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SOCIAL ASSETS*

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We present a model incorporating both social and economic components and analyze their interaction. The notion of a *social asset*, an attribute that has value only because of the social institutions governing society, is introduced. In the basic model, agents match on the basis of income and unproductive attributes. An attribute has value in some equilibrium social institutions (matching patterns), but not in others. We then show that productive attributes (such as education) can have their value increased above their inherent productive value by some social institutions, leading to the notion of the *social value of an asset*.

1. INTRODUCTION

Nearly all economic decisions have a social component. Individuals care about social characteristics such as status, honor, or popularity. Even when individuals only care about "economic" variables, the nature of interactions that facilitate economic activity are themselves at least partially social in nature. This social "context" must be taken into account when studying many economic problems. If status, honor, or popularity are important to an individual, predictions about labor market response to taxes or bonuses that ignore these considerations will be less accurate than predictions that consider these factors.

In this article, we continue our interest (begun in Cole et al., 1992, 1998) in understanding the impact of economic considerations on the social environment. These papers, which focused on equilibrium social institutions with personalized enforcement mechanisms, investigate social norms that solve social dilemmas in the presence of incentives for opportunistic behavior. In these social institutions, an individual who violated the accepted norms of behavior is "punished" (in an incentive compatible way).[2]

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[2] We discuss this work in more detail in Section 7.

Social institutions can also affect the value of assets without the use of personalized enforcement. More specifically, since there is a range of possible social institutions, assets no longer need have a value independent of governing institutions. Since different social institutions can create different incentives for a variety of economically relevant behaviors and decisions, assets that have little (or no) value under some social institutions may have positive value under others. The qualitative nature of these incentives differs from those that arise with personalized enforcement and give rise to the *social value* of an asset. An agent may have a number of assets, the possession of which leads to higher utility than would be possible without them. Examples include both alienable assets that can be transferred to other people, such as machines or money, and inalienable assets that cannot be transferred—human capital. For a fixed set of preferences and technology, an asset may or may not have productive value. A machine that can produce radios and a bel canto ("beautiful singing") voice both have productive value. Other assets have no direct productive value, yet their possession may lead to higher utility. For example, it may be that in a particular society agents with lighter skin or a particular accent enjoy higher consumption than those with darker skin or a less desirable accent, even when those attributes have no productive value per se. We refer to an attribute of an agent that has value only because of a social institution as a *social asset*. The *social value of an asset* is that part of an asset's value that stems from the social institutions, that is, the part of the value beyond that attributable to the asset's productive value.

We introduce a model that combines both the social and the traditional economic aspects of society in a manner that allows for an analysis of the interaction between the social environment and agents' decisions. We consider a generational model in which men and women match and have children. Income is random and nonstorable; men and women match and then jointly consume their income. People get utility from their own consumption and their descendants' consumption. An individual's sole decision is the identity of his or her partner.

Since consumption is joint, a wealthier partner leads to higher consumption. There will then be equilibria in which each person's wealth determines his or her match. In addition to equilibria of this kind, there may be additional equilibria in which nonproductive attributes affect matching. In particular, in equilibrium, attributes that have no fundamental value can have instrumental value. Individuals care about their children's consumption, which depends on the children's (random) income. We assume it is not possible to insure against this risk. There are thus two reasons why outcomes are not fully efficient: Parents would like to insure against the consumption risk their children face and rich parents would like to transfer consumption to their children, neither of which is possible. This allows the possibility that social institutions may arise that ameliorate the resulting inefficiency.

Suppose there is a heritable attribute that is independent of income, blue eyes for example, and suppose further the attribute does not enter people's utility functions. Suppose, nevertheless, that in this society people with blue eyes are considered desirable mates, that is, that people are willing to match with a person who has blue eyes with slightly less income than with a person without blue eyes.

In such a society, people will naturally prefer their children to have blue eyes since, all else equal, they consume more. But if they prefer children with blue eyes, and blue eyes are a heritable attribute, they will naturally prefer mates with blue eyes. In other words, a preference for blue eyes may be self-fulfilling. Note that this has nothing to do with any *intrinsic* desirability for blue eyes; within this same society it could equally well have been that brown eyes were a desirable attribute. *Any* heritable attribute might serve as a social asset in this way.[3]

If the social institutions make blue eyes a desirable attribute, we see that the degree of assortativeness of matching on wealth is decreased relative to the case that matching is on wealth alone. When there is no such desirable attribute, wealthy men match only with wealthy women and vice versa. When the social institutions value an attribute such as blue eyes, some wealthy brown-eyed people match with less wealthy people with blue eyes. The consequence of social institutions that value such assets is that the variance of consumption in society is lower. When people are risk averse, the social institutions that value attributes that are fundamentally extraneous can be welfare superior to institutions that ignore such extraneous attributes. Separately from the insurance value, the attribute also allows a wealthy parent to transmit something of value to his offspring. This is desirable from a parent's point of view if the child's expected wealth is lower than the parent's, whether the child's wealth is risky or not.

The discussion above focuses on the case in which the attribute is nonproductive, that is, the attribute is completely independent of anything that enters directly into peoples' utility functions. An analogous situation can arise for productive attributes. Suppose that height is the attribute in question, and that it has a productive component; for example, a tall person may be able to reach the top shelves in a storage closet without getting a ladder, thereby being able to do some tasks more quickly than a short person. In such a scenario, height leads to a higher expected income. All people would naturally prefer tall partners in such a world, even if height did not enter directly into utility functions, since people would realize that the children they have with tall partners are more likely to have high income.

Even when the attribute has a productive component, it still may be possible to identify a social component of its value. Since it is productive, people will prefer partners with the attribute to those without, all other things equal. But if the productive advantage is small, there may be two stable matchings corresponding to those described above for the unproductive attribute case. One will have high-income people without the attribute matching with like partners, and a second will

[3] The role of a social asset here is reminiscent of that of fiat money in a model of exchange. There are, however, several important differences. First, the social asset (attribute) is inalienable. A child who inherits this asset cannot dispose of it; the only use the attribute can be put to is the "purchase" of a higher-income mate than would otherwise be the case. Second, the child who inherits the asset cannot capture the full value of the asset, as he or she *must* bequeath the asset on to their offspring. In a sense the individual who inherits the attribute captures the present flow of value from it, but is unable to capture any of the future value.

This story also has some similarities with theories of sexual selection that explain, for example, peacock tails (see Ridley, 1993, Chapter 5). We discuss the relationship in Section 7.

have high-income people without the attribute matching with low-income people with the attribute. The situation is as before: It may be that the social institutions in the society are such that if others in society value the attribute above and beyond its productive value, then it is rational for each individual to do so as well.

We emphasize that we view this model as a parable rather than a serious model of marriage and investment in human capital. The model is designed to demonstrate how the social and economic components of society interact and the role of social assets in such a model. There are many models that accomplish these goals, and we chose the particularly simple one described above for expository ease.[4] The examples of social assets in the description and motivation above were characteristics or traits that were physically embodied in the individual such as accent or height. Some social assets may have such a physical manifestation, but it is not necessary. For example, our conception of an individual's social assets includes the set of people that one knows personally.

In the next section, we formalize the model described above. In Section 3 we consider the case where the characteristics appear in half the population and where the transmission is "balanced" (so that the fraction with the characteristic is always a half). We provide conditions under which there are equilibria with nontrivial social assets, and we consider how such social assets can arise. We emphasized in the discussion above that a central concern in this article was the analysis of the interaction between peoples' decisions and the social environment. In Section 4 we analyze how the social institutions within a society—that is, what assets have value—can endogenously change over time. Some attributes are transmitted socially, such as accents or manners, and there is no reason that such transmission need be balanced. Section 5 treats the more general case of nonbalanced transmission. Finally, we drop the restriction that individuals can only affect their future offsprings' chances of acquiring the attribute through the choice of a mate. We extend our analysis in Section 6 to allow individuals access to a market to influence the chance their children will have desirable attributes, such as education. We conclude with a discussion in Section 7.

2. MODEL

There is an infinite sequence of two-period lived agents, each of which consists of a continuum of men and women. There is a single nonstorable consumption good. In each period, old men and women match and consume their combined wealth (so that the good is a public good within couples).[5] In addition, each couple has two offspring (one of each gender). The common consumption utility function for old agents is concave and denoted $U:\Re\rightarrow\Re$. Individuals care about their descendants' welfare: The utility to any matched couple is their utility from consumption plus the discounted average utility of their children, with common discount rate β.

[4] An alternative, nonmatching, model is discussed in Section 7.

[5] The critical implication of the assumption that consumption is joint is that agents prefer richer partners, all else equal. We can allow for some private consumption, as long as this implication holds.

This means, of course, that their utility depends on the consumption of all future generations.

Although agents neither take actions nor receive utility in their first period of life, they may acquire an attribute. We assume (except in Section 6) that agents can only acquire this attribute through their parents: Both offspring will have the attribute for sure if both parents possess the attribute, they will surely not have it if neither have it, and they will have it with probability ρ if one parent had the attribute.[6] For simplicity, we assume that either both offspring have the attribute or neither does. Individuals with the attribute are y agents, whereas those without the attribute are n agents. This attribute does not enter into agents' utility functions. Education is an especially interesting attribute; we consider this attribute in Section 6 and allow parents to expend resources to increase the likelihood of their children having this attribute. At present, however, we assume the transmission is exogenous.

Each agent receives an endowment of the consumption good (income) at the beginning of their second period of life. This income is either high (H) or low (L). The attribute is possibly productive: The probability that a y agent has high income (H) is $\frac{1}{2}+k$, and the probability an n agent has high income is $\frac{1}{2}-k$, $k \geq 0$. The productivity of the attribute is captured by k; if $k=0$, the attribute is nonproductive, and agents are equally likely to have high or low income.

We assume an agent's income is independent of the parents' incomes. Possible consumption levels for matched pairs are $2H$, $2L$, and $H+L$. We normalize the utility function so that $U(2L)=0$ and $U(2H)=1$ and denote the utility of the third possible consumption level, $H+L$, by u; $u \in [\frac{1}{2}, 1)$ since U is concave. An agent's income level and the presence/absence of the attribute together constitute that agent's *characteristic*.

The only decision an agent makes in this economy (except in Section 6) concerns matching. A proposed matching is *stable* if no pair of agents can increase each of their utilities by matching, taking into account the consequences for their descendants (Roth and Sotomayer, 1990).[7] Any matching of agents induces a matching on agent characteristics in the obvious manner. A matching is *strictly stable* if, for each unmatched pair of agent characteristics, agents with these characteristics would strictly decrease their utilities by matching (taking into account the consequences for their descendants).

