Pay-as-you-go Social Security when Innovators Extract Rents

Christiane L. Roehler
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104
e-mail: croehler@ssc.upenn.edu
Phone: (215) 546-5429

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Abstract

In this paper I show that a pay-as-you-go Social Security system may be Pareto-improving if a group of agents has property rights in technological knowledge that lets firms produce more efficiently. This is true even if the economy does not over-accumulate capital, if the economy is dynamically efficient. The model is based on a standard deterministic two-period overlapping generations model with production and exogenous growth. In contrast to the standard model there are two groups of agents, workers and innovators. All agents are endowed with labor. In addition, innovators are endowed with technological knowledge.

Innovators charge fees for the right to use the improved technology which are proportional to the capital stock of the firm. This drives a wedge between the marginal product of capital and the return to savings. Thus, the return to savings may be below the growth rate of the economy. This can occur even if the marginal product of capital is above the growth rate and hence the economy does not over-accumulate capital. If the fee is sufficiently large, a pay-as-you-go Social Security system can be welfare improving for all agents by improving the intertemporal allocation of resources. However, the capital stock and thus total consumption in each period decrease.

1. Introduction

The current debate on the privatization of unfunded pay-as-you-go (henceforth PAYG) Social Security systems is centered around the issue of saving and capital accumulation. It is commonly believed that the PAYG Social Security system reduces savings and hence investment. Thus the capital stock and output in the economy are too low, which imposes substantial welfare losses.

However from a theoretical point of view, it is well-known that equilibria in in¯nite horizon economies with an overlapping generations structure need not be Pareto-e±cient. Samuelson (1958) showed in a very simple model without production or storage that if the interest rate in the economy is below the growth rate of the economy, the economy is dynamically ine±cient. A tax-transfer system that taxes the young and transfers the proceeds to the old can pay an implicit rate of return of the growth rate and hence is Pareto-improving. A pay-as-you-go Social Security system can be viewed as such a tax-transfer system.

Diamond (1965) explicitly introduced production and capital accumulation into overlapping generations economies. Again, competitive equilibria are dynamically ine±cient if the interest rate is below the growth rate. This is the case if and only if the economy over-accumulates capital. Consumption and hence utility for all generations could be increased by reducing the capital stock. This can be accomplished by a PAYG Social Security system. Abel et al. (1989) have shown that an assessment of whether economies over-accumulate capital can be based on an a cash °ow criterion. If there is always a net cash °ow from the production sector (in addition to wage payments) which is used for consumption, the economy does not over-accumulate capital. This criterion can be used not only in deterministic environments but also for economies which are subject to a variety of shocks. Abel et al. applied their cash °ow criterion to some OECD economies and concluded that these economies are dynamically e±cient. Hence a PAYG Social Security system seems undesirable.

In this paper I present a model in which a PAYG Social Security system may be welfare-improving even if the economy does not over-accumulate capital. Hence an assessment of dynamic e±ciency as done by Abel et al. (and others) is not su±cient to conclude that a Social Security System is undesirable. Other empirical criteria must be used to make that claim.

The basic idea can be illustrated with a simple storage technology. There are two groups of agents in the economy, workers and innovators. All agents live for two periods. Total endowment in each period is constant and equal to total wage income of the workers. The population does not grow. Agents can save through a storage technology. The rate of return to the technology is larger than one, i.e. the interest rate is positive. Workers and innovators alike earn income from labor in the first period only and have to provide for their old age through savings. They can choose to save through the storage technology. However, every agent who decides to do so has to pay a fee to the innovators based on the amount invested. If the fee is large enough, the net return to investors can be below one. In such a situation agents would prefer to forego saving through the storage technology even though it reduces total output and total consumption. They would be better off to make a contract with future generations to simply transfer part of the first period wages of the young to the old at a rate of one. Such a tax - transfer system can be interpreted as a PAYG Social Security system.

In the remainder of the paper I develop this idea for economies with productive capital. Innovators have technological knowledge that improves productivity. They sell this knowledge to firms. Because innovators are monopolists in their knowledge, they set a perfectly discriminating pricing scheme that maximizes their income from this knowledge. I show that innovators set a pricing scheme that has a component which is proportional to the capital stock of the firms. This fee on capital drives a wedge between the marginal product of capital and the rate of return to investment. Thus, the rate of return to owners of the firm may be pushed below the growth rate of the economy even though the marginal product of capital is above the growth rate, i.e. even though the economy is dynamically efficient. Wages and income grow at the growth rate of the economy. A PAYG Social Security system of some size with an implicit rate of return at the growth rate seems desirable.

However, in a dynamically efficient economy which produces with accumulated capital, the introduction of a Social Security system (under standard assumptions) reduces the capital stock and thus total consumption. Therefore a Social Security system is only welfare-improving if the utility derived from improved intertemporal allocation is larger than the loss of utility due to the reduction in total consumption.

In the next section I lay out an explicit overlapping generations model with production and two groups of agents, workers and innovators with monopolistic powers. In section 3 the decision problem of the innovator is analyzed in detail, and section 4 describes the equilibrium. Section 5 derives the main result - the welfare effects of the Social Security system - which is illustrated with an example in section 6. Section 7 discusses the results and concludes.

2. The model

Consider an overlapping generations model with no population growth. Each agent in this economy lives for 2 periods. Agents who become economically active in period t constitute generation t. The economy starts at time 0 with agents of generation -1 living in their second period.

Agents: Preferences of all individuals are given by their utility function

$$U = u(c_{1t}; c_{2;t+1})$$

with c_{ij}; $i = 1; 2$; $j = t; t+1$ denoting consumption spending at age i in period j.

Assumption 1: U is twice continuously differentiable, increasing in both variables, strictly quasi-concave on the interior of R^2_+ and homothetic.

Assumption 2: $\lim_{c_{1t} \to 0} u_1(c_{1t}; c_{2;t+1}) = \infty$ if $c_{2t} > 0$ and $\lim_{c_{2t} \to 0} u_2(c_{1t}; c_{2;t+1}) = \infty$ if $c_{1t} > 0$

Assumption 3: c_{1t} and c_{2t+1} are normal goods and gross substitutes.

There are N agents in the economy, N^W workers and N^I innovators. All workers are identical, hence there is a representative worker indicated by superscript W. Innovators are indexed by i, $i = 1; ...; N^I$. All agents are endowed with one unit of labor when young which they sell inelastically in the market for the wage w_t.

Each innovator, in addition, is endowed with some unique technological knowledge – his innovation. The innovation is valuable because it increases the productivity of the technology. Hence, the innovator may receive some payment z^i_t for his knowledge. The innovator has a monopoly on his knowledge for only one period, then the knowledge becomes part of the generally available technology.

Consumption in period 2 for either group of agents has to be financed out of savings or through transfers (if any).

Workers maximize their utility subject to their budget constraint, given the price of the consumption good, the wage rate, taxes and expected transfers, and the expected return to savings. The budget constraints (in real terms) for workers are

$$
\begin{aligned}
w_t - t_t &= c^W_{1t} + s^W_t \\
c^W_{2,t+1} &= \rho_{t+1} s^W_t + T_{t+1}
\end{aligned}
$$

where ρ_{t+1} indicates the rate of return to investment, t_t taxes paid when young, T_{t+1} transfers received when old. The equilibrium concept employed is the concept of perfect foresight equilibrium. Hence expected prices are confirmed in equilibrium. Therefore I do not introduce distinct notation for expected prices. Given the assumptions on the utility function, for each $(w_t, \rho_{t+1}, t_t, T_{t+1})$ the solution to the intertemporal allocation problem for workers exists and is unique. The indirect utility function for the representative worker is $U^{*W}(w_t, \rho_{t+1}, t_t, T_{t+1})$ and his savings function is $s^W(w_t, \rho_{t+1}, t_t, T_{t+1})$.

