards the unconditional distributions, we find that the distributions of the realized daily variances stocks are highly non-normal and skewed to the right, but that the logarithms of the realized variances are approximately normal. Similarly,

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[3] In their analysis of monthly U.S. stock market volatility, Campbell and Lettau (1999) augment the time series of monthly sample standard deviations with various alternative volatility measures based on the dispersion of the returns on individual stocks in the market index.

[4] As such, the use of high-frequency returns plays a critical role in justifying our measurements.

[5] Schwert (1990a), Hsieh (1991), and Fung and Hsieh (1991) also study daily standard deviations based on 15-minute equity returns. However, their analysis is strictly univariate and decidedly less broad in scope than ours.

although the unconditional distributions of the covariances are all skewed to the right, the realized daily correlations are approximately normal. Finally, although the unconditional daily return distributions are leptokurtic, the daily returns normalized by the realized standard deviations are also normal. Rather remarkably, the results hold for the vast majority of the *30* volatilities and *435* covariances/correlations associated with the *30* Dow Jones stocks.

Moving to conditional aspects of the distributions, all of the volatility measures fluctuate substantially over time, and our estimates suggest strong dynamic dependence, well-characterized by slowly mean reverting fractionally integrated processes with a degree of integration, *d*, around *0.35*, as underscored by the existence of very precise scaling laws under temporal aggregation, which we document. Although statistically significant, we find that the much debated leverage-effect, or asymmetry in the relationship between past negative and positive returns and future volatilities, is relatively unimportant from an economic perspective. Interestingly, the same type of asymmetry is present in the realized correlations. Finally, there is a systematic tendency for the variances to move together, and for the correlations among the different stocks to be high/low when the variances for the underlying stocks are high/low, and when the correlations among the other stocks are also high/low.

Although several of these features have been documented previously for U.S. equity returns, the existing evidence relies almost exclusively on the estimation of specific parametric volatility models. In contrast, stylized facts for the thirty DJIA stocks documented here are explicitly model-free. Moreover, the facts extend the existing results in important directions and both robustify and expand on the more limited set of results for two exchange rates in ABDL (1999a,b) and the DJIA stock index in Ebens (1999a). As such, our findings set the stage for the development of improved volatility models and corresponding out-of-sample volatility forecasting, consistent with the actual distributional characteristics of equity returns. In turn, this should allow for better risk management, portfolio allocation, and asset pricing decisions.[6]

The remainder of the paper is organized as follows. In section 2 we provide a brief account of the diffusion-theoretic underpinnings of our realized volatility measures, along with a discussion of the actual data and volatility calculations. We discuss the unconditional univariate return and volatility distributions in section 3, and we detail the dynamic dependence, including long-memory effects and

[6] Ebens (1999a), for example, makes an initial attempt at modeling univariate realized stock volatility for the DJIA index.

scaling laws, in section 4. In section 5 we assess the symmetry of responses of realized volatilities and correlations to unexpected shocks. We discuss important multivariate aspects of the unconditional distributions in section 6, and we conclude in section 7 with a brief summary of our main findings and some suggestions for future research.

2. Realized Volatility Measurement

2.1 Theory

Here we provide a discussion of the theoretical justification behind our volatility measurements. For a more thorough treatment of the pertinent issues within the context of special semimartingales we refer to ABDL (1999a) and the general discussion of stochastic integration in Protter (1992). To set out the basic idea and intuition, assume that the logarithmic $N \times 1$ vector price process, p_t, follows a multivariate continuous-time stochastic volatility model,

$$dp_t \;=\; \mu_t \, dt \;+\; \Omega_t \, dW_t \,, \tag{1}$$

where W_t denotes a standard N-dimensional Brownian motion, the process for the $N \times N$ positive definite diffusion matrix, Ω_t, is strictly stationary, and we normalize the unit time interval, or $h = 1$, to represent one trading day. Conditional on the sample path realization of μ_t and Ω_t, the distribution of the continuously compounded h-period returns, $r_{t+h,h} \equiv p_{t+h} - p_t$, is then

$$r_{t+h,h} \;\;/\;\; \sigma\{ \mu_{t+\tau} , \Omega_{t+\tau} \}_{\tau=0}^{h} \;\;\sim\;\; N(\textstyle\int_0^h \mu_{t+\tau} \, d\tau \,,\;\; \int_0^h \Omega_{t+\tau} \, d\tau \,) \,. \tag{2}$$

The integrated diffusion matrix thus provides a natural measure of the true latent h-period volatility. This notion of integrated volatility already plays a central role in the stochastic volatility option pricing literature, where the price of an option typically depends on the distribution of the integrated volatility process for the underlying asset over the life of the option.[7]

By the theory of quadratic variation, we have that under very general regularity conditions,

[7] Specifically, according to the the Hull and White (1987) formula, the price of a European call takes the form $\pi_t = E[BS(\int_0^h \sigma_{t+\tau} \, d\tau \mid \sigma\{ \sigma_{t+\tau} \}_{\tau=0}^{h}]$, where σ_t denotes the instantaneous volatility of the underlying asset, and $BS(\cdot)$ refers to the standard Black-Scholes option pricing formula.

$$\Sigma_{j=1,\ldots,[h/\Delta]}\, r_{t+j\cdot\Delta,\Delta} \cdot r'_{t+j\cdot\Delta,\Delta} \;-\; \int_0^h \Omega_{t+\tau}\, d\tau \;\rightarrow\; 0 \tag{3}$$

almost surely for all t as the sampling frequency of the returns increases, or $\Delta \rightarrow 0$. Thus, by summing sufficiently finely high-frequency returns, it is possible to construct ex-post *realized* volatility measures for the integrated latent volatilities that are asymptotically free of measurement error.[8] This contrasts sharply with the common use of the cross-product of the h-period returns, $r_{t+h,h} \cdot r'_{t+h,h}$, as a simple ex-post volatility measure. Although the squared returns over the forecast horizon provides an unbiased estimate for the realized integrated volatility, it is also an extremely noisy estimator. Consequently, any predictable variation in the true latent volatility process is dwarfed by measurement errors.[9] Moreover, if the horizon is lengthy the conditional mean will contaminate this variance measure. In contrast, by explicitly incorporating the variation in the high-frequency price movements the measurement error effectively vanishes, and the impact of the mean term is annihilated.

These assertions remain valid if the underlying continuous time process in equation (1) contains jumps, so long as the price process is a special semimartingale, which essentially means that it is arbitrage-free. In the general case the limit of the summation of the high-frequency returns will involve an additional jump component, but the interpretation of the sum as the realized h-period return volatility remains intact; for further discussion along these lines see ABDL (1999a). Of course, in the presence of jumps the conditional distribution of the returns in equation (2) is no longer Gaussian. As such, the corresponding empirical distribution of the standardized returns speaks directly to the importance of allowing for jumps in the underlying continuous time process when analyzing the returns over longer h-period horizons.[10]

2.2 Data

Our empirical analysis is based on data from the TAQ (Trade And Quotation) database. The TAQ

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[8] To build intuition, consider the case of univariate discretely sampled i.i.d. normally distributed mean-zero returns; i.e., $N = 1$, $\mu_t = 0$, and $\Omega_t = \sigma^2$. It follows then by standard arguments that $E(h^{-1}\cdot\Sigma_{j=1,\ldots,[h/\Delta]}\, r^2_{t+j\cdot\Delta,\Delta}) = \sigma^2$, while $Var(h^{-1}\cdot\Sigma_{j=1,\ldots,[h/\Delta]}\, r^2_{t+j\cdot\Delta,\Delta}) = (\Delta/h)\cdot 2\cdot\sigma^4 \rightarrow 0$, as $\Delta \rightarrow 0$.

[9] In empirically realistic situations, the variance of $r_{t+1,1}\, r'_{t+1,1}$ is easily twenty times the variance of the true daily volatility, or $\int_0^1 \Omega_{t+\tau}\, d\tau$; see Andersen and Bollerslev (1998).

[10] This idea underlies the recent test for jumps in Drost, Nijman and Werker (1998), based on a comparison of the sample kurtosis to the population kurtosis implied by a continuous time GARCH(1,1) model, and it is exploited by ABDL (1999b).

data files contain continuously recorded information on the trades and quotations for the securities listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and the National Association of Security Dealers Automated Quotation system (NASDAQ). The database is published monthly, and has been available on CD-ROM from the NYSE since January 1993; we refer the reader to the corresponding data manual for a more complete description of the actual data and the method of data-capture. Our sample extends from January 2, 1993 until May 29, 1998, for a total of *1,366* trading days, and consists of all the trades for the thirty DJIA firms, as of the reconfiguration of the index on March 17, 1997. A list of the relevant ticker symbols is contained in the tables below.

The DJIA stocks are among the most actively traded U.S. equities, yet the median length of the times between trades range from a low of *7* seconds for Merck & Co. Inc. (MRK) to a high of *54* seconds for United Technologies Corp. (UTX), with a median arrival time for all of the stocks across the full sample of *23.1* seconds. As such, it is not practically feasible to push the continuous record asymptotics and the length of the observation interval Δ in equation (3) beyond this level. Moreover, because of the organizational structure of the market, the available quotes and transaction prices are subject to discrete clustering and bid-ask bounce effects. Such market microstructure features are generally not important when analyzing longer horizon interdaily returns but can seriously distort the distributional properties of high-frequency intraday returns; see, e.g., the textbook treatment by Campbell, Lo and MacKinlay (1997). Following the analysis in Andersen and Bollerslev (1997), we rely on artificially constructed five-minute returns.[11] With the daily transaction record extending from 9:30 EST until 16:05 EST, there are a total 79 five-minute returns for each day, corresponding to $\Delta = 1/79 \approx 0.0127$ in the notation above. However, the five-minute horizon is short enough so that the accuracy of the continuous record asymptotics underlying our realized volatility measures work well, and long enough so that the confounding influences from market microstructure frictions are not overwhelming; see ABDL (1999c) for further discussion along these lines.[12]

2.3 Construction of Realized Equity Volatilities

[11] An alternative, and much more complicated approach, would be to utilize all of the observations by explicitly modeling the high-frequency frictions.

[12] As detailed below, the average daily variance of the "typical" DJIA stock equals *3.109*. Thus, in the case of i.i.d. normally distributed returns, it follows that a five-minute sampling frequency translates into a variance for the daily variance estimates of *0.245*.

The five-minute return series are constructed from the logarithmic difference between the prices recorded at or immediately before the corresponding five-minute marks. Although the limiting result in equation (3) is independent of the value of the drift parameter, μ_t, the use of a fixed discrete time interval may allow dependence in the mean to systematically bias our volatility measures. Thus, in order to purge the high-frequency returns of the negative serial correlation induced by the uneven spacing of the observed prices and the inherent bid-ask spread, we first estimate an MA(1) model for each of the five-minute return series. Consistent with the spurious dependence that would be induced by non-synchronous trading and bid-ask bounce effects, all estimated moving-average coefficients are negative, with a median value of -0.214 across the thirty stocks. We denote the corresponding thirty demeaned MA(1)-filtered return series of $79 \times 1{,}366 = 107{,}914$ five-minute returns by $r_{t+\Delta,\Delta}$.[13] Finally, to avoid any confusion, we denote the daily unfiltered raw returns by a single time subscript; i.e., r_t where $t = 1, 2, ..., 1{,}336$.

The realized daily covariance matrix for the thirty DJIA stocks is then,

$$Cov_t \equiv \Sigma_{j=1,..,1/\Delta}\, r_{t+j\cdot\Delta,\Delta} \cdot r'_{t+j\cdot\Delta,\Delta}\,, \tag{4}$$

where $t = 1, 2, ..., 1{,}366$ and $\Delta = 1/79$. For notational simplicity we refer to each of the 30 realized variances given by the diagonal elements as $v^2_{j,t} \equiv \{ Cov_t \}_{j,j}$, and the corresponding daily logarithmic standard deviations as $lv_{j,t} \equiv log(v_{j,t})$. Similarly, we denote the realized daily correlations by $Corr_{i,j,t} \equiv \{ Cov_t \}_{i,j} / (v_{i,t} \cdot v_{j,t})$. In addition to the daily measures, we also briefly consider the statistical properties of various multi-day volatility measures, whose construction follows in straightforward fashion from equation (4) by extending the summation to cover h/Δ intervals, where $h > 1$ denotes the multiday horizon.

Because volatility is now effectively observable, we may rely on conventional statistical procedures for characterizing its distributional properties. We now proceed to do so.

3. Univariate Unconditional Return and Volatility Distributions

3.1 Returns

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[13] We also experimented with the use of unfiltered and linearly interpolated five-minute returns, which produced very similar results.

A voluminous literature, seeking to characterize the unconditional distribution of speculative returns, has evolved over the past three decades.[14] Consistent with this literature, the summary statistics in Table 1 show that the daily DJIA returns analyzed here, $r_{j,t}$, have fatter tails than the normal and, for the majority of the stocks, are also skewed to the right.[15]

Quite remarkably, however, the next set of numbers in Table 1 indicate that all of the thirty standardized return series, $r_{j,t}/v_{j,t}$, are approximately unconditionally normally distributed.[16] In particular, the median value of the sample kurtosis is reduced from *5.416* for the raw returns to only *3.129* for the standardized returns. This is also evident from Figure 1, which plots the kernel density estimates for the mean-zero and unit-variance standardized returns for each of thirty stocks, along with a normal reference density.[17] The close approximations afforded by the normal densities are striking. This result stands in sharp contrast to the leptokurtic distributions for the standardized daily returns that typically obtain when relying on an estimate of the one-day-ahead conditional variance from a parametric ARCH or stochastic volatility model; see e.g., Bollerslev, Engle and Nelson (1994) for a general discussion, and Kim and Kon (1994) for explicit results related to the distributions of the DJIA stocks over an earlier time period. Of course, in the context of a continuous time diffusion, both of these results are to be expected, and thus indirectly suggest that for the sample period analyzed here, jumps in the underlying price processes may be relatively unimportant. The results in Table 1 also imply that the unconditional distribution for the returns should be well approximated by a continuous variance mixture of normals, as determined by the unconditional distribution for the mixing variable, $v_{j,t}^2$. The following section details this distribution.