An *allocation* in a period is a pair (μ, m), where $\mu \equiv (\mu_y, \mu_n)$ is the distribution of attributes in the economy (μ_y is the fraction of men, and of women, with the attribute, and $\mu_n = 1-\mu_y$), and m is the matching. Given μ, the distribution of characteristics is determined by the productivity of the attribute, so that, for example, the fraction of the population with high income and the attribute is $(\frac{1}{2}+k)\mu_y$. An *equilibrium* is a specification of $\{(\mu^t, m^t)\}_{t=0}^{\infty}$, where μ^t, the distribution of attributes in period t, is induced from the distribution of characteristics and

[6] Our model of attribute transmission is identical to the vertical transmission model of Cavalli-Sforza and Feldman (1981); they, however, do not consider the incentives agents have to match with different partners.

[7] Since there are no side payments, a matching will only be destabilized if *both* deviating agents strictly prefer to match.

matching in period $t-1$, and where the matching in each period is stable. Note that this notion of equilibrium is anonymous. Parents can only affect the utility of their children through the characteristics they receive.

3. A SPECIAL CASE

In this section and the next, we analyze the case where half the population has the attribute and transmission is balanced, i.e., $\rho = \frac{1}{2}$. This case is relatively straightforward to analyze for two reasons. First, with balanced transmission, the proportion of the population that has the attribute is independent of the matching. As a consequence, to analyze equilibrium, it is enough to describe the stable matchings. Second, when half the population has the attribute, the mixed matching is particularly simple, since all the *Hn*'s match with all the *Ly*'s. When the fraction of the population with the attribute is different from a half, there is rationing in the mixed matching and the calculations become more complicated; we analyze this case in Section 5.

3.1. *Stable Assortative Matching.* If the attribute is unproductive, and the distribution over offspring characteristics is independent of parents' characteristics, any matching positively assortative on income will clearly be stable. If the attribute is productive, a matching that is positive assortative on income but not on attribute cannot be stable (since an *Hy* agent can do better by matching with another *Hy* agent than with an *Hn* agent). The *assortative* matching has high-income men match with high-income women and men with the attribute match with women with the attribute:

Men		Women
Hy	$\longleftrightarrow$	Hy
Hn	$\longleftrightarrow$	Hn
Ly	$\longleftrightarrow$	Ly
Ln	$\longleftrightarrow$	Ln

It will be convenient to work with average discounted value functions. If an agent has discount factor β, the *average discounted value* of the stream of utilities $\{v_t\}_{t=1}^{\infty}$ is $\sum_{t=1}^{\infty} \beta^{t-1}(1-\beta)v_t$. By rescaling flow utility by the factor $(1-\beta)$, a constant sequence of flow utility v has average discounted value of v.

Denote by V_y^A the average discounted value function for agents who have the attribute when matching is assortative, evaluated before their income has been realized, and by V_n^A the value function of those who do not have the attribute. Note that although matching occurs after income is realized, since matching is assortative on attribute as well as income, if an agent has the attribute, then she/he will match with a partner who also has the attribute, and so their offspring has the attribute with probability 1. The uncertainty over current income translates into uncertainty only over current utility. Then,

$$V_y^A = \left(\frac{1}{2}+k\right)\left[(1-\beta)U(2H)+\beta V_y^A\right] + \left(\frac{1}{2}-k\right)\left[(1-\beta)U(2L)+\beta V_y^A\right]$$
$$= \left(\frac{1}{2}+k\right)(1-\beta)+\beta V_y^A$$

(recall our normalization that $U(2H) = 1$ and $U(2L) = 0$). Solving for V_y^A gives

$$V_y^A = \frac{1}{2} + k.$$

Similarly, we have

$$V_n^A = \frac{1}{2} - k.$$

The value of having the attribute in this equilibrium is $V_y^A - V_n^A = 2k$, which is the flow value of the productivity of the attribute.

Consider now an Hn agent. If he or she matches according to the prescribed assortative matching, the resulting utility will be $1 - \beta + \beta V_n^A$, since such a matching yields for sure children without the attribute. If this agent matches instead with an Ly agent, he or she gives up some current consumption utility, but has the chance of producing offspring with the attribute. The resulting utility is $(1-\beta)u + \frac{\beta}{2}(V_y^A + V_n^A)$, and consequently he or she would prefer to match with an Ly agent if

$$(1-u)(1-\beta) < \frac{\beta}{2}\left(V_y^A - V_n^A\right) = \beta k.$$

The constraint that an Ly prefers to match with Hn rather than another Ly is

$$u(1-\beta) + \frac{\beta}{2}\left(V_y^A + V_n^A\right) > \beta V_y^A,$$

that is,

$$u(1-\beta) > \frac{\beta}{2}\left(V_y^A - V_n^A\right) = \beta k.$$

Hence, the matching that is perfectly assortative on income and attributes is *not* stable if and only if

$$1 - u < \frac{\beta k}{1-\beta} < u. \tag{1}$$

As we indicated at the beginning of this section, it is clear that assortative matching is stable when the attribute is unproductive ($k = 0$). But the matching is also stable when the attribute is very productive ($k > u(1-\beta)/\beta$). In this case, an *Ly* agent would not want to match with an *Hn* agent, since the income gain is worth less than the expected sacrifice in terms of the matching prospects of the offspring. Here in a sense, matching is driven primarily by the attribute, and only secondarily by income.

Note also that if $u < \frac{1}{2}$ (i.e., agents are risk loving), then (1) must always be violated. In order for the assortative matching to fail to be stable, not only must a high-income agent without an attribute be willing to give up some current utility for the possibility of offspring with the attribute, but a low-income agent with the attribute must be willing to sacrifice future utility since the offspring may, as a result, not have the attribute.

It is clear that agents with the characteristic *Hy* never have an incentive to deviate, whereas *Ln* agents can never induce a matching from agents with other characteristics. Thus, we have the following proposition.

PROPOSITION 1. *Matching assortatively on income and attribute is stable if and only if either*

$$\beta k \leq (1-u)(1-\beta)$$

or

$$u(1-\beta) \leq \beta k.$$

3.2. *Stable Mixed Matching.* The second interesting matching is the *mixed matching*:

Men		Women
Hy	$\longleftrightarrow$	*Hy*
Hn	$\longleftrightarrow$	*Ly*
Ly	$\longleftrightarrow$	*Hn*
Ln	$\longleftrightarrow$	*Ln*

As in the assortative matching, *Hy*'s match with *Hy*'s and *Ln*'s match with *Ln*'s, but unlike that matching, *Hn*'s match with *Ly*'s. The question of stability of this mixed matching immediately arises when the attribute is unproductive. Why would an *Hn* give up current consumption by matching with an *Ly*, who contributes less to current consumption than an *Hn*? Clearly, if the discount factor β is 0, that is, if parents care only about their personal consumption, mixed matching is not stable: Two *Hn* agents would have higher utility by matching together than they would have if they followed the prescribed matching, whether or not other agents follow the mixed matching prescriptions.

However, if parents care about their children, there is a benefit to an *Hn* who matches with an *Ly* when all other agents are following the prescribed mixed matching. An *Hn*'s offspring will have the attribute with probability $\frac{1}{2}$ when matched with an *Ly*, but with probability 0 if he or she matches with another *Hn*. Even when the possession of this attribute does not affect the child's income, it *does* affect who they will match with. An *Ly* child will match with a high-income agent (*Hn*), whereas an *Ln* child matches with an *Ln*. Consequently, if other agents are following the prescriptions of mixed matching, the attribute has value in affecting offsprings' matching prospects (and, a fortiori, consumption prospects) even when the attribute is nonproductive. The fact that the attribute has value because of its affect on matching does not ensure that the mixed matching is stable of course. Stability will be determined by the trade-off that an *Hn* faces between the lower current consumption that matching with an *Ly* entails and the expected benefit it will confer on his or her offspring.

The value functions for agents with and without the attribute are denoted V_y^M and V_n^M (the superscript M denotes mixed matching). An agent with the attribute has income H with probability $(\frac{1}{2}+k)$; under mixed matching this agent then matches with an identical agent, jointly consumes $2H$, and has offspring who inherit the attribute with probability 1. An agent with the attribute has income L with probability $(\frac{1}{2}-k)$ and, under mixed matching, matches with an *Hn* agent. Jointly they consume $H+L$, and their offspring inherit the attribute with probability $\frac{1}{2}$. Thus, V_y^M is given by

$$V_y^M = \left(\frac{1}{2}+k\right)\left[1-\beta+\beta V_y^M\right] + \left(\frac{1}{2}-k\right)\left[u(1-\beta)+\frac{\beta}{2}\left(V_y^M+V_n^M\right)\right]. \tag{2}$$

Similarly, V_n^M is given by

$$V_n^M = \left(\frac{1}{2}-k\right)\left[u(1-\beta)+\frac{\beta}{2}\left(V_y^M+V_n^M\right)\right] + \left(\frac{1}{2}+k\right)\beta V_n^M, \tag{3}$$

and hence,

$$V_y^M - V_n^M = \left(\frac{1}{2}+k\right)\left[1-\beta+\beta\left(V_y^M-V_n^M\right)\right],$$

and so

$$V_y^M - V_n^M = \frac{(1+2k)(1-\beta)}{2-\beta(1+2k)}. \tag{4}$$

For the mixed matching to be stable, an *Hn* agent must prefer to match with an *Ly* agent rather than match with another *Hn* agent, and an *Ly* agent must prefer to match with an *Hn* agent rather than with another *Ly* agent. The incentive constraint for an *Hn* agent is

$$u(1-\beta)+\frac{\beta}{2}\left(V_y^M+V_n^M\right) \geq 1-\beta+\beta V_n^M. \tag{5}$$

Similarly, the incentive constraint for an Ly agent is

$$u(1-\beta)+\frac{\beta}{2}\left(V_y^M+V_n^M\right)\geq \beta V_y^M.$$

Combining these inequalities, a necessary and sufficient condition for the mixed matching to be an equilibrium is

$$(1-u)(1-\beta)\leq \frac{\beta}{2}\left(V_y^M-V_n^M\right)\leq u(1-\beta). \tag{6}$$

Rearranging the inequality and using (4), we have the following proposition.

PROPOSITION 2. *The mixed matching is stable if and only if*

$$1-u\leq \frac{\beta(1+2k)}{2(2-\beta(1+2k))}\leq u. \tag{7}$$

3.3. *Unproductive Attributes.* The polar case in which the attribute has no productive value, $k=0$, is of particular interest. The corresponding condition for mixed matching to be an equilibrium when the attribute is not productive is

$$1-u\leq \frac{\beta}{2(2-\beta)}\leq u.$$

Since $u\geq \frac{1}{2}$, the second inequality is satisfied for all $\beta\in[0,1]$. Hence, a sufficient condition for mixed matching to be stable when $k=0$ is the first inequality, which is equivalent to

$$u\geq \frac{4-3\beta}{2(2-\beta)}. \tag{8}$$

Figure 1 illustrates the combinations of u and β for which mixed matching is stable.

Recall that the assortative matching is necessarily stable in the case that the attribute is unproductive. Thus, in the unproductive attribute case, there are multiple stable matchings when this inequality is satisfied.

