Macro Implications of Household Finance

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Introduction

Portfolio Behavior

- non-participation: the majority of US households hold only safe assets, real estate and vehicles
- heterogeneity among participants: large variation in "investment styles"
- "sophisticated" investors earn higher Sharpe ratios on their investments and take on more risk
- Campbell(2006) infers that households voluntarily limit their set of assets for fear of making mistakes.

Wealth Distribution

- wealth distribution is more dispersed than income distribution
- wealth is highly correlated with equity fraction of portfolio

Asset Prices

- the equity premium is large and volitile
- the risk-free rate is low and stable.
- equity premia are counter-cyclical

Can limiting household trading options account for these features of the data?

Conjecture

- If many households are subject to both aggregate and idiosyncratic risk,
- but some households passively save via low risk portfolios without adjusting portfolio composition
- this could force a small number of more active traders to absorb a lot of aggregate risk.
- Interesting things might happen in such a model.

Objective of paper

- Propose to formalize this story and evaluate.
- develop "new" method for handling incomplete markets
- allows for differential portfolio restrictions with ease of analysis and computation
- plug in some numbers and evaluate story

Quantitative Findings

Introducing **heterogeneity in investment behavior**

- helps us understand the connections between
  - household portfolio behavior
  - household wealth distribution
  - asset prices
- reduces the gap between model and data:
  - model can match moments of asset prices
  - model delivers better match of wealth distribution
  - model delivers better match of risk asset holdings and wealth

Methodological Contribution

- develop **multiplier method** for incomplete market economies
- use measurability restrictions to capture portfolio restrictions
  - building on Aiyagari, Marcet, Sargent and Seppala et al. (2002) and Lustig, Sleet and Yeltekin (2002)

Methodological Contribution

- develop **multiplier method** for incomplete market economies
- use measurability restrictions to capture portfolio restrictions
- develop new state variable - recursive multiplier
  - building on Cuoco and He (2001), Basak and Cuoco (1998) , Marcet and Marimon (1999)

Methodological Contribution

- develop **multiplier method** for incomplete market economies
- use measurability restrictions to capture portfolio restrictions
- develop new state variable - recursive multiplier
- construct analytic consumption sharing rule and SDF
  - extends Lustig (2006) complete markets result

Methodological Contribution

- develop **multiplier method** for incomplete market economies
- use measurability restrictions to capture portfolio restrictions
- develop new state variable - recursive multiplier
- construct analytic consumption sharing rule and SDF
- develop simple quantiative method
  - avoid guessing the mapping from the wealth distribution to equilibrium asset prices
  - similar to Krusell and Smith (1997)
  - avoids complexities of Krusell and Smith (1998)

Literature

- literature: **participation vs non-participation**
- our paper: **active vs passive traders**
- literature: **preference heterogeneity**
  - EIS (Guvenen (2003), Vissing-Jorgensen (2001))
  - time discount rate (Krusell and Smith (1998))
- our paper: **heterogeneity in trading technologies**

Macro and Micro Risk

- Aggregate output $Y_t = \exp(z_t) Y_{t-1}$ comes in two forms
  - *tradeable output* $(1-\gamma) Y_t$ depends on z^t
  - *non-tradeable output* $\gamma Y_t \eta_t$ depends on η_t
- Idiosyncratic shocks
  - η are i.i.d. across households and $E\{\eta_t | z^t\} = 1$
  - $\pi(z^t, \eta^t)$ is probability of observing z^t and η^t

Households

- Continuum of **ex ante identical** households
  - $\pi(z^t, \eta^t)/\pi(z^t)$ is fraction of households with history η^t given z^t.
- Households have preferences given by:

$$E_0 \left\{ \sum_{t \geq 1}^{\infty} \beta^t \frac{c_t^{1-\alpha}}{1-\alpha} \right\}$$

  - identical CRRA preferences.

Leverage

- $(1-\gamma)Y_t$ comes in two forms
  - bond payouts: $R^f_t(z^{t-1})b(z^{t-1}) - b(z^t)$
  - dividend payouts: $d_t(z^t)$
- $\varpi(z^t)$: price of claim to tradeable output $(1-\gamma)Y_t$
- assume fixed leverage ψ, which determines $b(z^t)$ and $d_t(z^t)$
- bond/equity ratio:

$$b(z^t) = \psi\left[\varpi(z^t) - b(z^t)\right]$$

Heterogeneous Trading Technologies

- *active* traders
  1. complete market traders (c):
     - trade claims on both z_{t+1} and η_{t+1} realizations
  2. z-complete market traders (z):
     - trade claims only on z_{t+1} realizations
- *passive* traders
  1. diversified traders (div):
     - trade claims to $(1-\gamma)Y(z^t)$ only
     - equivalent to fixed portfolio: $1/(1+\psi)$ in levered equity and $\psi/(1+\psi)$ in risk-free bonds
  2. non-participants (np):
     - only a risk-free bond