3.2 Variances and Logarithmic Standard Deviations

The first four columns in Table 2 provide the same set of summary statistics for the unconditional

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[14] In early contributions, Mandelbrot (1963) and Fama (1965) argued that the Stable Paretian distributions provide a good approximation. Subsequently, however, Praetz (1972) and Blattberg and Gonedes (1974), among many others, found that finite variance-mixtures of normals, such as the student-t distribution, generally afford better characterizations.

[15] Under the null hypothesis of i.i.d. normally distributed returns, the sample skewness and kurtosis are asymptotically normal with means equal to *0* and *3*, respectively, and variances equal to *6/T* and *24/T*, respectively, where *T* denotes sample size. Thus for $T = 1{,}366$ the two standard errors equal *0.066* and *0.133*, respectively.

[16] This matches the results for exchange rates reported in ABDL (1999b).

[17] The kernel density estimates are based on a Gaussian kernel and Silverman's (1986) bandwidth.

distribution of the realized daily variances. The median value for the sample means is *3.109*, implying an annualized standard deviation for the typical stock of around *28* percent. However, there is considerable variation in the average volatility across the thirty stocks, ranging from a high of *42* percent for Walmart Stores Inc. (WMT) to a low of *22* percent for UTX. The standard deviations given in the second column also indicate that the realized daily volatilities fluctuate significantly through time. Lastly, it is evident from the third and the fourth columns that the distributions of the realized variances are extremely right-skewed and leptokurtic. This may seem surprising, as the realized daily variances are based on the sum of *79* five-minute return observations. However, as emphasized by Andersen, Bollerslev and Das (2000), intraday speculative returns are strongly dependent so that, even with much larger samples, standard Central Limit Theorem arguments provide poor approximations in the high-frequency data context.

The next part of Table 2 refers to the realized logarithmic standard deviations, $lv_{j,t}$. Interestingly, the median value of the sample skewness across all of the thirty stocks is reduced to only *0.192*, compared to *5.609* for the realized variances and, although the sample kurtosis for all but Union Carbide Corp. (UK) exceed the normal value of three, the assumption of normality is obviously much better in this case. This is also illustrated by Figure 2, in which we show estimates of the thirty unconditional densities of $lv_{j,t}$, along with the standard normal density. For ease of comparison, all distributions have been standardized to have zero mean and unit variance. With the exceptions of AT&T (T), WMT, and Exxon Corp. (XON), the normal approximations are very good.

This evidence is consistent with Taylor (1986) and French, Schwert and Stambaugh (1987), who find that the distribution of logarithmic monthly standard deviations constructed from the daily returns within the month is close to Gaussian. It is also directly in line with the recent evidence in ABDL (1999a) and Zumbach et al. (1999), which indicates that realized daily foreign exchange rate volatilities constructed from high-frequency data are approximately log-normally distributed. Taken together, the results in Tables 1 and 2 imply that the unconditional distribution for the daily returns should be well described by a continuous lognormal-normal mixture, as advocated by Clark (1973) in his seminal treatment of the Mixture-of-Distributions-Hypothesis (MDH).

Our discussion thus far has centered on univariate return and volatility distributions. However, asset pricing, portfolio selection, and risk management decisions are invariably multivariate, involving many assets, with correlated returns. The next section summarizes the unconditional distributions of the pertinent realized covariances and correlations.

3.3 Covariances and Correlations

The realized covariance matrix for the thirty DJIA stocks contains a total of *435* unique elements. Space constraints rule out providing a detailed characterization of each individual series. Instead, we report in Table 3 the median value of the sample mean, standard deviation, skewness, and kurtosis for the covariance and correlations for each of the thirty stocks with respect to all of the twenty-nine other stocks; i.e., the median value of the particular sample statistic across the *29* time series for stock i as defined by $Cov_{i,j,t}$ and $Corr_{i,j,t}$, where $j = 1, 2, ..., 30$, and $j \neq i$.

The median of the medians of the mean covariance across all of the stocks equals *0.373*, while the typical correlation among the DJIA stocks is around *0.113*. However, the realized covariances and correlations exhibit considerable variation across the different stocks and across time. For instance, the median of the average correlations for UK equals *0.080*, whereas the median for General Electric (GE) is as high as *0.150*.

As with the realized variances, the distributions for the covariances are extremely right skewed and leptokurtic. However, the realized correlations are approximately normally distributed. In particular, the median kurtosis for all of the *435* realized covariances equals *61.86*, whereas the median kurtosis for the realized correlations equals *3.037*. In order to better illustrate this result, Figure 3 graphs the unconditional distributions for the realized correlations for the first twenty-nine stocks with respect to XON, the alphabetically last ticker symbol of the thirty DJIA stocks.[18] With few exceptions, the normal reference densities afford very close approximations.

The unconditional distributions detailed above capture important aspects of the return generating process. However, the summary statistics in Tables 2 and 3 also indicate that all of the realized volatilities vary through time. In the next section, we explore the associated dynamic dependence. Again, the use of realized volatilities allows us to do so in a model-free environment, freed from reliance on complicated and intractable parametric latent volatility models.

4. Temporal Dependence, Long-Memory and Scaling

The conditional distribution of stock market volatility has been the subject of extensive research effort

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[18] Similar graphs were produced for all of the other stocks.

during the past decade. Here we robustify, solidify and extend the findings in that literature; in particular, we reinforce the existence of pronounced long-run dependence in volatility and show that it is also present in correlation. Motivated by the results of the previous section, we focus on logarithmic volatilities and correlations.

4.1 Logarithmic Standard Deviations

It is instructive first to consider the time series plots for $lv_{j,t}$ in Figure 4. It is evident that all thirty time series are positively serially correlated, with distinct periods of high and low volatility readily identifiable. This is, of course, a manifestation of the well documented volatility clustering effect, and directly in line with the results reported in the extant ARCH and stochastic volatility literatures; see, e.g., Lamoureux and Lastrapes (1990) and Kim and Kon (1994) for estimation of GARCH models for individual daily stock returns.

To underscore the significance of this effect, the first column in Table 4 reports the standard Ljung-Box portmanteau test for the joint significance of the first *22* autocorrelations of $lv_{j,t}$ (about one month of trading days). The hypothesis of zero autocorrelations is overwhelmingly rejected for all stocks. The correlograms in Figure 5 show why. With few exceptions, the autocorrelations are systematically above the conventional Bartlett ninety-five percent confidence error bands, even at the longest displacement of *120* days (approximately half a year). Similarly slow decay rates have been documented in the literature with daily time series of absolute or squared returns spanning several decades (e.g., Crato and de Lima, 1993, and Ding, Granger and Engle, 1993), but the results in Figure 5 are noteworthy in that the sample "only" spans five-and-a-half years. In spite of this slow decay, the augmented Dickey-Fuller tests, reported in the second column in Table 4, reject the null hypothesis of a unit root for all but four of the stocks when judged by the conventional *-2.86* five-percent critical value.

A number of recent studies argue that the long-run dependence in financial market volatility may be conveniently modeled by fractional integrated ARCH or stochastic volatility models; see, e.g., Baillie, Bollerslev and Mikkelsen (1996), Breidt, Crato and de Lima (1998) and Robinson and Zaffaroni (1998). The log-periodogram regression estimates for the degree of fractional integration, or *d*, for the realized logarithmic volatilities, given in the third column in Table 4, are directly in line with these studies (see Geweke and Porter-Hudauk, 1993, Robinson, 1995, and Deo and Hurvich, 1999, for formal discussion of the log-periodogram regression technique, often called the GPH technique after Geweke and Porter-Hudak). The reported regressions rely on the first $m = [1{,}366]^{3/5} = 76$ sample periodogram ordinates, implying an asymptotic standard error of $\pi\cdot(24\cdot m)^{-½} = 0.074$. All thirty estimates are very close to the median value of 0.349.[19] It is also evident that the implied hyperbolic decay rates, $j^{2\cdot d-1}$, superimposed in Figure 5, afford a good approximation to the long-run behavior of the autocorrelations for most of the thirty stocks.

An implication of the long-memory associated with fractional integration concerns the behavior of the variance of partial sums. In particular, let $[\,x_t\,]_h \equiv \Sigma_{j=1,...,h}\, x_{h\cdot(t-1)+j}$, denote the h-fold partial sum process for x_t. If the process for x_t is fractionally integrated, the partial sums will obey a *scaling law* of the form $Var(\,[\,x_t\,]_h\,) = c\cdot h^{2d+1}$. Thus, given d and the unconditional variance at one aggregation level, it is possible to calculate the implied variance for any other aggregation level. To explore this feature, Figure 6 plots the logarithm of the variance of the partial sum of the daily realized logarithmic standard deviations, $log(Var[lv_{j,t}\,]_h)$, against the logarithm of the aggregation level, $log(h)$, for $h = 1, 2,..., 30$. The accuracy of the fit of the thirty lines, $c + (2d+1)\cdot log(h)$, is striking.[20] Moreover, the corresponding regression estimates for d, reported in the fourth column in Table 4, are generally very close to the GPH estimates.

4.2 Correlations

The estimation of multivariate volatility models is notoriously difficult and, as a result, relatively little is known about the temporal behavior of individual stock return correlations. The last four columns in Table 4 provide our standard menu of summary statistics for the *435* series of daily realized correlations. In accordance with our convention in section 3.3 above, each entry gives the median value of that particular statistic across the thirty stocks, while the corresponding graphs are restricted to the *29* representative correlations for XON.

Turning to the results, the time series plots in Figure 7 suggest important dependence and hence predictability in the XON correlations, $Corr_{XON,j,t}$. This impression is confirmed by the correlograms in Figure 8 and the Ljung-Box portmanteau statistics for up to 22^{nd} order serial correlation reported in column 5 of Table 4. Moreover, as with the ADF tests for $lv_{j,t}$, the tests for

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[19] This particular choice of m was motivated by Deo and Hurvich (1999), who show that the log-periodogram regression estimator for d for the long-memory stochastic volatility model with non-Gaussian errors is consistent and asymptotically normal provided that $m = O(T^{-\delta})$, where $\delta < 4d\cdot(1+4d)^{-1}$. For $d = 0.35$-0.40 this implies a value of δ around 0.6.

[20] LeBaron (1999) has recently demonstrated that apparent scaling laws may arise for short-memory, but highly persistent processes. In the present context, the hyperbolic decay in Figure 5 further buttresses the long-memory argument.

$Corr_{i,j,t}$ reported in the sixth column systematically reject the unit root hypothesis. The GPH estimates for d are significantly different from zero (and unity), with typical values around 0.35. The d estimates are consistent with the hyperbolic decay rates superimposed in Figure 8, which afford good approximations at long lags, and with the apparent scaling laws for XON shown in Figure 9, in which we plot $log(Var[Corr_{XON,,j,t}]_h)$ against $log(h)$, for $h = 1, 2,..., 30$.

Overall, our correlation results suggest that the *univariate* unconditional and conditional distributions for realized correlations among the DJIA stocks closely mimic the qualitative characteristics of the realized volatilities, discussed earlier. We now turn to multivariate aspects.

5. Asymmetric Responses of Volatilities and Correlations

A number of studies have noted an asymmetry in the relationship between equity volatility and returns, i.e., negative returns have a bigger impact on future volatility than positive returns. Two explanations have been put forth to account for this phenomenon. According to the so-called leverage effect, a large negative return increases financial and operating leverage, in turn raising equity return volatility (e.g., Black, 1976, and Christie, 1982). Alternatively, if the market risk premium is an increasing function of volatility, large negative returns increase future volatility more than positive returns due to a volatility feedback effect (e.g., Campbell and Hentschel, 1992). Here we evaluate the evidence on the basis of our realized volatility measures.

5.1 Logarithmic Standard Deviations

The use of realized volatilities allows for direct tests of asymmetries in the impact of past returns. However, in order to avoid confusing the influence of the past returns with the own dynamic dependence documented in the previous section, it is imperative that the own dynamic dependence be properly modeled. The first four columns in Table 5 therefore report the regression estimates based on the fractionally differenced series,

$$(1-L)^{d_i} lv_{i,t} = \omega_i + \gamma_i \cdot |z_{i,t-1}| + \phi_i \cdot |z_{i,t-1}| I(z_{i,t-1}<0) + u_{i,t}, \qquad (5)$$

where $I(\cdot)$ refers to the indicator function, $z_{i,t} \equiv r_{i,t}/v_{i,t}$, and the values for d_i are fixed at the d_{GPH} estimates reported in Table 4. Also, to account for any additional short-run dynamics, the t-statistics are based on a Newey-West covariance matrix estimator using 22 lags.

Only five of the t-statistics for γ_i are statistically significantly greater than zero, when judged by the standard *95*-percent critical value of *1.645*. All but one of the thirty t-statistics for ϕ_i, however, exceed that value. These results are broadly consistent with the EGARCH model estimates for daily individual stock returns reported by Cheung and Ng (1992) and Kim and Kon (1994), indicating an asymmetry in the impact of past negative and positive returns.

However, while statistically significant, the economic importance of this effect is limited. Consider Figure 10, which displays the scatterplots for the logarithmic standard deviations, $lv_{i,t}$, against the lagged standardized returns, $r_{i,t-1}/v_{i,t-1}$, with the corresponding regression lines for the negative and positive returns superimposed. This provides a direct empirical analogy to the news impact curves for parametric ARCH models analyzed by Engle and Ng (1993). Although all the news impact curves in Figure 10 are more steeply sloped to the left of the origin, the systematic effect is not very strong. This parallels the findings for four individual stocks in Tauchen, Zhang and Liu (1996), who also note that while asymmetry is a characteristic of the point estimates, the magnitude is quite small. In contrast, the parametric volatility model estimates reported in Nelson (1991), Glosten, Jagannathan and Runkle (1993) and Hentschel (1995), among others, all point toward important asymmetries in market-wide equity index returns. As such, this calls into question the leverage explanation, and instead suggests that the significant asymmetries for the aggregate market returns reported in these studies are most likely due to a volatility feedback effect (see also the recent discussion of Bekaert and Wu, 1999).

