

A Algorithm

Implementation The algorithm works as follows. We start with a matrix $\widehat{g}(s, s')$. Given this matrix, we can compute the aggregate state price in the stationary version of the economy, denoted $\widehat{P}(s')/\widehat{P}(s)$. In computing the equilibrium, we find it more convenient to keep track of agents by their consumption share c rather than their (normalized) multiplier ζ . Note that $c^{-\alpha} = \zeta$.

To come up with an initial guess for household savings $\widehat{a}_0(c, s, \eta)$, we assume that c is unchanged and we simply use the savings equation to compute \widehat{a}_0 . To compute the household savings function \widehat{a}_{j+1} in stage $j + 1$ of the iteration given \widehat{a}_j we implement the following algorithm:

1. For each savings grid point $a_i \in \mathbf{A}$, we can compute the implied consumption shares $c'(s', \eta')$, where $a_i = \widehat{a}_j(c'(s', \eta'), s', \eta')$. Since \widehat{a}_j is piecewise linear, it is trivial to invert this function.
2. Given a_i and $\tilde{c}'(s', \eta')$, we can compute recursively the consumption share today from the optimality condition for state today (s, η) . This is given by

$$E \{ \tilde{c}'(s', \eta')^{-\alpha} | s, \eta \} \widehat{g}(s, s')^\alpha = c(s, \eta)^{-\alpha},$$

If we do this for every grid point savings grid tomorrow, fixing the state today (s, η) , this yields a vector of current consumption shares \mathbf{c} and their future associated net savings levels \mathbf{a}' for each possible transition (η, s, s') . We can then fit linear piecewise linear functions to the $[\mathbf{c}, \mathbf{a}']$ for each transition (η, s, s') . Hence we have constructed $a'(c; \eta, s, s')$.

3. Given these piecewise linear functions $a'(c; \eta, s, s')$, we can compute $\widehat{a}_{j+1}(c, s, \eta)$ for each current consumption share c in our grid using the recursive saving equation in equation(4.2) since c is the consumption share today and we have already computed the future savings levels via our piecewise linear function for each possible future transition (η, s, s') :

$$\widehat{a}_{j+1}(c, s, \eta) = (c - \eta) + PDV [a'(c; \eta, s, s')]$$

In doing so, we compute a vector of current consumption shares \mathbf{c} and their associated current net savings levels $\widehat{\mathbf{a}}_{j+1}$. We can then fit linear piecewise linear functions to the $[\mathbf{c}, \widehat{\mathbf{a}}_{j+1}]$ for each (s, η) . In so doing we have constructed the function $\widehat{a}_{j+1}(c, s, \eta)$. The iterations continues until the \widehat{a}_j functions converge.

4. To simulate our economy and update \widehat{g} , we take a single panel draw of aggregate and idiosyncratic shocks. We then compute the updated consumption shares, where each period

we normalize the consumption shares to average 1, and use the normalization factor g to generate a revised estimate $\widehat{g}'(s, s') = g$. Given this revised estimate, we continue iterating until \widehat{g}' converges.

B The Separability of Aggregate and Idiosyncratic Risk

In this section, we show that the equilibrium distribution of the household multipliers does not depend on the realization of the aggregate shocks provided that all agents can trade a claim to all diversifiable income, and provided that

Condition B.1. *The aggregate shocks are i.i.d. : $\phi(z_{t+1}|z_t) = \phi(z_{t+1})$.*

Condition B.2. *The idiosyncratic shocks are independent of the aggregate shocks:*

$$\pi(\eta_{t+1}, z_{t+1}|\eta_t, z_t) = \varrho(\eta_{t+1}|\eta_t)\phi(z_{t+1}|z_t).$$

Proposition B.1. *If condition (B.2) and (B.1) are satisfied, in any economy without non-participants the equilibrium values of the multipliers ζ and the equilibrium consumption shares are independent of z^t .*

This result is an (obvious) extension of Krueger and Lustig (2006) to the case of segmented markets. In the absence of non-participants, the degree of consumption smoothing within and among different trading groups only affects the risk-free rate, not the risk premium. To prove this result, all we need to show is that the multiplier updating functions T^i do not depend on the aggregate history z^t .

