Visualizing VAR's: Regularization and Network Tools for High-Dimensional Financial Econometrics

Francis X. Diebold
University of Pennsylvania

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DGP: N-Variable $VAR(p)$, $t = 1, ..., T$

$$\Phi(L)x_t = \varepsilon_t$$

$$\varepsilon_t \sim iid(0, \Sigma)$$

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If you understand the VAR, you understand *everything*.

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(1) Estimate the *VAR*

(2) Identify the estimated *VAR*

(3) Understand the estimated *VAR*
– Examine variance decompositions, etc.

(1) Background Motivation

Financial/Economic Connectedness Measurement

A Natural Financial/Economic Connectedness Question:

What fraction of the H-step-ahead prediction-error variance of x_i is due to shocks in x_j, $j \neq i$?

Non-own elements of the variance decomposition: $d_{ij}^H, \ j \neq i$

Variance Decomposition / Connectedness Table

D

	x_1	x_2	...	x_N
x_1	d_{11}^H	d_{12}^H	$\cdots$	d_{1N}^H
x_2	d_{21}^H	d_{22}^H	$\cdots$	d_{2N}^H
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_N	d_{N1}^H	d_{N2}^H	$\cdots$	d_{NN}^H

Reading and Web Materials

Diebold, F.X. and Yilmaz, K. (2014), "On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms," *Journal of Econometrics*, 182, 119-134.

Diebold, F.X. and Yilmaz, K. (2015), *Financial and Macroeconomic Connectedness: A Network Approach to Measurement and Monitoring*, Oxford University Press. With K. Yilmaz.

www.FinancialConnectedness.org

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"High dimensionality"
"Big Data"

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(1) Estimate the *VAR*

Key theme: One way or another, we need to recover d.f.

"Regularization": Shrinkage, Selection, Combinations

Leading example: adaptive elastic net (lasso variant)

$$\hat{\beta}_{AEnet} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_i \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} w_i \left(\alpha|\beta_i| + (1-\alpha)\beta_i^2 \right) \right)$$

where $w_i = 1/|\hat{\beta}_i|^{\nu}$, $\hat{\beta}_i$ is OLS or ridge, and $\nu > 0$.

Lasso is $\alpha = 1$, $w_i = 1 \forall i$
Adaptive lasso is $\alpha = 1$
Elastic net is $w_i = 1 \forall i$

Opportunities with Shrinkage, Selection, Combinations

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We's like system estimation, and system lasso
(i.e., single system λ)

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First-round coefficient lasso with second-round D thresholding?

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– Christian Hansen two-step lasso?

"Regularization": Dimensionality Reduction

Leading example: dynamic factor model

$$y_t = \lambda f_t + \varepsilon_t$$

$$\varepsilon_t \sim WN(0, \Sigma)$$

$$\Phi(L) f_t = v_t$$

$$v_t \perp \varepsilon_{t-\tau}, \ \forall \tau$$

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– Exact vs. approximate vs. switching factor structure

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– Estimate Σ (and its Cholesky factor)

(2) Identify the Estimated *VAR*

Key theme: *SVAR*-style identification is hopeless

We need to return to basics, like Cholesky

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But it's hard to understand the *VAR*.

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– Staring at variance decompositions (VD's)
is adequate and maybe productive
But how will you identify them?
And how will you stare at them?

VD's Require Identification

Intricate theory identification

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But perhaps there is a natural Cholesky ordering
in many finance applications
(e.g., put large-cap institutions first)

And, Perhaps Graphical Modeling can Help

Cholesky Orthogonalization:

$$(I - \Phi_1 L - ... - \Phi_p L^p) y_t = P v_t$$

$$v_t \sim WN(0, I),$$

where $\Sigma = PP'$ (Cholesky factorization)

Moving-average representation:

$$y_t = (I + \Theta_1 L + \Theta_2 L^2 + ...) P v_t$$

$$= P v_t + \Theta_1 P v_{t-1} + ...$$

Recursive Structural System

Structural Simultaneous-Equations Model (SEM):

$$Ay_t = \Phi_1 y_{t-1} + ... + \Phi_p y_{t-p} + \varepsilon_t$$

$$\varepsilon_t \sim (0, \Sigma)$$

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Recursive SEM: A triangular and Σ diagonal

$$Ty_t = \Phi_1 y_{t-1} + ... + \Phi_p y_{t-p} + \varepsilon_t$$

$$\varepsilon_t \sim (0, D)$$

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$$Ty_t = \Phi_1 y_{t-1} + ... + \Phi_p y_{t-p} + \varepsilon_t$$

$$\varepsilon_t \sim (0, D)$$

The MA representation of the reduced form is:

$$y_t = T^{-1}\varepsilon_t + ...$$

DAG's, Bayes Nets, and all That...

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Lauritzen, S. (1996). *Graphical Models*, Clarendon Press.

Pearl, J. (2009), *Causality: Models, Reasoning, and Inference*, Cambridge University Press (second edition).