5.2 Correlations

As noted above, little is known about the temporal behavior of individual stock return correlations.[21] However, if the volatility asymmetry at the individual stock level is caused by a leverage effect, then the change in financial leverage should also affect the covariances with other stocks, which in turn is likely to impact the correlations. On estimating several different ARCH type models, Kroner and Ng (1998) report statistically significant asymmetries in the conditional covariance matrices for the weekly returns on a pair of well diversified small- and large-stock portfolios. At the same time, the bivariate EGARCH models in Braun, Nelson and Sunier (1995) indicate that while market volatility responds asymmetrically to positive and negative shocks, monthly conditional (time-varying) betas for size- and

[21] In the context of international equity markets, Erb, Harvey and Viskanta (1994) and Longin and Solnik (1998) have argued that the correlations tend to be higher when the returns are negative.

industry-sorted portfolios are generally symmetric.[22]

We now extend the analysis above to test for asymmetries in the realized daily correlations. In particular, the last four columns in Table 5 report the results from the regressions,

$$(1-L)^{d_i}Corr_{i,j,t} = \omega_i + \gamma_i \cdot \{|z_{i,t-1}| + |z_{j,t-1}|\} + \theta_i \cdot \{|z_{i,t-1} + z_{j,t-1}| I(z_{j,t-1} \cdot z_{i,t-1} > 0)\}$$

$$+ \phi_i \cdot \{|z_{i,t-1} + z_{j,t-1}| I(z_{j,t-1} < 0, z_{i,t-1} < 0)\} + u_{i,j,t} , \tag{6}$$

where, as before, the d_i are fixed at the d_{GPH} estimates reported in Table 4, and the t-statistics are based on a Newey-West covariance matrix estimator using *22* lags. Note that γ_i captures the impact of the past absolute returns, θ_i gives the additional influence when the past returns are of the same sign, and ϕ_i measures the additional impact if both of the returns are negative, which facilitates a direct series of asymmetry tests. The entries in the table give the median value of the *29* t-statistics for each of the thirty stocks, along with the number of test statistics that exceed the 95-percent critical value of *1.645*.

Interestingly, ϕ_i indeed appears to be the most important parameter in equation (6). However, only about half, or *203* of the *435* unique regressions, have significant t_ϕ - statistics at the *95*-percent level. This relatively weak asymmetry is underscored by Figure 11, which plots the daily realized correlations for XON, or $Corr_{XON,j,t}$, against the sum of the lagged standardized returns, or $r_{XON,t-1}/v_{XON,t-1} + r_{j,t-1}/v_{j,t-1}$, for each of the *29* stocks. As with the individual news impact curves in Figure 10, there is a tendency for the lines corresponding to the sum of the two returns being negative to be slightly more steeply sloped, but this asymmetry effect remains minor.

6. Multivariate Unconditional Volatility Distributions

Here we investigate various aspects of the multivariate unconditional volatility distributions. In Figure 12 we provide scatterplots of the realized daily logarithmic standard deviation for XON, $lv_{XON,t}$, against the logarithmic standard deviations for each of the twenty-nine other stocks; i.e., $lv_{j,t}$ for $j \neq$

[22] Recently, however, Cho and Engle (1999) report significant asymmetries in the conditional daily betas for nine individual stocks, which suggests that the results for monthly portfolio betas in Braun, Nelson and Sunier (1999) may be due to cross-sectional and temporal aggregation.

XON. It is evident that the volatilities are all positively correlated. In fact, the median correlation across the twenty-nine panels in Figure 12 is as high as *0.266*. In the first column in Table 6, we report the corresponding median correlations for all thirty stocks. They range from a low of *0.121* for the Aluminum Company of America (AA) to a high of *0.296* for the Boeing Corporation (BA). This tendency of return volatility to vary in tandem across individual stocks is consistent with a joint dynamic factor structure in volatility along the lines suggested by Diebold and Nerlove (1989) and Tauchen and Tauchen (1999), among others.[23]

Next, in Figure 13 and the second column of Table 6 we document the presence of a volatility-in-correlation effect. In particular, in Figure 13 we plot the realized daily correlations for XON, $Corr_{XON,j,t}$, against the logarithmic standard deviations of the corresponding returns for each of the twenty-nine stocks in the DJIA; i.e., $\frac{1}{2}\cdot(lv_{XON,t} + lv_{j,t})$ for $j \neq XON$. As in ABDL (1999a), a strong positive association is evident. This is further underscored by the results in Table 6; the median of the median of the corresponding correlations is *0.143*. Moreover, our direct model-free measurement of realized correlation is very different from the procedures previously entertained in the literature, so our findings provide additional empirical support for the phenomenon. Of course, a positive relationship is not surprising, see, e.g., Ronn, Sayrak, and Tompaidis, 1998. At the same time, the specific manifestation of the effect is model dependent, which renders direct predictions about magnitudes impossible within our nonparametric setting. Nonetheless, the strength of the effect is noteworthy and provides a benchmark measure that candidate models should be able to accommodate. At the very least, it suggests that standard mean-variance efficiency calculations based on constant correlations may be misguided.[24]

Finally, in Figure 14 we show the scatter of the average realized daily correlations for XON plotted against the average realized correlations for stock j; i.e., $(1/28)\cdot\Sigma_i Corr_{XON,i,t}$ for $i \neq XON$ and $i \neq j$ versus $(1/28)\cdot\Sigma_i Corr_{j,i,t}$ for $i \neq j$ and $i \neq XON$. The strong association between the realized daily

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[23] The use of correlation as a measure for interdependence is generally not unproblematic. However, given that all of the distributions we explore in Table 6 are approximately Gaussian, the correlation should provide a meaningful measure; see, e.g., Embrechts, McNeil and Strauman (1999).

[24] Similar observations have recently been made in the context of international equity index returns by Solnik, Boucrelle and Le Fur (1996). This also motivates the switching ARCH model estimated by Ramchand and Susmel (1998), who argue that the correlations between the U.S. and other world markets are on average 2 to 3.5 times higher when the U.S. market is in a high variance state as compared to a low variance state.

correlations is truly striking. Clearly, there is a powerful commonality in the return movements across the individual stocks. The last column of Table 6 tells the same story. Again, this seems to suggest that there is a lower dimensional factor structure driving the second moment characteristics of the joint distribution.

7. Conclusions

We exploit direct model-free measures of realized daily volatility and correlation obtained from high-frequency intraday stock prices to confirm, solidify and extend existing characterizations. Our findings are remarkably consistent with existing work such as ABDL (1999a, b) and Ebens (1999a). This is true of the right-skewed distributions of the variances and covariances, the normal distributions of the logarithmic standard deviations and correlations, the normal distributions of daily returns standardized by realized standard deviations, the strongly persistent dynamics of the realized volatilities and correlations, well-described by a stationary, fractionally integrated process and conforming to scaling laws under temporal aggregation. The striking congruence of all findings across asset classes (equity vs. forex) and underlying method of price recording (averages of logarithmic bid and ask quotes versus transaction prices) suggests that the results reflect fundamental attributes of speculative returns.

Although confirmation and robustification of existing results are certainly laudable goals, our analysis is also noteworthy in that it achieves significant extensions, facilitated throughout by the model-free measurement of realized volatility and correlation afforded by high-frequency data, and the simplicity of our methods, which enable straightforward high-dimensional correlation estimation. We shed new light on some distinct properties of equity return dynamics and illustrate them, for example, via the news impact curve. We confirm the existence of an asymmetric relation between returns and volatility, with negative returns being associated with higher volatility innovations than positive returns of the identical magnitude. However, the effect is much weaker at the individual stock level than reported at the aggregate market level, thus lending support to a volatility risk premium feedback explanation rather than a financial leverage effect. Moreover, we find a pronounced volatility-in-correlation effect, rendering portfolio diversification less effective when it is needed most. The strength of this relation suggests that suboptimal decisions will likely result from analysis based on a variance-covariance structure assumed to be constant. Finally, the strong positive association between individual stock volatilities and the corresponding strong relationship between contemporaneous stock correlations motivate the development of parsimonious factor models for the covariance structure of stock returns.

We envision numerous potential applications of the approach adopted in this paper. For example, the direct measurement of volatilities and correlations should alleviate the errors-in-variables problem that plagues much work on the implementation and testing of the CAPM, because realized betas may be constructed directly from the corresponding realized covariances and standard deviations. Multi-factor models based on factor replicating portfolios are similarly amenable to direct analysis. To take another example, the effective observability of volatilities and correlations will facilitate direct time-series modeling of portfolio choice and risk management problems under realistic and testable distributional assumptions. Work along these lines is currently being pursued in Andersen, Bollerslev, Diebold and Labys (1999d). Finally, our methods will also facilitate direct comparisons of volatility forecasts generated by alternative models and procedures. Such explorations are underway in Ebens (1999b) and Ebens and de Lima (1999).

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Table 1
Unconditional Daily Return Distributions

	$r_{i,t}$				$r_{i,t}/v_{i,t}$			
Stock	Mean	St.Dev.	Skew.	Kurt.	Mean	St.Dev.	Skew.	Kurt.
AA	0.022	1.530	0.100	4.109	-0.003	0.960	0.089	2.814
ALD	0.024	1.504	0.564	11.98	0.007	0.780	0.157	3.044
AXP	0.063	1.481	0.180	3.810	0.031	0.711	0.113	3.268
BA	0.011	1.363	0.160	8.565	-0.003	0.731	0.097	3.222
CAT	0.080	1.553	0.133	5.036	0.045	0.907	0.129	2.900
CHV	0.056	1.225	0.163	4.009	0.045	0.826	0.069	3.175
DD	0.140	1.407	0.082	6.587	0.099	0.802	0.152	2.964
DIS	0.016	1.385	0.422	6.363	0.007	0.772	0.097	3.251
EK	0.077	1.418	0.073	5.334	0.048	0.852	0.082	3.396
GE	0.055	1.238	0.500	8.331	0.053	0.824	0.164	3.078
GM	-0.036	1.530	0.176	3.916	-0.015	0.862	0.223	3.087
GT	-0.028	1.387	0.038	4.236	-0.011	0.751	0.141	3.402
HWP	-0.020	1.833	0.225	4.497	-0.001	0.948	0.113	2.734
IBM	0.007	1.703	0.123	6.404	0.005	0.955	0.104	2.829
IP	-0.059	1.499	0.158	6.536	-0.033	0.820	0.116	3.342
JNJ	0.011	1.421	0.034	3.919	0.015	0.796	0.045	3.029
JPM	0.031	1.261	0.035	4.420	0.017	0.848	0.050	3.103
KO	0.077	1.275	0.275	5.874	0.064	0.766	0.212	3.411
MCD	0.051	1.258	0.215	5.499	0.024	0.682	0.081	3.155
MMM	0.052	1.151	-0.221	7.038	0.034	0.789	0.103	3.302
MO	-0.006	1.539	-0.100	10.32	0.033	0.797	0.233	3.052
MRK	0.065	1.452	0.163	4.233	0.056	0.818	0.215	3.283
PG	0.109	1.336	0.466	5.960	0.079	0.809	0.251	3.005
S	0.063	1.705	0.019	4.379	0.032	0.865	0.030	2.778
T	-0.014	1.325	0.231	8.359	-0.016	0.779	0.211	3.418
TRV	0.024	1.777	-0.025	4.619	0.010	0.854	0.059	2.928
UK	0.088	1.599	0.506	5.150	0.023	0.654	0.158	3.578
UTX	0.080	1.219	-0.018	4.097	0.051	0.849	-0.054	3.007
WMT	-0.015	1.617	0.473	7.186	0.002	0.623	0.322	3.848
XON	0.071	1.149	0.006	6.472	0.055	0.794	-0.008	3.284
Median	0.041	1.419	0.159	5.416	0.024	0.806	0.113	3.129
Mean	0.036	1.438	0.172	5.908	0.025	0.808	0.125	3.156
Min.	-0.059	1.149	-0.221	3.810	-0.033	0.623	-0.054	2.734
Max.	0.140	1.833	0.564	11.98	0.099	0.960	0.322	3.848

Note: The summary statistics are based on the daily returns for each of the thirty DJIA stocks, $r_{i,t}$. The sample covers the period from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The realized daily volatilities, $v_{i,t}$, are calculated from the five-minute returns within the day as detailed in the main text.

Table 2
Unconditional Daily Volatility Distributions

	$v_{j,t}^2$				$lv_{i,t}$			
Stock	Mean	St.Dev.	Skew.	Kurt.	Mean	St.Dev.	Skew.	Kurt.
AA	2.472	1.785	6.970	118.7	0.370	0.280	0.164	3.458
ALD	3.389	3.965	19.11	517.9	0.514	0.290	0.314	4.551
AXP	4.609	2.712	3.020	23.28	0.692	0.274	-0.336	3.787
BA	3.232	3.066	19.74	567.8	0.509	0.271	-0.062	4.718
CAT	2.617	1.864	4.706	42.15	0.403	0.269	0.282	4.043
CHV	2.263	1.320	3.681	34.32	0.343	0.256	-0.011	3.157
DD	2.785	1.923	5.800	62.83	0.447	0.238	0.636	4.972
DIS	3.119	2.176	7.280	113.9	0.500	0.260	0.205	3.839
EK	2.539	1.665	5.504	55.47	0.406	0.228	0.646	5.254
GE	2.161	2.189	18.64	518.1	0.304	0.264	0.482	4.758
GM	3.088	1.583	3.042	23.01	0.512	0.227	-0.072	4.026
GT	3.365	1.698	1.161	8.718	0.546	0.253	-0.394	3.711
HWP	3.577	2.745	4.209	34.58	0.544	0.293	0.382	3.411
IBM	2.905	2.325	4.741	43.50	0.437	0.295	0.391	3.602
IP	3.198	2.254	7.258	125.4	0.501	0.280	-0.015	3.553
JNJ	3.131	2.052	5.714	81.74	0.497	0.267	0.065	3.475
JPM	1.994	1.159	3.463	24.24	0.287	0.230	0.465	4.167
KO	2.790	1.631	5.465	69.49	0.454	0.239	-0.015	4.064
MCD	3.450	2.069	4.168	46.33	0.555	0.246	0.286	3.438
MMM	2.023	1.376	7.303	132.0	0.272	0.286	-0.280	3.682
MO	3.242	6.319	20.70	542.8	0.452	0.308	1.023	6.620
MRK	3.390	2.458	8.760	168.0	0.531	0.279	-0.028	3.741
PG	2.679	1.877	11.85	264.1	0.435	0.227	0.478	4.921
S	3.444	1.776	1.980	8.964	0.565	0.225	0.370	3.177
T	2.564	1.789	6.987	98.03	0.408	0.230	0.907	5.018
TRV	4.021	2.219	2.632	14.99	0.637	0.239	0.096	3.932
UK	6.552	4.727	1.451	5.789	0.818	0.353	0.021	2.282
UTX	1.899	1.278	3.516	27.84	0.239	0.279	0.180	3.313
WMT	6.854	5.165	15.38	392.3	0.894	0.264	-0.537	5.002
XON	1.961	1.474	8.322	142.6	0.270	0.235	1.005	5.363
Median	3.108	1.988	5.609	66.16	0.476	0.264	0.192	3.885
Mean	3.178	2.355	7.433	143.6	0.478	0.263	0.222	4.101
Min.	1.899	1.159	1.451	5.789	0.239	0.225	-0.537	2.282
Max.	6.854	6.319	20.70	567.8	0.894	0.353	1.023	6.620

Note: The sample covers the period from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The realized daily variances, $v_{j,t}^2$, and logarithmic standard deviations, $lv_{i,t} \equiv log(v_{i,t})$, are calculated from the five-minute returns within the day as detailed in the main text.