Proof. We start out by noting the borrowing constraints are proportional to aggregate income. From our definition (5.1) and our asset pricing result (3.11), it follows that

$$\underline{M}(\eta^t, z^t) = -\psi \sum_{\{z^\tau \succeq z^t, \eta^\tau \succeq \eta^t\}} \gamma Y(z^\tau) \eta_\tau \frac{\pi(z^\tau, \eta^\tau) \beta^\tau C(z^\tau)^{-\alpha} h(z^\tau)^\alpha}{\pi(z^{t+1}, \eta^{t+1}) \beta^{t+1} C(z^{t+1})^{-\alpha} h(z^{t+1})^\alpha}.$$

Since the growth rate of $C(z^t)$ is i.i.d. by assumption, it follows that $\underline{M}(\eta^t, z^t)/C(z^t)$ is independent of z^t , and hence

$$\underline{M}(z^t, \eta^t) = \underline{M}(\eta^t) C(z^t).$$

Then, we define the ratio of savings to aggregate consumption \tilde{a} as follows:

$$a(\zeta(z^t, \eta^t); z^t, \eta^t) = C(z^t) \tilde{a}(\zeta(z^t, \eta^t); z^t, \eta^t). \quad (\text{B.1})$$

Our recursive relationship for $a(\zeta(z^t, \eta^t); z^t, \eta^t)$ implies that

$$\tilde{a}(\zeta(z^t, \eta^t); z^t, \eta^t) = \gamma \eta_t - \frac{\zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}}{h(z^t)} + \beta \sum_{z_{t+1}} \widehat{\phi}(z_{t+1}|z^t) \sum_{\eta_{t+1}} \varrho(\eta_{t+1}|\eta_t) \tilde{a}(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1}).$$

where $\widehat{\phi}(z_{t+1}|z^t) = \phi(z_{t+1}) \left[\frac{h(z^{t+1})}{h(z^t)} \right]^\gamma \exp((1-\gamma)z_{t+1})$. In addition, our debt constraint in terms of the savings/consumption ratio \tilde{S} is simply given by:

$$\tilde{a}(\zeta(z^{t+1}, \eta^{t+1}); z^t, \eta^t) \leq \underline{M}(\eta^{t+1}). \quad (\text{B.2})$$

The reason behind the independence result is straightforward. Start by conjecturing that $h(z^{t+1})/h(z^t)$ does not depend on z^{t+1} , and conjecture that the savings/consumption ratio $\tilde{a}(\zeta(z^t, \eta^t); z^t, \eta^t)$ does not depend on z^t . This being the case, nothing else in the recursive equation depends on the realization of the aggregate shock z_t , because $\widehat{\phi}(z_{t+1})$ does not depend on z^t , in the measurability constraints active traders or in the debt constraint. That verifies our conjecture about the savings consumption ratio. So, the measurability constraint for the active traders is independent of z_t :

$$\tilde{a}^z(\zeta(\eta^{t+1}), \eta^{t+1}) = \tilde{a}^z(\zeta(\tilde{\eta}^{t+1}); \tilde{\eta}^{t+1}) \text{ for all } \eta^{t+1}, \tilde{\eta}^{t+1} \text{ and } z^{t+1}, \quad (\text{B.3})$$

and this implies that the updating function does not depend on z^t either:

$$T^z(\eta^{t+1}|\eta^t)(\zeta(\eta^t)) = \zeta(\eta^{t+1}).$$

What about the buy-and-hold investors? Let pd_t denote the price/dividend ratio on a claim to consumption. For the buy-and-hold investors, the measurability constraint reads as:

$$\frac{\tilde{a}^{bh}(\zeta(\eta^{t+1}), \eta^{t+1})}{[(1-\gamma) + pd_{t+1}]} = \frac{\tilde{a}^{bh}(\zeta([\eta^t, \tilde{\eta}_{t+1}]); \eta^t, \tilde{\eta}_{t+1})}{[(1-\gamma) + pd_{t+1}]}$$

for all $\eta^{t+1}, [\eta^t, \tilde{\eta}_{t+1}], z^{t+1}$ and $[z^t, \tilde{z}_{t+1}]$. Since the pd_t can only evolve deterministically, given the i.i.d. shocks and the conjecture about h_{t+1}/h_t , the buy-and-hold trader faces the same measurability constraints as the z -complete traders. Hence, the buy-and-hold investor's updating function does not depend on z^{t+1} :

$$T^{bh}(\eta^{t+1}|\eta^t)(\zeta(\eta^t)) = \zeta(\eta^{t+1}).$$

This being the case, it is easy to show that h_{t+1}/h_t does not depend on z^{t+1} either, as long as there are no non-participants, simply because nothing on the right hand side depends on z^{t+1} :

$$h_{t+1} - h_t = \sum_{j \in T} \int \sum_{\eta^{t+1} \succ \eta^t} \left\{ [T^j(\eta^{t+1}|\eta^t)(\zeta(\eta^t))]^{\frac{-1}{\alpha}} \varrho(\eta_{t+1}|\eta_t) - \zeta(\eta^t)^{\frac{-1}{\alpha}} \right\} d\Phi_t^j \quad (\text{B.4})$$

where $T = \{c, z, bh\}$. □

Corrolary B.1. *Independent of the market segmentation, if all households can trade a claim to diversifiable income, the (conditional) equity risk premium is the Breeden-Lucas one.*

When $\{h_{t+1}/h_t\}$ is non-random, market incompleteness only affects the risk-free rate, not the risk premium. The consumption shares of all households do not depend on the aggregate shocks. There is no time variation in expected returns, and households only want to trade a claim to aggregate consumption to hedge against aggregate risk. All the asset market participants face the same measurability condition if $\{h_{t+1}/h_t\}$ is non-random. The distinction between active and passive traders is irrelevant, because there is no spread between state prices other than that in a representative agent model. Households all hold fixed portfolios (i.e. the market) in equilibrium, and there exists a stationary equilibrium with an invariant wealth distribution. This result implies that the multipliers are not affected by the aggregate shocks.

C Trading Redundant Assets

We need some additional notation for the sequence economy with debt, equity and state-contingent bonds (hence forth the *multiple-asset economy*). The total supply of equity shares is one. We denote the price of equity by $\omega^e(z^t)$. Of course, the price of equity is just the price of a claim to diversifiable wealth less the bonds outstanding, or

$$\omega^e(z^t) = \omega(z^t) - b(z^t).$$

The number of equity shares that an agent holds at the end of the period is denoted $e(z^t, \eta^t)$ and the number of state-contingent bonds for consumption delivered in (z^{t+1}, η^{t+1}) is denoted as $\hat{a}_t(z^{t+1}, \eta^{t+1})$. The budget constraint for a trader in the spot market in state (z^t, η^t) reads as follows:

$$\begin{aligned} & \gamma Y(z^t) \eta_t + \hat{a}_{t-1}(z^t, \eta^t) - c(z^t, \eta^t) + R_t^f(z^{t-1}) b(z^{t-1}, \eta^{t-1}) + e(z^{t-1}, \eta^{t-1}) (\omega^e(z^t) + d(z^t)) \\ & \geq \sum_{z^{t+1} \succ z^t} \sum_{\eta^{t+1} \succ \eta^t} [q(z_{t+1}, z^t) \hat{a}_t(z^{t+1}, \eta^{t+1}) \pi(\eta^{t+1} | z^{t+1}, \eta^t)] + b(z^t, \eta^t) + e(z^t, \eta^t) \omega^e(z^t) \quad (\text{C.1}) \end{aligned}$$

$\forall (z^t, \eta^t)$.