These two matchings have different economic consequences. For the case of balanced transmission, the number of agents with the attribute is independent of the matching, and hence, the distribution of income in the society is independent of the social institutions: Half of the society has income H and half has L. However, since consumption within couples is joint, different matchings may lead to different distributions of *consumption*. That the distribution of consumption differs for the two stable matchings described above is clear. Under assortative matching, high-income agents always match with high income, and hence, half of the people consume $2H$, whereas the other half consume $2L$. On the other hand, in the mixed matching, half of the low-income people (those with the attribute) match with high-income agents (the high-income people without the attribute). Hence,

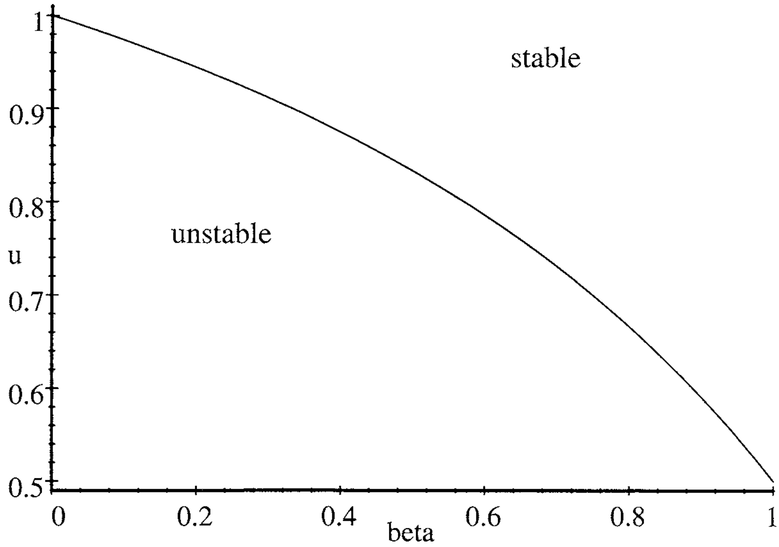

FIGURE 1

FOR ANY β, IF u IS ABOVE THE CURVED LINE, MIXED MATCHING WILL BE STABLE

only a quarter of the people consume $2H$, a quarter consume $2L$, and the remaining half consume $H + L$.

We now argue that mixed matching Pareto dominates assortative matching when the mixed matching is stable. The binding incentive constraint for the mixed matching to be stable is given by (5): an *Hn* agent prefers to match with an *Ly* agent rather than match with another *Hn* agent in this period, assuming that his offspring follow the mixed matching. Since the environment is stationary, this constraint is equivalent to the constraint that an *Hn* agent prefers to follow the mixed matching rather than have all his *Hn* offspring as well as himself match with *Hn* agents. This latter behavior is almost assortative matching by the *Hn* agent and his descendants (since the matching behavior of *Hy* and *Ln* agents in the mixed and assortative matchings agree). Assortative matching differs in that the *Ly* descendants are matching with *Hn* agents, whereas in the assortative matching they match with *Ly* agents. Consider now the payoff implications of all *Ly* descendants matching with *Ly* agents rather than with *Hn* agents (when descendants of the other characteristics, *Hy*, *Hn*, and *Ln*, match assortatively). Since the descendants with the other characteristics are ignoring the attribute in matching, this change must lower payoffs. Hence, an agent with characteristic *Hn* must (weakly) prefer the mixed matching to the assortative matching, when the mixed is stable. This then implies that an agent with characteristic *Ly* also (weakly) prefers the mixed matching to the assortative matching, when the mixed is stable. (By the above argument, such an agent's payoff is reduced if all his *Hn* descendants match assortatively and reduced even further when all *Ly* descendants, including himself, match assortatively). This then implies that agents with characteristics *Hy* and *Ln* (weakly) prefer the mixed matching to the assortative matching.

It is straightforward to verify by direct computation of the value functions that the converse is also true (see the Appendix). We summarize this in the following proposition.

PROPOSITION 3. *Suppose the attribute is unproductive* ($k = 0$). *The mixed matching (weakly) Pareto dominates the assortative matching if and only if the mixed matching is stable, i.e., inequality (8) holds.*

The interpretation is straightforward. Higher u corresponds to a more concave utility function over consumption. As in most models similar to ours, the concavity of the utility function is doing double duty, both representing agents' attitude toward risk and their rate of intertemporal substitution. To the extent that more concave utility functions reflect higher risk aversion, the value of the insurance associated with the asset has higher value. Treating concavity as measuring the rate of intertemporal substitution, more concave utility functions increase the benefit to a high-income agent of transferring consumption to his child. In both ways, more concave utility functions are associated with a greater value of the asset. For a fixed β, if agents' utility functions are sufficiently concave, regardless of income, agents obtain higher utility under mixed matching than under assortative matching.

3.4. *The Emergence of Mixed Matching.* Although the mixed matching on an unproductive attribute is welfare superior when it is stable, it is natural to ask how or why a society might end up with a matching that depends on nonproductive characteristics. Here we briefly outline one possibility: The attribute was at one time productive, and its productivity initially *requires* mixed matching for stability. At a later time, the attribute is no longer productive, and consequently, matching that ignores the attribute may become stable. Nonetheless, the existing mixed matching for which the attribute matters remains stable.

There is a simple intuition why matchings that do not pair Ly agents with Hn agents may not be stable for with productive attributes ($k > 0$). When the attribute is sufficiently productive, an Hn agent may find that the increase in expected income for his offspring more than compensates for the decrease in current consumption *independent of any change in matching prospects that might also ensue.*

We now argue that there are configurations of k, β, and u for which (7), (8), and (1) all hold. Recall that (8) is equivalent to $1 - u \leq \beta/[2(2-\beta)]$. Fix $k < \min\{(4-3\beta)/(6\beta), (1-\beta)/\beta\}$. Since $k < (4-3\beta)/(6\beta)$,

$$\frac{\beta}{2}\left[\frac{1+2k}{2-\beta(1+2k)}\right] < 1.$$

Moreover,

$$0 < \frac{\beta}{2(2-\beta)} < \frac{\beta}{2}\left[\frac{1+2k}{2-\beta(1+2k)}\right].$$

Thus, by choosing u large enough all three inequalities will be satisfied.

Consider a world in which the attribute is productive (with parameter k), and that all three inequalities are satisfied. Suppose that in every period there is a small probability p that the attribute becomes unproductive. If p is sufficiently small, the matching must be mixed in every equilibrium before the attribute becomes unproductive. Moreover, after the attribute becomes unproductive, the mixed matching remains stable. One can interpret this as an explanation as to how nonproductive attributes can be valued. They once had productive value, and the environment was such that matching *must* take this attribute into account. The eventual disappearance of the productiveness of the attribute does not upset the stability of the mixed matching.[8]

4. ENDOGENOUSLY CHANGING SOCIAL INSTITUTIONS

One of our primary interests is how social institutions within a society can change over time. It is often suggested that within some societies, values *do* change through time, evidenced by the common lament that "people just do not care about the things that used to be important." The analysis above showed that in the unproductive attribute case, both assortative and mixed matching can be stable for some values of the parameters u and β. One could simply assert that the change in values is captured by a switch from one equilibrium matching to another, but there are objections to this approach.

First, we would like the change in norms in a society to be endogenous, that is, we would like the change to arise from the underlying characteristics of the society. Explanations that simply assume that a society switches from one equilibrium to another rely on explanations that are outside the model. Since the explanations do not come from the model itself, they provide no insight into *why* the change took place.

A second objection is less conceptual but more serious. For a matching to be stable, there is an incentive constraint that no unmatched pair of agents would prefer to match rather than follow the suggested matching. The calculations in the determination of the incentive constraints assume the matching is permanent. If agents understand that the matching may change in the future, this should be incorporated into the incentive constraints if we wish to maintain our assumption that agents are fully rational.

More concretely, in the mixed matching, Hn agents prefer to match Ly's. An Hn is trading off the present period utility cost of not matching with another Hn (and getting higher consumption) with the benefit of matching with an Ly (and getting a positive probability of offspring with the desirable attribute, which will assure those offspring higher consumption). The higher expected consumption of offspring that compensates for the immediate lower consumption is less valuable

[8] Under the configuration of the parameters for which (7), (8), and (1) all hold, any equilibrium must have mixed matching while the asset is productive. We focus on the equilibrium in which mixed matching continues subsequent to the productivity change. There is, however, another equilibrium in which the necessarily initially mixed matching when the asset is productive abruptly changes to assortative matching when the asset becomes nonproductive.

if there is a chance that future generations will not "honor" the claim to higher consumption expected for agents with attribute y.

Our approach is to construct an equilibrium in which the matching specification is stochastic, with the change in matching arising from changes in the environment. The basic idea is that, as we showed above, the possibility that a mixed matching is stable depends on the relationship between u and β. The discount factor β is fixed, but we introduce income growth into the basic model. A high-income agent who matches with a low-income agent has lower utility from consumption than if he or she had matched with another high-income agent. The utility difference, however, will generally depend on the two income levels. If there is rising income, the "risk premium" an agent will pay to ameliorate the riskiness in future generations' consumption may decrease. If this risk premium *does* decrease, it may destabilize mixed matching. We illustrate next how this may occur in equilibrium. For simplicity, the discussion in this section is confined to the case of unproductive attributes.

We maintain the two-point income distribution analyzed above, but allow the possibility of a one-time income increase that occurs at a random time.[9] (Any change in the income process occurs at the end of a period after matching and before the next period's income is realized; the change is assumed to be common knowledge.) As above, there are initially two income levels, $L < H$. In each period, with probability p, the income levels increase from (L, H) to $(\alpha L, \alpha H)$, $\alpha > 1$. Once the higher-income level is reached, it remains at that level permanently.

This particular income growth process preserves relative incomes; only the level changes. If the utility function U exhibits constant relative risk aversion, the incentive constraint for stability of the mixed matching will be satisfied at the initial income level if and only if it is satisfied at the higher level.[10] This implies that if we introduce stochastic income growth as above with p sufficiently small, the mixed matching remains stable.

Suppose, however, that the utility function U exhibits decreasing relative risk aversion. In this case the risk premium associated with the random consumption of future generations will be smaller after the income increase than before, and the incentive constraint requiring a type Hn to prefer matching with a type Ly to matching with another Hn may not be satisfied after the income increase. If this is the case, only assortative matching will be stable after the income increase.

Can it be the case that prior to the income increase the mixed matching can be stable? As mentioned, we maintain rational expectations in the sense that prior to the income increase, the mixed matching must be stable when the agents *know* that there is a chance that the norm will break down in any period, and hence, that it *must* break down eventually. Recall that a matching is *strictly stable* if, for each unmatched pair of agent characteristics, agents with these characteristics would

[9] We discuss the possibility of perpetually increasing incomes below.