Sequential Trading

- all households trade a *complete* menu of contingent claims $a_t(z_t, \eta_t)$ and $\sigma_{t-1}(z_{t-1}, \eta_{t-1})$ shares in the Lucas tree at price $\varpi(z^t)$
- net wealth :

$$\hat{a}(z^t, \eta^t) \equiv a(z^t, \eta^t) + \sigma(z^{t-1}, \eta^{t-1}) \left[(1-\gamma) Y(z^t) + \varpi(z^t) \right].$$

- subject to
  - a budget constraint
  - a debt bound:

$$\widehat{a}_t(z_t, \eta_t) \geq M_t(\eta_t, z_t)$$

  - *measurability restrictions* on $\widehat{a}_t(z_t, \eta_t)$ which replicaes portfolio restriction

Measurability Restrictions

1. active traders:
   - *complete* traders: no restrictions

$$\hat{a}_t(z_t, \eta_t)$$

   - *z-complete* traders:

$$\hat{a}_t(z_t, \eta_t) = \hat{a}_t(z_t, \tilde{\eta}_t),$$

   for all t and η_t, $\tilde{\eta}_t \in N$.

Measurability Restrictions

1. active traders:
2. passive traders:

$$\frac{\hat{a}_t(z_t, \eta_t)}{R_t^{port}(z_t)} = \frac{\hat{a}_t(\tilde{z}_t, \tilde{\eta}_t)}{R_t^{port}(\tilde{z}_t)},$$

for all t, z_t, $\tilde{z}_t \in Z$, and η_t, $\tilde{\eta}_t \in N$.

- $R_t^{port}(z_t)$ is return on a passive trader's total portfolio:
  - *non-participants*: $R_t^{port}(z_t) = R_{t-1}^f$
  - *diversified* traders: $R_t^{port}(z_t) = R_t(z_t)$

Solving the **z-Complete Trader's** Problem

- present-value of net savings from state (z^t, η^t) onwards:

$$S(z^t, \eta^t) = E_t \left[\sum_{\tau \geq t} P(z^\tau) \left(\gamma Y(z^\tau) \eta_\tau - c(z^\tau, \eta^\tau) \right) \right].$$

  - $P(z^t)$ denotes state prices
  - Implicitly, present value price $p(z^t, \eta^t) = \pi(\eta^t | z^t) \pi(z^t) P(z^t)$
    - arbitrage condition when trading is allowed on η^t
    - innocuous when not

-

$$S(z^t, \eta^t) + \hat{a}_t(z_t, \eta_t) = 0$$

so measureability restrictions on $\hat{a}$ also apply to S.

Solving the **z-Complete Trader's** Problem

$$L = \min_{\{\gamma,\nu,\varphi\}} \max_{\{c,\hat{a}\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} u(c(z^t,\eta^t))\pi(z^t,\eta^t)$$
$$+\gamma\left\{S(z^0,\eta^0)+\varpi(z^0)\right\}$$

γ is multiplier on budget constraint

$$+\sum_{t\geq 1}\sum_{z^t,\eta^t}\nu(z^t,\eta^t)\left\{\begin{array}{c} S(z^t,\eta^t) \\ -P(z^t)\pi(z^t,\eta^t)\hat{a}(z^t,\eta^{t-1}) \end{array}\right\}$$

$\nu(z^t,\eta^t)$ measurability multiplier

$$+\sum_{t\geq 1}\sum_{(z^t,\eta^t)}\varphi(z^t,\eta^t)\left\{\begin{array}{c} \underline{M}_t(z^t,\eta^t)P(z^t)\pi(z^t,\eta^t) \\ -S(z^t,\eta^t) \end{array}\right\}.$$

$\varphi(z^t,\eta^t)$ debt constraint multiplier.

First-order condition for $c(z^t, \eta^t)$

$$\beta^t u'(c(z^t, \eta^t))\pi(z^t, \eta^t) = \left\{ \gamma_0 - \sum_{(z^\tau, \eta^\tau) \preceq (z^t, \eta^t)} [v(z^\tau, \eta^\tau) - \varphi(z^\tau, \eta^\tau)] \right\} P(z^t)\pi(z^t, \eta^t)$$

First-order condition for $\hat{a}(z^t, \eta^{t-1})$

$$\sum_{\eta_t} v(z^t, [\eta^{t-1}, \eta_t]) P(z^t)\pi(z^t, \eta^t) = 0$$

Multipliers

- Define recursive multipliers $\zeta_0 = \gamma$ and law of motion

$$\zeta_t = \zeta_{t-1} + \nu_t - \varphi_t .$$

- f.o.c. for consumption is given by:

$$\frac{\beta^t u'(c_t)}{P(z^t)} = \zeta_t .$$

- f.o.c. for $\hat{a}_t(z_t, \eta_{t-1})$ is:

$$E\left[\nu_{t+1} | z^{t+1}\right] = 0$$

in each z^{t+1}.