The period 0 spot budget constraint is given by

$$\omega(z^0) \geq \sum_{z_1} \sum_{\eta_1} q(z_1, z^0) \hat{a}_0(z^1, \eta^1) \pi(\eta^{t+1} | z^{t+1}, \eta^t) \quad (\text{C.2})$$

$$+ e_0(z_0) \omega^e(z^0, \eta^0) + b(z^0, \eta^0), \quad (\text{C.3})$$

where z^0 and η^0 are states representing the initial position in the planning state at time 0 before any of the shocks have been realized and where $q(z_1, z^0)$ denotes the price in this stage of a claim to consumption in period 1. This initial budget constraint allows an agent to adjust his holdings of equities, which some of our traders will use in order to conform to their portfolio limitations. In addition to their spot budget constraint, these traders also face a lower bound on the value of their net asset position

$$\hat{a}_t(z^{t+1}, \eta^{t+1}) + R_{t+1}^f(z^t)b(z^t, \eta^t) + e(z^t, \eta^t) (\omega^e(z^{t+1}) + d(z^{t+1})) \geq \underline{M}(\eta^{t+1}, z^{t+1}). \quad (\text{C.4})$$

The *active traders* can trade risk-free bonds, equities, and a complete menu of claims whose payoffs are contingent on the aggregate state z^{t+1} . This means that their state contingent bond positions must satisfy

$$\hat{a}_t(z^{t+1}, [\eta^t, \eta_{t+1}]) = \hat{a}_t(z^{t+1}, [\eta^t, \tilde{\eta}_{t+1}]) \text{ for all } z^{t+1}, \eta^t, \eta_{t+1} \text{ and } \tilde{\eta}_{t+1}. \quad (\text{C.5})$$

The other households are *passive traders* who can only trade a fixed-weighted portfolio of bonds and stocks. The buy-and-hold traders hold the fraction ω of their financial wealth in equities and $1 - \omega$ in risk-free bonds, and no state-contingent bonds. These constraints take the form

$$\frac{e(z^t, \eta^t)\omega^e(z^t)}{b(z^t, \eta^t) + e(z^t, \eta^t)\omega^e(z^t)} = \varpi \text{ and } \hat{a}_{t+1}(z^{t+1}, \eta^{t+1}) = 0 \text{ for all } z^{t+1}, \eta^{t+1}, \quad (\text{C.6})$$

The buy-and-hold traders have $\varpi > 0$, and when they set $\varpi = 1/(1 + \phi)$, they are holding the market. The nonparticipants have $\varpi = 0$, and hold only risk-free bonds.

The problem of an active trader is choosing sequences $\{c(z^t, \eta^t), e(z^t, \eta^t), b(z^t, \eta^t), \hat{a}_t(z^{t+1}, \eta^{t+1})\}$ so as to maximize her total expected utility subject to their budget constraints and debt bounds, (C.2-C.4), and the portfolio restriction in (C.5). A passive trader with equity share ϖ chooses sequences $\{c(z^t, \eta^t), e(z^t, \eta^t), b(z^t, \eta^t), \hat{a}_t(z^{t+1}, \eta^{t+1})\}$ so as to maximize her total expected utility subject to their budget constraints and debt bounds, (C.2-C.4) and their portfolio restriction (C.6). A fraction μ_a of households are active traders. A fraction μ_{bh} are passive equity traders who hold equity share $\varpi > 0$ in their portfolio, and a fraction μ_{np} are passive nonparticipants with $\varpi = 0$ (hold only risk-free bonds).