[10] To see this, note that a constant relative risk aversion utility function is either $U(x) = cx^{\gamma}$, or $U(x) = \ln x$. In the first case, $U(\alpha x) = \alpha^{\gamma} U(x)$, whereas in the second $U(\alpha x) = \ln \alpha + \ln x$. In either case, it is straightforward that the relevant inequalities are identical.

strictly decrease their utilities by matching (taking into account the consequences for their descendants).

PROPOSITION 4. *Suppose the attribute is unproductive and the mixed matching is strictly stable for income levels* (L, H) *and is not stable for* $(\alpha L, \alpha H)$. *Suppose income levels begin at* (L, H) *and in each period there is a probability* p *of a permanent increase to* $(\alpha L, \alpha H)$. *There exists* $\bar{p} > 0$ *such that for* $p \in (0, \bar{p})$, *it is an equilibrium for matching to be mixed whereas income is at the level* (L, H), *and for it to be assortative once income increases to* $(\alpha L, \alpha H)$.

PROOF. We denote by $V_i^M(\alpha)$, $i \in \{Hy, Hn, Ly, Ln\}$, the value functions for the agents of each type under mixed matching when incomes start at the level (L, H) and in any period there is a probability p that incomes increase to $(\alpha H, \alpha L)$, and matching changes to assortative matching at that time (with complementary probability, incomes do not increase and matching remains mixed). Denote by V_i^A (p), $i \in \{H, L\}$, the value functions under assortative matching with the higher incomes $(\alpha H, \alpha L)$. The equations for the initial value functions $V_t^M(p)$ are

$$V_{Hy}^M(\alpha) = (1-\beta) + \beta(1-p)\left[\frac{1}{2}V_{Hy}^M(\alpha) + \frac{1}{2}V_{Ly}^M(\alpha)\right] + \beta p\left[\frac{1}{2}V_H^A(\alpha) + \frac{1}{2}V_L^A(\alpha)\right],$$

$$\begin{aligned} V_{Hn}^M(\alpha) = V_{Ly}^M(\alpha) \equiv V_m^M(\alpha) &= u(1-\beta) + \beta(1-p) \\ &\quad \times \left[\frac{1}{4}V_{Hy}^M(\alpha) + \frac{1}{2}V_m^M(\alpha) + \frac{1}{4}V_{Ln}^M(\alpha)\right] + \beta p\left[\frac{1}{2}V_H^A(\alpha) + \frac{1}{2}V_L^A(\alpha)\right], \end{aligned}$$

and

$$V_{Ln}^M(\alpha) = \beta(1-p)\left[\frac{1}{2}V_{Hn}^M(\alpha) + \frac{1}{2}V_{Ln}^M(\alpha)\right] + \beta p\left[\frac{1}{2}V_H^A(\alpha) + \frac{1}{2}V_L^A(\alpha)\right].$$

Since the assortative matching value functions are bounded, as $p \to 0$, each value function $V_t^M(\alpha)$ converges to the value function $V_t^M(0)$, that is, the value functions calculated in the previous section. Hence, since the incentive constraint for the case in which income is unchanging is satisfied with strict inequality, for sufficiently low probability p, it will be satisfied for the case in which incomes increase with probability p. ■

To summarize: If at the initial income levels, the mixed matching is stable with a strict inequality in the incentive constraint, and if at the increased income level the incentive constraint is not satisfied, there will be an equilibrium in which matching is based on the mixed matching until incomes increase, at which point the matching must change to the income only ranking.

We can easily generalize this observation to perpetually (stochastically) increasing incomes. In each period there are two income levels. In the first period, the incomes are $H_1 = H$ and $L_1 = L$. In period t, the incomes are $(\alpha_t H, \alpha_t L)$, $\alpha_t \geq 1$. As before, the relative wealth levels stay the same but the incomes grow over time. The income factors α_t are stochastic with $\alpha_t = \alpha_{t-1}$ with probability

$1 - p \in (0, 1)$, and $\alpha_t = \alpha_{t-1} + \gamma$ with probability p.[11] Suppose that the utility function U exhibits decreasing relative risk aversion. Then the value of the insurance to an *Hn* agent from a match with a high attribute partner is decreasing, and the opportunity cost in terms of forgone current consumption to obtain that insurance is increasing. If at some point, it is not sufficient to offset the immediate utility loss from consumption that results from a match with an *Ly* agent, mixed matching is not stable, and matching will be assortative. However, if the initial income levels are such that the incentive constraint for stability of mixed matching is satisfied with strict inequality, then for sufficiently small p, there will be an equilibrium characterized by mixed matching that will be stable as long as that incentive constraint is satisfied, and assortative matching after that. Furthermore, if $R(x) = -x\frac{U''(x)}{U'(x)} \to 0$ as $x \to 0$, then the incentive constraint for mixed matching will eventually be violated with probability 1.

This result can be interpreted as the sure eventual demise of social institutions that depend on non-payoff-relevant criteria when there is asymptotically vanishing relative risk aversion. It is interesting to note that at the point at which the matching regime changes, there may be only a small change in the income distribution, but a large change in the distribution of consumption. Under assortative matching, all high-income agents match with other high-income people, whereas in mixed matching, half the high-income agents match with low-income agents. The collapse of mixed matching is accompanied by a large increase in the variance of consumption.

5. THE GENERAL CASE

We have thus far focused on the special case where half the population always had the attribute. We now extend our analysis to more general situations. This includes both balanced transmission (where the probability of transmission in a couple in which one parent has the attribute, ρ, equals one half) with nonuniform distribution of attributes (i.e., $\mu = (\mu_y, \mu_n) \neq (\frac{1}{2}, \frac{1}{2})$), as well as unbalanced transmission where ρ may be different from one half. For simplicity, we continue to assume the attribute is nonproductive, i.e., $k = 0$.

When matching is assortative, either both parents have the attribute or both parents do not have the attribute. Consequently, the values of ρ and μ are irrelevant, and the fraction of the population that has the attribute is stationary. Moreover, when the attribute is nonproductive, the assortative matching is stable.

When matching is mixed, on the other hand, there are many matched pairs in which only one parent has the attribute. Moreover, when $\mu_y \neq \frac{1}{2}$, there will be different numbers of agents of characteristic *Hn* and *Ly*, so that the larger set is

[11] We assume that the increases in income, γ, do not depend on the period or the current income level for expositional ease only. We could allow the size of the increases to depend on these without changing any of the analysis as long as the increases are bounded above. Similarly, the probability that incomes may rise at any time may depend on the period and the current income level; the constraint will be on the maximum probability of an income change in any period.

rationed. In the mixed matching, all agents in the smaller group match with agents from the other group, and the remaining agents from the larger group are matched with agents with the same characteristics.

We calculate the difference equation describing the evolution of the fraction of the population with the attribute for general ρ. Observe first that agents with the attribute in period t, in proportion μ_y^t, are equally likely to have high income, H, or low income, L. In terms of keeping track of μ^t, we can think of each agent being replaced by a single child in the next period. Each agent with the attribute and income H will have a child with the attribute, since the parent is matched with an agent who also has the attribute. Hence, each Hy agent (in proportion $\frac{1}{2}\mu_y^t$) has a child with the attribute. Similarly, Ln agents match with the same type and contribute no children with the attribute the next period. If $\mu_y^t \geq \frac{1}{2}$, there are more agents with the attribute than without. Hence, there are fewer Hn agents than Ly agents. Consequently, all Hn agents will be in mixed attribute matches, and this group will contribute $\frac{1}{2}\mu_n^t\rho$ children to the pool of agents with the attribute in the next period. The Ly agents who are in mixed matches with an Hn agent also have probability ρ of having a child with the attribute; hence the contribution from this type to the pool of agents with the attribute next period is $\frac{1}{2}\mu_n^t\rho$. Finally, the Ly agents who do not match with Hn agents (there are $\frac{1}{2}(\mu_y^t - \mu_n^t)$ of these) will instead match with other Ly agents and have a child with the attribute with probability 1. Summing over these, the proportion of agents with the attribute in period $t+1$, when the proportion in period t is μ_y^t, is $\frac{1}{2}\mu_y^t + \frac{1}{2}\mu_n^t\rho + \frac{1}{2}\mu_n^t\rho + \frac{1}{2}(\mu_y^t - \mu_n^t) = \mu_y^t + (\rho - \frac{1}{2})\mu_n^t$. When $\mu_y^t < \frac{1}{2}$, there are more Hn agents than Ly agents, and so not all Hn agents match with an Ly. Only $\frac{1}{2}\mu_y^t$ will do so, and only these have the possibility of having a child with the attribute. Agents with characteristic Ln match with agents of the same type and contribute no children possessing attribute y in the next period. Hence the proportion of children with the attribute in period $t+1$, μ_y^{t+1}, is $\mu_y^t(\frac{1}{2} + \frac{1}{2}\rho) + \frac{1}{2}\mu_y^t\rho = \mu_y^t(\frac{1}{2} + \rho)$. Combining these two cases, we have

$$\mu_y^{t+1} = \begin{cases} \mu_y^t\left(\frac{1}{2} + \rho\right), & \text{if } \mu_y^t < \frac{1}{2} \\ \mu_y^t + \left(\rho - \frac{1}{2}\right)\mu_n^t, & \text{if } \mu_y^t \geq \frac{1}{2} \end{cases}. \tag{9}$$

We first consider the case $\rho = \frac{1}{2}$. Any fraction of the population with the attribute is now clearly a steady state of the dynamic (9). We argued at the beginning of Subsection 3.3 that inequality (8) was a necessary and sufficient condition for the mixed matching to be stable when $\mu_y = \frac{1}{2}$. An important consideration comes into play when $\mu_y \neq \frac{1}{2}$: If the fraction with the attribute is too large, then an Ly cannot be guaranteed a match with an Hn agent, lowering the value of having the attribute. Suppose $\mu_y > \frac{1}{2}$ and let $\alpha = \mu_n/\mu_y$ be the probability an Ly agent matches with an Hn agent (so that with probability $1-\alpha$, an Ly matches with another Ly). Then the value of having the attribute is (compare with (2) when $k = 0$)

$$V_y^M = \frac{1}{2}\big[1 - \beta + \beta V_y^M\big] + \frac{1}{2}\left[\alpha\left\{u(1-\beta) + \frac{\beta}{2}\big(V_y^M + V_n^M\big)\right\} + (1-\alpha)\beta V_y^M\right].$$

The value of not having the attribute is still given by (3), since *Hn* are not rationed. Hence,

$$V_y^M - V_n^M = \frac{1}{2}\big[1 - \beta + \beta\big(V_y^M - V_n^M\big)\big] - \frac{(1-\alpha)}{2}\left[u(1-\beta) + \frac{\beta}{2}\big(V_y^M - V_n^M\big)\right],$$

and so

$$V_y^M - V_n^M = \frac{2(1-\beta)[1 - u + \alpha u]}{4 - \beta - \alpha\beta}.$$

Stability of the mixed matching still requires that an *Hn* agent must prefer to match with an *Ly* agent rather than match with another *Hn* agent, and an *Ly* agent must prefer to match with an *Hn* agent rather than with another *Ly* agent. Consequently, inequality (6) (with $k = 0$) is still the relevant condition. Not surprisingly, since the attribute is not productive, an *Ly* agent always wants to match with an *Hn* agent, irrespective of the value of α. This is not true of *Hn* agents. Simplifying (5) in this case gives the relevant inequality for stability as

$$u \geq 1 - \frac{\alpha\beta}{2(2-\beta)},$$

which is an increasingly severe restriction on u as α becomes small (i.e., as μ_y approaches 1). Intuitively, if μ_y is too close to 1 (so that α is close to 0), the matching with respect to *income* is almost the same as in assortative matching. That is, nearly all *Ly*'s match with agents of the same type, in particular with low-income agents. Hence the difference between the expected utility for a child with the attribute and without is arbitrarily small. There is little insurance value in having the attribute, so an *Hn* agent will prefer to match with another *Hn* agent to matching with an *Ly* agent. That is, mixed matching is not stable.