- form of f.o.c. for $\hat{a}_t$ changes with portfolio restrictions
- For the **diversified** investors:

$$+\sum_{t\geq 1}\sum_{z^t,\eta^t}\widetilde{P}(z^t,\eta^t)\left[\begin{array}{c}\zeta(z^t,\eta^t)\left(\gamma\eta^t Y(z^t)-c(z^t,\eta^t)\right)+\nu\left(z^t,\eta^t\right)\sigma(z^{t-1},\eta^{t-1})\\ \left[(1-\gamma)Y(z^t)+\varpi(z^t)\right]-\varphi(z^t,\eta^t)\underline{M}(z^t,\eta^t)\end{array}\right]$$
$$+\gamma\varpi(z^0).$$

- The first order condition with respect to $\sigma(z^t,\eta^t)$ is given by:

$$\sum_{z^{t+1}\succ z^t,\eta^{t+1}\succ\eta^t}\nu\left(z^{t+1},\eta^{t+1}\right)\left[(1-\gamma)Y(z^{t+1})+\varpi(z^{t+1})\right]\pi(z^{t+1},\eta^{t+1})P(z^{t+1})$$
$$=0.$$

- f.o.c. for $\hat{a}_t(z_{t-1},\eta_{t-1})$ implies ζ is a "Martingale" when the borrowing constraint does not bind since:

$$\nu_t=\zeta_t-\zeta_{t-1}+\varphi_t.$$

Martingales

- **General Form of the Martingale condition**

$$E\left[\zeta_t m_t R_t \middle| \cdot\right] = \zeta_{t-1} E\left[m_t R_t \middle| \cdot\right]$$

- where
  - R_t is the (possibly state-contingent) return
  - $m_t = P(z^t)/P(z^{t-1})$
  - z-complete traders conditioned on (z^t, η^{t-1}) and $R_t = 1$
  - passive traders conditioned on (z^{t-1}, η^{t-1}) and R_t is portfolio return
- active traders' multipliers are *conditional martingales* and passive traders' are *twisted martingales*.

Euler inequalities

- martingale conditions imply standard Euler inequalities:
  - *np* :
$$u'(c_t) \geq R_t^f \beta E_t \left\{ u'(c_{t+1}) \right\}$$
  - *div* :
$$u'(c_t) \geq \beta E_t \left\{ R_{t+1} u'(c_{t+1}) \right\}$$
  - *z* :
$$u'(c_t) \geq \beta E_t \left\{ u'(c_{t+1}) \frac{P(z^t)}{P(z^{t+1})} | z^{t+1} \right\}$$
  - *c* :
$$u'(c_t) \geq \beta \left\{ u'(c_{t+1}) \frac{P(z^t)}{P(z^{t+1})} \right\}$$

Consumption sharing rule

- f.o.c. for consumption implies that

$$c_t = u'^{-1}\left[\frac{\zeta_t P(z^t)}{\beta^t}\right].$$

- aggregate consumption is:

$$C_t = \sum_{\eta^t} c(z^t, \eta^t)\pi(\eta^t | z^t).$$

- the household consumption share is

$$\frac{c_t}{C_t} = \frac{u'^{-1}\left[\zeta(z^t, \eta^t)P(z^t)\right]}{\sum_{\eta^t} u'^{-1}\left[\zeta(z^t, \eta^t)P(z^t)\right]\pi(\eta^t | z^t)}.$$

Characterizing Equilibrium with Multipliers

- For **all traders**, consumption share is:

$$c_t / C_t = \zeta_t^{-\frac{1}{\alpha}} / h(z^t),$$

with the aggregate multiplier:

$$h(z^t) = \sum_{\eta^t} \zeta(z^t, \eta^t)^{\frac{-1}{\alpha}} \pi(\eta^t | z^t).$$

- The stochastic discount factor is:

$$m_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{h_{t+1}}{h_t} \right)^{+\alpha}.$$

with the aggregate multiplier:

$$h(z^t) = \sum_{\eta^t} \zeta(z^t, \eta^t)^{\frac{-1}{\alpha}} \pi(\eta^t | z^t).$$

- no need to *clear* each market separately

What do these Multipliers do?

- present value of future savings S is recursive in ζ:

$$\begin{aligned} S_t(\zeta; z^t, \eta_t) &= C_t \left(\gamma \eta_t - \frac{\zeta_t^{\frac{-1}{\alpha}}}{h(z^t)} \right) \\ &+ E_t \left[\frac{P(z^{t+1})}{P(z^t)} S_{t+1}(\zeta_{t+1}; z^{t+1}, \eta_{t+1}) \right] \end{aligned}$$

What do these Multipliers do?

$$S_t(\zeta; z^t, \eta_t) \quad = \quad C_t \left(\gamma\eta_t - \frac{\zeta_t^{\frac{-1}{\alpha}}}{h(z^t)} \right) + E_t \left[\frac{P(z^{t+1})}{P(z^t)} S_{t+1}(\zeta_{t+1}; z^{t+1}, \eta_{t+1}) \right]$$