In equilibrium, we need to clear the contingent bond markets:

$$\sum_{\eta^t} \mu_a \hat{a}_{t-1}^a(z^t, \eta^t) \pi(\eta^t | z^t) = 0, \quad (\text{C.7})$$

the equity markets:

$$\sum_{\eta^t} [\mu_a e_t^a(z^t, \eta^t) + \mu_{bh} e^{bh}(z^t, \eta^t)] \pi(\eta^t | z^t) = 1, \quad (\text{C.8})$$

the non-contingent bond markets:

$$\sum_{\eta^t} [\mu_a b^a(z^t, \eta^t) + \mu_{bh} b^{bh}(z^t, \eta^t) + \mu_{np} b^{np}(z^t, \eta^t)] \pi(\eta^t | z^t) = b(z^t), \quad (\text{C.9})$$

and the goods markets:

$$\sum_{\eta^t} [\mu_a c^a(z^t, \eta^t) + \mu_{bh} c^{bh}(z^t, \eta^t) + \mu_{np} c^{np}(z^t, \eta^t)] \pi(\eta^t | z^t) = Y(z^t). \quad (\text{C.10})$$

One of these is redundant. In the multiple-asset economy, there are redundant assets, so the distribution of assets and the asset positions of the traders are not uniquely determined. However, this redundancy occurs because some traders can hold bonds and equities *and* state-contingent bonds. The only traders who can do this are the active traders. Hence *only* their asset positions are not pinned down by optimization given prices.

To see this, define the net financial wealth of a trader as

$$a_t(z^{t+1}, \eta^{t+1}) = \hat{a}_t(z^{t+1}, \eta^{t+1}) + R_{t+1}^f(z^t) b(z^t, \eta^t) + e(z^t, \eta^t) (\omega^e(z^{t+1}) + d(z^{t+1})), \quad (\text{C.11})$$

(where we are anticipating using this condition to determine the equivalent position in state-contingent bond economy with state-contingent bonds holds denoted by $a_t(z^{t+1}, \eta^{t+1})$). Then, so long as there is more than one possible realization of z_{t+1} in period $t + 1$, and the returns on the bonds and stocks are not identical across these realizations, $a_t(z^{t+1}, \eta^{t+1})$ uniquely determines the asset composition of the passive traders. Or, perhaps more appropriately, $b(z^t, \eta^t)$ and $e(z^t, \eta^t)$ uniquely determines $a_t(z^{t+1}, \eta^{t+1})$. Clearly this not the case for active traders since we can simply alter $b(z^t, \eta^t)$ or $e(z^t, \eta^t)$, and make an offsetting adjustment in $\hat{a}_t(z^{t+1}, \eta^{t+1})$.

We show that any sequence of financial allocations for each of our trading types in the multiple-asset economy will uniquely determine the equivalent financial allocation in the economy with measurability restrictions where agents only trade state-contingent bonds (here after the *contingent-bond economy*). We will show that the reverse is true for our passive traders: a financial allocation in the contingent-bond economy will uniquely determine their portfolio allocation in the original multiple-asset economy. However, it will only determine the net asset positions for the active traders in the multiple-asset economy. This last statement is, of course, unsurprising since it was only determined in the multiple-asset economy up to this extent.

To show, starting from the financial allocation in the multiple-asset economy, that there exists an equivalent financial allocation in the contingent-bond economy, simply take the trader's net

financial wealth in node (z^{t+1}, η^{t+1}) as his state contingent bond purchases in the prior node $(z^t, \eta^t) \prec (z^{t+1}, \eta^{t+1})$. It follows trivially that this sequence of bond purchases will satisfy the traders sequence budget constraint since (C.2) implies that

$$\gamma Y(z^t)\eta_t + a_t(z^t, \eta^t) - c(z^t, \eta^t) \geq \sum_{z^{t+1} \succ z^t} \sum_{\eta^{t+1} \succ \eta^t} [q(z_{t+1}, z^t)a(z^{t+1}, \eta^{t+1})\pi(\eta^{t+1}|z^{t+1}, \eta^t)]. \quad (\text{C.12})$$