A similar calculation for the case $\mu_y < \frac{1}{2}$ (so that *Hn* is now rationed, whereas *Ly* is not), shows that (8) is the relevant inequality for stability in this case (as it is when $\mu_y = \frac{1}{2}$). The logic on Pareto dominance from Proposition 3 applies here as well, so we have:

PROPOSITION 5. *Suppose $k = 0$ and $\rho = \frac{1}{2}$. If $\mu_y \leq \frac{1}{2}$, the mixed matching is stable if and only if (8) holds. If $\mu_y > \frac{1}{2}$, the mixed matching is stable if and only if*

(10) $$u \geq 1 - \frac{\mu_n\beta}{2\mu_y(2-\beta)}.$$

Moreover, when the mixed matching is stable, it Pareto dominates assortative matching.

5.1. *Unbalanced Transmission.* We now turn to unbalanced transmission of attributes, which does not preserve the population fractions, i.e., $\rho \neq \frac{1}{2}$. There are two cases with very different properties, corresponding to whether ρ is smaller or larger than $\frac{1}{2}$. If $\rho < \frac{1}{2}$, then both parents having the attribute results in a more than proportionate increase in the probability that offspring will have the attribute. Consequently, we say that we have *economies of scale* in the transmission of the attribute. Conversely, if $\rho > \frac{1}{2}$, we have *diseconomies of scale*.

An example of an attribute that displays economies of scale might be the ability to converse intelligently. If both parents have this attribute, it will be passed on to the children because of the social interactions that occur within the family, whereas if only one parent has the attribute, then it may be less likely that the child acquires the attribute. On the other hand, if the attribute is the ability to play the piano, it may make little difference in the probability that a child acquires the attribute whether one or two parents possess the attribute.

When $\rho < \frac{1}{2}$, couples in which only one parent has the attribute have a less than even chance of producing offspring with the attribute; the fraction of the population with the attribute will decline. The value $\mu = 0$ is the globally stable rest point of (9). Moreover, in comparison with balanced transmission, the expected value of the insurance received by matching with an *Ly* agent is reduced, since offspring have a smaller probability of acquiring the attribute. Denote by $\bar{\mu}$ the value of μ_y for which (10) holds as an equality. We then have:

PROPOSITION 6. *Suppose $\rho < \frac{1}{2}$, $k = 0$, and (8) holds strictly. For all $\mu_y^0 \in (0, \bar{\mu})$, there exists $\underline{\rho} \in (0, \frac{1}{2})$ such that if $\rho \in (\underline{\rho}, \frac{1}{2})$, then the mixed matching is stable. Moreover, the fraction of the population with the attribute converges to 0 and the utility of the representative agent in this matching converges to that of the representative agent in the assortative matching.*

The analysis is a straightforward variant of that above, and we simply discuss the intuition for stability. Clearly if the probability of transmission, ρ, is too small, the mixed attribute matching cannot be stable. An *Hn* can match with another *Hn* and get higher utility from consumption, and the continuation payoffs will be nearly the same as if he matches with an *Ly* agent.

However, similarly to the stochastically increasing income case analyzed in the previous section, when ρ is sufficiently close to $\frac{1}{2}$, the value functions will be nearly the same as in the case with $\rho = \frac{1}{2}$. Thus, since the incentive constraint when $\rho = \frac{1}{2}$ is satisfied with strict inequality (by assumption, (8) holds strictly), the analogous incentive constraint will be satisfied when ρ is close to $\frac{1}{2}$.

Note that the proportion of agents with the attribute does not go to 0 under all matchings. In particular, under the assortative matching (which guarantees that attributes match with attributes), all children born to parents with the attribute will have the attribute, and the proportion is unchanged over time.

We turn now to $\rho > \frac{1}{2}$, that is, the expected number of children with high attribute coming from mixed matches (matches with exactly one parent with high attribute) is greater than 1. In contrast to the case where $\rho < \frac{1}{2}$, now the value $\mu = 1$ is the globally stable rest point of (9) and so mixed matching is no longer stable.

PROPOSITION 7. *Suppose* $\rho > \frac{1}{2}$ *and* $k = 0$. *The mixed matching is not stable.*

In this case, asymptotically all agents possess the attribute, and we have already seen that the mixed matching cannot be stable in this case.[12] But then the prescribed matching will "unravel," that is, in the period prior to that in which the incentive constraint is violated, no *Hn* will match with an *Ly*, and hence the same in the period prior to this, and so on. In other words, mixed matching cannot be stable.

It is worth mentioning that it is inconsistent with equilibrium to have initially mixed matching, and then at some time t (when μ_y^t has become sufficiently close to 1) switch to assortative matching. The large population means that the dynamics on the fraction of the population with the attribute are deterministic, and so the last possible trigger date is common knowledge. But then, as we have just argued, mixed matching will break down in the previous period, and so on.

It is easy to see why mixed matching cannot be stable, since forward-looking agents will see that it cannot forever be stable; hence it will unravel. Here, as in most unraveling arguments, the unraveling is highly sensitive to particular features of the model. If it were not common knowledge that the matching would unravel, it may be stable for a long time and eventually break down in a manner analogous to the situation with bubbles in finite horizon rational expectations models. Similarly, we could have mixed matching stable if there was a small stochastic component similar to that introduced in the case of endogenously changing social institutions above.

6. ENDOGENOUS ATTRIBUTE CHOICE

A central feature of the analysis above is that an attribute may have value in matching both because it has direct productive value and it has social value, that is, because it enhances matching prospects. If the attribute is productive, agents who possess the attribute are, of course, more attractive mates. Although the productive value of the attribute is the same across different matching rules, the social value is not. If there are investment opportunities available to parents that can affect the chances their children will possess the attribute, the return to the investment will then depend on the social institutions governing matching. Our aim in this section is to demonstrate that social institutions can affect the proportion of agents who possess the attribute, and a fortiori, average income.

We return to the productive attribute case, but allow parents to purchase the attribute for their children if they did not inherit it. We first modify the process by which a new generation inherits the attribute from the previous generation. We assume that if both parents have the attribute, both children inherit the attribute with probability $2p < 1$, and if one parent has the attribute, both children inherit the attribute with probability p. This specification ensures that the proportion of

[12] When $k > 0$, the fact that all agents asymptotically possess the attribute under mixed matching does not necessarily ensure that mixed matching will be unstable. Although the insurance value of the asset asymptotically disappears in this case, it still has direct productive value.

people in any generation who inherit the attribute is $2p$ times the proportion of people in the previous generation that had the attribute.[13] If no couples purchase the attribute for their children, the attribute asymptotically disappears from the population.

In addition to the possibility of inheriting the attribute, we allow parents to make investments that allow their children to acquire the attribute with positive probability in the event that the child does not inherit the attribute. Specifically, we assume that if offspring do not inherit the attribute, they obtain the attribute with probability q if parents pay a cost $c(q)$, where $c'(q) \geq 0$ and $c''(q) > 0$, and $c'(0) = 0$. Parents must fund the investments from current income. The choice of expenditure on attribute is made *after* the realization of whether the child has inherited the attribute from his or her parents. As before, agents with the attribute have probability $\frac{1}{2} + k$ of having high income and those without the attribute have probability $\frac{1}{2} - k$.

We are interested in the proportion of parents who purchase education in any period in a stationary equilibrium, and in how that proportion is affected by matching. We provide conditions under which the value of the attribute is higher under mixed matching than under assortative matching. When the value of the attribute is higher, all parents whose children did not inherit the attribute will invest more to increase the probability that their offspring will acquire the attribute, and consequently, the proportion with the attribute will be higher. Since the attribute is productive, this implies that aggregate income will be higher under mixed matching than under assortative matching.

We first describe the steady state under assortative matching. In this case, Hn agents do not match with Ly agents; hence, the value of having the attribute is independent of the proportion of people in the population with the attribute, and so of expenditures on the attribute by other agents. As before, we denote the continuation values of children with the attribute and without the attribute (prior to the realization of their income) by V_y^A and V_n^A. The benefit to parents who purchase probability q of acquiring the attribute for their children is then $q(V_y^A - V_n^A)$, and marginal benefit is $(V_y^A - V_n^A)$. For $k > 0$, the marginal benefit is strictly positive. The marginal cost to parents is the marginal utility of forgone consumption. Hence, for a couple with total income $2H$, the marginal cost is $U'(2H - c(q))c'(q)$. This expression is clearly increasing in q, and the couple's optimal q, denoted q_{HH}^A, solves

$$(1-\beta)U'(2H - c(q))c'(q) = \beta\big(V_y^A - V_n^A\big).$$

Similarly, the optimal purchases for families with one high and one low income (if there were any) satisfies

$$(1-\beta)U'\big(H + L - c\big(q_{HL}^A\big)\big)c'\big(q_{HL}^A\big) = \beta\big(V_y^A - V_n^A\big),$$

[13] If $p = \frac{1}{2}$ and a proportion of parents bounded away from 0 purchase the attribute, asymptotically, all agents will have the attribute. Consequently, for reasons analogous to those outlined in the diseconomies-of-scale case, mixed matching is not stable.

and the choice for families with two low incomes solves

$$(1-\beta)U'\big(2L-c(q^A_{LL})\big)c'\big(q^A_{LL}\big)=\beta\big(V^A_y-V^A_n\big),$$

where q^A_{xy} denotes the optimal q for a couple whose respective incomes are x and y. It is straightforward to show that, if $k>0$, $1>q^A_{HH}>q^A_{HL}>q^A_{LL}>0$.

These values imply the steady-state fraction of the population with the attribute, μ^A_y:

$$\mu^A_y=\frac{q^A_{HH}(1-2k)+q^A_{LL}(1+2k)}{2\{(1-2p)(1-q^A_{LL})+q^A_{HH}-(1+2k)(1-p)(q^A_{HH}-q^A_{LL})\}}.$$

Just as in the productive exogenous attribute case, assortative matching may or may not be stable.