- non-participants: S_{t+1} cannot depend on η_{t+1} (or z_{t+1})
  - multipliers change in response to shocks

$$\begin{array}{lll} \eta_{t+1} = hi & \nu_{t+1} \swarrow & \zeta_{t+1} \swarrow \\ \eta_{t+1} = lo & \nu_{t+1} \nearrow & \zeta_{t+1} \nearrow \end{array}$$

    enforces measurability constraint
  - on average zero change in multiplier:

$$E_t[\zeta_{t+1} m_t / E[m_t] | z^t, \eta^t] = \zeta_t \Leftrightarrow u'(c_t) = R_t^f \beta E_t \left\{ u'(c_{t+1}) \right\}$$

    enforces Euler equation

Full Participation

- If
  1. there are no non-participants,
  2. z_t is i.i.d., $\phi(z_{t+1})$, and
  3. η_t is independent of z_t
  4. $\underline{M}(z^t, \eta^t) = \underline{M}(\eta^t) Y(z^t)$.
- household consumption shares are independent of the aggregate history z^t, only depend on η^t. (Krueger and Lustig, 2005)

Full Participation

Simple Proof Step 1

- define $\tilde{S}$ as:

$$S(\zeta_t; z^t, \eta_t) = C(z^t)\tilde{S}(\zeta_t; \eta_t).$$

- recursive rule for $\tilde{S}$ does not depend on z^t

Recursive rule for $S(\zeta_t; z^t, \eta_t)$ implies that

$$\tilde{S}(\zeta(z^t, \eta^t); z^t, \eta^t) = \gamma \eta_t - \frac{\zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}$$
$$+\beta \sum_{z_{t+1}} \widehat{\phi}(z_{t+1}|z^t) \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta_t) \tilde{S}(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1}).$$

where

$$\widehat{\phi}(z_{t+1}|z^t) = \phi(z_{t+1}) \left[\frac{h(z^{t+1})}{h(z^t)} \right]^{\gamma} e^{(1-\gamma) z_{t+1}}.$$

If $h(z^{t+1})/h(z^t)$ doesn't depend upon z^t, then z^t drops out of this recursion.

Full Participation

Simple Proof Step 3

- define $\tilde{S}$ as:

$$S(\zeta_t; z^t, \eta_t) = C(z^t)\tilde{S}(\zeta_t; \eta_t).$$

- recursive rule for $\tilde{S}$ does not depend on z^t
- borrowing constraint does not depend upon z^t:

$$\tilde{S}(\zeta(z^{t+1}, \eta^{t+1}); z^t, \eta^t) \leq \underline{M}(\eta^{t+1}). \tag{1}$$

- for *div*, z **and** c traders their measurability constraints do not depend on z^t

Full Participation

- for *div*, z **and** c traders their measurability constraints do not depend on z^t:

  z-complete traders:

$$\tilde{S}^z(\zeta_{t+1};\eta_{t+1}) = \tilde{S}^z(\zeta_{t+1};\tilde{\eta}_{t+1})$$
$$\text{for all } \eta_{t+1},\ \tilde{\eta}_{t+1} \text{ and } z^{t+1},$$

  diversified traders:

$$\omega(z^{t+1}) + (1-\gamma)Y(z^{t+1}) \sim C(z^{t+1})$$

$$\tilde{S}^{div}(\zeta_{t+1};\eta_{t+1}) = \tilde{S}^{div}(\zeta_{t+1};\tilde{\eta}_{t+1})$$
$$\text{for all } \eta_{t+1},\ \tilde{\eta}_{t+1}\ \ z_{t+1} \text{ and } \tilde{z}_{t+1},$$

Full Participation

Simple Proof - complete

- define $\tilde{S}$ as:

$$S(\zeta_t; z^t, \eta_t) = C(z^t)\tilde{S}(\zeta_t; \eta_t).$$

- recursive rule for $\tilde{S}$ does not depend on z^t
- borrowing constraint does not depend upon z^t:

$$\tilde{S}(\zeta(z^{t+1}, \eta^{t+1}); z^t, \eta^t) \leq \underline{M}(\eta^{t+1}). \qquad (2)$$

- for *div*, *z* **and** *c* traders their measurability constraints do not depend on z^t
- hence updating functions $T^j, j \in \{div, z\}$ are **independent** of z^t.
- hence h_t does not depend on z^t

Irrelevance of Aggregate History

- if no non-participants, h_{t+1} **does not depend** on z^{t+1}.
- all have same portfolio (regardless of y^t) of $1/(1+\psi)$ in equity and remainder in risk-free
- consumption shares do not depend on z^t
- Breeden-Lucas risk premium obtains (small and constant risk premia)

$$m_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} \left(\frac{h_{t+1}}{h_t}\right)^{+\alpha}.$$

with the aggregate multiplier growth rate independent of z^{t+1}

  - segmentation of households in other trading technologies is irrelevant

Non-Participation and Relevance of Aggregate History

- for **non-participants**: measurability constraints does depend on z_{t+1}

$$\frac{\tilde{S}_{t+1}(\zeta_{t+1}; z^{t+1}, \eta_{t+1})}{C(z^t, z_t)/C(z^t)} = \frac{\tilde{S}_{t+1}(\zeta_t; \tilde{z}^{t+1}, \tilde{\eta}_{t+1})}{C(z^t, \tilde{z}_{t+1})/C(z^t)}.$$

- updating functions T^{np} **does depend** of z^{t+1}.
- h_{t+1} **does depend** on z^{t+1}.