Table 3
Unconditional Daily Covariance and Correlation Distributions

	$Cov_{i,\cdot,t}$				$Corr_{i,\cdot,t}$			
Stock	Mean	St.Dev.	Skew.	Kurt.	Mean	St.Dev.	Skew.	Kurt.
AA	0.290	0.499	2.659	19.24	0.099	0.139	0.192	3.129
ALD	0.382	0.886	10.44	204.7	0.105	0.148	0.266	3.081
AXP	0.412	0.799	5.076	61.86	0.113	0.155	0.337	3.137
BA	0.364	0.808	7.229	136.1	0.104	0.148	0.274	3.122
CAT	0.360	0.668	5.931	69.85	0.114	0.146	0.166	2.999
CHV	0.360	0.651	4.759	52.94	0.118	0.155	0.301	3.050
DD	0.428	0.737	4.862	44.37	0.130	0.153	0.250	2.975
DIS	0.390	0.769	7.948	133.1	0.123	0.155	0.265	3.083
EK	0.314	0.577	4.143	44.80	0.112	0.146	0.203	2.998
GE	0.468	0.778	5.961	68.60	0.159	0.163	0.265	2.933
GM	0.335	0.664	5.192	63.39	0.103	0.143	0.231	3.129
GT	0.282	0.544	3.053	25.62	0.091	0.139	0.226	3.036
HWP	0.473	0.848	5.073	52.94	0.127	0.151	0.212	3.007
IBM	0.402	0.777	6.254	78.34	0.130	0.152	0.224	3.004
IP	0.342	0.668	5.416	61.86	0.098	0.141	0.232	3.151
JNJ	0.419	0.831	6.748	95.83	0.132	0.155	0.222	2.986
JPM	0.356	0.606	4.163	34.37	0.131	0.158	0.274	3.035
KO	0.417	0.789	6.448	89.77	0.137	0.159	0.252	2.948
MCD	0.330	0.732	8.223	130.1	0.098	0.140	0.157	3.078
MMM	0.350	0.557	4.054	34.74	0.126	0.153	0.197	2.952
MO	0.381	0.687	4.248	39.40	0.111	0.149	0.258	3.130
MRK	0.404	0.789	5.192	59.47	0.130	0.163	0.310	2.935
PG	0.457	0.842	7.386	120.9	0.145	0.160	0.228	2.875
S	0.366	0.679	4.085	42.44	0.110	0.145	0.195	3.056
T	0.347	0.735	8.509	132.3	0.108	0.143	0.242	3.152
TRV	0.403	0.780	3.883	34.00	0.106	0.155	0.295	3.039
UK	0.317	0.706	2.953	33.00	0.080	0.138	0.233	3.212
UTX	0.272	0.508	3.768	35.76	0.107	0.145	0.167	2.991
WMT	0.427	1.091	7.678	103.1	0.093	0.150	0.417	3.472
XON	0.413	0.763	5.879	70.58	0.141	0.164	0.310	3.025
Median	0.373	0.736	5.192	61.86	0.113	0.150	0.237	3.037
Mean	0.375	0.726	5.574	72.45	0.116	0.150	0.247	3.057
Min.	0.272	0.499	2.659	19.24	0.080	0.138	0.157	2.875
Max.	0.473	1.091	10.44	204.7	0.159	0.164	0.417	3.472

Note: Each entry in the table reports the median value of the relevant summary statistics for the realized covariance or correlation for that particular stock with respect to the twenty-nine other stocks included in the DJIA. The realized daily covariances, $Cov_{i,j,t}$, and correlations, $Corr_{i,j,t}$, are calculated from the five-minute returns within the day as detailed in the main text. The sample covers the period from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations.

Table 4
Dynamic Volatility Dependence

	$lv_{i,t}$				$Corr_{i,\cdot,t}$			
Stock	Q_{22}	*ADF*	d_{GPH}	d_S	Q_{22}	*ADF*	d_{GPH}	d_S
AA	2966	-3.105	0.298	0.359	544	-3.997	0.357	0.263
ALD	7041	-2.955	0.410	0.414	1174	-3.395	0.353	0.308
AXP	6801	-3.010	0.377	0.409	2224	-2.710	0.413	0.352
BA	4018	-3.532	0.350	0.375	1211	-3.713	0.361	0.315
CAT	2019	-4.313	0.279	0.335	741	-3.814	0.327	0.274
CHV	4745	-3.164	0.317	0.390	1954	-3.047	0.435	0.346
DD	2694	-4.141	0.323	0.350	1388	-3.322	0.385	0.318
DIS	5885	-2.808	0.394	0.400	1527	-3.220	0.391	0.325
EK	982	-4.620	0.263	0.286	953	-3.866	0.343	0.297
GE	5519	-3.207	0.388	0.397	2661	-2.820	0.460	0.360
GM	3335	-3.359	0.342	0.362	605	-4.344	0.354	0.267
GT	4988	-3.137	0.317	0.392	708	-3.672	0.320	0.274
HWP	3580	-3.337	0.288	0.365	940	-3.687	0.346	0.297
IBM	2499	-4.738	0.392	0.340	1662	-3.236	0.368	0.327
IP	6150	-2.992	0.349	0.407	435	-4.298	0.337	0.246
JNJ	6665	-2.905	0.409	0.410	1760	-3.210	0.387	0.327
JPM	1148	-4.850	0.338	0.287	1818	-3.172	0.434	0.334
KO	5086	-3.595	0.409	0.387	2367	-3.184	0.435	0.349
MCD	6074	-2.544	0.372	0.398	496	-4.260	0.330	0.257
MMM	5521	-3.059	0.369	0.398	1414	-3.169	0.424	0.325
MO	3722	-3.778	0.402	0.367	1050	-3.976	0.361	0.299
MRK	7791	-2.540	0.383	0.417	2802	-2.708	0.411	0.363
PG	3302	-3.545	0.328	0.366	2357	-2.952	0.411	0.348
S	2699	-3.918	0.311	0.346	725	-3.770	0.345	0.278
T	4684	-2.969	0.280	0.385	573	-4.276	0.383	0.266
TRV	2142	-4.118	0.300	0.334	2075	-2.885	0.455	0.350
UK	14254	-2.178	0.406	0.463	466	-4.039	0.292	0.255
UTX	3677	-3.772	0.327	0.374	1178	-3.501	0.368	0.310
WMT	6749	-3.984	0.416	0.407	1387	-3.329	0.397	0.318
XON	5124	-3.317	0.356	0.393	2201	-3.075	0.442	0.349
Median	4715	-3.327	0.349	0.386	1299	-3.362	0.375	0.317
Mean	4729	-3.450	0.350	0.377	1380	-3.488	0.381	0.310
Min.	982	-4.850	0.263	0.286	435	-4.344	0.292	0.246
Max.	14254	-2.178	0.416	0.463	2802	-2.708	0.460	0.363

Note: We report the Ljung-Box portmanteau test for up to 22nd order autocorrelation, Q_{22}, the Augmented Dickey-Fuller test for a unit root involving 22 augmentation lags, *ADF*, the Geweke-Porter-Hudak estimate for the degree of fractional integration, d_{GPH}, and the estimate for the degree of fractional integration based on the scaling-law, d_S. The realized correlations are the median value across the twenty-nine statistics for each of the thirty stocks. The realized volatilities and correlations are calculated from five-minute returns within the day from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations.

Table 5
News Impact Functions

	$lv_{i,t}$				$Corr_{i,\cdot,t}$		
Stock	γ	t_γ	ϕ	t_ϕ	t_γ	t_θ	t_ϕ
AA	0.025	2.080	0.033	2.150	0.486 / 2	-0.188 / 1	1.325 / 12
ALD	0.006	0.350	0.040	2.331	1.005 / 6	0.164 / 3	1.583 / 13
AXP	-0.013	-0.848	0.055	2.820	0.829 / 6	0.807 / 4	1.847 / 21
BA	-0.006	-0.372	0.048	2.358	0.194 / 0	0.544 / 6	1.771 / 17
CAT	0.018	1.507	0.054	3.178	0.531 / 4	0.186 / 2	2.430 / 25
CHV	0.010	0.786	0.042	2.589	0.088 / 2	0.255 / 2	1.509 / 12
DD	0.033	2.387	0.048	2.732	0.700 / 5	0.118 / 2	0.757 / 7
DIS	0.012	0.969	0.043	2.659	0.802 / 8	0.133 / 1	1.494 / 14
EK	0.028	3.288	0.025	1.776	1.550 / 12	0.399 / 2	0.719 / 4
GE	-0.001	-0.068	0.070	4.193	1.051 / 2	0.769 / 3	1.834 / 17
GM	0.006	0.667	0.028	2.103	0.002 / 4	0.834 / 5	2.109 / 18
GT	0.001	0.085	0.052	2.909	0.178 / 6	0.909 / 5	1.621 / 14
HWP	0.043	3.763	0.034	2.408	1.551 / 11	0.186 / 2	1.509 / 13
IBM	0.022	1.577	0.073	4.592	1.287 / 9	0.326 / 2	1.441 / 9
IP	0.014	1.154	0.058	3.190	1.222 / 12	-0.099 / 4	1.273 / 9
JNJ	0.019	1.262	0.054	3.223	0.886 / 5	0.734 / 5	1.771 / 15
JPM	0.005	0.473	0.071	4.354	0.593 / 6	0.440 / 4	1.521 / 13
KO	0.007	0.704	0.053	2.975	0.829 / 5	-0.071 / 2	1.325 / 12
MCD	-0.009	-0.603	0.035	2.037	0.787 / 6	-0.016 / 0	1.605 / 14
MMM	0.017	1.258	0.030	1.997	0.899 / 7	-0.166 / 1	1.267 / 9
MO	-0.001	-0.051	0.036	2.281	1.297 / 7	-0.002 / 2	2.024 / 21
MRK	-0.004	-0.327	0.041	2.975	0.896 / 5	0.549 / 3	1.683 / 15
PG	-0.002	-0.188	0.036	2.354	0.487 / 4	0.086 / 3	1.183 / 9
S	0.016	1.134	0.036	2.456	1.017 / 7	0.637 / 2	1.921 / 17
T	0.005	0.444	0.024	1.575	-0.173 / 1	0.896 / 5	1.812 / 16
TRV	0.013	1.079	0.024	2.899	-0.462 / 0	0.981 / 8	1.706 / 15
UK	-0.014	-0.859	0.038	1.817	0.639 / 4	0.425 / 3	1.241 / 10
UTX	0.025	2.075	0.030	2.039	0.124 / 5	0.055 / 3	1.167 / 8
WMT	0.001	0.048	0.078	4.079	1.381 / 12	0.592 / 4	1.787 / 15
XON	0.008	0.818	0.041	3.208	0.231 / 3	0.137 / 3	1.379 / 12
Median	0.007	0.745	0.042	2.624	0.795 / 5	0.290 / 3	1.552 / 14
Mean	0.010	0.820	0.045	2.742	0.697 / 6	0.354 / 3	1.554 / 14
Min.	-0.014	-0.859	0.024	1.575	-0.462 / 0	-0.188 / 0	0.719 / 4
Max.	0.043	3.763	0.078	4.592	1.551 / 12	0.981 / 8	2.430 / 25

Note: We report OLS estimates for the news impact functions for the fractionally differenced series, $(1-L)^{d_i} lv_{i,t} = \omega_i + \gamma_i \cdot |z_{i,t-1}| + \phi_i \cdot |z_{i,t-1}| I(z_{i,t-1}<0) + u_{i,t}$, and $(1-L)^{d_i} Corr_{i,j,t} = \omega_i + \gamma_i \cdot \{|z_{i,t-1}| + |z_{j,t-1}|\} + \theta_i \cdot \{|z_{i,t-1} + z_{j,t-1}| I(z_{j,t-1} \cdot z_{i,t-1}>0)\} + \phi_i \cdot \{|z_{i,t-1} + z_{j,t-1}| I(z_{j,t-1}<0, z_{i,t-1}<0)\} + u_{i,j,t}$, where $z_{i,t} \equiv r_{i,t}/v_{i,t}$, and the values for d_i are fixed at the d_{GPH} estimates reported in Table 4. The t-statistics are based on Newey-West covariance matrix estimation with 22 lags. The t-statistics for the correlations refer to the median value across the twenty-nine regressions for each of the thirty stocks, along with the number that exceed the five-percent critical value of 1.646. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations.