Consider now mixed matching. Under mixed matching, the values of having the attribute are no longer independent of the proportion of people in the population who have the attribute. With mixed matching, an Hn agent matches with an Ly agent *if possible*. The "if possible" modifier is necessary since there may not be equal numbers of the two types. When there are more of one than another, some of those on the long side of the market will not be able to participate in a mixed match, and instead will be matched with others of the same type as themselves.

We are interested in understanding when each of the different matchings is an equilibrium. For mixed matching to be an equilibrium, it must be in the interests of the Hn and Ly agents to match with each other. The incentive constraints for each type of agent to prefer this match to a match with a partner of the same type will depend on the proportion of agents with the attribute and the expenditures the different matches will make on the attribute should their children *not* inherit the attribute. We first state the following proposition that there are steady-state proportions and expenditures; the proof of the proposition is left to the Appendix.

PROPOSITION 8. *Let μ_y be the fraction of the population with the attribute and q^M_ℓ, $\ell\in\{HH,HL,LL\}$, be the probability that income pair ℓ has purchased, assuming mixed matching. There exist steady-state values of μ_y and $q^M\equiv(q^M_{HH},q^M_{HL},q^M_{LL})$.*

As with assortative matching, we must address the stability of mixed matching. We next present two examples in which mixed matching is stable with productive attributes. In the first example, assortative matching is also stable and gives a lower per capita income than mixed matching, whereas it is not stable in the second.

EXAMPLE 1. The cost function is $c(q)=\alpha q^2$ and $k=0$. Since $k=0$, under assortative matching, no agents will purchase a positive probability of their children acquiring the attribute. We denote by $\mu_y^A(0)$ the proportion of agents who have the attribute under assortative matching when k equals 0; since $p < \frac{1}{2}$, $\mu_y^A(0) = 0$.

If mixed matching is stable, the attribute has value and, because $c'(0) = 0$, all couples whose children have not inherited the attribute will purchase positive probabilities of their children acquiring the attribute. The proportion of agents who have the attribute under mixed matching when k equals 0, $\mu_y^M(0)$, is strictly positive.

Mixed matching may not, however, be stable. For example, if it is very inexpensive for couples to purchase the attribute, an Hn agent will prefer to match with another Hn agent and use the additional family income to purchase high probability of acquiring the attribute to getting probability p of children having the attribute by matching with an Ly agent. However, for sufficiently high α, matching with an Ly agent will be more cost effective for an Hn agent to secure a given probability of offspring with the attribute than relying on the "after market."

To summarize, if α is sufficiently high and $k=0$, both positive assortative matching and mixed matching will be stable and $\mu_y^M > \mu_y^A = 0$. Furthermore, it is easy to see that μ_y^M is larger for larger p since larger p implies a higher expected number of descendants who will have the attribute.

The value functions V_y^M and V_n^M are continuous in k at $k=0$. Consequently, if the incentive constraints for mixed matching to be stable are satisfied with strict inequality when $k=0$, they will still be satisfied for k small enough. Thus, if mixed matching is stable with strict inequalities on matching for $k=0$, mixed matching will be stable for positive, but small, k. Since V_y^M, V_n^M, V_y^A, and V_n^A are continuous in k at $k=0$, $\mu_y^A(\cdot)$ and $\mu_y^M(\cdot)$ are continuous; hence, for k small, $\mu_y^A(\cdot) < \mu_y^M(\cdot)$. In words, more agents have the productive attribute under mixed matching than under assortative matching. In the case of nonproductive attributes, the matching affected the distribution of income in the society, but not the aggregate income. For the productive example described above, matching affects both the total societal income and its distribution.

It is easy to see why mixed matching leads to greater number of agents with the attribute by looking at the problem facing a couple whose child has not inherited the attribute, $\max_q (1-\beta)U(l-c(q)) + q\beta(V_y - V_n)$ (l is the pair's combined income). The first-order conditions for this problem are

$$(1-\beta)U'(l-c(q))c'(q) = \beta\big(V_y^S - V_n^S\big), \quad S \in \{A, M\}.$$

The left-hand side is the marginal utility cost of q, and the right-hand side is the marginal benefit. The marginal benefit is close to 0 under assortative matching when k is small, but bounded away from 0 for small k under mixed matching due to the "social" benefits of the attribute (i.e., the insurance benefits the attribute provides). The greater marginal value of the attribute under mixed matching naturally leads to higher investment in the attribute.

As noted, this example is driven by the higher marginal value of the attribute in the mixed matching equilibrium than in the assortative matching equilibrium, and the attendant higher investments that result from this. We now present a second example in which mixed matching is stable, whereas assortative is not.

EXAMPLE 2. We first consider the incentive constraints describing a mixed pairing under either assortative or mixed matching (these should be satisfied for the mixed matching to be stable, and one must be violated for the assortative matching to be stable). Fix a matching, assortative or mixed, and let $V_y(V_n)$ denoted the expected utility of an agent with (without) the attribute under that matching. Children from an *HnLy* match acquire the attribute in two ways: They either inherit the attribute with probability p, or failing to inherit, their parents invest $c(q_{HL})$ toward this end. Thus the (unconditional) probability that the offspring of matched pairs *HnLy* have the attribute is $p + (1-p)q_{HL}$, which we denote by p_{HL}. Analogously, we denote by $p_{LL} = 2p + (1-2p)q_{LL}$ the (unconditional) probability that the offspring of matched pairs *LyLy* have the attribute. An *Hn* agent prefers to match with an *Ly* agent if

$$\begin{aligned}(11)\qquad & p\{(1-\beta)U(H+L)+\beta V_y\}+(1-p)\{(1-\beta)U(H+L-c(q_{HL})) \\ &\qquad +\beta[q_{HL}V_y+(1-q_{HL}V_n)]\} \\ &\geq (1-\beta)U(2H-c(q_{HH}))+\beta(q_{HH}V_y+(1-q_{HH})V_n).\end{aligned}$$

Analogously, an *Ly* agent prefers matching with an *Hn* rather than another *Ly* agent if

$$\begin{aligned}(12)\qquad & p\{(1-\beta)U(H+L)+\beta V_y\}+(1-p)\{(1-\beta)U(H+L-c(q_{HL})) \\ &\qquad +\beta[q_{HL}V_y+(1-q_{HL}V_n)]\} \\ &\geq 2p\{(1-\beta)U(2L)+\beta V_y\}+(1-2p)\big\{(1-\beta)U\big(2L-c\big(q_{LL}^M\big)\big) \\ &\qquad +\beta\big(q_{LL}^M V_y+\big(1-q_{LL}^M\big)V_n\big)\big\}.\end{aligned}$$

Rearranging (11) yields

$$\begin{aligned}(13)\qquad & \beta(p_{HL}-q_{HH})(V_y-V_n) \\ &\geq (1-\beta)\{U(2H-c(q_{HH}))-[pU(H+L)+(1-p)U(H+L-c(q_{HL}))]\},\end{aligned}$$

whereas (12) yields

$$\begin{aligned}(14)\qquad & \beta(p_{HL}-p_{LL})(V_y-V_n) \\ &\geq (1-\beta)\{[2pU(2L)+(1-2p)U(2L-c(q_{LL}))] \\ &\qquad -[pU(H+L)+(1-p)U(H+L-c(q_{HL}))]\}.\end{aligned}$$

Mixed matching is stable (and assortative matching unstable) if the two inequalities (13) and (14) are satisfied.

For this example, we assume $L = 0$, $H > \frac{p}{(1-p)}$, and $U'(0) = \infty$, so that two matched low-income agents have no money to purchase the attribute, implying $q_{LL} = 0$ and $H - c(q_{HL}) > 0$. Fix ε, $\eta > 0$ small and assume the cost function satisfies

$$c(q) = \begin{cases} \eta q, & q < p/(1-p) \\ \bar{c}, & q = 2p - \varepsilon \end{cases},$$

with $c'(2p - \varepsilon) = \infty$, $\bar{c} < H$, and c convex on $[0, 2p - \varepsilon]$. These assumptions on the cost function ensure that (for sufficiently small η; see below) the investment choices for the matched pairs HL and HH satisfy the following inequality:

$$\frac{p}{(1-p)} < q_{HL} < q_{HH} < 2p - \varepsilon. \tag{15}$$

Note that these inequalities hold under both mixed and assortative matching. Consequently,

$$\begin{aligned} p_{HL} - p_{LL} &= p + (1-p)q_{HL} - 2p \\ &= (1-p)q_{HL} - p > 0 \end{aligned}$$

and

$$\begin{aligned} p_{HL} - q_{HH} &= p + (1-p)q_{HL} - q_{HH} \\ &> 2p - q_{HH} > \varepsilon. \end{aligned}$$

Hence, the left-hand sides of the inequalities (13) and (14) are positive. Since $L = 0$, and consequently $q_{LL} = 0$, the right-hand side of (14) equals (ignoring the $(1 - \beta)$ term)

$$U(0) - [pU(H) + (1-p)U(H - c(q_{HL}))],$$

which is negative, since $U(H) > 0$ and $U(\cdot) \geq 0$. This immediately gives us (14).

Next consider (13). We first obtain a lower bound on $V_y - V_n$ that is independent of the behavior of the utility function above $H - \bar{c}$. Note that every paired term in the expression

$$\begin{aligned} V_y - V_n = \frac{1}{2}&\{(V_{Hy} - V_{Hn}) + (V_{Ly} - V_{Ln})\} \\ &+ k\{(V_{Hy} - V_{Ly}) + (V_{Hn} - V_{Ln})\} \end{aligned}$$

is nonnegative, so that any term can serve as a lower bound for $V_y - V_n$.

Suppose the matching under consideration is the mixed matching. The fraction of the population with the attribute μ_y in steady state is bounded above by $4p(1-p)$ (from (15)). Note that this bound is independent of the utility function. Thus the probability that an Ly agent is matched with a Hn agent is at least

$$\min\left\{1, \frac{1-4p(1-p)}{4p(1-p)}\right\} \equiv \xi,$$

and with probability p the offspring of that match will have the attribute, and so

$$\begin{aligned} V_y - V_n &\geq \frac{1}{2}(V_{Ly} - V_{Ln}) \\ &\geq \frac{1}{2}\xi p(1-\beta)U(H). \end{aligned}$$

Thus, (13) is implied by

$$\beta\varepsilon\frac{1}{2}\xi pU(H) \geq U(2H - c(q_{HH})) - U(H - c(q_{HL})).$$

But this is clearly satisfied by any utility function that displays sufficient risk aversion. It remains to provide the appropriate upper bound on η (to ensure $q_{HL} > p/(1-p)$). The first-order condition determining q_{HL} is

$$(1-\beta)c'(q_{HL})U'(H + c(q_{HL})) = \beta(V_y - V_n),$$

so it enough that

$$\eta < \frac{\beta\frac{1}{2}\xi pU(H)}{U'(2H)}.$$

Turning to assortative matching, observe that

$$V_y - V_n \geq k(V_{Hy} - V_{Ly}) \geq k2p(1-\beta)U(2H)$$

and so (13) is satisfied (and η is sufficiently small) when

$$2k > \frac{\xi}{2}.$$

To summarize, mixed matching is stable for this configuration of cost function and incomes in the example if the agents are sufficiently risk averse. At the same time, if the attribute is sufficiently productive, the assortative matching is not stable. The cost function in the second example was deliberately chosen to have a particular "threshold" form: The cost of acquiring the attribute was relatively low until a point at which it increased steeply. These characteristics of the cost function guarantee that HL couples invest nearly as much as HH couples in the event that their offspring do not inherit the attribute. On the other hand, by setting L very low (0 in the extreme case), LL couples can invest little (or none) in the attribute. Under assortative matching, even though there is no social value to the attribute, a mixed matching is profitable because the concavity of the utility function together with the structure of the cost function imply that the short-run cost for an Hn agent of matching with an Ly is dominated by the productiveness of the attribute.

