Generalized Dynamic Factor Models and Volatilities: Recovering the Market Volatility Shocks

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Background

This paper is concerned with a fundamental problem in modern time series econometrics:

- It's hard to estimate large covariance matrices.
- It's REALLY hard to estimate large time-varying covariance matrices.

Key Problem: Dimensionality. Estimating a covariance matrix typically requires $\sim n^2$ parameters. Estimating a full model of covariance dynamics typically requires $\sim n^4$ parameters.

Background

Recent successful venture: Fan et al (2013) estimate a large covariance matrix Ω_Y of a series Y_t by decomposing

$$\Omega_Y = \Omega_x + \Omega_\eta$$

with Ω_{x_t} low rank and Ω_{η_t} sparse.

- Problem: This decomposition is constructed from a static factor model for levels

$$\begin{aligned} Y_{it} &= b_i f_t + \eta_{it} \\ \Omega_x &= b \Omega_f b' \end{aligned} \tag{1}$$

- This assumes that the common component of volatilities is precisely the volatility of the level-common component.

This Paper's Contribution

This paper uses the same "low rank plus sparse" decomposition in a stochastic volatility setting

$$\begin{aligned} \Omega_{Y_t} &= \Omega_{x_t} + \Omega_{\eta_t} \\ &= b\Omega_{f_t} b' + \Omega_{\eta_t} \end{aligned} \tag{2}$$

- Contribution: Uses the observation that the optimal Ω_{x_t} for a "low rank plus sparse" decomposition is not necessarily $b\Omega_{f_t} b'$.
- Approach: Begin with (2), then measure co-movement of the resulting $\Omega_{x_t} = b\Omega_{f_t} b'$ and Ω_{η_t}.

General method

Let $Y := Y_{it} | i \in 1 : n, t \in 1 : T$ be the data of interest.

- Fit $Y_{it} = b_i f_t + \eta_{it}$
- Obtain volatily proxies for market component and idiosyncratic component s_{it} and w_{it}
- Fit factor models to s_{it} and w_{it} separately, then to the panel of both.
- Difference between number of factors in combined panel and sum of factors in separate panels = number of factors in common.

Description of Full Method

Begin with data $Y := Y_{it} | i \in 1:n, t \in 1:T$ and dynamic factor model with q factors:

$$Y_{it} = X_{it} + Z_{it} = \sum_{k=1}^{q} b_{ik}(L) u_{kt} + Z_{it}$$

- u_{kt} are the market shocks, giving rise to the common component X_{it}.

Forni and Lippi (2011) and (2014) show that with mild additional assumptions, there exist a set of block-diagonal filters $A_n(L)$ and a full rank constant matrix H such that

$$(I - A_n(L))Y_t = Hu_t + (I - A_n(L))Z_t$$

Residuals for volatility analysis

The above lets us easily obtain residuals for the level-common component: $e = \{e_{it}\} = \{(Hu_t)_i | i \in 1:n, t \in 1:T\}$. Since the remainder is idiosyncratic, residuals are obtained by fitting univariate AR regressions, yielding idiosyncratic residuals v_{it}.

- To conduct volatility analysis, take $s_{it} = log(e_{it}^2)$ and $w_{it} = log(v_{it}^2)$ as volatility proxies. Assume these proxies are demeaned.
- We now conduct factor analysis on s_{it} and w_{it}.

Technical Issue - Factor Structure of Common Volatility

Suppose we find 1 factor in the filtered data for the levels (so that u_t is scalar). Then

$$s_{it} = log(H_i^2 u_t^2) = 2log(H_i) + 2\log(u_t)$$

After demeaning, we will find a single factor for s_{it}, given by $\log(u_t)$.

- We should not need to estimate factor structure on the volatilities of the common component.
- We DO need to estimate factor structure on volatility of $u_t's$ in the case where there are many factors for the levels.