Table 6
Multivariate Unconditional Volatility Distributions

Stock	$Corr(lv_{i,t}, lv_{\cdot,t})$	$Corr(Corr_{i,\cdot,t}, lv_{\cdot,t})$	$Corr(Corr_{i,\cdot,t}, Corr_{\cdot,\cdot,t})$
AA	0.121	0.131	0.274
ALD	0.149	0.128	0.307
AXP	0.192	0.146	0.334
BA	0.296	0.148	0.302
CAT	0.168	0.134	0.288
CHV	0.185	0.137	0.334
DD	0.230	0.139	0.326
DIS	0.265	0.155	0.331
EK	0.239	0.144	0.290
GE	0.230	0.144	0.357
GM	0.216	0.160	0.289
GT	0.178	0.119	0.274
HWP	0.149	0.152	0.305
IBM	0.285	0.154	0.315
IP	0.129	0.122	0.252
JNJ	0.257	0.158	0.334
JPM	0.225	0.139	0.336
KO	0.275	0.157	0.348
MCD	0.174	0.144	0.271
MMM	0.165	0.132	0.315
MO	0.186	0.162	0.306
MRK	0.295	0.161	0.354
PG	0.260	0.141	0.340
S	0.165	0.147	0.299
T	0.213	0.141	0.280
TRV	0.225	0.123	0.329
UK	0.128	0.124	0.262
UTX	0.205	0.115	0.297
WMT	0.165	0.130	0.296
XON	0.266	0.148	0.356
Median	0.209	0.143	0.306
Mean	0.208	0.141	0.310
Min.	0.121	0.115	0.252
Max.	0.296	0.162	0.357

Note: The column labeled $Corr(lv_{i,t}, lv_{\cdot,t})$ gives the median value of the 29 sample correlations for stock i with respect to each of the other stocks. The column labeled $Corr(Corr_{i,\cdot,t}, lv_{\cdot,t})$ refers to the median value of the corresponding 28·29 sample correlation for stock i, while the last column denoted $Corr(Corr_{i,\cdot,t}, Corr_{\cdot,\cdot,t})$ gives the median value across the 29·28·27 different sample correlations for stock i. The sample period for the time series of realized volatilities extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations.

Figure Titles and Notes

Figure 1

Unconditional Distributions of Daily Standardized Returns

The figure shows the unconditional distributions for the standardized daily returns, $r_{i,t}/v_{i,t}$, for each of the thirty stocks in the DJIA. All of the distributions have been standardized to have mean zero and unit variance. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The realized volatilities are calculated from the sum of the squared five-minute returns within each day. The dotted lines refer to the standard normal density.

Figure 2

Unconditional Distributions of Daily Logarithmic Standard Deviations

The figure shows the unconditional distributions for the realized daily logarithmic standard deviations, $lv_{i,t} \equiv log(v_{i,t})$, for each of the thirty stocks in the DJIA. All of the distributions have been standardized to have mean zero and unit variance. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The volatilities are calculated from the sum of the squared five-minute returns within each day. The dotted lines refer to the standard normal density.

Figure 3

Unconditional Distributions of Daily Correlations

The figure shows the unconditional distributions for the daily realized correlations for XON, $Corr_{XON,j,t}$, with respect to each of the twenty-nine other stocks included in the DJIA. All of the distributions have been standardized to have mean zero and unit variance. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The correlations are calculated from the five-minute returns within each day. The dotted lines refer to the standard normal density.

Figure 4

Time Series of Daily Logarithmic Standard Deviations

The figure shows the time series of the daily realized logarithmic standard deviations, $lv_{i,t} \equiv log(v_{i,t})$, for each of the thirty stocks in the DJIA. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The volatilities are calculated from the sum of the squared five-minute returns within each day.

Figure 5

Sample Autocorrelations of Daily Logarithmic Standard Deviations

The figure shows the sample autocorrelations for the daily realized logarithmic standard deviations, $lv_{i,t} \equiv log(v_{i,t})$, for each of the thirty stocks in the DJIA out to a displacement of 120 days. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The volatilities are calculated from the sum of the squared five-minute returns within each day. The dotted lines give the minimum-distance estimates of the hyperbolic decay rates, $c \cdot h^{2d-1}$.

Figure 6

Volatility Scaling Plots for Daily Logarithmic Standard Deviations

The figure shows the logarithm of the variance of the partial sum of the daily realized logarithmic standard deviations, $log(Var[lv_{i,t}]_h)$, plotted against the logarithm of the aggregation level, $log(h)$, for $h = 1, 2,..., 30$. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 observations at the daily level. The daily volatilities are calculated from the sum of the squared five-minute returns within each day. The dotted lines refer to the least-squares estimates of the regression lines $c + (2d+1) \cdot log(h)$.

Figure 7

Time Series of Daily Correlations

The figure shows the time series of daily realized correlations for XON, $Corr_{XON,j,t}$, with respect to each of the twenty-nine other stocks included in the DJIA. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The volatilities are calculated from the sum of the squared five-minute returns within each day.

Figure 8

Sample Autocorrelations of Daily Correlations

The figure shows the sample autocorrelations for the daily realized correlations for XON, $Corr_{XON,j,t}$, with respect to each of the twenty-nine other stocks included in the DJIA. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The volatilities are calculated from the sum of the squared five-minute returns within each day. The dotted lines give the minimum-distance estimates of the hyperbolic decay rates, $c \cdot h^{2d-1}$.

Figure 9

Volatility Scaling Plots for Daily Correlations

The figure shows the logarithm of the variance of the partial sum of the daily realized correlations for XON, $Corr_{XON,j,t}$, with respect to each of the twenty-nine other stocks included in the DJIA; i.e., $log(Var[Corr_{XON,,j,t}]_h)$, plotted against the logarithm of the aggregation level, $log(h)$, for $h = 1, 2, ..., 30$. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 observations at the daily level. The daily volatilities are calculated from the sum of the squared five-minute returns within each day. The dotted lines refer to the least-squares estimates of the regression lines $c + (2d+1)\cdot log(h)$.

Figure 10

News Impact Functions for Daily Logarithmic Standard Deviations

The figure shows scatterplots of daily realized logarithmic volatilities, $lv_{i,t}$, against the lagged standardized returns, $r_{i,t-1}/v_{i,t-1}$, for each of the thirty stocks in the DJIA. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The daily volatilities are calculated from the sum of the squared five-minute returns within each day. The solid lines refer to the estimated regression lines for lagged negative and positive returns.

Figure 11

News Impact Functions for Daily Correlations

The figure shows the scatterplots for the realized daily correlations for XON, $Corr_{XON,j,t}$, against the sum of the lagged standardized returns, $r_{XON,t-1}/v_{XON,t-1} + r_{j,t-1}/v_{j,t-1}$, for each of the twenty-nine other stocks included in the DJIA. The solid lines refer to the estimated regression lines for the sum of the lagged returns being negative or positive. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The daily volatilities are calculated from the sum of the squared five-minute returns within each day.

Figure 12

Scatterplots of Daily Realized Logarithmic Standard Deviations

The figure shows scatterplots of realized daily logarithmic standard deviations for XON, $lv_{XON,t}$, against the logarithmic standard deviations for each of the twenty-nine other stocks included in the DJIA; i.e., $lv_{j,t}$ for $j \neq XON$. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The daily volatilities are calculated from the sum of the squared five-minute returns within each day.

Figure 13

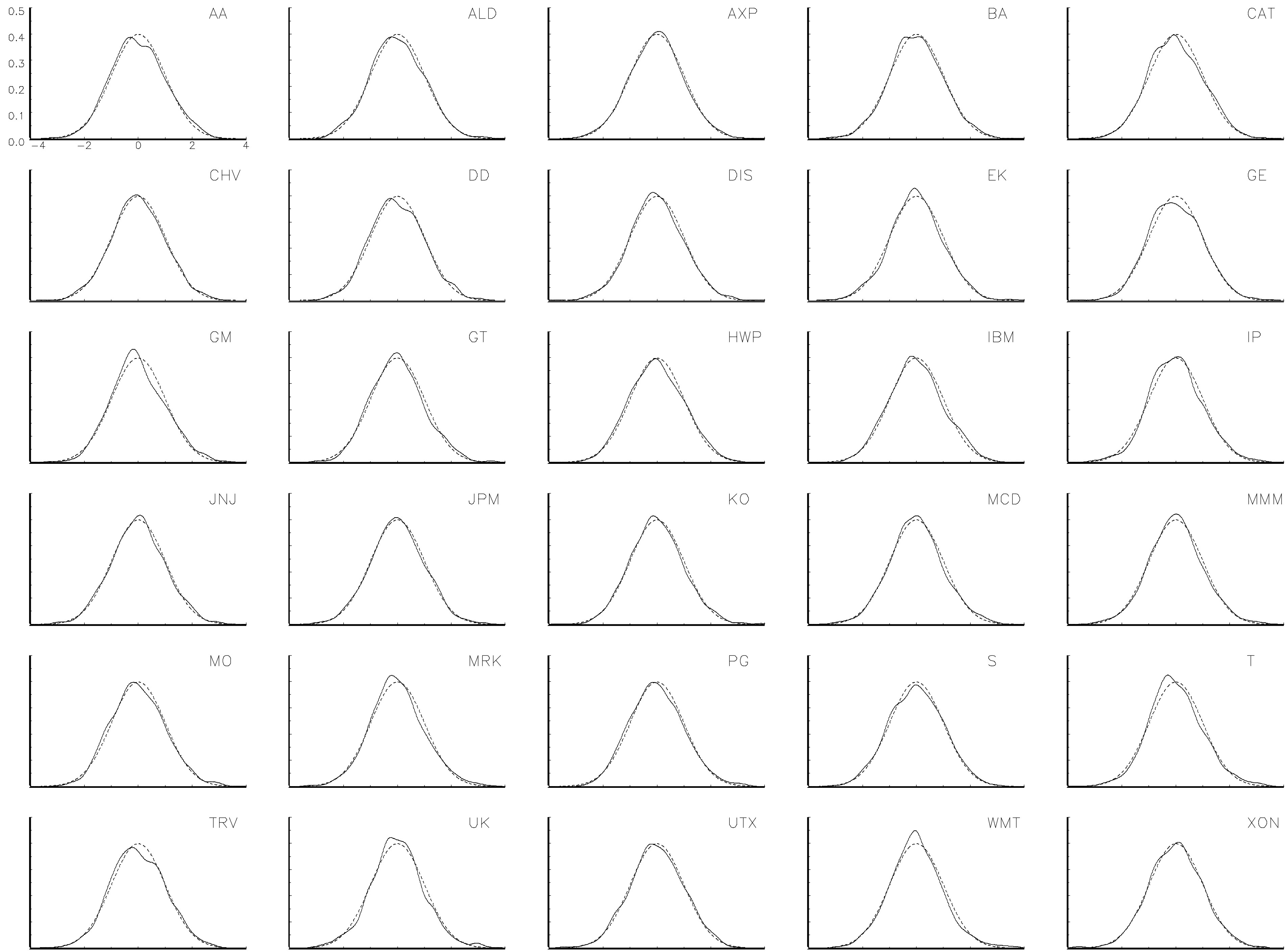
Scatterplots of Daily Realized Correlations versus Logarithmic Standard Deviations

The figure shows scatterplots of realized daily correlations of XON, $Corr_{XON,j,t}$, against the average logarithmic standard deviations for each of the twenty-nine stocks included in the DJIA; i.e., $\frac{1}{2}\cdot(lv_{XON,t} + lv_{j,t})$ for $j \neq XON$. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The daily volatilities are calculated from the sum of the squared five-minute returns within each day.

Figure 14

Daily Realized Correlations

The figure shows scatterplots of the average realized daily correlations of XON against the average realized correlations for stock j; i.e., $(1/28)\cdot \Sigma_i Corr_{XON,i,t}$ for $i \neq XON$ and $i \neq j$ against $(1/28)\cdot \Sigma_i Corr_{j,i,t}$ for $i \neq j$ and $i \neq XON$. The sample period extends from January 2, 1993 through May 29, 1998, for a total of 1,366 daily observations. The daily volatilities are calculated from the sum of the squared five-minute returns within each day.



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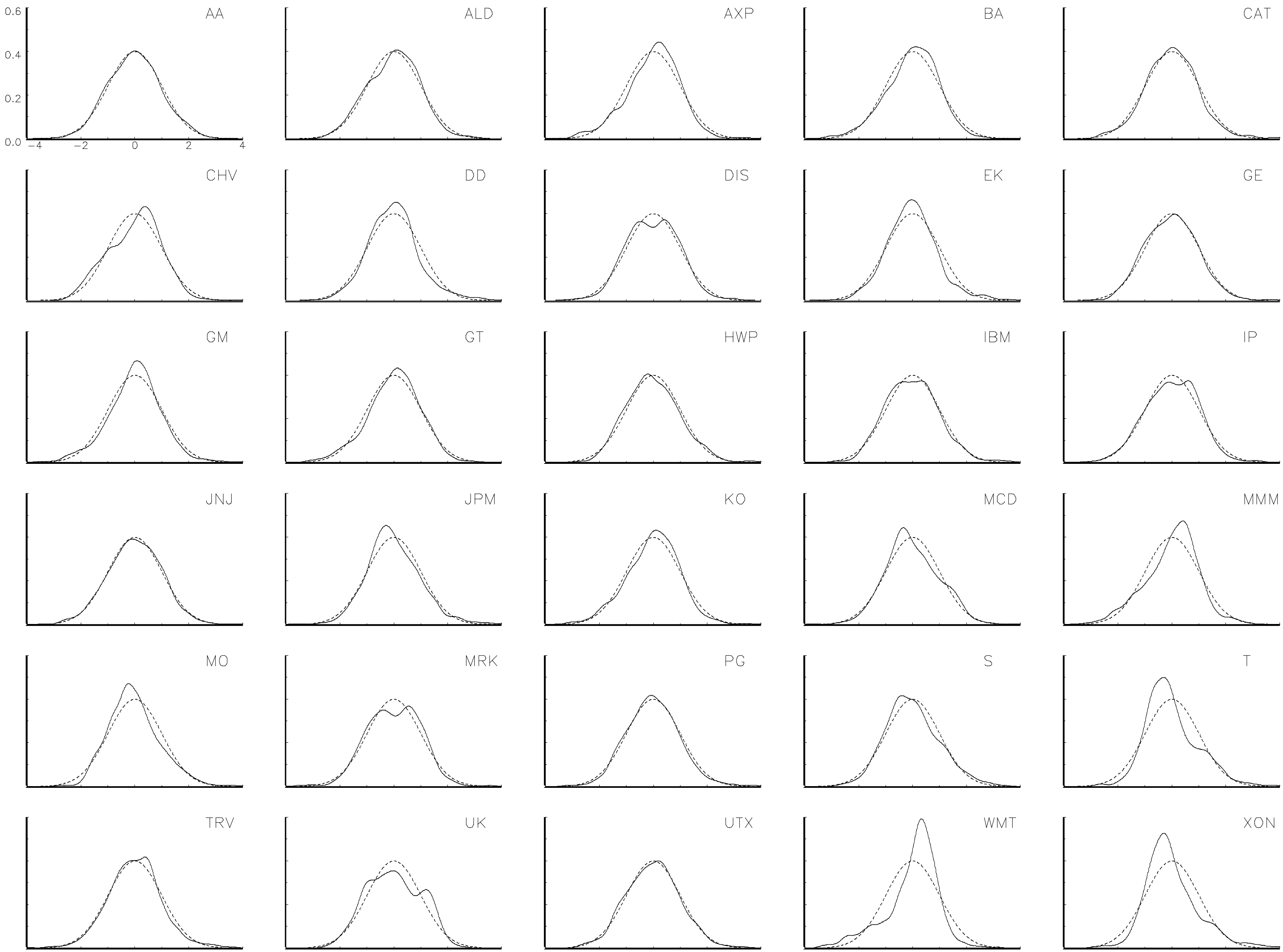