7. DISCUSSION

1. *Related literature.* Our earlier papers, Cole et al. (1992) and Cole et al. (1998) (hereafter collectively CMP), analyze a growth model incorporating matching between men and women. In that model, there are multiple equilibria characterized by different matchings between men and women. The current paper shares with those papers the feature that different matching arrangements lead to different economic choices—attribute choice in this paper and savings/bequests there. As we indicated in the Introduction, in contrast to the model here, CMP is interested in matching arrangements that can be supported using personalized punishments.[14] Corneo and Jeanne (1998) analyze a variant of the model in CMP in which each agent's characteristics were not perfectly observable to other agents. In addition to results similar to those in CMP, Corneo and Jeanne show the possibility of societal segmentation in which agents are affected by the characteristics of only a subset of other agents. Heller (2003) applies the principles of the model in this paper to investigate when the value of tradable assets may depend on the social institutions of a society.

Cozzi (1998) analyzes an overlapping generations model in which individuals can invest in a social asset that he calls "culture." Culture has no consumption value in itself, but has positive external effects on the growth rate of an economy. An individual learns culture from an individual who currently has culture and who charges to teach it. An individual who invests in culture gets no direct benefit, but is to be able to charge people in the future who wish to invest in culture. In Cozzi's (1998) model, there is an opportunity cost to learn culture in addition to the transfer to the teacher, and thus, the price of culture must be growing over time. There are equilibria in which culture has a constant price 0 and no one invests in it, but in addition there may be equilibria with "culture bubbles," that is, equilibria in which culture has a positive value, increasing over time. The increase in the price of culture is sustainable because wage incomes rise over time, which in turn is linked to the technological innovations that are generated as an externality of culture. If in some generation no one bought culture, technological progress would cease, and the growth rate of the economy would decrease. The social asset in Cozzi (1998) differs qualitatively from ours in that it has direct productive value. Although any single individual does not benefit from the asset in his model, there is a higher rate of growth in economies in which people invest in culture than in economies without such investment. In particular, the culture

[14] Mailath and Postlewaite (2003) discuss these papers in more detail.

could *not* have positive value in Cozzi (1998) if it did not lead to technological progress.

Fang (2001) analyzes a model in which nonproductive attributes can have value in equilibrium because agents have imperfectly observed ability. Fang demonstrates that high ability agents may acquire a costly nonproductive asset that (in equilibrium) serves as a signal even though the cost of acquiring the attribute is independent of ability. Social assets play an entirely different role in our model as there is no asymmetric information, hence no role for signaling.

2. We have analyzed a model in which there is an interaction between the social environment and agents' decisions. Differing social institutions can lead to differences in important economic decisions, and, conversely, agents decisions have important consequences for the stability of the social institutions. Many of the insights the model generates stem from the multiplicity of equilibria. Of course, this is not the first paper to point to the importance of multiple equilibria characterized by different economic choices by agents. Diamond (1970), for example, demonstrated the link between different equilibria and the level of aggregate economic activity. The nature of multiplicity in this article, however, differs in an important way. Diamond's multiplicity stems from a complementarity in the production technology: Each agent has little incentive to produce when few other agents produce. There is no analogous production complementarity in our model: In the productive attribute case, the productive value of the attribute is independent of the social institutions. The *social* value, however, is not independent of the institutions. Hence, the economic consequences of the multiplicity in our model result from a change in the social return to the attribute rather than through the technology.

3. The example we have analyzed in some detail focuses on income uncertainty and insurance as the conduit through which an asset may have social value. Our point is more general, and to illustrate this, we now describe a simple nonmatching example in which a nonproductive attribute can have social value.

In the example, there are overlapping generations of lawyers, each lawyer living two periods. There is a single nonstorable good over which lawyers have identical utility functions, $u(c_1, c_2) = c_1 \cdot c_2$, where c_i is consumption at age i. There is a continuum of lawyers born in each period indexed by $i \in [0, 2]$. Each young lawyer generates an output of 2 in his first period but they differ in their output in period 2. In period 2, lawyer i can produce i. Each lawyer can go into practice on his own and consume his own output, which generates total utility $2i$ for lawyer i. Alternatively, a new lawyer can apprentice himself to a "white-shoe" lawyer, who in addition to being a lawyer, has social skills.[15] The social skills have no use in and of themselves, but can be transmitted to others. Each white-shoe lawyer can take on at most one apprentice, and a fraction α of the lawyers are white shoe. At the beginning of each period, each young lawyer who had apprenticed himself to a white-shoe lawyer in the previous period becomes a white-shoe lawyer himself and makes an offer of an apprenticeship to a new lawyer. The offers are take it or

[15] The phrase "white-shoe" refers to the white buck shoes that were a fashion requirement within U.S. elite social organizations in the 1950s.

leave it offers which will be accepted by young lawyers if and only if their utility will be higher from accepting the offer than rejecting the offer and practicing on their own. If a second period white-shoe lawyer is able to hire an apprentice for a wage $w \leq 2$ (the output of all young lawyers), the older lawyer will have a profit of $2 - w$ from the younger lawyer, in addition to his own output.

Consider steady-state outcomes, where the fraction of white-shoe lawyers is constant at α. Since each old lawyer can take on exactly one apprentice, a fraction α of the young lawyers must accept apprenticeships. Clearly, a wage offer of 2 yields an equilibrium in which the social skills have no value, all young lawyers are indifferent between accepting and rejecting the offers, and all old white-shoe lawyers are indifferent between successfully hiring an apprentice. Consider now a wage of $w = \alpha$. We claim that all young lawyers $i \leq \alpha$ will accept the offers, and the remainder reject. If a young lawyer with index i accepts the offer, his income will be α in period 1 and his income in period 2 from his apprentice will be $2 - \alpha$ (output of 2 less wage of α). His own output is i (his index); hence his total income in period 2 is $2 - \alpha + i$, yielding a utility of $\alpha(2 - \alpha + i)$. If he rejects the apprenticeship, his income will be 2 in the first period and i in the second period, which yields utility $2i$. Hence, young lawyer i accepts the offer if and only if $i \leq \alpha$.

Thus, there exist qualitatively different equilibria for this problem. First, there is an equilibrium in which the social skills that are the mark of a white-shoe lawyer has no value. In addition, there is an equilibrium in which social skills have value, and this equilibrium Pareto dominates the equilibrium in which the social skills have no value.

In this example, social value is not driven by risk aversion since there is no uncertainty. Rather, it is driven by the desire to transfer consumption from the first period of an agent's life to the second period. As in the model we analyzed, missing markets are key. This example is similar to Cozzi's (1998) model in that individuals make an investment in learning the social skills that constitute being a white-shoe lawyer, which they can then sell to another lawyer in the future. Our example differs from Cozzi's (1998) model in that the social skills that are the social asset in the example truly have no productive value. The output of every lawyer in every generation is independent of whether there are any white-shoe lawyers or not. This example also highlights the connection to models of money, since the social skills play the role of a record device, similar to money as memory as described by Kocherlakota (1998).

4. There is a large literature arguing that institutions such as a functioning legal system and respected property rights can usefully be thought of as examples of social capital; more specifically, social capital is typically viewed as a "community-level attribute" (Putnam, 1994; Glaeser et al., 2001). From a formal perspective, social capital is best viewed as a characteristic of equilibrium in an infinite horizon game. The social norms as described in Okuno-Fujiwara and Postlewaite (1995) and Kandori (1992) are prototypical examples. A common feature is that the incentives in these equilibria require the punishment of deviators. Consequently, equilibria exhibiting social capital will not be "Markov."

In contrast, the equilibria we study are Markov. Although the particular matching structure (e.g., mixed or assortative) could be thought as an instance of social capital, we think it is useful to distinguish between social institutions that are necessarily sustained by sanctions (which we would term social capital) and those which do not require sanctions. As we noted in the previous point, the stability of both the mixed and assortative matching is driven by anonymous considerations: The attribute has a certain value (which may be zero) and it is irrelevant how that attribute was acquired.

5 The mixed matching equilibrium is reminiscent of some versions of the theory of sexual selection (Ridley, 1993). These theories have been motivated by the existence of animals such as peacocks. A nontrivial amount of the peacock's biological resources are invested in long and elegant tail feathers, which serve no productive purpose. Since natural selection selects for the fittest peacocks (those with shorter tails), there is a puzzle. Why do peahens prefer less fit males (those who have devoted scarce resources to feathers), as they must in order for the less fit males to dominate the population? The explanation is similar to the logic of the stability of the mixed matching in our model. Suppose peahens prefer long-tailed peacocks; then so long as long tails are genetically transmitted to male offspring, the male offspring of a peahen matched with a long-tailed mate will fare better in the market for mates in the next generation. Hence, there is an advantage to peahens that match with long-tailed peacocks that offsets the resources associated with long tails.

Peacocks' long tail feathers are similar to social assets in our model, but the incentives associated with the matching differ. A peahen that mates with a short-tailed peacock will have more offspring that survive. However, although a peahen that mates with a long-tailed mate may have fewer surviving offspring, the male offspring will themselves have more surviving offspring due to the advantage long tails confer in the matching process. Thus, selection, which is driving the peahen's choice of mates, is essentially balancing between the genetic advantage of more surviving offspring in the next generation and more surviving offspring in the subsequent generation. In our model, by contrast, a rich person without an attribute considering matching with a poor person with the attribute is balancing current consumption against descendants' consumption.