Each innovator maximizes his utility. The budget constraints (in real terms) for innovator i are

$$
\begin{aligned}
w_t + z^i_t - t_t &= c^i_{1t} + s^i_t \\
c^i_{2,t+1} &= \rho_{t+1} s^i_t + T_{t+1}
\end{aligned}
$$

The innovator takes ρ_{t+1}, t_t and T_{t+1} as given. z^i_t and possibly w_t depend on how the innovator sells his technological knowledge in the market or otherwise uses it. However, for any $w_t + z^i_t$ and given $(\rho_{t+1}, t_t, T_{t+1})$ the solution to the intertemporal allocation problem for innovators exists and is unique. Denote the indirect utility function for innovator i given some w_t and z^i_t by $U^{*i}(w_t, \rho_{t+1}, z^i_t, t_t, T_{t+1})$ and his savings function by $s^i(w_t, \rho_{t+1}, z^i_t, t_t, T_{t+1})$. Standard arguments show that the innovator's utility is strictly increasing in his income, i.e.

$$
\frac{dU^{*i}(w_t, \rho_{t+1}, z^i_t, t_t, T_{T+1})}{d(w_t + z^i_t)} > 0 \, . \tag{2.1}
$$

By Assumption 3 savings are increasing in ρ_{t+1}, i.e. $\frac{\partial s^W(w_t, \rho_{t+1}, t_t, T_{t+1})}{\partial \rho_{t+1}} > 0$ and $\frac{\partial s^i(w_t, \rho_{t+1}, z^i_t, t_t, T_{t+1})}{\partial \rho_{t+1}} > 0$.

Production: At the end of $t - 1$ the young found Φ_t firms indexed by $\varphi = 1, ..., \Phi_t$. Firms exist for one period. The investment decisions of the young determine the amount of capital K^φ which firm φ owns. Hence total capital stock in period t is $K_t = \sum_{\varphi=1}^{\Phi_t} K^\varphi$.

Firms maximize their value to their owners. Firms do so by producing the single good of the economy using their capital K^j and hiring labor. Alternatively, firms sell their productive capacity. Firms are small compared to the economy and hence act as price taker when making their decisions. At the end of period t firms distribute their profits or their selling price to their owners.

The technology of the economy is described by the constant return to scale production function $F(K, \gamma L)$. γ is a productivity coefficient which can differ from firm to firm and also changes over time.

Assumption 4: F is twice continuously differentiable, increasing, strictly concave and $F_{KL} > 0$.

Assumption 5: The Inada conditions are satisfied, i.e. $\lim_{K \to 0} F(K, \gamma L) = 0$, $\lim_{K \to 0} F_K(K, \gamma L) = 0$ and $\lim_{K \to \infty} F_K(K, \gamma L) = \infty$

Assumption 6: $F_L(K, \gamma L) + \gamma L F_{LL}(K, \gamma L) > 0$.

Capital depreciates fully, hence $F(K, \gamma L)$ is total output. In each period technological progress takes place and increases the productivity coefficient γ. In each period all available new knowledge combined can increase productivity by a factor $\lambda \geq 1$. Hence in period t, $F(K, \lambda^t L)$ is the best available technology. However, some of the knowledge necessary to achieve λ^t is privately owned by innovators and can only be used if the innovator permits it.

Assume that in period t firms by their own efforts can utilize the technology according to a productivity coefficient $\mu \lambda^{t-1}$. The factor μ with $\mu \leq \lambda$ measures how well firms can utilize the available knowledge which permits a productivity coefficient λ^{t-1}. If $\lambda \geq \mu > 1$ firms make a positive contribution to technological progress in each period. Innovator i owns knowledge that permits a productivity increase measured by the factor μ_i. If a firm obtained the right to use the knowledge of innovator i, it would produce with a coefficient $\mu_i \mu \lambda^{t-1}$. Hence $\lambda = \mu \prod_{i=1}^{N} \mu_i$.

Without access to the privately owned knowledge, firm j's profits are

$$\max_{L^j} \left\{ F\left(K^j, \mu \lambda^{t-1} L^j\right) - w L^j \right\}^{1} \tag{2.2}$$

This is the minimum profit that firm j can generate, its "reservation-profit".

[1] Time subscripts are omitted for simplicity because the decision probem of the firm is not intertemporal.

If innovator i offers to license knowledge to firm ϕ, the firm makes its decision according to

$$\max_{1;2}\left\{\begin{array}{l}\max_{L^\phi}\left\{F\left(K^\phi;\gamma^i\prod_J\gamma^j\gamma\lambda^{t-1}L^\phi\right)-wL^\phi-z^{\phi i}\right\},\\ \max_{L^\phi}\left\{F\left(K^\phi;\prod_J\gamma^j\gamma\lambda^{t-1}L^\phi\right)-wL^\phi\right\}\end{array}\right\}. \tag{2.3}$$

$\prod_J \gamma^j$ is the knowledge that firm ϕ has licensed from innovators $J \subset \{j \mid j = 1, \ldots, N^I; j \neq i\}$ before i's offer. $z^{\phi i}$ denotes the payment of firm ϕ to the innovator i. $z^{\phi i}$ can be a function of K^ϕ, L^ϕ and other characteristics of firm ϕ. Note that firms take wages as being independent of their choice of technology because they are small relative to the market.

If innovator i offers to purchase firm ϕ, the firm makes its decision taking into account its reservation profit (2.2) and expected profits due to offers by other innovators.

Social Security: A pay-as-you-go Social Security system taxes the young for an amount t_t to pay transfers T_t to the old. The Social Security system taxes wage income at rate $\tau \geq 0$ for a total amount of $t_t = \tau w_t$ per agent. It must balance its budget every period, hence each agent receives $T_{t+1} = \tau w_{t+1}$ when old. As labor is supplied inelastically the tax on wages is a lump sum tax.

3. Pricing Decisions by Innovators

From the discussion of the innovators' optimization problem (see (2.1)) it is clear that innovators want to maximize (real) income . The strategy to achieve that goal depends on how innovators perceive their environment. I present two scenarios. First, innovators set prices for their knowledge but take the real wage rate as given. Second, innovators realize that their monopoly power over the technology may have spill-over effects in the labor market and they take this into account. In addition, I briefly discuss the interaction between multiple innovators.

3.1. Innovators as Price Takers

To simplify the presentation I assume for the remainder of this section that there is only one innovator I. Then firms produce with production coefficient λ^t if they use the knowledge of the innovator, and with $\gamma\lambda^{t-1}$ without the knowledge of the innovator. Equation (2.1) shows that the innovator maximizes his utility when he maximizes z_t^I for given w_t.

Suppose the innovator licenses his knowledge to firms. Then z_t^I is the sum of payments from each firm, i.e. $z_t^I = \sum_{\phi=1}^{\Phi_t} z_t^{\phi I}$. The decision problem of each firm is independent of the decision problem of all the other firms. Hence the innovator maximizes his income by perfectly price discriminating between the firms and maximizing his income from each firm. This in particular implies that the innovator licenses his knowledge to all firms. The innovator solves the following problem for each firm. For ease of notation I omit time subscripts and the superscript I denoting the innovator.

$$\max z^\phi$$

$$\text{s.to.} \ \max_{L^\phi} \left\{ F\left(K^\phi, \lambda^t L^\phi\right) - wL^\phi - z^\phi \right\} = \max_{L^\phi} \left\{ F\left(K^\phi, \mu\lambda^{t-1} L^\phi\right) - wL^\phi \right\} \quad (3.1)$$

where z^ϕ can be a function of the characteristics of the firm in particular its capital stock and labor input. The constraint is a participation constraint for the firm derived from the maximization problem of the firm (2.3). The right hand side is the "reservation-profit" for the firm, the profit the firm can earn without the innovator.

Define $L^{\phi\mu}$ and $L^{\phi\lambda}$.

$$\begin{aligned} L^{\phi\mu} &= \arg\max_{L^\phi} \left\{ F\left(K^\phi, \mu\lambda^{t-1} L^\phi\right) - wL^\phi \right\} \\ \text{and} \quad L^{\phi\lambda} &= \arg\max_{L^\phi} \left\{ F\left(K^\phi, \lambda^t L^\phi\right) - wL^\phi - z^\phi \right\} \end{aligned} \quad (3.2)$$

Note that $L^{\phi\lambda}$ depends on the the function z^ϕ.

Proposition 3.1. The innovator sets a licensing fee for each firm ϕ which is proportional to the capital stock of the firm, i.e. $z^\phi = \zeta K^\phi$.