It also follows that the debt bound will hold with respect to net financial wealth, or

$$a_t(z^{t+1}, \eta^{t+1}) \geq \underline{M}(\eta^{t+1}, z^{t+1}). \quad (\text{C.13})$$

We turn next to the market clearing conditions. The market clearing conditions in our original multi-asset economy imply that the market clearing condition in the state-contingent bonds will hold, or

$$\sum_{\eta^t} [\mu_a a_t^a(z^{t+1}, \eta^{t+1}) + \mu_{bh} a_t^{bh}(z^{t+1}, \eta^{t+1}) + \mu_{np} a_t^{np}(z^{t+1}, \eta^{t+1})] \pi(\eta^{t+1}|z^{t+1}) = \omega(z^{t+1}).$$

Hence we can always construct an equivalent state-contingent bond allocation that replicated the portfolio allocation coming from our multiple-asset economy with state-contingent bonds, risk-free bonds, and equities.

To show that the reverse is also true, first let the bond choices in the contingent-bond economy determine the net asset positions in the multiple-asset economy via (C.11), where $a_t(z^{t+1}, \eta^{t+1})$ is the bond position in contingent-bond economy. Then, note that for the nonparticipants in the multiple-asset economy, we can determine their bond positions by the requirement that

$$\frac{a_t^{np}(z^{t+1}, \eta^{t+1})}{R^f(z^t)} = b^{np}(z^t, \eta^t) \text{ for } (z^t, \eta^t) \prec (z^{t+1}, \eta^{t+1}).$$

To determine the portfolio positions for the buy-and-hold traders start with the requirement that

$$\frac{a_t^{bh}(z^{t+1}, \eta^{t+1})}{\varpi R^e(z^{t+1}) + (1 - \varpi)R^f(z^t)} = e^{bh}(z^t, \eta^t)\omega^e(z^t) + b^{bh}(z^t, \eta^t) \text{ for } (z^t, \eta^t) \prec (z^{t+1}, \eta^{t+1}),$$

where $R^e(z^{t+1})$ is the return on equities and is given by

$$R^e(z^{t+1}) = \frac{\omega^e(z^{t+1}) + d(z^{t+1})}{\omega^e(z^t)}.$$

This condition along with the requirement that the ratio of equities to debt is $\varpi/(1 - \varpi)$ in (C.6) determines the equity holders portfolio. We can then determine the asset positions for the active

traders that by choosing the sequence of positions $[\hat{a}_t(z^{t+1}, \eta^{t+1}), b(z^t, \eta^t), e(z^t, \eta^t)]$ so that (C.11) and the market clearing conditions holds. This is, of course, an undetermined system of equations, but any selection from the set of possible solutions will do.

Finally, in the special case in which there are only two aggregate growth shocks, we can restrict our active traders to only hold bonds and equities in the multiple-asset economy since these securities are sufficient. In this case, the active traders' portfolio is uniquely determined. We exploit this fact when we present the debt-equity composition of the active trader's portfolio.

We summarize our findings in the following proposition.

Proposition C.1. *Given an equilibrium in the sequence economy with multiple assets, there exists an equivalent equilibrium in the sequence economy in which agents trade state-contingent bonds subject to measurability constraints, and vice versa.*

D Calibration

Our calibration strategy is based on Alvarez and Jermann (2001). The first 4 moments pin down the aggregate shocks and the aggregate transition matrix. The final 3 moments identify the idiosyncratic shocks and the idiosyncratic transition matrix. transition.