APPENDIX

Proof of Proposition 3. Since $k = 0$, the attribute has no value under assortative matching, and so $V_y^A = V_n^A = \frac{1}{2}$. Hence, the utility of a high-income agent with the attribute, V_{Hy}^A, equals that of a high-income agent without the attribute, V_{Hn}^A:

$$V_{Hy}^A = V_{Hn}^A = (1-\beta) + \beta\frac{1}{2} = \frac{2-\beta}{2},$$

and the utility of a low-income agent with the attribute, V_{Ly}^A, equals that of a low-income agent without the attribute, V_{Ln}^A:

$$V_{Ly}^A = V_{Ln}^A = \frac{\beta}{2}.$$

Under mixed matching, an Hy matches with a similar agent, consumes $2H$, and has two children, each of whom has attribute y. Hence, the utility of such an agent is

$$V_{Hy}^M = 1 - \beta + \beta V_y^M.$$

Similarly, an Ln agent will match with an agent of the same type, consume $2L$, and have two children without the attribute. The utility is

$$V_{Ln}^M = \beta V_n^M.$$

Finally, Hn agents and Ly agents will have the same utility, since they are matched with each other and jointly consume $H + L$ and have children that are equally likely to have attribute y or n. Denoting their utility by V_m^M, we have

$$V_m^M = u(1 - \beta) + \frac{\beta}{2}\big(V_y^M + V_n^M\big).$$

Solving (2) and (3) when $k = 0$ yields

$$V_y^M = \frac{4 - 3\beta + 2u(2 - \beta)}{4(2 - \beta)} = \frac{(4 - 3\beta)}{4(2 - \beta)} + \frac{u}{2}$$

and

$$V_n^M = \frac{\beta + 2u(2 - \beta)}{4(2 - \beta)} = \frac{\beta}{4(2 - \beta)} + \frac{u}{2}.$$

Thus,

$$\begin{aligned} V_{Hy}^M &= 1 - \beta + \beta\left(\frac{(4 - 3\beta)}{4(2 - \beta)} + \frac{u}{2}\right) \\ &= \frac{8 - 8\beta + \beta^2}{4(2 - \beta)} + \frac{\beta u}{2}, \\ V_{Ln}^M &= \frac{\beta^2}{4(2 - \beta)} + \frac{\beta u}{2}, \end{aligned}$$

and

$$\begin{aligned} V_m^M &= u(1 - \beta) + \frac{\beta}{2}\left(\frac{1}{2} + u\right) \\ &= u\left(1 - \frac{\beta}{2}\right) + \frac{\beta}{4}. \end{aligned}$$

Now,

$$V_m^M \geq V_H^A \Longleftrightarrow \frac{\left(1-\frac{1}{2}\beta\right)u+\frac{\beta}{4}}{1-\beta} \geq \frac{2-\beta}{2(1-\beta)}$$
$$\Longleftrightarrow u \geq \frac{4-3\beta}{2(2-\beta)}.$$

That is, Hn agents and Ly agents have (weakly) higher utility under mixed matching than under assortative matching, if and only if (8) holds. Since $V_{Hy}^M > V_m^M$, Hy agents have strictly higher utility under mixed matching when (8) holds.

Finally, Ln agents are also better off if

$$V_{Ln}^M \geq V_L^A \Longleftrightarrow$$
$$\frac{\beta^2}{4(2-\beta)}+\frac{\beta u}{2} \geq \frac{\beta}{2} \Longleftrightarrow$$
$$u \geq \frac{4-3\beta}{2(2-\beta)}. \qquad \blacksquare$$

PROOF OF PROPOSITION 8. We define a mapping $\Phi : [0,1]^4 \to [0,1]^4$ with the property that stationary values of (μ_y, q^M) are precisely the fixed points of $\Phi \equiv (\Phi_1, \Phi_2)$. The function $\Phi_1(\mu_y, q^M)$ is the fraction of the population that will have the attribute next period if the current population has fraction μ_y and parents purchase according to q^M: There are two cases we need to deal with, $\mu_y \geq \frac{1}{2}$ and $\mu_y < \frac{1}{2}$. Suppose that $\mu_y \geq \frac{1}{2}$, so that there are at least as many Ly as Hn agents. The fraction of the population with the attribute in the next period is then

$$\begin{aligned}
&\mu_y\left\{\left(\frac{1}{2}+k\right)(2p+(1-2p)q_{HH}^M)+\left(\frac{1}{2}-k\right)\right.\\
&\quad\left.\times\left(\frac{(1-\mu_y)}{\mu_y}\big(p+(1-p)q_{HL}^M\big)+\frac{(2\mu_y-1)}{\mu_y}\big(2p+(1-2p)q_{LL}^M\big)\right)\right\}\\
&\quad+(1-\mu_y)\left\{\left(\frac{1}{2}-k\right)\big(p+(1-p)q_{HL}^M\big)+\left(\frac{1}{2}+k\right)q_{LL}^M\right\}\\
&=\mu_y\left(\frac{1}{2}+k\right)(2p+(1-2p)q_{HH}^M)+(1-2k)(1-\mu_y)\big(p+(1-p)q_{HL}^M\big)\\
&\quad+\left(\frac{1}{2}-k\right)(2\mu_y-1)(2p+(1-2p)q_{LL}^M)+(1-\mu_y)\left(\frac{1}{2}+k\right)q_{LL}^M.
\end{aligned}$$

On the other hand, if $\mu_y < \frac{1}{2}$, there are more *Hn* than *Ly* agents, and so

$$\begin{aligned}
&\mu_y\left\{\left(\frac{1}{2}+k\right)\left(2p+(1-2p)q^M_{HH}\right)+\left(\frac{1}{2}-k\right)\left((p+(1-p)q^M_{HL})\right)\right\}\\
&\quad+(1-\mu_y)\left\{\left(\frac{1}{2}-k\right)\left(\frac{\mu_y}{(1-\mu_y)}(p+(1-p)q^M_{HL})+\frac{(1-2\mu_y)}{(1-\mu_y)}q^M_{HH}\right)\right.\\
&\qquad\left.+\left(\frac{1}{2}+k\right)q^M_{LL}\right\}\\
&=\mu_y\left(\frac{1}{2}+k\right)\left(2p+(1-2p)q^M_{HH}\right)+(1-2k)\mu_y\big(p+(1-p)q^M_{HL}\big)\\
&\quad+\left(\frac{1}{2}-k\right)(1-2\mu_y)q^M_{HH}+(1-\mu_y)\left(\frac{1}{2}+k\right)q^M_{LL}.
\end{aligned}$$

Clearly, Φ_1 is continuous.

In order to define Φ_2, we first need to calculate a value for the attribute, given μ_y and q^M. In this calculation, it is important to note that we are not requiring that μ_y be consistent with q^M (though at the fixed point, they will be consistent). Denote by $\tilde{V}_i$ the expected value to agent $i \in \{Hy, Ly, Hn, Ln\}$ of being in a situation characterized by (μ_y, q^M) *in each period*. Then, setting

$$\tilde{V}_y \equiv \left(\frac{1}{2}+k\right)\tilde{V}_{Hy}+\left(\frac{1}{2}-k\right)\tilde{V}_{Ly}$$

and

$$\tilde{V}_n \equiv \left(\frac{1}{2}-k\right)\tilde{V}_{Hn}+\left(\frac{1}{2}+k\right)\tilde{V}_{Ln},$$

we have

$$\begin{aligned}
\tilde{V}_{Hy} &= 2p\{(1-\beta)U(2H)+\beta\tilde{V}_y\}\\
&\quad+(1-2p)\big\{(1-\beta)U\big(2H-c(q^M_{HH})\big)+\beta\big(q^M_{HH}\tilde{V}_y+\big(1-q^M_{HH}\big)\tilde{V}_n\big)\big\}
\end{aligned}$$

and

$$\tilde{V}_{Ln} = (1-\beta)U\big(2L-c\big(q^M_{LL}\big)\big)+\beta\big(q^M_{LL}\tilde{V}_y+\big(1-q^M_{LL}\big)\tilde{V}_n\big).$$

Moreover, if $\mu_y \geq \frac{1}{2}$ (the reverse inequality is an obvious modification), we have

$$\begin{aligned}
\tilde{V}_{Hn} &= p\{(1-\beta)U(H+L)+\beta\tilde{V}_y\}\\
&\quad+(1-p)\big\{(1-\beta)U\big(H+L-c\big(q^M_{HL}\big)\big)+\beta\big(q^M_{HL}\tilde{V}_y+\big(1-q^M_{HL}\big)\tilde{V}_n\big)\big\}
\end{aligned}$$

and

$$\begin{aligned}\tilde{V}_{Ly} = \frac{(1-\mu_y)}{\mu_y}\big[&p\{(1-\beta)U(H+L)+\beta\tilde{V}_y\}\\&+(1-p)\big\{(1-\beta)U\big(H+L-c\big(q_{HL}^M\big)\big)\\&\qquad+\beta\big(q_{HL}^M\tilde{V}_y+\big(1-q_{HL}^M\big)\tilde{V}_n\big)\big\}\big]\\+\frac{(2\mu_y-1)}{\mu_y}\big[&2p\big\{(1-\beta)U(2L)+\beta\tilde{V}_y\big\}\\&+(1-2p)\big\{(1-\beta)U\big(2L-c\big(q_{LL}^M\big)\big)\\&\qquad+\beta\big(q_{LL}^M\tilde{V}_y+\big(1-q_{LL}^M\big)\tilde{V}_n\big)\big\}\big].\end{aligned}$$

These equations have a unique solution. Moreover, this solution is continuous in μ_y and q^M.

Note that at this point there is no reason to expect $\tilde{V}_y > \tilde{V}_n$. We define $\Phi_2(\mu_y, q^M) = (\tilde{q}_{HH}^M, \tilde{q}_{HL}^M, \tilde{q}_{LL}^M)$ by

$$\tilde{q}_\ell^M = \operatorname*{argmax}_{q\in[0,c^{-1}(\ell)]} (1-\beta)U(\ell - c(q)) + \beta(q\tilde{V}_y + (1-q)\tilde{V}_n).$$

The maximizer is unique since U is concave and c is strictly convex. Since the maximizer is unique, it is a continuous function of (μ_y, q^M), through the continuity of $\tilde{V}_y$ and $\tilde{V}_n$.

Since Φ is a continuous function on a compact convex subset of $\Re^4$, there is a fixed point by Brouwer. Note also that in the fixed point, $\tilde{V}_y > \tilde{V}_n$: Suppose that $\tilde{V}_y \le \tilde{V}_n$. Optimization then implies $q_\ell^M = 0$ for all ℓ. But then $\mu_y = 0$ (since $p < \frac{1}{2}$), and so $\tilde{V}_n = \frac{1}{2} - k$ (recall our normalization $U(2L) = 0$). Moreover, if $\tilde{V}_y \le \tilde{V}_n$, $\tilde{V}_y > (1-\beta)\{\frac{1}{2} + k + (\frac{1}{2} - k)u\} + \beta\tilde{V}_y$ (where $U(H+L) = u$), contradicting $\tilde{V}_y \le \tilde{V}_n$. Since we have assumed $c'(0) = 0$, $q_\ell^M > 0$ for all ℓ and so $\mu_y > 0$. Note also that $\mu_y < 1$: For suppose $\mu_y = 1$, then a fraction $\frac{1}{2} - k$ of agents are Ly's and since $2p + (1-2p)\, q_{LL}^M < 1$, not all the population in the next period can have the attribute. ∎

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