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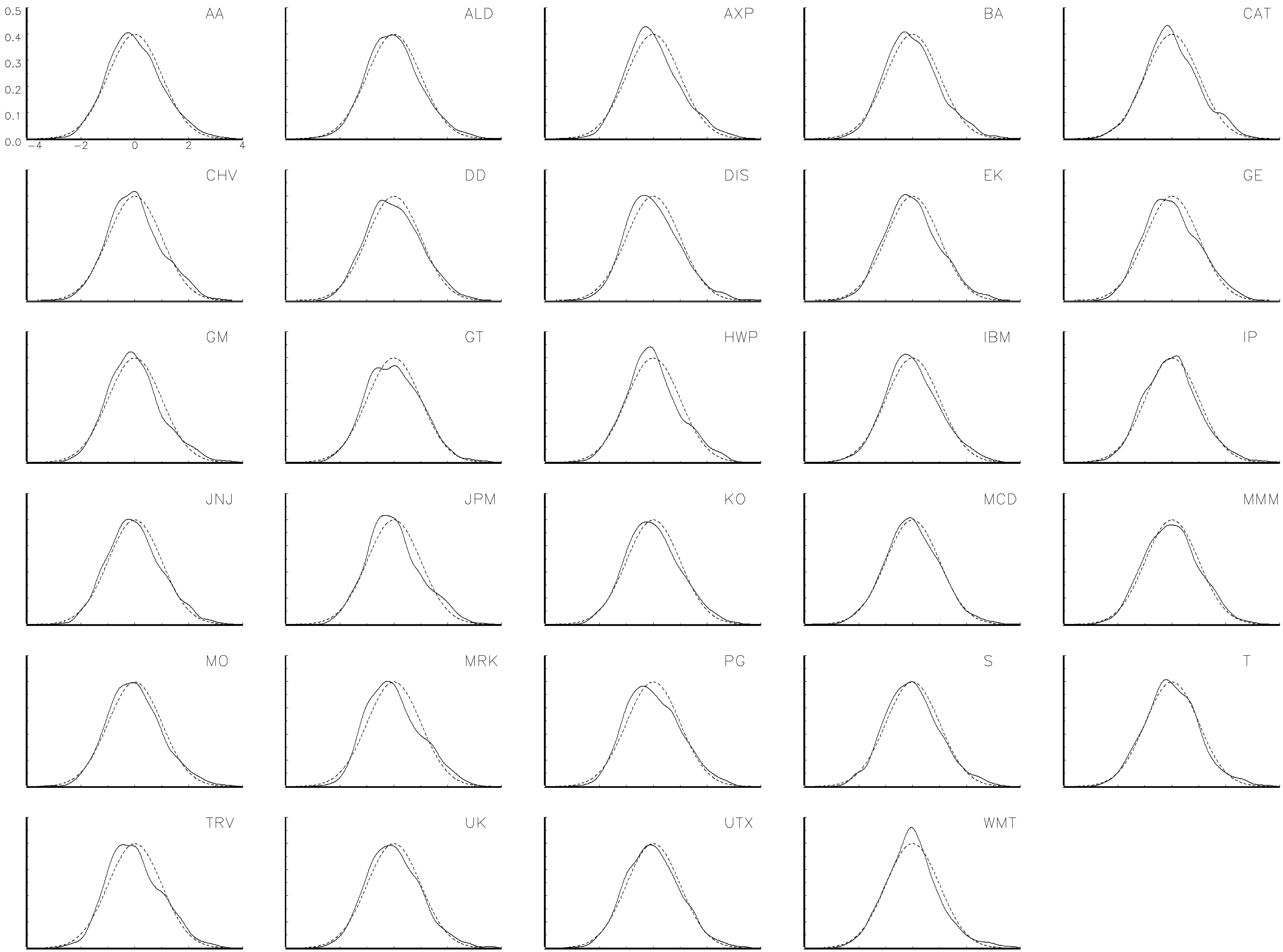

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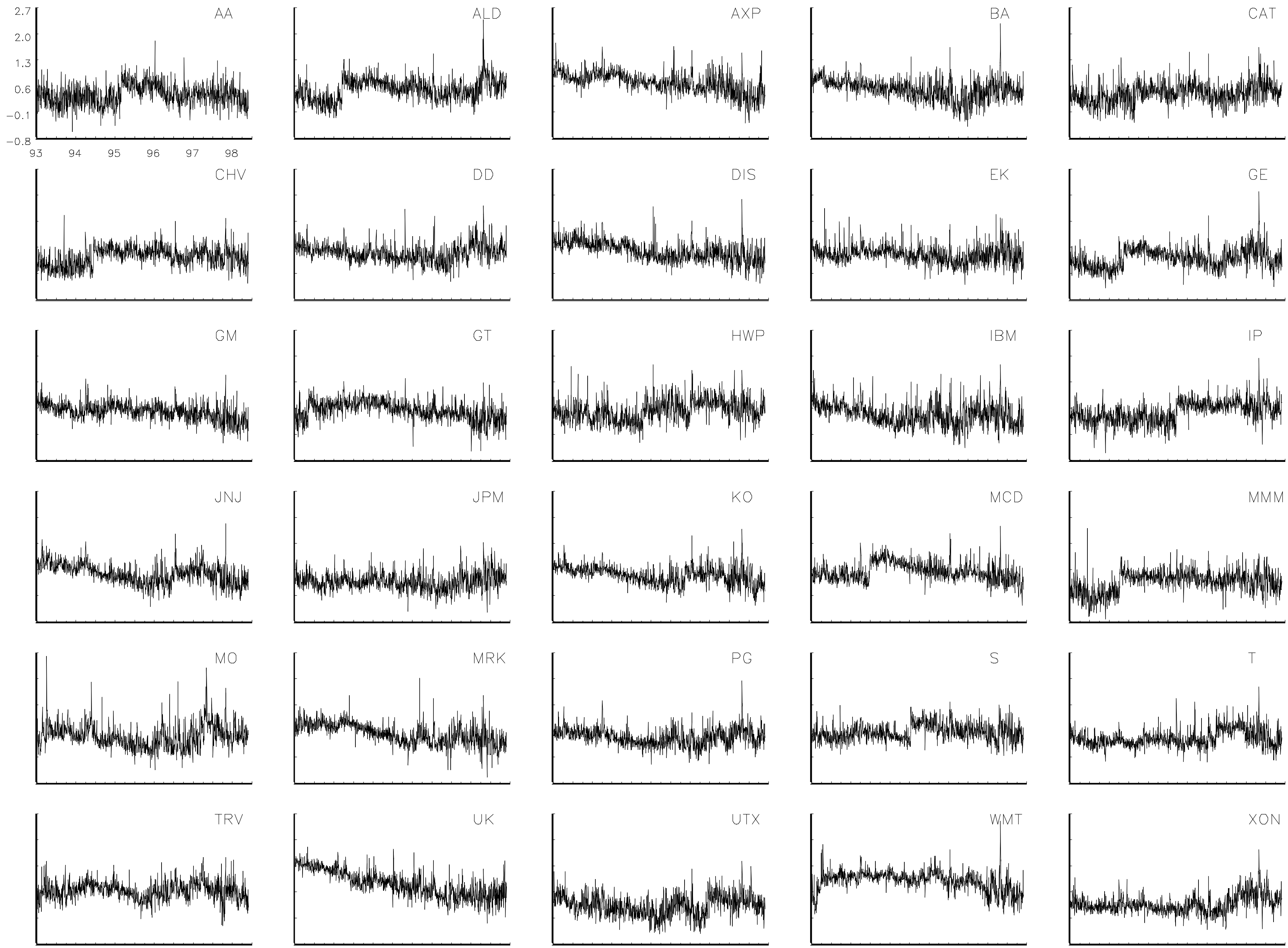

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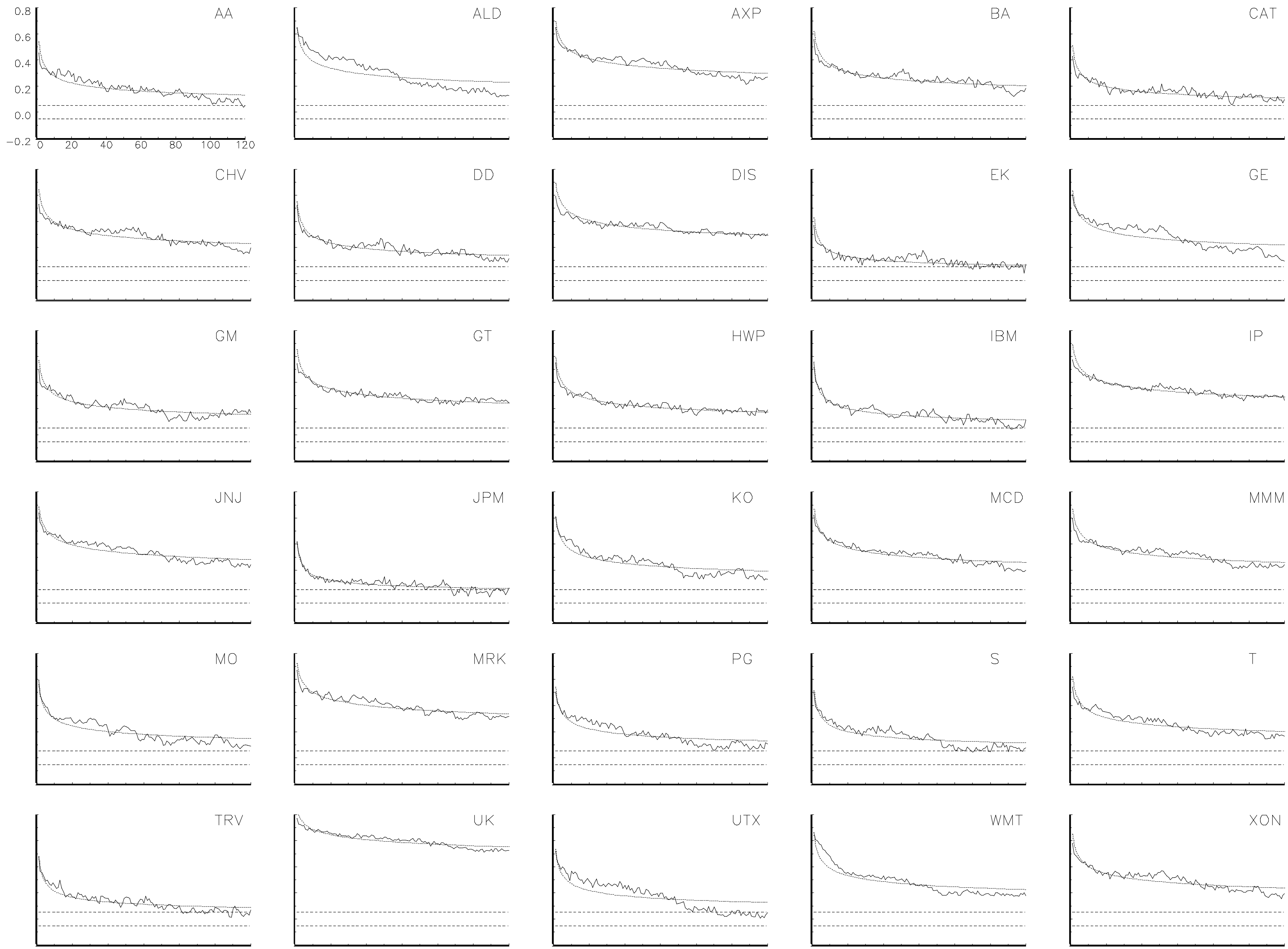

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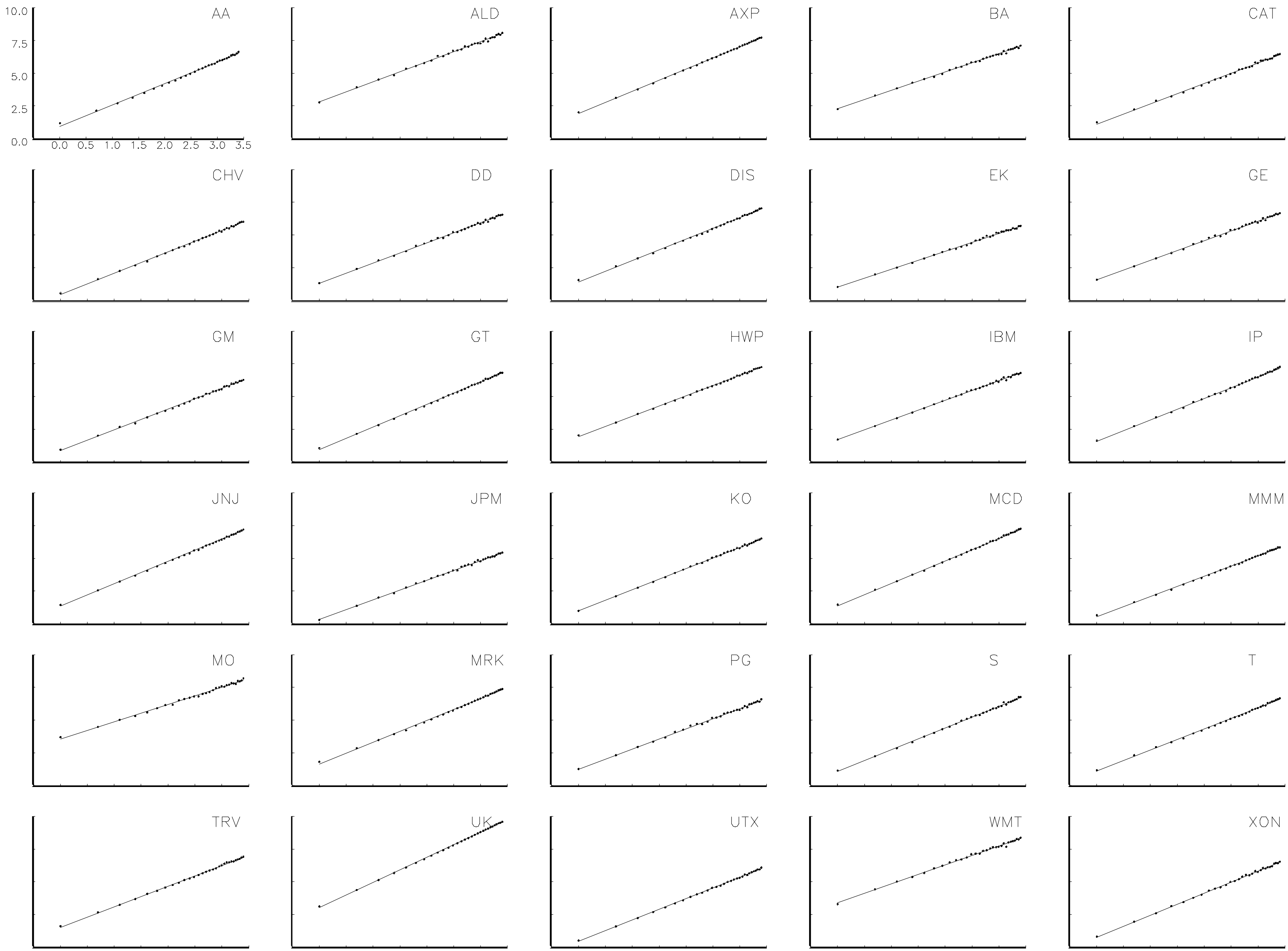