Aggregate Savings

- The aggregate multiplier for each group $h^j(z^t)$ is given by

$$h^j(z^t) = \int_{\eta^t} \zeta(z^t, \eta^t)^{-1/\alpha} \pi(\eta^t | z^t) d\eta^t$$

  - $\frac{h^j(z^t)}{h(z^t)}$ is consumption share of trader segment j
- the aggregate savings function for each group of traders:

$$S_a^j(z^t) \quad = \quad C(z^t) \left[\gamma \mu^j - \frac{h^j(z^t)}{h(z^t)} \right] + \sum_{z^{t+1}} \frac{\pi(z^{t+1}) P(z^{t+1})}{\pi(z^t) P(z^t)} S_a^j(z^{t+1}).$$

  - market clearing implies:

$$\sum_j S_a^j(z^t) \quad = \quad -[\varpi(z^t) + (1-\gamma) Y(z^t)]$$

- the **same measurability restrictions** apply to the aggregate savings function $S_a^j(z^t)$

Dumping Aggregate Risk?

$$S_a^{np}(z^t) \quad = \quad C(z^t)\left[\gamma\mu^{np} - \frac{h^{np}(z^t)}{h(z^t)}\right] + \sum_{z^{t+1}} \frac{\pi(z^{t+1})P(z^{t+1})}{\pi(z^t)P(z^t)} S_a^{np}(z^{t+1}).$$

- for **non-participants** $S_a^{np}(z^t)$ cannot not depend on z_t
  - follows directly from aggregating their measurability restrictions
  - implies **counter-cyclical** savings share $S_a^{np}(z^t) / \sum_j S_a^j(z^t)$
  - generating **aggregate risk**

Dumping Aggregate Risk?

$$S_a^{div}(z^t) \quad = \quad C(z^t)\left[\gamma\mu^{div} - \frac{h^{div}(z^t)}{h(z^t)}\right] + \sum_{z^{t+1}} \frac{\pi(z^{t+1})P(z^{t+1})}{\pi(z^t)P(z^t)} S_a^{div}(z^{t+1}).$$

- for **diversified traders** the ratio $S_a^{div}(z^t)/\sum_j S_a^j(z^t)$ cannot not depend on z_t
  - follows directly from aggregating their measurability restrictions
  - bear **none** of the residual aggregate risk

Dumping Aggregate Risk on active traders.

- for **non-participants** $S_a^{np}(z^t)$ cannot not depend on z_t
- for **diversified traders** the ratio $S_a^{div}(z^t)/\sum_j S_a^j(z^t)$ cannot not depend on z_t
- for **active** traders the ratio $S_a^{div}(z^t)/\sum_j S_a^j(z^t)$ does depend on z_t and is procyclical
  - bear all residual aggregate risk of non-participants
  - active traders induced to do so by the spread in state prices

System of equations

Table: System of Equations

	Martingale Conditions
np	$\widetilde{E}_t\left[\nu_{t+1}\right] = 0.$
div	$\widetilde{E}_t\left[\nu_{t+1}\right] = 0$
z	$E_t\left[\nu_{t+1} \mid z^{t+1}\right] = 0$
c	$\nu_{t+1} = 0$
	Measurability Conditions
np	$S_t(z_t, \eta_t) = S(\tilde{z}_t, \tilde{\eta}_t)$
div	$\frac{S_t(z_t, \eta_t)}{R_t(z_t)} = \frac{S_t(\tilde{z}_t, \tilde{\eta}_t)}{R_t(\tilde{z}_t)}$
z	$S(z_t, \eta_t) = S(z_t, \tilde{\eta}_t)$

Solving system of equation

- *updating function* T^i for ζ:

$$\zeta_{t+1} = T^i(z^{t+1}, \eta_{t+1} | z^t, \eta_t)(\zeta_t)$$

- T^i is solution to system of equations defined by:
  1. **measurability** conditions using recursive expression for S
  2. **martingale** conditions
  3. **borrowing constraint** using recursive expression for S
- $h(z^t)$ is defined as

$$h(z^{t+1}) = \sum_{j \in T} \int \sum_{\eta^{t+1} \succ \eta^t} \left[T^j(z^{t+1}, \eta_{t+1} | z^t, \eta_t)(\zeta_t) \right]^{\frac{-1}{\alpha}} \varphi(\eta_{t+1} | \eta_t) d\Phi^j_t,$$