Factors of Volatilities

As with the levels, we have the decompositions

$$s_{it} = \chi_{s;it} + \xi_{s;it} = \sum_{k=1}^{q_s} d_{s;ik}(L)\varepsilon_{s;kt} + \xi_{s;it}$$

$$w_{it} = \chi_{w;it} + \xi_{w;it} = \sum_{k=1}^{q_w} d_{w;ik}(L)\varepsilon_{w;kt} + \xi_{w;it}$$

- Now by construction, $\varepsilon_{s;kt}$ and $\xi_{s;it}$ are independent, as with $\varepsilon_{w;kt}$ and $\xi_{w;it}$.
- But $\varepsilon_{s;kt}$ and $\varepsilon_{w;kt}$ may not be, as with $\xi_{s;it}$ and $\xi_{w;it}$, etc, etc.

Factors of Volatilities

Moving forward, we can decompose further:

$$s_{it} = \sum_{k=1}^{q_s} d_{s;ik}(L)(\phi_{kt} + \psi_{s;kt}) + (\zeta_{s;it} + \xi^*_{s;it})$$

$$w_{it} = \sum_{k=1}^{q_s} d_{w;ik}(L)(\phi_{kt} + \psi_{w;kt}) + (\zeta_{w;it} + \xi^*_{s;it})$$

Common and Idiosyncratic Volatility Shocks

These decompositions give rise to 4 categories:

- Strongly common: The shocks common to both s and w.
- Weakly common: The shocks $\psi_{s;kt}$ and $\psi_{w;kt}$. These are the common shocks of each block not driven by the strongly common shocks.
- Weakly idiosyncratic: $\zeta_{s;it}$ and $\zeta w; it$. These are the shocks idiosyncratic to the common shocks of their own block, but not necessarily to the common shocks of the opposite block. Arguably the most nebulous of the four.
- Strongly idiosyncratic: $\xi^*_{s;it}$ and $\xi^*_{w;it}$. These shocks are independent of each other and independent of all market-driven shocks. Essentially iid white noise.

Weakly Idiosyncratic Shocks

A useful way to think of the idiosyncratic shocks: We understand $\phi_{kt}, \psi_{s;kt}$ and $\psi_{w;kt}$. Consider

$$\eta_{it} = \begin{cases} s_{it} = \chi^{s}_{\eta;it} + \xi^{s}_{\eta;it} = \sum_{k=1}^{Q} d^{s}_{\eta;ik}(L)\varepsilon_{kt} + \xi^{s}_{\eta;it} \\ w_{it} = \chi^{w}_{\eta;it} + \xi^{w}_{\eta;it} = \sum_{k=1}^{Q} d^{w}_{\eta;ik}(L)\varepsilon_{kt} + \xi^{w}_{\eta;it} \end{cases}$$

Then ξ_η's are strongly idiosyncratic by construction, so we conclude:

$$\zeta_{s;it} = \chi^{s}_{\eta;it} - \chi_{s;it}$$

And similarly

$$\zeta_{w;it} = \chi^{w}_{\eta;it} - \chi_{w;it}$$

A revealing example

Suppose $Q = q_s = q_w$ (as is found to be the case). Then there are no weakly common or weakly idiosyncratic components, just a strongly common shock and strongly idiosyncratic shocks.

- We may thus write

$$\log(e_{it}^2) = d_{s;i}(L)\phi_{kt} + \xi_{s;it}$$

$$\log(v_{it}^2) = d_{w;i}(L)\phi_{kt} + \xi_{w;it}$$

- Here $\phi_{kt}, \xi_{s;it}$, and $\xi_{w;it}$ are mutually orthogonal at all leads and lags.

This is often the case found in financial empirical work, and is the case found in the empirical section of this paper, but in general need not be.

The S&P100 Panel

- The above method is applied to a panel of 90 times series for which 3457 trading days are observed.
- Method successfully picks up periods of high market volatility by estimating market shocks u_t. They find overall 1/3 of total variance of returns is driven by variation in level-common shocks.
  - Moreover, u_t has a correlation of .95 with average daily returns, consistent with interpretation that common shocks are "market return" shocks.
- Factor structure: They find $Q = q_s = q_w = 1$.