Proof. Firm ϕ's optimal labor demand $L^{\phi\mu}$ for technology μ is determined by $\mu\lambda^{t-1} F_L\left(K^\phi, \mu\lambda^{t-1} L^\phi\right) = w$. Given w the reservation profit is determined. This implies that z^ϕ can attain its maximal value only when the firm's profit $F\left(K^\phi, \lambda^t L^\phi\right) - wL^\phi$ is maximal. Like the firm, the innovator takes w as given. Hence, the innovator wants the optimal labor choice $L^{\phi\lambda}$ to be governed by $\lambda^t F\left(K^\phi, \lambda^t L^\phi\right) = w$. The innovator does not want to distort the labor input decision $L^{\phi\lambda}$ by the firm. Optimal z^ϕ does not depend on labor.

Thus, optimal choices of $L^{\phi\lambda}$ and $L^{\phi\mu}$ combined with the fact that F is constant return to scale implies

$$\begin{aligned} F\left(K^\phi, \lambda^t L^{\phi\lambda}\right) - wL^{\phi\lambda} &= F_K\left(K^\phi, \lambda^t L^{\phi\lambda}\right) K^\phi \\ \text{and} \quad F\left(K^\phi, \mu\lambda^{t-1} L^{\phi\mu}\right) - wL^{\phi\mu} &= F_K\left(K^\phi, \mu\lambda^{t-1} L^{\phi\mu}\right) K^\phi \end{aligned} \quad (3.3)$$

Hence,

$$z^{\iota} = \left[F_K\left(K^{\iota}; \lambda^t L^{\iota\lambda}\right) - F_K\left(K^{\iota}; \mu\lambda^{t-1} L^{\iota\mu}\right)\right] K^{\iota}$$

The constant return to scale property also implies that the marginal product of labor and capital only depend on the capital labor ratio $K^{\iota}/L^{\iota\lambda}$ and $K^{\iota}/L^{\iota\mu}$, respectively. Given w these capital labor ratios are constant across firms. Thus, the difference in the marginal products of capital for the different technologies is constant across firms. Hence, the innovator sets the fee $z^{\iota} = \zeta K^{\iota}$ proportional to the capital stock of each firm with

$$\zeta = \left[F_K\left(K^{\iota}; \lambda^t L^{\iota\lambda}\right) - F_K\left(K^{\iota}; \mu\lambda^{t-1} L^{\iota\mu}\right)\right] \tag{3.4}$$

∎

The fee of the innovator creates a wedge between the marginal product of capital and the return to saving. This fee has an effect similar to a tax on capital.

Lemma 3.2. If innovator I contributes to technical progress $\left(\text{i.e. } \lambda > \mu \text{ or } \mu^I > 1\right)$ the fee is positive. It increases in μ^I.

Proof. To show that $\zeta > 0$ analyze $dG/d\gamma$ with

$$G \equiv F_K\left(K^{\iota}; \gamma L^{\iota\gamma}\right)$$

$$\text{s.to. } \gamma F_L\left(K^{\iota}; \gamma L^{\iota\gamma}\right) - w = 0 .$$

For any capital stock K^{ι} and any wage rate w the marginal product of capital is determined by the optimal labor choice $L^{\iota\gamma}$. If $dG/d\gamma > 0$, a better technological coefficient γ increases the marginal product of capital. The firm with the higher coefficient (λ^t versus $\mu\lambda^{t-1}$) has a higher marginal product of capital.

Total differentiation yields

$$dG = F_{KL} \cdot \left[L^{\iota\gamma} d\gamma + \gamma dL^{\iota\gamma}\right]$$

$$\left[F_L + \gamma L^{\iota\gamma} F_{LL}\right] d\gamma + \left[\gamma^2 F_{LL}\right] dL^{\iota\gamma} = 0 \tag{3.5}$$

Rearranging (3.5) shows that effective labor input $\gamma L^{\iota\gamma}$ increases in γ

$$L^{\iota\gamma} d\gamma + \gamma dL^{\iota\gamma} = -\frac{F_L}{\gamma F_{LL}} d\gamma \tag{3.6}$$

By the constant return to scale property of F, in particular $-F_{KL}/F_{LL} = \gamma L^{\varphi\gamma}/K^{\varphi}$, the result

$$\frac{dG}{d\gamma} = \frac{F_L L^{\varphi\gamma}}{K^{\varphi}} > 0$$

is obtained. Hence $\zeta > 0$ if $\mu^l > 1$. As $\zeta = \left[F_K\left(K^{\varphi}; \mu^l \mu^{\lambda^{t-1}} L^{\varphi\lambda}\right) - F_K\left(K^{\varphi}; \mu^{\lambda^{t-1}} L^{\varphi\mu}\right)\right]$, ζ increases in μ^l. ■

Remark 1: Even though effective labor input $\gamma L^{\varphi\gamma}$ increases in γ for a given w, in general $L^{\varphi\gamma}$ need not increase in γ. $L^{\varphi\gamma}$ increases in γ if (from (3.5))

$$\gamma L^{\varphi\gamma} F_{LL} + F_L > 0$$

By Assumption 6 the technology has this property.

Remark 2: The above result depends on the assumption that firms believe they are small compared to the market. They believe that their own choice of technology does not influence the price system. This however means that the prices they assume to be prevailing if they do not adopt the technology are not equilibrium prices if a non-negligible fraction of firms does not adopt the technology.

The next lemma shows that licensing the knowledge in the above way is an optimal strategy.

Lemma 3.3. Given the price taking assumption, the innovator is indifferent between licensing his knowledge to firms and purchasing the productive capacity of firms and operating them himself.

Proof. As both the firms and the innovator take prices as given, the optimal choice of labor input for an amount K^{φ} is identical. Hence output and labor costs are the same. The licensing fee z^{φ} is so large that firm φ just earns its reservation profit. But if the innovator purchased the firm he would have to pay the firm at least its reservation profit. Hence the innovator cannot do better by producing himself. ■

Denote total capital stock in the economy by

$$K = \sum_{\varphi=1} K^{\varphi} \tag{3.7}$$

and total hypothetical labor demand if technology $\mu\lambda^{t-1}$ were used by

$$L^{\mu} = \sum_{\phi=1}^{\Phi} L^{\phi\mu}. \tag{3.8}$$

With these definitions ζ can be expressed in terms of aggregate variables.

Lemma 3.4. In equilibrium $\zeta = [F_K(K; \lambda^t N) - F_K(K; \mu\lambda^{t-1}L^{\mu})]$ with L^{μ} determined by $\lambda^t F_L(K; \lambda^t N) = \mu\lambda^{t-1}F_L(K; \mu\lambda^{t-1}L^{\mu})$.

Proof. If the innovator charges the optimal licensing fee, firms are indifferent between licensing the technology and not licensing. Therefore, in equilibrium all firms license the technology. Then by the constant return of scale property of the production function, the market clearing wage is $w = \lambda^t F_L(K; \lambda^t N)$. Hence for each firm the optimal capital labor ratio is $K^{\phi}/L^{\phi\lambda} = K/N$. This ratio determines the marginal product of capital in the first part of ζ.

In equilibrium the reservation profit for firms is determined at the equilibrium wage rate for technology λ. Again by constant return to scale, all firms choose an identical capital labor ratio $K^{\phi}/L^{\phi\mu} = K/L^{\mu}$ with L^{μ} determined as stated above. Thus ζ has the structure as stated. ∎

The analysis in this section showed the crucial role of the wage rate in determining the market value of the innovation. If the innovator realizes that he may be able to influence the wage rate if he purchases firms and hence acquires market power in the labor market, the above licensing structure is not optimal. The next section discusses this issue in detail.

3.2. An Innovator as Price Leader

As in the previous section assume that there is only one innovator I. Because I want to discuss how the monopoly power of the innovator leads to imperfect competition in the labor market, this assumption intensifies any potential effects.

Suppose that the innovator purchases firms outright and operates them himself instead of licensing his knowledge to these firms. If he buys enough firms he becomes large relative to the market. However, as long as there is any other firm in the market – which may produce with technology $\mu\lambda^{t-1}$ – the innovator has to compete for labor because workers work for the firm that pays the highest wage. But by assumption, existing firms act as price takers. Thus, the innovator can establish himself as a price leader in the labor market; he chooses the wage rate w.

The innovator announces a wage rate w, purchases some firms at the reservation profit to their owners and licenses his technology to others.[2]

Proposition 3.5. The optimal licensing schedule is linear in K^ϕ and L^ϕ.