Algorithm

1. in stage i, guess an aggregate weight forecasting function $\{h^i(z^k)\}$ with truncated history z^k
2. solve system of equations for updating functions $T^j, j \in \{z, c, div, np\}$
3. updating functions define new $\{h^{i+1}(z^k)\}$, computed by simulating long panels ($N = 3,000$ and $T = 10,000$) and averaging:

$$h^{i+1}(z^{k+1}) = \sum_{j \in T} \int \sum_{\eta^{t+1} \succ \eta^t} \left[T^j(z^{k+1}, \eta_{t+1} | z^k, \eta_t)(\zeta_t) \right]^{\frac{-1}{\alpha}} \varphi(\eta_{t+1}|\eta_t) d\Phi_t^j,$$

4. iterate until convergence of $\{h^{i+1}(z^k)\}$

Algorithm

Without analytic asset pricing result:

- Need to guess asset pricing relationship
    - asset prices can be a complicated function of the wealth distribution.
- Need to compute market clearing prices in each period of the panel simulation
    - solve for continuation payoff given asset pricing rules in recursion stage.
    - use continuation payoff to compute portfolio demands in simulation stage.
    - compute new prices by solving for market clearing prices in simulation stage.
    - use new prices to update pricing rules.
- **Tough Problem**

Calibration

- Preferences: $\alpha = 5$ and $\beta = .95$
- Endowments:
  - aggregate consumption growth: MP
    - aggregate consumption growth $E[\lambda(z)] = 1.0183$ and $std[\lambda(z)] = .035$
    - but i.i.d. growth to focus on internal propagation - report other results in paper.
  - no concentration of idio. risk in recessions
    - $\log \eta : \rho[\log \eta] = .92$

Calibration

- choose γ to match wealth-to-income ratio
  - Total Collateralizable wealth to income ratio is 4.30 in postwar US data
  - we (conservatively) choose $\gamma = .90$.
- choose segment sizes to match asset prices
  - 10% in z-complete
  - 20% diversified
  - 70% non-participants.
- adjust segment per capita income to match income distribution
  - So long as shares of nontradeable wealth don't change, prices don't change.

Accuracy

- The simulation moments are generated by 10000 draws from an economy with 3000 agents.
- For the benchmark calibration max $\frac{\sigma([h'/h])}{E([h'/h])} = 0.579$.
- History works better than Wealth moments
  - R^2 results for h'/h in state-by-state regressions are low
  - with $E(W)$ and $E(W_z/W)$ range from .45 - .55
  - with additional lagged z_{t-1} increase to .59 - .67
  - with four lags range from .98 - .99

Market Price of Risk and Risk-Free Rate

- large and volatile market price of risk $\sigma(m)/E(m)$

$$\sigma(m)/E(m) \geq E\left[R_s - R_f\right]/\sigma\left[R_s - R_f\right] = .44$$

- low and stable risk-free rate

	HTT Model		HTT Model	Data	RA Model
$\sigma(m)/E(m)$	0.440	$E(R_f)$	1.737	1.049	13.04
Std$[\sigma(m)/E(m)]$	0.033	$\sigma(R_f)$	0.066	1.56	3.144

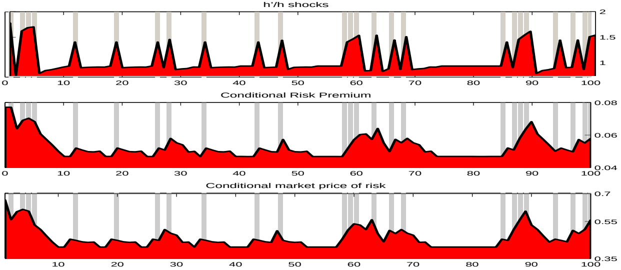
Equity Risk Premium

- large equity premium and Sharpe ratio
  - dividends are a leveraged claim to consumption
  - the leverage parameter ψ is 3

	HTT Model	Data	RA Model
$E[R_s - R_f]$	6.702	7.53	2.32
$\sigma[R_s - R_f]$	15.27	16.94	13.34
$E[R_s - R_f] / \sigma[R_s - R_f]$	0.44	0.44	.174

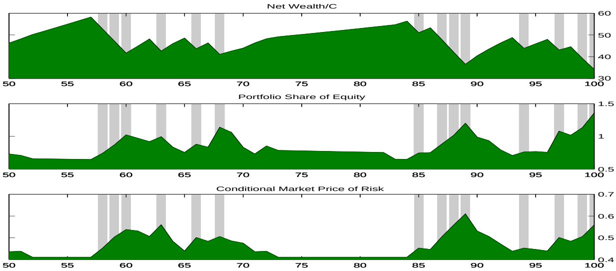
Time Variation

- large and volatile market price of risk $\sigma(m)/E(m)$
- counter-cyclical market price of risk $\sigma_t(m)/E_t(m)$
- ditto for conditional risk premium



Notes: Market Segmentation: 10% in z-complete, 20% diversified and 70% non-participants.