D.1 Aggregate Risk

We adopt $M1$ through $M4$, the first 4 moments, directly from Alvarez and Jermann (2001) and Mehra and Prescott (1985). The first aggregate state is a recession and the second aggregate state is an expansion. The transition probability matrix is denoted:

$$\phi = \begin{bmatrix} q_s & 1 - q_s \\ 1 - p_s & p_s \end{bmatrix}$$

This implies the following invariant distribution π :

$$\begin{bmatrix} a & b \\ a & b \end{bmatrix} = \begin{bmatrix} a & b \\ a & b \end{bmatrix} \begin{bmatrix} q_s & 1 - q_s \\ 1 - p_s & p_s \end{bmatrix}$$

The relative frequency of recessions and expansions is

$$\frac{a}{b} = \frac{1 - p_s}{1 - q_s}$$

Since $a + b = 1$

$$b = \frac{1 - q_s}{2 - p_s - q_s}$$
$$a = \frac{1 - p_s}{2 - p_s - q_s}$$

The first order autocorrelation of the aggregate endowment growth shocks is given by:

$$\rho = p_s + q_s - 1$$

From the first two moments (M1 and M2):

$$M1 = \rho = -0.14$$
$$M2 = \frac{b}{a} = 2.65$$

we can pin down transition probabilities:

$$p_s = 0.1723$$
$$q_s = 0.6877$$

using these two conditions:

$$p_s + q_s - 1 = -0.14$$
$$\frac{1 - p_s}{1 - q_s} = 2.65$$

The second two moments M3 and M4 identify the aggregate consumption growth shocks. The moments are given by:

$$M3 = E(z) = 0.0183$$
$$M4 = Std(z) = 0.0357,$$

which implies that the following two conditions need to be satisfied:

$$az_R + bz_E = 1.0183$$
$$a(z_R - E(z))^2 + b(z_E - E(z))^2 = 0.0357^2.$$

Solving for z_R and z_H , we end up with these two aggregate consumption growth shocks:

$$z_R = 0.9602$$

$$z_H = 1.0402$$

and the following aggregate transition probability matrix:

$$\phi = \begin{bmatrix} 0.1723 & 0.8277 \\ 0.3123 & 0.6877 \end{bmatrix}$$

In the IID case, we replace condition M1 with $\rho = 0$. We end up with the same aggregate endowment growth shocks

$$z_R = 0.9602$$

$$z_H = 1.0402$$

but a different transition probability matrix:

$$\phi = \begin{bmatrix} 0.2740 & 0.7260 \\ 0.2740 & 0.7260 \end{bmatrix}$$

D.2 Idiosyncratic Risk

We assume that the transition matrix can be written multiplicatively:

$$\pi(z', \eta' | z, \eta) = \varrho(\eta' | \eta) \phi(z' | z)$$

We impose the following symmetry condition:

$$\varrho(\eta_L | \eta_L) = \varrho(\eta_H | \eta_H)$$

Hence, we have 1 unknown in the transition matrix and 2 unknowns in the idiosyncratic shocks η .

We use the following conditions from Alvarez and Jermann (2001) (see table 1):

$$\begin{aligned} M5 &= \text{std}(\ln \eta) = 0.71 \\ M8 &= \frac{\text{std}(\ln \eta' | z' = R)}{\text{std}(\ln \eta' | z' = E)} = 1.88 \end{aligned}$$

to identify the idiosyncratic shocks. Finally, M6 pins down the transition probability:

$$M6 = \rho(\ln \eta) = 0.89.$$

We end up with the following idiosyncratic shocks:

$$\eta = [0.1178 \quad 0.2552 \quad 0.8822 \quad 0.7448]$$

and the transition probability matrix:

$$\boldsymbol{\rho} = \begin{bmatrix} 0.945 & 0.055 \\ 0.055 & 0.945 \end{bmatrix}$$

In the IID case, we set this ratio to one instead:

$$M8 = \frac{\text{std}(\ln \eta' | z' = R)}{\text{std}(\ln \eta' | z' = E)} = 1$$

and we obtain the following η

$$\eta = [0.1947 \quad 0.1947 \quad 0.8053 \quad 0.8053]$$

and the same transition matrix

$$\boldsymbol{\rho} = \begin{bmatrix} 0.945 & 0.055 \\ 0.055 & 0.945 \end{bmatrix}$$