Proof. Suppose the innovator purchased all firms outright. Then the innovator controls total production in the economy. Hence all production decisions are made to maximize the innovator's real income and thus his utility. The innovator considers the following problem:

$$\max_{\{L^\phi; L^{\phi\mu}\}} w + \sum_{\phi=1} \left[F(K^\phi; \lambda^t L^\phi) - wL^\phi \right] - \sum_{\phi=1} \left[F(K^\phi; \mu\lambda^{t-1} L^{\phi\mu}) - wL^{\phi\mu} \right] \quad [3]$$

$$\text{s.to. } \mu\lambda^{t-1} F_L(K^\phi; \mu\lambda^{t-1} L^{\phi\mu}) = w \quad \forall\phi; \phi = 1; \ldots; \Phi \qquad (3.9)$$

$$\text{and } \sum_{\phi=1} L^\phi = N$$

Again, define K as total capital stock (see (3.7)) and L^μ as total hypothetical labor demand (see (3.8)).

First, consider the L^ϕ's. The optimal choices of L^ϕ require that the marginal product of labor $\lambda^t F_L(K^\phi; \lambda^t L^\phi)$ be equal for all firms and that all resources be used. The constant return to scale property and the resource constraints imply $\lambda^t F_L(K^\phi; \lambda^t L^\phi) = \lambda^t F_L(K; \lambda^t N)$ $\forall\phi$. Hence,

$$\sum_{\phi=1} \left[F(K^\phi; \lambda^t L^\phi) - wL^\phi \right] = F(K; \lambda^t N) - wN$$

Second, (3.9) implies that $K^\phi / L^{\phi\mu}$ is constant across all firms. Thus,

[2] I assume the innovator cannot 'cheat' on his wage announcement. If he were able to purchase all firms and only then commence production, he would not want to pay the wage he announced but a wage rate of 0. But then previous owners would regret their sale.

[3] Recall that the innovator supplies one unit of labor himself. Also note, that a positive tax on wages alters the decision of the innovator. His after tax income from labor is $(1 - \tau)w$.

$\mu\lambda^{t-1}F_L(K^\phi, \mu\lambda^{t-1}L^{\phi\mu}) = \mu\lambda^{t-1}F_L(K, \mu\lambda^{t-1}L^\mu)\ \forall\phi$ and

$$\sum_{\phi=1}\left[F(K^\phi, \mu\lambda^{t-1}L^{\phi\mu}) - wL^{\phi\mu}\right] = F(K, \mu\lambda^{t-1}L^\mu) - wL^\mu$$

Moreover,

$$w = \mu\lambda^{t-1}F_L(K, \mu\lambda^{t-1}L^\mu) \tag{3.10}$$

Thus, the decision problem of the innovator can be expressed as a choice problem over L^μ.

$$\max_{L^\mu} w + F(K, \lambda^t N) - wN - F(K, \mu\lambda^{t-1}L^\mu) + wL^\mu$$

with w as in (3.10). Taking the derivative with respect to L^μ gives the condition

$$\mu^2\lambda^{2(t-1)}F_{LL}(K, \mu\lambda^{t-1}L^\mu)[1 - N + L^\mu] = 0$$

The second derivative at $[1 - N + L^\mu] = 0$ is $\mu^2\lambda^{2(t-1)}F_{LL}(K, \mu\lambda^{t-1}L^\mu) < 0$. Hence, the innovator maximizes his real income when he sets wages such that $L^\mu = N - 1$.

The innovator is indifferent between purchasing a firm and licensing his knowledge to that firm if the firm uses a capital labor ratio $K^\phi/L^\phi = K/N$ and pays

$$z^\phi = F(K^\phi, \lambda^t L^\phi) - wL^\phi - \left[F(K^\phi, \mu\lambda^{t-1}L^{\phi\mu}) - wL^{\phi\mu}\right].$$

The price taking firm chooses the right capital labor ratio if the innovator charges a fee on labor

$$\zeta_L = \lambda^t F_L(K, \lambda^t N) - \mu\lambda^{t-1}F_L\left(K, \mu\lambda^{t-1}(N-1)\right) \tag{3.11}$$

The remainder of the desired fee z^ϕ can be extracted if the innovator charges a fee on capital

$$\zeta_K = F_K(K, \lambda^t N) - F_K\left(K, \mu\lambda^{t-1}(N-1)\right) \tag{3.12}$$

The argument is analogous to (3.3)

Hence,

$$z^{\phi} = \zeta_L L^{\phi} + \zeta_K K^{\phi}$$

■

As in the price taking case, the innovator wants to license his knowledge to all firms. However the wage rate which maximizes his income is not equal to the optimal marginal product of labor. The licensing fee on labor equalizes the effective wage rate to firms with the marginal product of labor. If the innovator did not charge such a fee on labor, firms in the market would demand more labor than optimal because of the low wage rate. The actions of the innovator have to support his announced wage rate. Hence, he would have to reduce the labor demand for the firms he owns below the optimal amount. The marginal product of labor for the innovators' own firms would be higher than the marginal product of labor for firms in the market.

Lemma 3.6. ζ_L and ζ_K are positive.

Proof. ζ_L is positive by Assumption 6, ζ_K is positive by $F_{KL} > 0$. ■

The following lemma compares the fee on capital for the price taking scenario and the price leader scenario.

Lemma 3.7. The fee on capital is higher if the innovator is a price taker than if he is a price leader on the labor market.

Proof. Compare ζ_K (see (3.12)) and ζ in the price taking equilibrium (see Lemma 3.4). The marginal product of capital of actual production is identical. However, hypothetical labor input for production with technology $\mu\lambda^{t-1}$ is lower in the price taking scenario. The wage rate in the price taking case is $w = \lambda^t F_L(K; \lambda^t N)$ and thus by Assumption 6 higher than $w = \mu\lambda^{t-1} F_L(K; \mu\lambda^{t-1}(N-1))$ in the price leading case. As $F_{KL} > 0$ the hypothetical marginal product of capital with technology $\mu\lambda^{t-1}$ is lower in the price taking scenario. ■

Remark 3: It is a well-known result that in models with imperfectly competitive profit maximizing firms, the price-normalization rule for the economy can influence the equilibrium allocation[4]. Imperfectly competitive firms manipulate the relative prices in the economy and use the profit objective to evaluate various relative price systems. But this implies that the price level at each of the relative price systems is relevant for the comparison and hence influences the ultimate allocation in the economy.

If the innovator acts as a price taker, he does not actively influence the relative price system. Thus, the price-normalization rule has no influence on the equilibrium allocation even though the innovator sets his fees according to an income criterion.

If the innovator acts as a price leader he manipulates the relative price system and the price-normalization rule could potentially be relevant for the equilibrium allocation. However, the primary objective of the innovator is to maximize his utility. Income maximization is only a derived objective. As the innovator only derives utility from the single (consumption) good in the economy maximization of his utility is equivalent to maximization of his real income in that good.

3.3. Many Innovators

If many innovators contribute to technical knowledge, these innovators may compete among themselves for the right to sell their innovation to firms. However, in this model knowledge is cumulative. Any increase in the production coefficient increases profit to the firm. Hence, no matter how productive the firm already is, the firm would like to use the knowledge of the innovator and conversely the innovator can sell his knowledge for a positive price to that firm. If innovators act as price takers in the labor market, they sell their knowledge to all firms. However, an issue arises about the fee the innovator can charge. The order in which innovators sell their knowledge to firms matters.

Suppose the firm produces with production coefficient γ which is determined by the generally available coefficient and the productivity increases due to knowledge already purchased from other innovators. The innovator currently negotiating with the firm can improve the production coefficient by a factor μ^i. He can charge the fee

$$\zeta = \left[F_K\left(K^{\phi}, \mu^i \gamma L^{\phi(\mu\gamma)}\right) - F_K\left(K^{\phi}, \gamma L^{\phi\gamma}\right) \right]$$

with $L^{\phi(\mu\gamma)}$ and $L^{\phi\gamma}$ the optimal labor choices at the wage rate w. The first

[4] The earliest reference for this class of results I know of is Gabszewicz and Vial (1972).

derivative with respective to γ

$$\mu i L^{\phi(\mu\gamma)} F_{KL}\left(K^{\phi}; \mu i \gamma L^{\phi(\mu\gamma)}\right) - L^{\phi\gamma} F_{Kl}\left(K^{\phi}; \gamma L^{\phi\gamma}\right)$$

shows that this ζ is increasing in γ because $\mu i > 1$, $L^{\phi(\mu\gamma)} > L^{\phi\gamma}$ by Assumption 6, and $F_{KL} > 0$. Additional knowledge is more valuable the more productive the firm already is. But this implies that an innovator would want to be the last one to sell his knowledge to firms. Thus, given the price taking assumption no innovator has an incentive to purchase firms outright. Once he has done so he would want to license knowledge from other innovators; however the profit he realizes from his own innovation would be the lowest one given the opportunities in the economy.