Minor Aside

- The authors worked hard to construct the decompositions into strongly common, weakly common, weakly idiosyncratic, and strongly idiosyncratic shocks.
- Are there datasets for which this analysis is more fruitful?
  - Large collections of macroeconomic indicators in a DSGE model unlikely to have single factor volatility.

Analyzing Volatility Shocks

- There are two volatilities to consider: Volatility of the returns factor s_{it} and volatility of the returns idiosyncratic component w_{it}.
- Market volatility shocks accounts for $2/3$ of the volatility of the returns factor, and $1/10$ of the volatility of the returns idiosyncratic component.
  - Most of that $1/10$ is observed during the 2008-2009 financial crisis, during which market-driven volatility shocks account for closer to $1/5$ of the volatility of the level-idiosyncratic component.
  - Outside of that time period, generally closer to $1/20$.

Analyzing Volatility Shocks

Combine the fact that the factor for volatility accounts for 2/3 of the variation of the returns factor, and 1/10 of the variation of the returns idiosyncratic component with the following:

- The common component explains an average 1/3 of the volatility of returns, the idiosyncratic component explaining the remaining 2/3.
- Doing some bad math...

$$2/3^*(1/3) + 1/10^*(2/3) = .29$$

This suggests that the extracted factor for volatility explains around 30% of the variation in returns.

Level-Common and Volatility-Common Shocks

FIGURE 1: *The market shocks* u_t^T *on returns, period 2000–2013.*

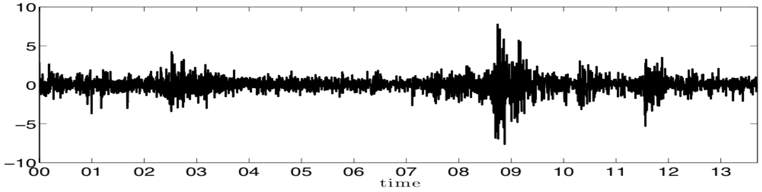
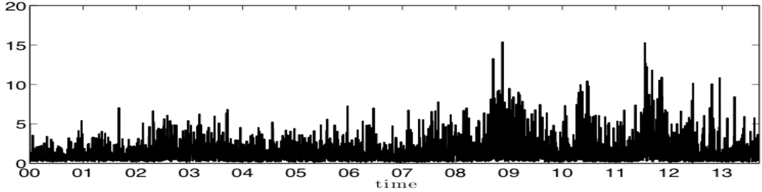


FIGURE 5: *The market shock* $\exp(\varepsilon_t^T)$ *on volatilities, period 2000–2013.*



Analyzing Volatility Shocks

The extracted factor for volatility explains little of the variation in idiosyncratic volatility.

- This is somewhat at odds to their original motivation.

This returns us to the technical issue: Is the extracted factor for volatility to close to the volatility of the level-common component (u_t^2) because the estimation procedure effectively used this series 90 times? (Recall the technical issue we raised).

- If the full panel of volatilities consisted of $\log(u_t^2)$ once, then the 100 idiosyncratic volatilities, would we get a more accurate estimate?
- Or does this procedure just ruin the asymptotics without solving the above problem?

Issues And Extensions

There are several key issues:

- Covariance estimation (as separate from volatility estimation).
- Forecasting ability - point versus density forecast.
- Volatility estimators - proxies instead of realized measures.

Covariance Estimation

This paper assumes that all covariance in Y_t is from joint loading off of factor - assume exactly diagonal idiosyncratic covariance $\forall t$.

- How realistic is this?

Let's consider a small exercise with 11 stocks (GE, AXP, Coca Cola, etc, etc). Fit a static factor model to them (scree plot suggests 1 factor) via principal components.