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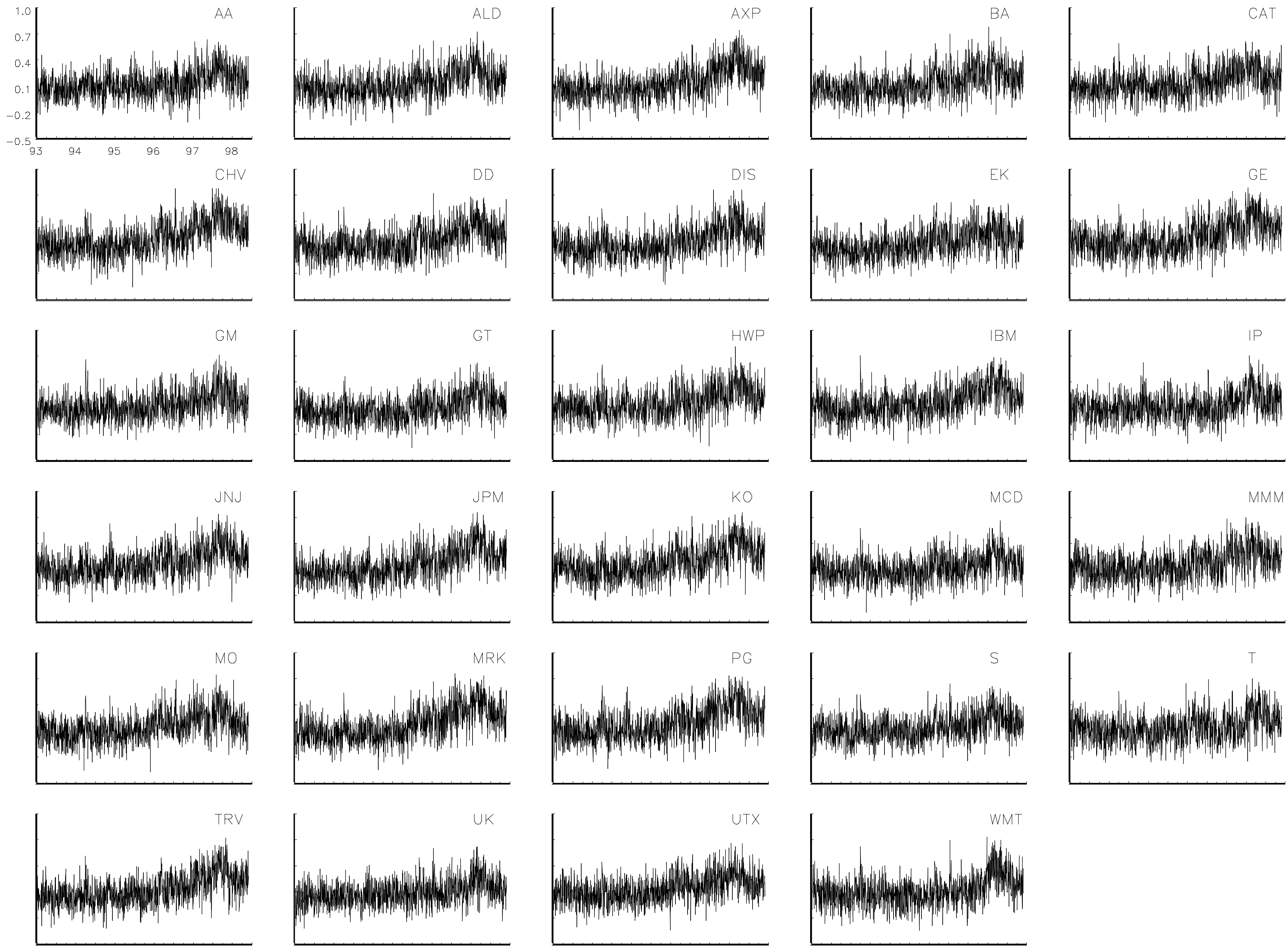

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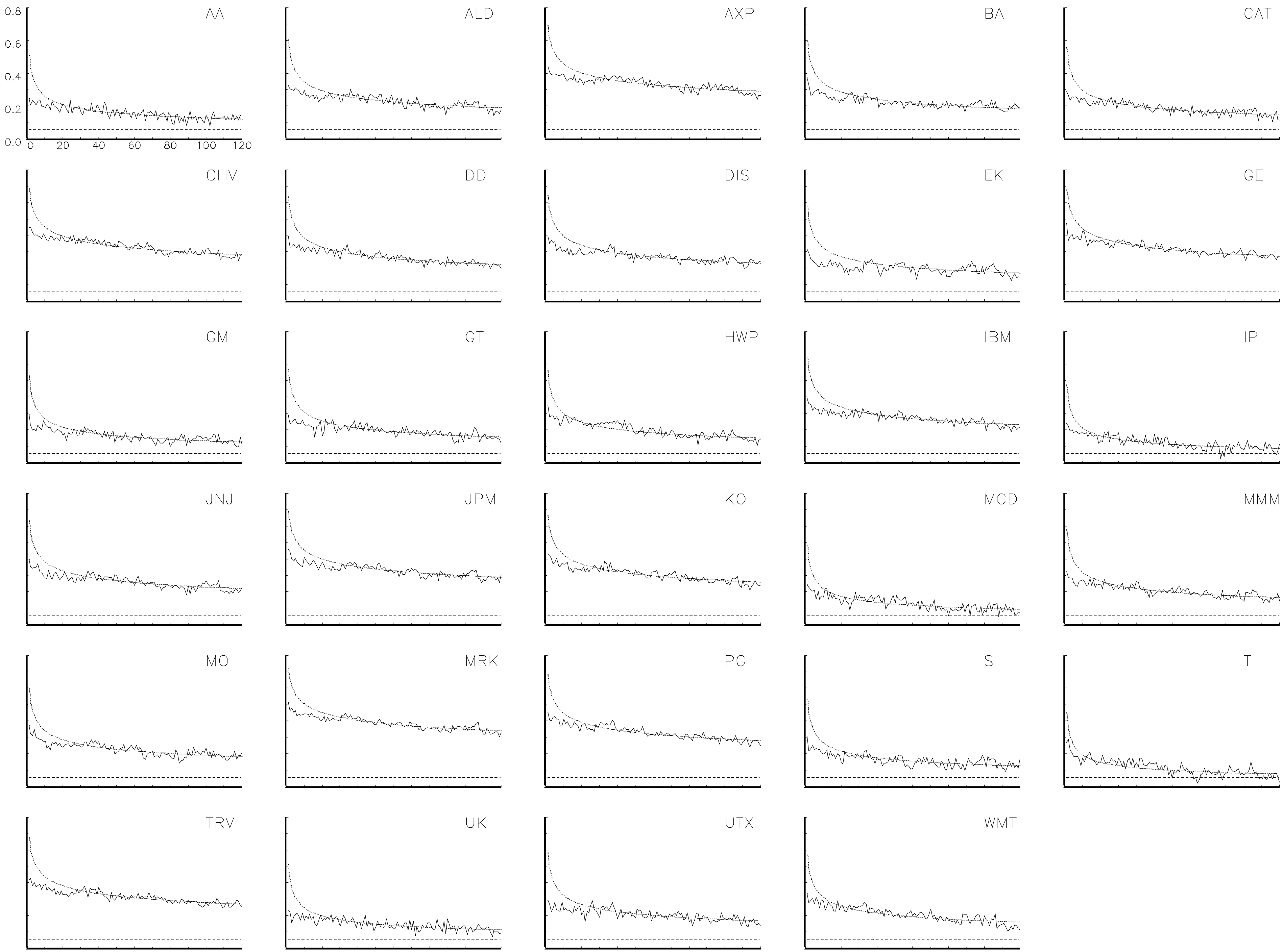
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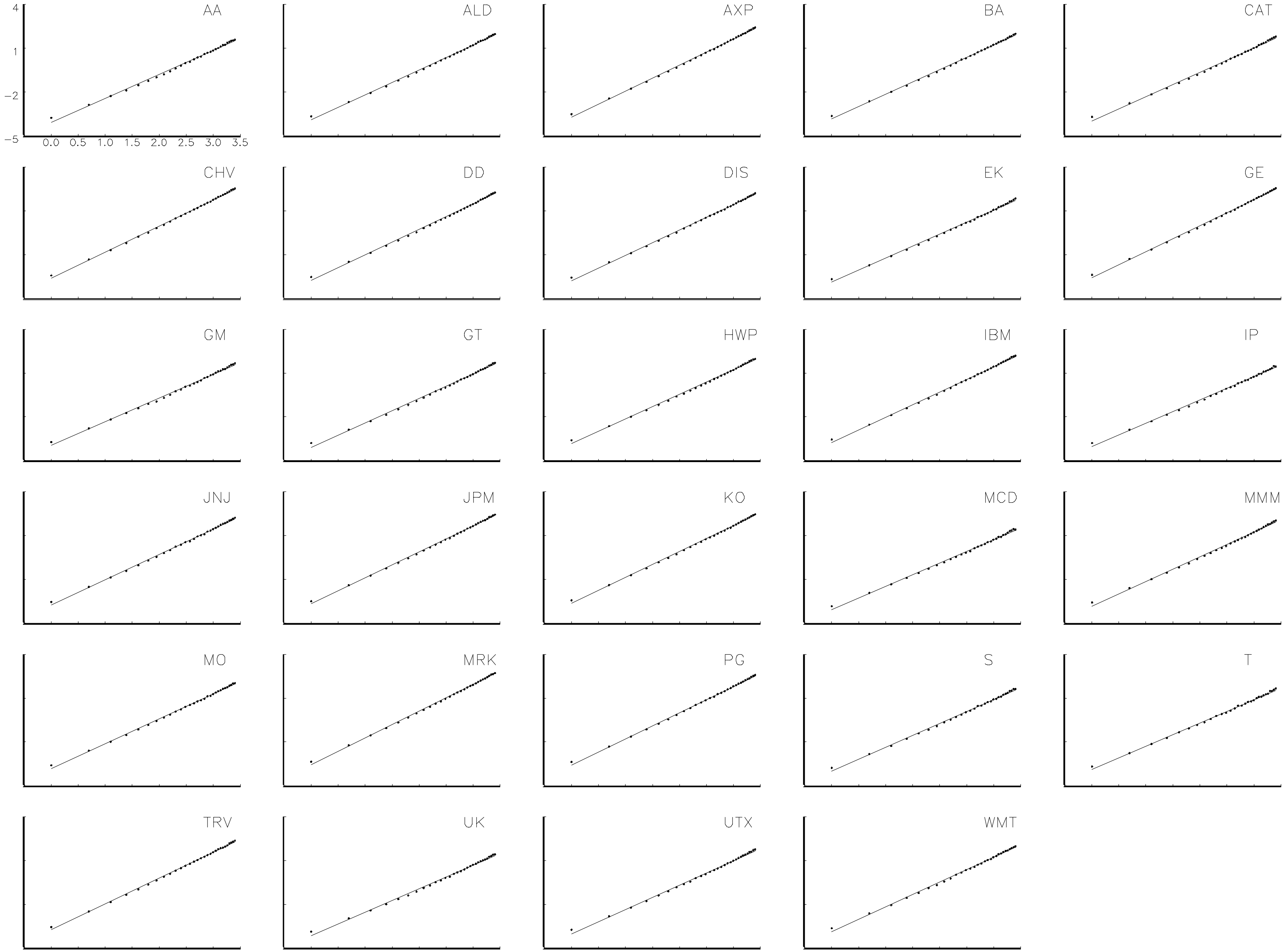
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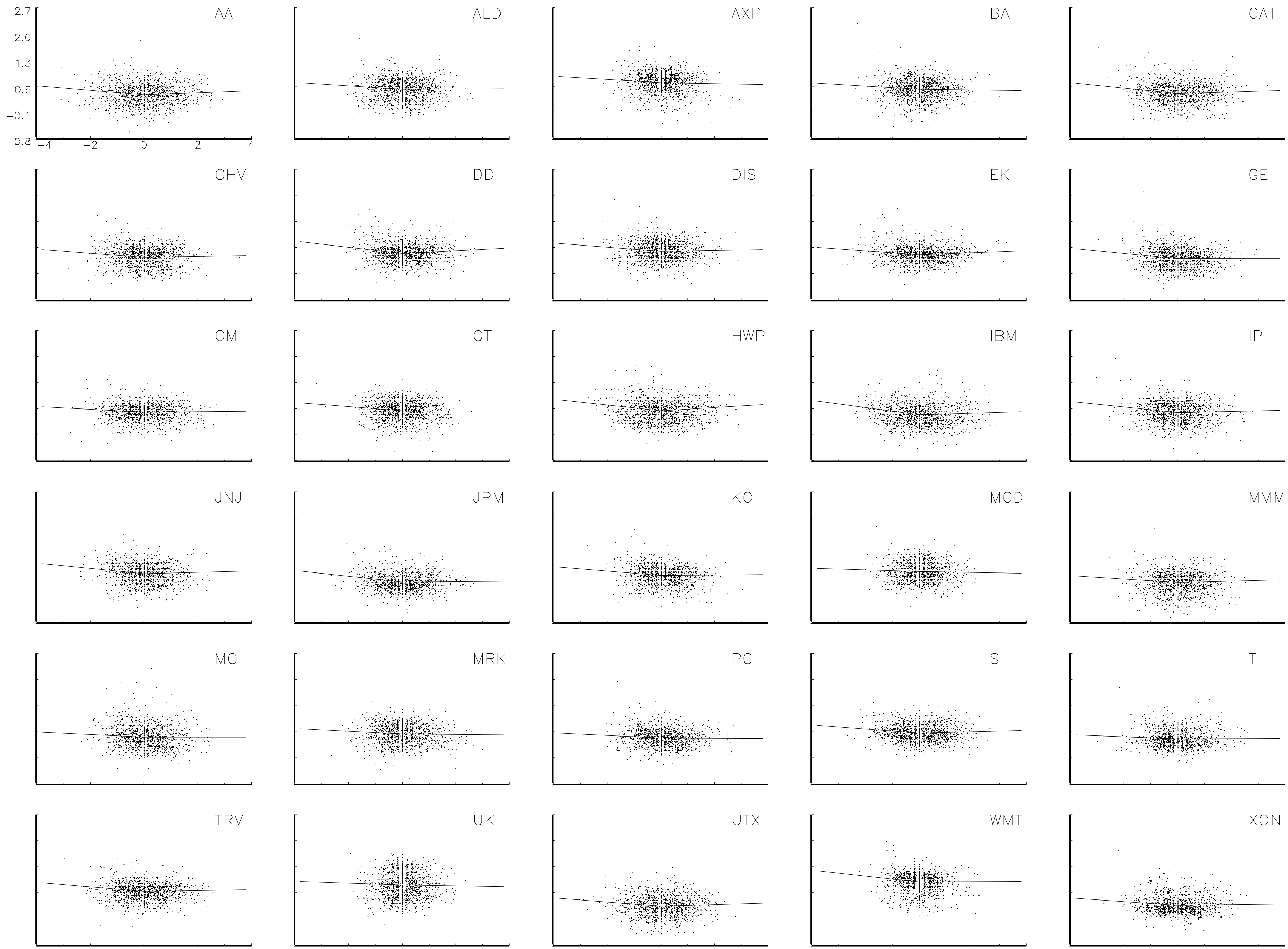