The total fee the innovator is able to charge depends on the order in which he and other innovators sell their knowledge to firms. I do not make any specific assumptions on how this order is determined.

If innovators realize that they may establish market power on the labor market, a variety of imperfectly competitive outcomes become a possibility. However, it should be clear that innovators always want to charge some fee related to the capital stock of the firm in order to extract the increased profit from the increase in the marginal product of capital.

To summarize, the analysis of the decision problem of the innovators has shown that irrespective of the assumption on price taking behavior, innovators charge some fee that is proportional to the capital stock of the firm. The presence of the innovators creates a wedge between the marginal product of capital and the return to investment. This wedge drives the result about the benefits of a Social Security system.

Thus, for the remainder of the paper I focus on the case with many innovators who are price takers in the labor market. To simplify the presentation, I assume that all innovators earn the same income from their knowledge and hence treat them as if there were a representative innovator. However, it is straightforward to adapt the notation to treat innovators individually.

4. Equilibrium

The following equations describe an equilibrium with many price taking innovators who (by simplifying assumption) earn the same income from their innovation. The demand functions arising from the maximization problem for workers are denoted

$c^W_{1t}(w_t,\rho_{t+1},w_{t+1},\tau)$, $c^W_{2t}(w_t,\rho_{t+1},w_{t+1},\tau)$ and $s^W_t(w_t,\rho_{t+1},w_{t+1},\tau)$, the demand functions for the innovators by $c^I_{1t}(w_t,\rho_{t+1},z_t,w_{t+1},\tau)$, $c^I_{2t}(w_t,\rho_{t+1},z_t,w_{t+1},\tau)$ and $s^I_t(w_t,\rho_{t+1},z_t,w_{t+1},\tau)$. This notation incorporates the balanced budget assumption of the Social Security system.

Definition: Given K_0, s^W_{-1} and s^I_{-1} an equilibrium is a sequence of allocations $\left\{\left(c^W_{1t},c^W_{2t},s^W_t\right);\left(c^I_{1t},c^I_{2t},s^I_t\right)\right\}_{t=0}^{\infty}$ and of prices $\{w_t,\rho_t,\zeta_t\}_{t=0}^{\infty}$ such that for all $t=0,1,2,\ldots$

(a) $\left(c^W_{1t},c^W_{2t},s^W_t\right)$, $\left(c^I_{1t},c^I_{2t},s^I_t\right)$ solve the agents maximization problem.

(b) $w_t = \lambda^t F_L\left(K_t,\lambda^t \hat{L}_t\right)$

(c) $\rho_{t+1} = F_K\left(K_t,\lambda^t \hat{L}_t\right) - \zeta_{t+1}$

(d) $\zeta_t = F_K\left(K_t,\lambda^t \hat{L}_t\right) - F_K\left(K_t,\mu\lambda^{t-1}L^{\mu}_t\right)$

(e) L^{μ}_t defined by $w_t = \mu\lambda^{t-1}F_L\left(K_t,\mu\lambda^{t-1}L^{\mu}_t\right)$

(f) $K_{t+1} = N^W s^W_t + N^I s^I_t$

(g) $\hat{L}_t = N$

(h) Firms adopt technology λ.

(i) The Social Security system balances its budget.

(a)-(g) and the budget constraints imply that materials balance:

$K_{t+1} + N^W c^W_t + N^I c^I_t = F(K_t,\lambda^t L_t)$

I use R to indicate the marginal product of capital. Hence, $R_t = \rho_t + \zeta_t$ and $z_t = \zeta_t K_t / N^I$.

The existence of the old technology gives rise to some unusual features of these equilibrium conditions. Plugging (d) into (c) shows that

$$\rho_{t+1} = F_K\left(K_t,\mu\lambda^{t-1}L^{\mu}_t\right).$$

The return to investment is fundamentally determined by the marginal product of capital associated with the "reservation-profit" of firms and not the realized marginal product of capital.

This implies that ρ_{t+1} (and similarly z_t via ζ) depends on the hypothetical labor input decision L^{μ}_t at wage rate w_t. In general L^{μ}_t is not equal to $\hat{L}_t = N$.

For the welfare analysis it proves useful to restate the equilibrium conditions, detrending all variables and using the production function in its intensive form.

The economy grows exogenously at rate λ. With the homogeneity assumptions on preferences and technology all variables can be detrended by dividing by λ^t. All detrended variables are indicated by a tilde $\tilde{}$, i.e. $\tilde{s}^W_t = s^W_t/\lambda^t$, $\tilde{s}^I_t = s^I_t/\lambda^t$, $\tilde{c}^W_{1,t} = c^W_{1,t}/\lambda^t$, $\tilde{c}^W_{2,t} = c^W_{2,t}/\lambda^t$, $\tilde{c}^I_{1,t} = c^I_{1,t}/\lambda^t$, $\tilde{c}^I_{2,t} = c^I_{2,t}/\lambda^t$, $\tilde{K}_t = K_t/\lambda^t$, $\tilde{w}_t = w_t/\lambda^t$, $\tilde{z}_t = z_t/\lambda^t$.

To transform the production function into its intensive form, output and input variables are divided by N to restate them in terms of "per person of generation t".

Define the capital labor ratio $\hat{k}_t \equiv \frac{K_t/\lambda^t}{N}$ and the "employment ratio for technology μ" $\ell_t^\mu \equiv \frac{L_t^\mu}{N}$[5]. By analogy, the "employment ratio for technology λ" is $\ell_t^\lambda \equiv \frac{L_t^\lambda}{N}$ which is equal to 1 in equilibrium. Define the intensive form of technology F by $f(x) \equiv F(x;1)$. For a technology μ the following relations hold:

$$\frac{F\left(K_t;\mu\lambda^{t-1}L_t^\mu\right)}{\lambda^t N} = \frac{\mu}{\lambda}\ell_t^\mu F\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t;1\right) = F_K\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t;1\right)\hat{k}_t + \frac{\mu}{\lambda}\frac{F_L\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t;1\right)}{\lambda^t}\ell_t^\mu$$

Hence, in particular, competitive firms choose the optimal labor input according to

$$\hat{w}_t = \frac{\mu}{\lambda}\frac{F_L\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t;1\right)}{\lambda^t} = \frac{\mu}{\lambda}f\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t\right) - f'\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t\right)\frac{\hat{k}_t}{\ell^\mu}$$

Below are the equilibrium conditions restated in their detrended per person form. The set of equilibrium conditions has been expanded to include one explicitly stating $\hat{e}_t$.

$$\hat{w}_t = f\left(\hat{k}_t\right) - f'\left(\hat{k}_t\right)\hat{k}_t \tag{4.1}$$

$$\rho_t = f'\left(\hat{k}_t\right) - \zeta_t \tag{4.2}$$

$$\hat{e}_t = \frac{N}{N^I}\zeta_t\hat{k}_t \tag{4.3}$$

$$\zeta_t = f'\left(\hat{k}_t\right) - f'\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t\right) \tag{4.4}$$

$$\hat{w}_t = \frac{\mu}{\lambda}f\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t\right) - f'\left(\frac{\lambda}{\mu}\frac{1}{\ell_t^\mu}\hat{k}_t\right)\frac{\hat{k}_t}{\ell_t^\mu} \tag{4.5}$$

$$\lambda\hat{k}_{t+1} = \frac{N^W}{N}\hat{s}^W\left(\hat{w}_t;\rho_{t+1};\hat{w}_{t+1};\tau\right) + \frac{N^I}{N}\hat{s}^I\left(\hat{w}_t;\hat{e}_t;\rho_{t+1};\hat{w}_{t+1};\tau\right) \tag{4.6}$$

Definition: A balanced growth equilibrium is an equilibrium in which $\hat{k} = \hat{k}_t$ for all t. This implies that $\hat{w}_t = \hat{w}$, $\hat{e}_t = \hat{e}$, $\rho_t = \rho$, $\zeta_t = \zeta$, $\ell^\mu = \ell_t^\mu$, $\hat{s}_t^W = \hat{s}^W$,

[5] Recall that this 'employment ratio for technology μ' is an out of equilibrium choice not an actually realized employment ratio. If some or all firms were actually operating with technology μ, the equilibrium wage rate would change until full employment is achieved.