Covariance Estimation

Construct a realized measure $RM\Omega_{Y_t}$ from high-frequency data. With the linear factor model in hand

$$Y_{it} = b_i f_t + \eta_{it}$$

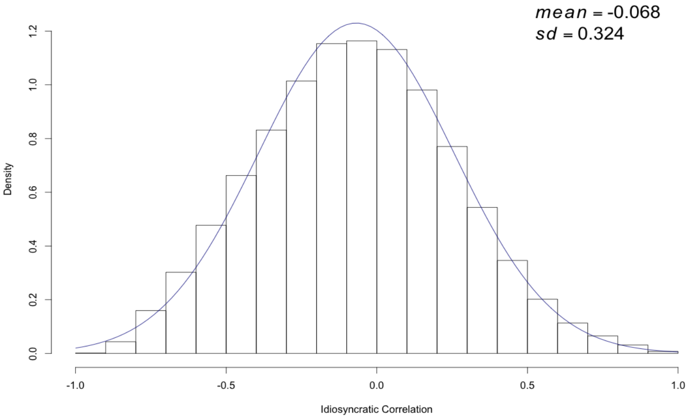
we can construct a high-frequency factor as

$$\hat{f}_t^{HF} = b^{'} Y_t^{HF}$$

From this we can construct an estimate of $bRM\Omega_{f_t} b'$, and a corresponding estimate of idiosyncratic covariance.

- What does idiosyncratic correlation look like?

Idiosyncratic Correlation



Idiosyncratic Correlation

Idiosyncratic correlation is mean zero (which we expect from factor model identification assumptions).

- But with static factor loadings and a 1-factor model (so that Ω_{f_t} is scalar), if you assume Ω_{η_t} diagonal $\forall t$, you arrive at a constant correlation model. This is very at odds with the data.
- Extending the model to capture structure of idiosyncratic correlation would be a great extension.

We see that at least for some datasets we may be able to do this parametrically/easily.

- Semi-open question: What idiosyncratic correlation structures still leave the model identified? Is long-run-mean ~ 0 necessary? Sufficient?

Density Forecasts

Consider a general stochastic volatility model

$$Y_t \sim F(\Sigma_t)$$

$$\Sigma_t \sim G(\Sigma_{t-1})$$

This yields a density forecast via:

$$\hat{P}(Y_{t+1}|\mathcal{F}_t) = \int \hat{F}(Y_{t+1}|\Sigma_{t+1})\hat{G}(\Sigma_{t+_1}|\mathcal{F}_t)d\Sigma_{t+1}$$

Most of the work on density forecasts in stochastic volatility settings has only focused on the performance of the total density forecast $\hat{P}(Y_{t+1}|\mathcal{F}_t)$.

- Extending the approach in this paper lets us address accuracy of $\hat{G}(\Sigma_t|\Sigma_{t-1})$.

Forecasting: Model-free setup

Problem: A model-free approach means only point forecasts.

- Can maintain model-free setup and estimate distribution of volatility shocks.
- Covariance densities require at least some structure (to maintain positive-definiteness).

Forecasting - Volatility proxies?

The estimation procedure gives rise to impulse response functions which are useful for point forecasts of volatility.

- How robust is the forecast to the choice of volatility proxy?
- Patton (2010) shows that the answer unfortunately depends on your choice of loss function. Only particular classes of loss functions are assured to be robust to choice of volatility proxy.
    - Readers familiar with Patton will not be surprised to hear that this class is precisely Bregman loss functions.
    - Bregman loss functions are quite general so this is not devastating, but worth keeping in mind.

Conclusions

- Key observation: that common component of volatilities is not the volatility of the common component.
  - This is an exciting and important observation that paves the way for accurate volatility estimation.
  - Empirical result that large numbers of common and idiosyncratic volatilities load off of a single factor is crucial for future tractable modeling.
- This paper is an extremely successful first pass at the approach, so it comes with the caveat that all first passes have: a LOT of room for forward progress. The two big ones are:
  - Restricts to volatility estimation, not covariance estimation.
  - Leaves open the question of how to estimate the densities of extracted shocks.