Equity Share of z-trader



Notes: Market Segmentation: 10% in z-complete, 20% diversified and 70% non-participants.

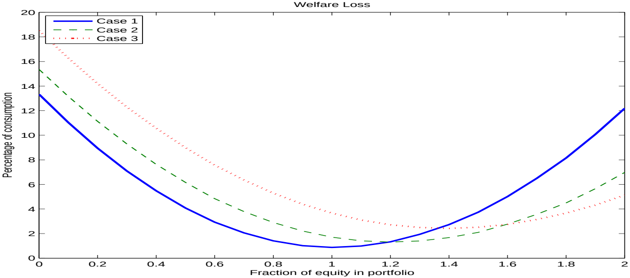
Household Portfolio Returns

z	$E[R_z^W - R_f]$	0.056
div	$E[R_{div}^W - R_f]$	0.015
np	$E[R_{np}^W - R_f]$	0.000
z	$E[R_z^W - R_f]/\sigma[R_z^W - R_f]$	0.413
div	$E[R_{div}^W - R_f]/\sigma[R_{div}^W - R_f]$	0.413
np	$E[R_{np}^W - R_f]/\sigma[R_{np}^W - R_f]$	0.000

- more sophisticated investors (CCS, 06):
  - higher Sharpe ratios
  - take on more risk $\sigma[W_z/W] = 1.390$
- median Swedish household gives up 1/3 of market's SR

Welfare Costs of Passivity and Nonparticipation

Figure: Equity Share



Notes: Case 1: 0/10/20 (complete/z-complete/diversified) composition of trader segments. Case 2: 5/5/20 composition. Case 3: 10/0/20 composition.

Household Consumption Growth

Standard Deviation

	Individual	Group Avg.
z	10.17	7.438
div	12.21	3.622
np	13.11	2.554

- Volatility of household consumption growth declines with trading sophistication.
- But sophisticated household load up on aggregate risk - which leads to group average volatility rising with trading sophistication.
  - Consistent with findings of Malloy, Moskowitz and Vissing-Jorgensen (2007)

Household Consumption-Returns Correlation

		Correlation
Part	$\rho\left[R_s, (\Delta \log(c_p)\right]$	0.431
z	$\rho\left[R_s, (\Delta \log(c_z)\right]$	0.712
div	$\rho\left[R_s, (\Delta \log(c_{div})\right]$	0.291
np	$\rho\left[R_s, (\Delta \log(c_{np})\right]$	0.200

- higher correlation for participants
  - consistent with Mankiw and Zeldes (91) and Brav, Constantinides and Geczy (02)
- lower risk aversion estimates off the Euler equation for stock returns for wealthier households
  - diversifed traders don't satisfy stockmarket Euler condition

Household Consumption Shares

		Correlation
Part	$\rho\left[R_s, (\Delta \log(\widehat{c}_p)\right]$	0.163
z	$\rho\left[R_s, (\Delta \log(\widehat{c}_z)\right]$	0.482
div	$\rho\left[R_s, (\Delta \log(\widehat{c}_{div})\right]$	0.003
np	$\rho\left[R_s, (\Delta \log(\widehat{c}_{np})\right]$	-0.071

- **z** traders: procyclical consumption shares
- **div** traders: constant consumption shares
- **np** traders: countercyclical consumption shares

Household Wealth

z	$E[W^{z}/W]$	3.213
div	$E[W^{div}/W]$	0.979
np	$E[W^{np}/W]$	0.689

- more sophisticated investors:
  - accumulate more wealth
  - by taking on more risk
- high wealth to income ratio for z traders suggest we can match skewness in wealth distribution

Matching the Income Distribution

First we try to better match the income distribution, we

- reduced the population share of the z traders to 7%
- reduced the population share of the div. traders to 17 %.

Income Distribution

Percentile Ratio	**Model**	**US Data**
90/50	3.353	2.908
75/25	4.136	3.449
80/25	4.369	3.943
85/25	4.624	4.663
90/25	5.063	5.618
80/20	4.613	4.710
90/10	11.42	11.64

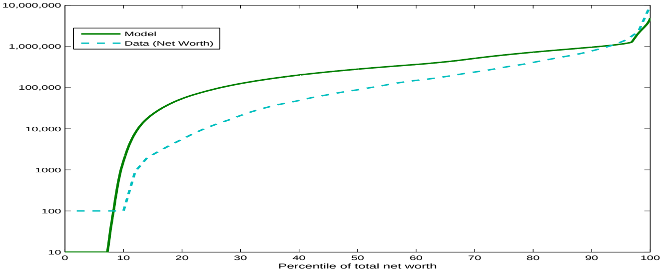
Wealth Distribution

	Bewley	*HTT*	Net Asset	Total Asset
kurtosis	2.842	10.43	15.78	48.85
skewness	0.882	3.189	3.60	6.25
Gini	0.486	0.587	0.793	0.697
W_{75} / W_{25}	5.529	6.967	25.09	10.64
W_{80} / W_{20}	9.409	13.31	65.41	33.42
W_{85} / W_{15}	19.12	36.15	211.9	55.75
W_{90} / W_{10}	82.20	472.6	999.1	580.5

Notes: The Bewley model is one in which there are no nonparticipants.