$\mathbf{s}^l_t = \mathbf{s}^l$, $\mathbf{e}^w_{1;t} = \mathbf{e}^w_1$, $\mathbf{e}^w_{2;t} = \mathbf{e}^w_2$, $\mathbf{e}^l_{1;t} = \mathbf{e}^l_1$, $\mathbf{e}^l_{2;t} = \mathbf{e}^l_2$ for all t. The capital stock, consumption and wages grow at the constant rate ¸. Hence $k_{t+1} =$ ¸k_t and $w_{t+1} =$ ¸w_t.

The following properties for the equilibrium can be established.

Existence: A perfect foresight balanced growth equilibrium exists for this economy. The proof follows Galor and Ryder (1989).

By $\frac{@\mathbf{e}(\mathbf{e}_t;½_{t+1};\mathbf{e}_t;¿)}{@½_{t+1}} > 0$, for every $\mathbf{k}_t$ there is a unique $\mathbf{k}_{t+1}$ and by (4.5) a unique $\mathbf{k}_{t+1}$=`$^1_{t+1}$ which solve (4.6). Hence $½_{t+1}$ is a perfect foresight price. Given the assumption on the production function there exist a maximal sustainable capital stock. Hence every path enters the range $(\mathbf{k}^{max}; 0]$ and remains there. With the assumption on preferences a balanced growth path exists with $\mathbf{k}$ in $(\mathbf{k}^{max}; 0]$.

Dynamic e±ciency: Moreover, a non-trivial balanced growth path is dynamically e±cient if R = ¸. Then economy does not over-accumulate capital; it is not possible to increase total balanced growth consumption.

Following Phelps (1965), total consumption measured per person of generation t is $\mathbf{e}_t = f(\mathbf{k}_t)$ ¡ $\mathbf{k}_{t+1} = f(\mathbf{k}_t)$ ¡ ¸$\mathbf{k}_t$. Di®erentiating this equation gives $\frac{@\mathbf{e}_t}{@\mathbf{k}_t} = f^0(\mathbf{k}_t)$ ¡ ¸ which implies the result.

5. Welfare e®ects of the Social Security System

I am now turning to the main result of the paper:

Proposition 5.1. There exist economies which do not over-accumulate capital in which a pay-as-you-go Social Security system can be welfare improving for all agents.

Proof. (Outline) First, the conditions are derived for which a marginal tax increase in the Social Security tax increases the utility for both the workers and the innovators in the economy on a balanced growth path. A necessary condition is that the growth rate for the economy ¸ is larger than the rate of return to savings ½. To show that there exist economies for which this marginal utility is positive, I study the balanced growth equilibria of economies with a Cobb-Douglas technology and log-linear preferences. For some parameter values these economies have a marginal product of capital R which is greater than the growth rate ¸ while at the same time the return to savings is below the growth rate. Moreover, I show that for some of these economies the marginal utility of a Social Security tax increase (from a level of 0) is positive. ■

In the remainder of this section I derive the condition for which a marginal increase in the tax rate τ increases the utility on the balanced growth path.[6]

The indirect utility functions are given by

$$U^{W*} = U^{W*}(\hat{w}; \rho; \tau) \text{ and } U^{I*} = U^{I*}(\hat{w}; \hat{r}; \rho; \tau)$$

As the decision problem for the workers and the innovators only differs in their budget constraint, the welfare analysis is similar for both. Therefore, I only state the conditions for the problem of the innovator. The difference between the problem of the worker and the innovator is that innovators have a second source of income and hence their utility depends on $\hat{r}$.

Totally differentiating the indirect utility function, recognizing that the budget constraint could be written as $\hat{c}_1 + \lambda/\rho\hat{c}_2 = \hat{I}^I$ with $\hat{I}^I = \left(1 - \tau + \frac{\tau\lambda}{\rho}\right)\hat{w} + \hat{r}$ gives:

$$\begin{aligned} dU^{I*}(\hat{w}; \hat{r}; \rho; \tau) &= \frac{\delta U^{I*}}{\delta \hat{I}}\frac{\delta \hat{I}^I}{\delta \hat{w}} d\hat{w} + \left[\frac{\delta U^{I*}}{\delta 1/\rho}\frac{\delta 1/\rho}{\delta \rho} + \frac{\delta U^{I*}}{\delta \hat{I}}\frac{\delta \hat{I}^I}{\delta \rho}\right] d\rho \\ &\quad + \frac{\delta U^{I*}}{\delta \hat{I}}\frac{\delta \hat{I}^I}{\delta \hat{r}} d\hat{r} + \frac{\delta U^{I*}}{\delta \hat{I}^I}\frac{\delta \hat{I}^I}{\delta \tau} d\tau \end{aligned} \tag{5.1}$$

$\frac{\delta U^{I*}}{\delta \hat{I}^I}$ is positive. By Roy's identity $\frac{\delta U^{I*}}{\delta 1/\rho} \Big/ \frac{\delta U^{I*}}{\delta \hat{I}^I} = -\lambda\hat{c}_2 = -\left(\rho\hat{s}^I + \tau\lambda\hat{w}\right)$. Then equation (5.1) can be rewritten as

$$\frac{1}{\frac{\delta U^{I*}}{\delta \hat{I}^I}} dU^{I*}(\hat{w}; \hat{r}; \rho; \tau) = \left[\left(1 - \tau + \frac{\tau\lambda}{\rho}\right); \frac{\hat{s}^I}{\rho}; 1; \left(\frac{\lambda}{\rho} - 1\right)\hat{w}\right] \begin{bmatrix} d\hat{w} \\ d\rho \\ d\hat{r} \\ d\tau \end{bmatrix} \tag{5.2}$$

To determine $[d\hat{w}; d\rho; d\hat{r}; d\tau]$ the system of equilibrium conditions is totally differentiated. $\hat{w}$, ρ, $\hat{r}$ depend on $\hat{k}$; in addition ρ and $\hat{r}$ depend on ℓ^1 via ζ.

$$D_1 \cdot \begin{bmatrix} d\hat{w} \\ d\rho \\ d\hat{r} \\ d\zeta \\ d\ell^1 \end{bmatrix} = v \cdot \left[\frac{\partial \hat{s}}{\partial \hat{w}}; \frac{\partial \hat{s}}{\partial \rho}; \frac{\partial \hat{s}}{\partial \hat{r}}; \frac{\partial \hat{s}}{\partial \tau}\right] \cdot \begin{bmatrix} d\hat{w} \\ d\rho \\ d\hat{r} \\ d\tau \end{bmatrix}$$

[6] This analysis is similar to Galor/ Ryder (1988).

with

$$D_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{N}{N^I}\tilde{K} & 0 \\ 0 & 0 & 0 & 1 & -f''\left(\frac{\lambda}{\mu}\frac{1}{\ell\mu}\tilde{K}\right)\frac{\lambda}{\mu}\frac{\tilde{K}}{(\ell\mu)^2} \\ 1 & 0 & 0 & 0 & -f''\left(\frac{\lambda}{\mu}\frac{1}{\ell\mu}\tilde{K}\right)\frac{\lambda}{\mu}\frac{\tilde{K}^2}{(\ell\mu)^3} \end{bmatrix}$$

and

$$v = \begin{bmatrix} -f''\left(\tilde{K}\right)\tilde{K} \\ f''\left(\tilde{K}\right) \\ \frac{N}{N^I}\left[f''\left(\tilde{K}\right) - f''\left(\frac{\lambda}{\mu}\frac{1}{\ell\mu}\tilde{K}\right)\frac{\lambda}{\mu}\frac{1}{\ell\mu}\right] \\ -f''\left(\frac{\lambda}{\mu}\frac{1}{\ell\mu}\tilde{K}\right)\frac{\lambda}{\mu}\frac{1}{(\ell\mu)^2}\tilde{K} \end{bmatrix}$$

where $\tilde{s}$ denotes total saving and the partial derivatives $\frac{\partial \tilde{s}}{\partial \tilde{w}}; \frac{\partial \tilde{s}}{\partial \rho}; \frac{\partial \tilde{s}}{\partial \tilde{z}}; \frac{\partial \tilde{s}}{\partial \tau}$ are derived from (4.6) the law of motions for capital

$$\tilde{K} = \tilde{s}(\tilde{w}; \tilde{z}; \rho; \tau) = \frac{1}{\lambda}\left[\frac{N^W}{N}\tilde{s}^W(\tilde{w}; \rho; \tau) + \frac{N^I}{N}\tilde{s}^I(\tilde{w}; \tilde{z}; \rho; \tau)\right].$$