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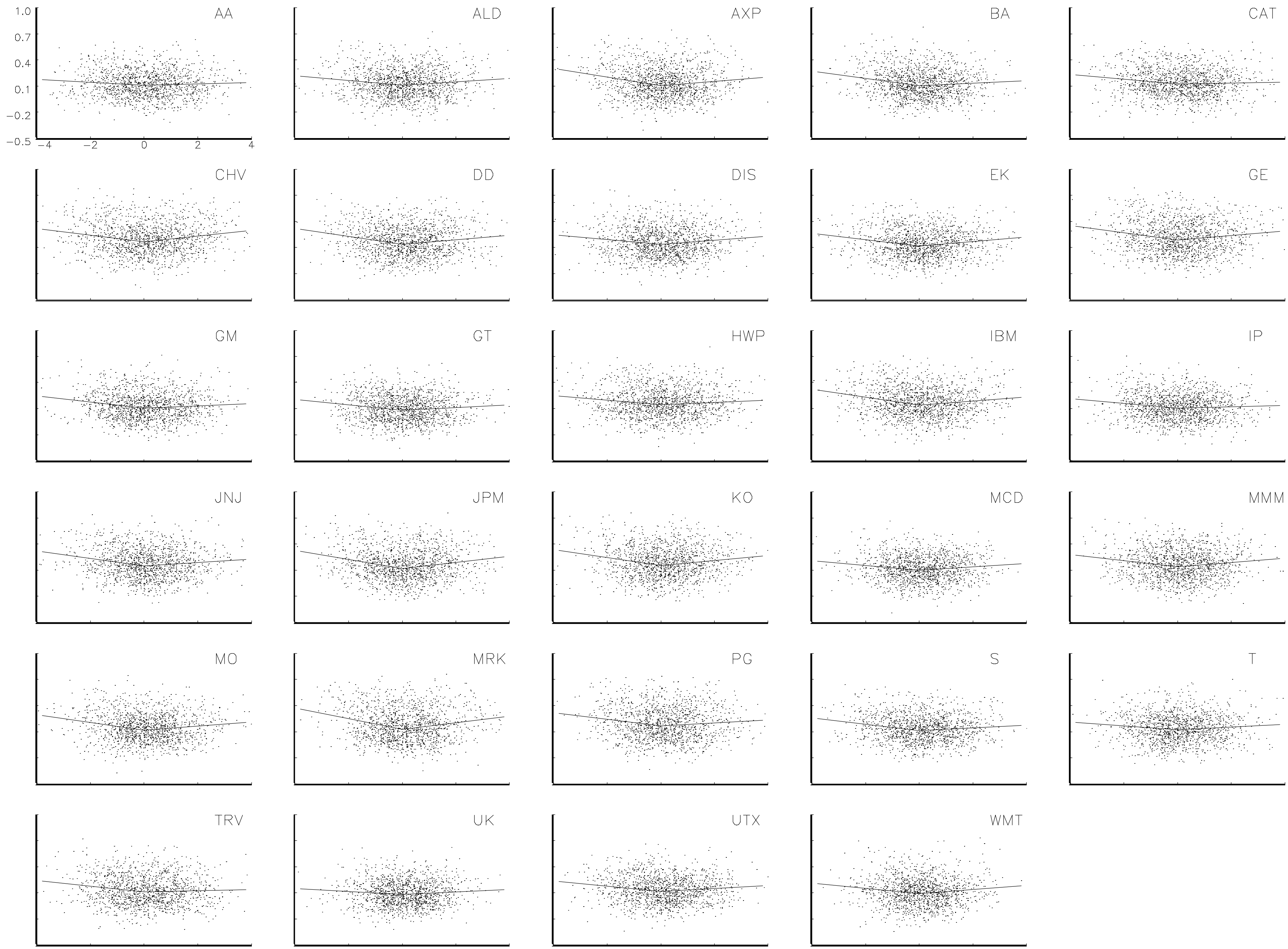
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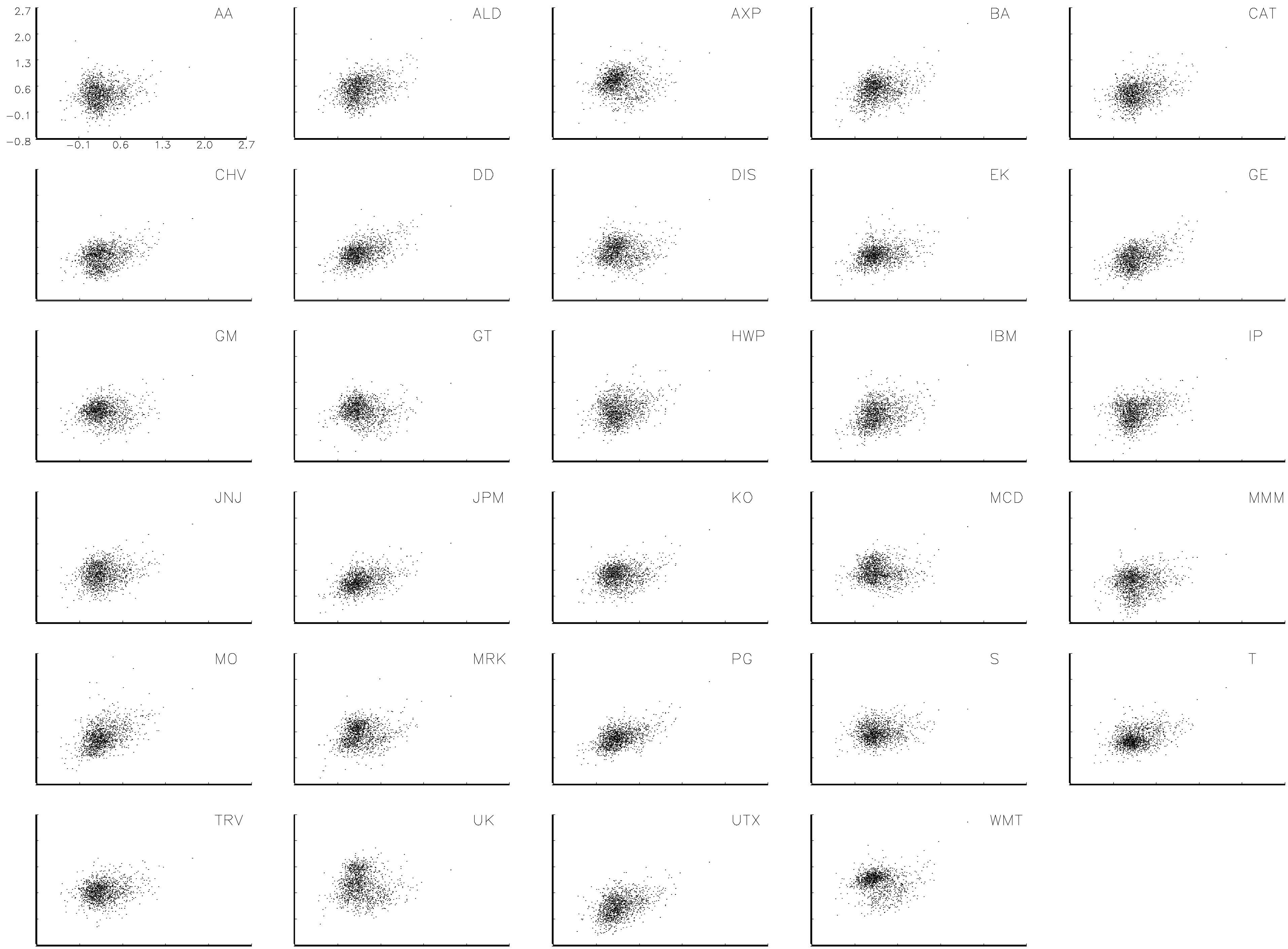
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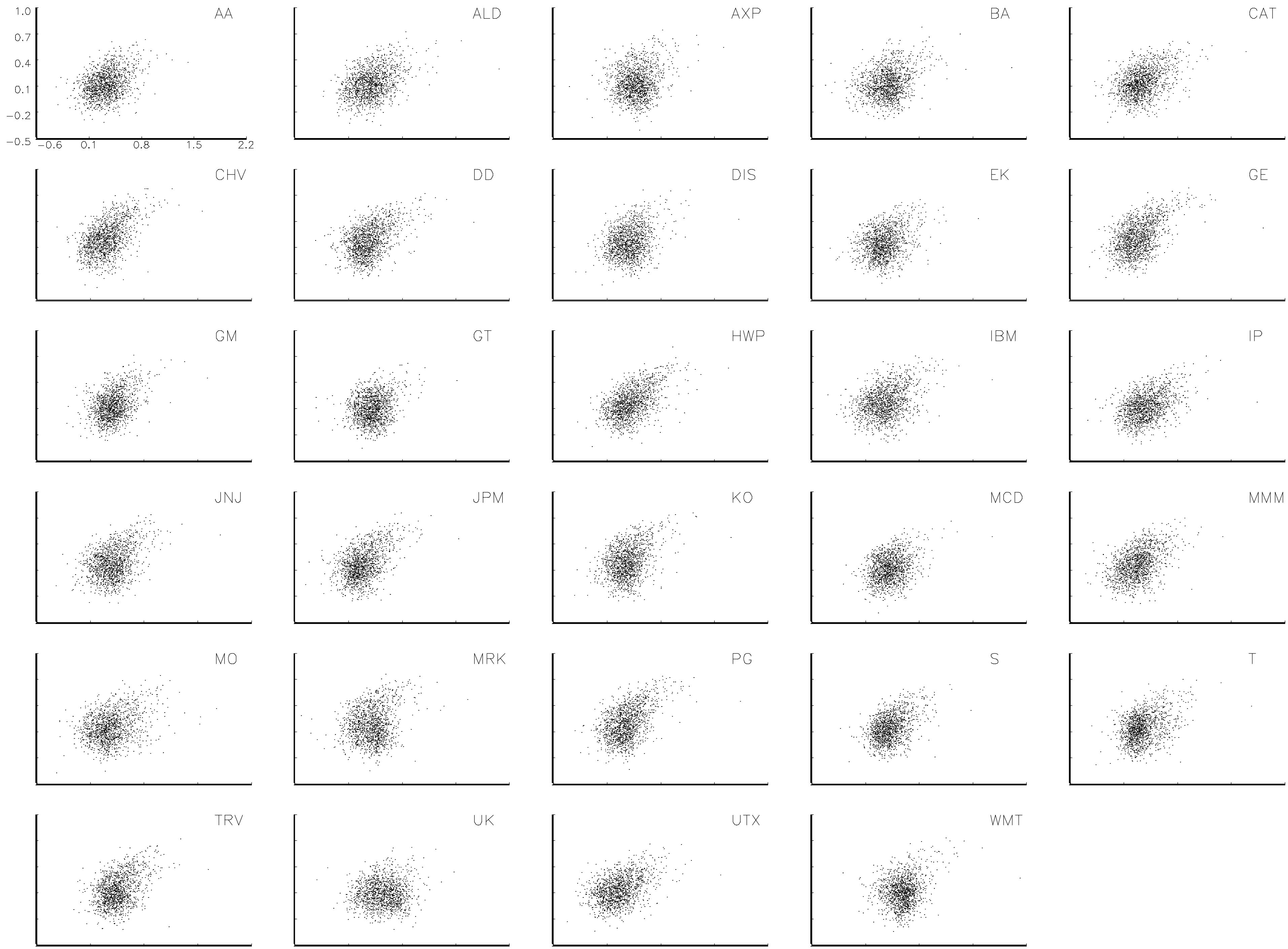

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