Wealth Distribution

- we match income distribution by changing fraction of population in each group j



Notes: Nondiversified wealth shares 10% in z-complete, 20% diversified and 70% non-participants, but adjusted population shares.

Table: Wealth Distribution Over Time -Data and Model

	Data Std	**Model Std**
kurtosis	**1.277**	**4.198**
skewness	**0.128**	**0.713**
Gini Coefficient	**0.009**	**0.039**
Percentile Ratio		
W_{80} / W_{20}	**15.84**	**2.24**
W_{85} / W_{15}	**89.9**	**11.42**
W_{90} / W_{10}	**1201**	–

Notes: The wealth data are from the SCF (all available years). The statistics shown are for Household Net Worth.

Asset Shares in Model and Data

- can we replicate asset holdings distribution in the data?

	Data		Model
Percentile	2001	2004	
15 %	4.512	2.633	3.942
50 %	8.077	2.762	3.115
65 %	11.09	10.16	8.207
75 %	19.04	10.12	11.02
85 %	24.16	16.56	9.263
95 %	34.30	25.37	41.86
100 %	42.67	34.19	59.80

Conclusion

- Methods
  1. Develop a multiplier method for incomplete markets.
  2. Very easy to solve and implement
- Findings
  1. Big distinction between passive and active traders
  2. Help us to understand asset prices,consumption and wealth
  3. Move closer to matching many features of the data

Literature

- basic risk-shifting mechanism similar to Guvenen (2003)
  - **Guvenen:** had two representative agents: participants and nonparticipants
  - **our paper:** continuum of each type of active and passive investor
  - **Guvenen:** production economy with no growth
  - **our paper:** endowment economy with stochastic growth

Comparing CCL to Krusell and Smith

- KS (1996).
  - only traded asset is capital - which is a very good hedge
  - labor is exogenous and stochastic
  - the return on capital is given by

$$z' F_K(K', L') + (1 - \delta),$$

    which only depends upon 1 endo variable: K'
  - leads to simple procedure - forecast K'

KS (1996) Procedure

- Assume that there exists a simple forecasting function $G(M_t, z_t)$ for K' where M_t are current moments
  - current capital turns out to work well
- Compute solution to individual's problem given forecasting rule and wealth w.
- can use capital savings policy function $s(w, z)$, to compute the total savings level

$$S = \int_w s(w, z)\omega_t(w)dw,$$

given pdf of worker's states.

KS (1996) Procedure

- Simulation methods used to construct pdf of the workers states in each period.
  - draw stochastic panel of aggregate and idio shocks
  - use individual savings function to construct savings choices
  - sum to get S_t
  - iterate on forecasting rule for K'

Comparing CCL to Krusell and Smith

- KS (1997)
  - traded assets expanded to include risk-free bond and capital
  - labor is still exogenous and stochastic
  - the return on capital is still given by

$$z' F_K(K', L') + (1 - \delta),$$

  - but now need a rule for the bond price q

KS (1997) Procedure

- Let $G_b(M, z)$ denote the market clearing bond price function
- let $G_K(M, z)$ denote the forecasting function for K'
  - which may depend upon q itself.
- Taking implied return function for capital and the price function for bonds as given,
  - one can solve the individual's problem to construct their continuation payoff function
  - given this continuation payoff tomorrow, one can solve for the individual's asset demand functions today given their individual state and q today.

KS (1997) Procedure

- Let s_k and s_b denote their respective savings functions today.
- draw stochastic panel of aggregate and idio shocks
- in panel solve for the q today that clears the bond market

$$q_t = \int_w s_b(w, z, q)\omega_t(w)dw = 0,$$

and with that q, determine the level of capital for tomorrow

$$K_{t+1} = \int_w s_k(w, z, q)\omega_t(w)dw,$$

- Given a sequence of implied returns and capital stocks, construct a new forecasting and a pricing rule
- iterate until the rules converge.

Comparing CCL to Krusell and Smith

- In KS the need to compute a market clearing price in each period comes with the addition a new asset whose return or price is not directly implied by some simple aggregate state variable.
- Our asset pricing result buys us is essentially the same thing as having only capital in the original KS model.
- We know all prices as a function of a single moment of the distribution of multiplier distribution.
- However, our forecasting problem may be larger since we must forecast h'/h for each possible combination of current and future aggregate states.
- Though, in our particular application, since we are pricing a one-period Arrow security, the number of market prices that would have to be determined is the same.