Define D to be the following matrix:

$$D = D_1 - v \cdot \left[\frac{\partial \tilde{s}}{\partial \tilde{w}}; \frac{\partial \tilde{s}}{\partial \rho}; \frac{\partial \tilde{s}}{\partial \tilde{z}}\right].$$

Then provided that D is invertible the equilibrium conditions can be written as:

$$\begin{bmatrix} d\tilde{w} \\ d\rho \\ d\tilde{z} \\ d\zeta \\ d\ell\mu \end{bmatrix} = \frac{\text{adj } D}{|D|} v \cdot \frac{\partial \tilde{s}}{\partial \tau} d\tau \tag{5.3}$$

with adj D the transpose of the cofactor matrix of D. The effect of parameter changes on the endogenous variables is reflect by the matrix E.

$$
e \cdot E \equiv \operatorname{adj} D \cdot v = e \cdot \begin{bmatrix} -f''_{\zeta} \, k \, k \\ f''_{\zeta} k \, \ell^{\mu} \\ \frac{N}{N^l} \left(\zeta + f''_{\zeta} k \, k (1 - \ell^{\mu}) \right) \\ \left[\zeta f'' \, k \, (1 - \ell^{\mu}) \right] \\ f'' \, k \, k - \frac{\lambda}{\mu(\ell^{\mu})^2} f'' \, \frac{\lambda}{1} \frac{1}{\ell^{\mu}} k \;\; / e \end{bmatrix}
$$

with

$$
e = -f'' \left(\frac{\lambda}{1} \frac{1}{\ell^{\mu}} k \right) \frac{\lambda}{1} \frac{k^2}{(\ell^{\mu})^3}
$$

$|D|$ evaluates to

$$
|D| = e - e \left(-f'' \left(k \, k \right) \frac{\partial s}{\partial w} + f'' \left(k \right) \ell^{\mu} \frac{\partial s}{\partial \rho} + \frac{N}{N^l} \left(\zeta + f'' \left(k \, k (1 - \ell^{\mu}) \right) \right) \frac{\partial s}{\partial z} \right)
$$

The sign of $|D|$ is related to the stability property of the equilibrium. If

$$
\left| \frac{k_{t+1}}{k_t} \right| = \frac{-f'' \left(k \, k \right) \frac{\partial s}{\partial w_t} + \frac{N}{N^l} \left(\zeta + f'' \left(k \, k (1 - \ell^{\mu}) \right) \right) \frac{\partial s}{\partial z}}{1 - f'' \, k \, \ell^{\mu} \frac{\partial s}{\partial \rho_{t+1}}} < 1 \tag{5.4}
$$

the equilibrium is stable. The denominator is positive by $\frac{\partial s}{\partial \rho_{t+1}} > 0$. Rearranging (5.4) shows that if the equilibrium is stable, $|D| > 0$ as $e > 0$. However, no general statement about the size and sign of $|D|$ can be made.

With the changes in the endogenous variables determined by (5.3) the marginal utility of a tax rate change using (5.2) can be expressed in the following form:

$$
M^l \equiv \frac{1}{\frac{\delta U^{l*}}{\delta \omega^l}} \frac{dU^{l*}(w; \rho; z; \tau)}{d\tau} \frac{|D|}{e}
$$

$$
= \left[\left(1 - \tau + \frac{\tau \lambda}{\rho} \right); \frac{s^l}{\rho}; 1; \left(\frac{\lambda}{\rho} - 1 \right) w \right] \begin{bmatrix} -f''_{\zeta} \, k \, k \frac{\partial s}{\partial \tau} \\ f''_{\zeta} k \, \ell^{\mu} \frac{\partial s}{\partial \tau} \\ \frac{N}{N^l} \left(\zeta + f'' \, k \, k (1 - \ell^{\mu}) \right) \frac{\partial s}{\partial \tau} \\ |D| / e \end{bmatrix} \tag{5.5}
$$

M^W can be defined analogously. M^W does not depend on $\bar e$, and the relevant saving is $\bar s^W$.

If the equilibrium is stable and $\left(M^I - M^W\right)$ is positive, then the marginal utility of a tax rate increase is positive for innovators (workers).

The last term $\left(\frac{\lambda}{\rho} - 1\right)\bar w\frac{|D|}{e}$ in $\left(M^I - M^W\right)$ shows the direct effect of the tax rate change. It is positive for stable equilibria when the growth rate in the economy λ is larger than the rate of return to savings ρ.

The first three terms of $\left(M^I - M^W\right)$ capture the general equilibrium effect of a change in tax rates on the capital stock. At $\tau = 0$ the first part of M^I evaluates to

$$\left[-f''\left(\bar k\right)\bar k\cdot 1+f''\left(\bar k\right)\ell^{-1}\frac{\bar s^I}{\rho}+\frac{N}{N^I}\left(\left(3+f''\left(\bar k\right)\bar k\right)(1-\ell^{-1})\right)\cdot 1\right]\frac{\delta\bar s}{\delta\tau}$$

and the first three terms of M^W to

$$\left[-f''\left(\bar k\right)\bar k\cdot 1+f''\left(\bar k\right)\ell^{-1}\frac{\bar s^W}{\rho}\right]\frac{\delta\bar s}{\delta\tau}$$

This implies that

$$M^I = M^W + \left[f''\left(\bar k\right)\ell^{-1}\frac{\bar s^I-\bar s^W}{\rho}+\frac{N}{N^I}\left(\left(3+f''\left(\bar k\right)\bar k\right)(1-\ell^{-1})\right)\cdot 1\right]\frac{\delta\bar s}{\delta\tau} \qquad (5.6)$$

As $\frac{\delta\bar s^I}{\delta\tau} < 0$ and $\frac{\delta\bar s^W}{\delta\tau} < 0$, total saving declines in τ, i.e. $\frac{\delta\bar s}{\delta\tau} < 0$. For $\left(M^I - M^W\right)$ to be positive, the utility gain from the Social Security system has to outweigh the effect of the decline in balanced growth capital stock.

6. Example

Assume that $U = \ln c_{1;t} + \beta \ln c_{2;t+1}$. Then savings for the workers is

$$s^W(\bar w;\rho;\tau) = \frac{\beta}{1+\beta}\left[\left(1-\tau-\frac{\tau\lambda}{\rho}\right)\bar w\right]$$

and for the innovators

$$s^I(\bar w;\bar e;\rho;\tau) = \frac{\beta}{1+\beta}\left[\left(1-\tau-\frac{\tau\lambda}{\rho}\right)\bar w+\bar e\right]:$$

Using (4.6), balanced growth capital stock is

$$\tilde{k} = \frac{\beta}{\lambda(1+\beta)}\left[\left(1 - \tau - \frac{\tau\lambda}{\rho}\right)\tilde{w} + \frac{N^{l}}{N}\tilde{z}\right] = \frac{\beta}{\lambda(1+\beta)}\left[\left(1 - \tau - \frac{\tau\lambda}{\rho}\right)\tilde{w} + \zeta\tilde{k}\right]:$$

Thus, at $\tau = 0$

$$\frac{\partial\tilde{s}}{\partial\tilde{w}} = \frac{\beta}{\lambda(1+\beta)}$$

$$\frac{\partial\tilde{s}}{\partial\rho} = 0$$

$$\frac{\partial\tilde{s}}{\partial\tilde{z}} = \frac{\beta}{\lambda(1+\beta)}\frac{N^{l}}{N}$$

$$\frac{\partial\tilde{s}}{\partial\tau} = \frac{-\beta}{\lambda(1+\beta)}\left(1 + \frac{\lambda}{\rho}\right)\tilde{w}$$