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Pearl, J. (2009), *Causality: Models, Reasoning, and Inference*, Cambridge University Press (second edition).

Potentially good primers:

Hjsgaard, S. "Graphical Models and Bayesian Networks with R" `www.people.math.aau.dk/~sorenh/misc/2014-useR-GMBN/`

Rebonato, R. (2010), *Coherent Stress Testing: A Bayesian Approach to the Analysis of Financial Stress*, Wiley.

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– Not fully simultaneous. Instead, *recursive*!

Key DAG Insights

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– Local Markov condition: In a DAG, a variable is independent of its non-descendants conditional on its ancestors.

Opportunities with DAG's

Potentially a very big step:

Local Markov / conditional independence can be used to help determine recursive orderings. "Likelihood information"

and

Bayesian analysis can blend likelihood and prior information.

(3) Learn From the Identified Estimated *VAR*

Key theme: Staring at massive D is just as hopeless as staring at massive coefficient matrices

But graph-theoretic tools come to the rescue

How?

Can't stare productively at coefficient and covariance matrices.

When $N = 5$ all is well.
We can stare productively at D.

When $N = 5000$ we're in trouble.
We can no longer stare productively at D!

The key tool for "digesting" VAR info
(i.e., examination of D)
is itself now indigestible!

Part I: Graph-Theoretic *D* Distillation

Interpret D as a Network Adjacency Matrix
Distill Using the Degree Distribution

Variance Decomposition / Connectedness Table

	x_1	x_2	...	x_N	From Others
x_1	d^H_{11}	d^H_{12}	$\cdots$	d^H_{1N}	$\sum_{j\neq 1} d^H_{1j}$
x_2	d^H_{21}	d^H_{22}	$\cdots$	d^H_{2N}	$\sum_{j\neq 2} d^H_{2j}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d^H_{N1}	d^H_{N2}	$\cdots$	d^H_{NN}	$\sum_{j\neq N} d^H_{Nj}$
To Others	$\sum_{i\neq 1} d^H_{i1}$	$\sum_{i\neq 2} d^H_{i2}$	$\cdots$	$\sum_{i\neq N} d^H_{iN}$	$\sum_{i\neq j} d^H_{ij}$

Total directional connectedness "from," $C^H_{i\leftarrow\bullet} = \sum_{\substack{j=1 \\ j\neq i}}^{N} d^H_{ij}$: "from-degrees"

Total directional connectedness "to," $C^H_{\bullet\leftarrow j} = \sum_{\substack{i=1 \\ i\neq j}}^{N} d^H_{ij}$: "to-degrees"

Systemwide connectedness, $C^H = \frac{1}{N} \sum_{\substack{i,j=1 \\ i\neq j}}^{N} d^H_{ij}$: mean degree

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– Examine aspects of the degree distribution *across* H

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The degrees of A track 1-step connectedness.
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(b) ∞-step connectedness ("eigenvalue centrality")
Examine second smallest eigenvalue λ_2 of $L = M - A$
(M is a diagonal matrix containing the node degrees)
(A is the adjacency matrix.)

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(c) But is any of this necessary/desirable for us?
Should we not simply vary H?

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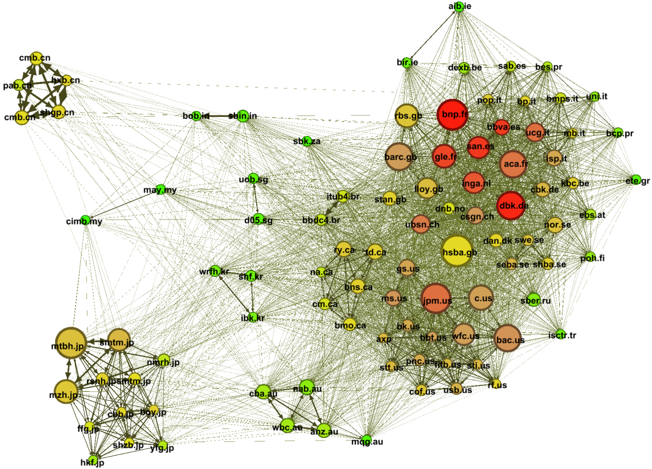
Part II: Graph-Theoretic *D Visualization*

Spring Graph



Spring Graph Detail

- Node size: Asset size
- Node color: Total directional connectedness "to others"
- Node location: Average pairwise directional connectedness (Equilibrium of repelling and attracting forces, where (1) nodes repel each other, but (2) edges attract the nodes they connect according to average pairwise directional connectedness "to" and "from.")
- Edge thickness: Average pairwise directional connectedness
- Edge arrow sizes: Pairwise directional connectedness "to" and "from"

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(Multiple comparisons or a single animation across H)

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across colors from “cool” to “hot”
(So use layering rather than colors)

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– Animate over time for fixed H,
for a variety of H

Concluding Perspective

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(Actually no one even *thought* about high dimensions)

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Still *VAR*'s, but:

Important new tools for estimation and analysis in high dimensions
are opening important new research areas