The technology is Cobb-Douglas with coefficient α; i.e. $f(\tilde{k}) = \tilde{k}^{\alpha}$.
From equation (4.5) ℓ^{μ} is determine as

$$\ell^{\mu} = \left(\frac{1}{\lambda}\right)^{\frac{1-\alpha}{\alpha}} = \left(\frac{1}{\lambda}\ell^{\mu}\right)^{1-\alpha}$$

The fee by the innovators is

$$\zeta = \alpha\tilde{k}^{\alpha-1}(1 - \ell^{\mu}) \tag{6.1}$$

and the balanced growth capital stock is

$$\tilde{k} = \left(\frac{\beta}{\lambda(1+\beta)}\left[1 - \alpha\left(\frac{1}{\lambda}\right)^{\frac{1-\alpha}{\alpha}}\right]\right)^{\frac{1}{1-\alpha}} \tag{6.2}$$

At any balanced growth equilibrium $|D|/e = 1 - \alpha$.
Hence at $\tau = 0$

$$M^{W} = -\left(1 + \frac{\lambda}{\rho}\right)\tilde{w}\left[\frac{\beta}{(1+\beta)\lambda}\alpha(1-\alpha)\tilde{k}^{\alpha-1} - \frac{\tilde{s}^{W}}{\rho}\ell^{\mu}\alpha(1-\alpha)\tilde{k}^{\alpha-2}\right.$$
$$\left. + \left(\frac{\lambda}{\rho} - 1\right)\tilde{w}\cdot\left[1 - \frac{\beta}{(1+\beta)\lambda}\right]\Big(\alpha(1-\alpha)\tilde{k}^{\alpha-1} + \zeta\alpha\Big)\right]$$

Using the balanced growth capital stock (6.2), ζ from (6.1), and the facts that $R = \alpha \tilde{k}^{\alpha-1}$ and $\rho = \ell^{\mu} R$ gives

$$\lambda(1+\beta)\frac{M^W}{\tilde{w}} = -\left(1+\frac{\lambda}{\rho}\right)\beta(1-\alpha)R\left[1-\frac{\beta(1-\alpha)}{(1+\beta)\alpha}\right] + \left(\frac{\lambda}{\rho}-1\right)(1-\alpha)\lambda(1+\beta) \tag{6.3}$$

M^I is derived analogously.

$$\frac{M^I}{\tilde{w}} = \frac{M^W}{\tilde{w}} - \left(1+\frac{\lambda}{\rho}\right)\left[\frac{\beta}{(1+\beta)\lambda}\alpha(\alpha-1)\tilde{k}^{\alpha-2}\frac{\ell^{\mu}}{\rho}\frac{\beta}{1+\beta}\frac{N}{N^I}\zeta\tilde{k} + \frac{N}{N^I}\alpha\zeta\right]$$

or

$$\lambda(1+\beta)\frac{M^I}{\tilde{w}} = \lambda(1+\beta)\frac{M^W}{\tilde{w}} - \left(1+\frac{\lambda}{\rho}\right)\frac{\beta}{1+\beta}\frac{N}{N^I}(2\alpha\beta+\alpha-\beta)\zeta \tag{6.4}$$

Note that $(2\alpha\beta+\alpha-\beta) > 0$ if the economy is dynamically efficient at $\mu = \lambda$ respectively $\zeta = 0$. As the capital stock increases in μ, this is a necessary condition for any economy to be dynamically efficient. Hence $M^I < M^W$ for the economies relevant for this analysis.

The innovators' utility is more adversely affected by a reduction in the capital stock than the workers' because their income depends directly on the total amount of capital in the economy. This effect is stronger the smaller the group of innovators, i.e. the larger N/N^I. Thus if the Social Security system is welfare improving for the innovators it is welfare improving for workers as well.

The balanced growth equation can be rewritten as

$$R = \frac{\lambda(1+\beta)-\beta\zeta}{\beta}\frac{\alpha}{1-\alpha} \tag{6.5}$$

As $\rho = R - \zeta$ the two equations (6.4) and (6.5) determine the sign of $\lambda(1+\beta)\frac{M}{\tilde{w}}$ and hence the sign of $dU^W(\tilde{w};\rho;\tau)/d\tau$ for a balanced growth economy described by Cobb-Douglas technology, log-linear utility and parameters α, β, λ, μ. $\zeta = \alpha\tilde{k}^{\alpha-1}\left(1-\left(\frac{1}{\lambda}\right)^{\frac{1-\alpha}{\alpha}}\right)$ reflects the importance of the knowledge of the innovators for that economy or alternatively is a measure for their monopoly power.

The graph in the Appendix shows in a $R-\zeta$ diagram (a) the loci of $R-\zeta$ combinations for which $M^I = 0$ and $M^W = 0$, respectively, (b) the locus of balanced growth economies which di®er by their parameter ¹ as captured by ³, and (c) the locus of economies for which the marginal product of capital R = ¸.

In steady state economies to the left of the line $M^I = 0$ innovators and hence all agents bene¯t from a Social Security system. The diagram shows the economies with ¯ = 1, ® = 0:38; $N^I = N$ = 0:2 and ¸ = 3:2 or an annual growth rate of 1:04 over 30 years. The locus of balanced growth economies and $M^I = 0$ intersect at R = 3:275 and ³ = 0:577. The associated ½ = 2:698 implies an annual rate of return of 1:033. Hence, for example the balanced growth economy with ³ = :65, R = 3:234 and an interest rate for savings ½ = 2:585 or annual interest of 1:032 has positive marginal utility for a Social Security system. In this economy ¯rms contribute ¹ = 2:8213 and innovators $¹^I$ = 1:134 to technological progress.

7. Discussion and Conclusion

This model shows that property rights of innovators in technological knowledge can make a PAYG Social Security system desirable for all agents in the long run. The transfer of wealth from the in¯nite future via the Social Security system can Pareto-improve the allocation. The utility gains from changes in the intertemporal allocation of consumption are so large that they outweigh utility losses due to reduced total consumption.

However it should be noted that the Social Security system does not achieve a Pareto-optimal allocation. The reason is that the economy is not e±cient in the short run. Since the marginal product of capital is larger than the growth rate of the economy, higher investment in the technology would increase total available consumption. Moreover, the additional investment could receive a rate of return at the marginal product of capital instead of the lower current rate of return to savings. Hence agents' intertemporal allocation could be improved as well. To achieve that goal innovators would have to be taxed. If this is not possible because innovators cannot be identi¯ed, a PAYG Social Security system may be the second best choice.

The example suggests that the costs of a Social Security system are larger to innovators than to workers. Inspection of M^I (see (5.6)) reveals that innovators are less in favor of a social security system ($M^I < M^W$) if the direct reduction in income due to a reduction in the capital stock is large enough relative to o®setting e®ects on the marginal product of capital. The e®ect of this income loss on innovators is stronger the fewer innovators there are in the economy. Hence it seems likely that innovators prefer no Social Security system or only a very small one, whereas workers are in favor of a larger system. Thus a Social Security system may be too large relative to the Pareto-criterion but still provide welfare gains for a large part of the population.

Whether the e®ects of monopolistic ownership of technology are large enough to make a Social Security system desirable will ultimately be an empirical question. In a stochastic environment it is not clear what interest rate should be used to compare rates of return to investment and growth rates of the economy. Moreover, whether a PAYG Social Security system is welfare improving depends not only on technological parameters but also on preferences.

A direct attempt to measure the size of payments to owners of technological information or individuals who can otherwise increase productivity could use stock market information. If company shares are issued to individuals without payment or with only partial payment, these individuals acquire claims against the future proceeds from the ¯rm. In other words, they are paid a fraction of the return to capital investment by previous owners. The ratio of new shares to new total shares gives the fraction of the return to capital that is paid to the new owners. Combined with information on the marginal product of capital, a measure for the return to investement is obtained.

As this paper shows, property rights in technological knowledge can potentially warrant the existence of a PAYG Social Security system. The development of empirical measures remains for future research.

8. References

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