

Information, Liquidity, Asset Prices and Monetary Policy*

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Abstract

What determines which assets are used in transactions? We develop a framework where the extent to which assets are recognizable determines the extent to which they are acceptable in exchange – i.e., it determines their liquidity. Recognizability and liquidity are endogenized by allowing agents to invest in information. We analyze the effects of monetary policy. There can be multiple equilibria, with different transaction patterns, and these patterns are not invariant to policy. We show small changes in information may generate large responses in asset prices, allocations and welfare. We also discuss some issues in international economics, including exchange rates and dollarization.

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Aringosa walked back to his black briefcase, opened it, and removed one of the bearer bonds. He handed it to the pilot. “What’s this?” the pilot demanded. “A ten-thousand-euro bearer bond drawn on the Vatican Bank.” The pilot looked dubious. “It’s the same as cash.” “Only cash is cash,” the pilot said, handing the bond back. Dan Brown, *The Da Vinci Code*.

1 Introduction

What determines which assets are used in transactions? We study economies in which assets are valued for both their rate of return and liquidity, by which we mean their usefulness in the exchange process. In our model, some trades are conducted in markets where certain frictions make credit imperfect. Sellers in these markets are unwilling to give buyers unsecured loans, and this makes assets essential for trade: buyers must either hand over assets to sellers directly, or use them to collateralize debt. Hence, assets facilitate exchange. This much is standard in modern monetary theory, what the recent surveys by Williamson and Wright [58], [59] and Nosal and Rocheteau [47] call New Monetarist Economics. The novel feature emphasized here is that some assets are not as good as others at facilitating transactions due to asymmetric information.

In particular, it can be difficult for some sellers to distinguish or recognize good- and bad-quality versions of certain assets, which makes these sellers reluctant to accept them, either as a means of payment or as collateral. We develop a general framework, with arbitrary numbers of real and monetary assets, in which these information frictions are made explicit, and use it to discuss applications in finance and monetary economics. Many of the basic ideas go back a long way. Classic discussions can be found in Jevons [22] and Menger [46], and the idea that the intrinsic properties of objects make them more or less well suited for use in payments can be found in many textbooks (see Nosal and Rocheteau [47]). These properties include portability, storability, divisibility and recognizability. It is recognizability that we emphasize here.¹

¹Other work on information and liquidity includes Alchian [2], Brunner and Meltzer [7], Freeman [11], and Banerjee and Maskin [6]. We are closer to the search-based literature on information frictions going back to Williamson and Wright [57] (see the surveys mentioned above for more references). An alternative approach to liquidity in finance, going back to Glosten and Milgrom [16] and Kyle [32], also considers exchange between asymmetrically-informed agents, but has little to say about the substantive issues addressed here.

As a simple example, it is typically thought that currency, or at least domestic currency, is recognizable to virtually everyone active in the economy, while alternatives are less so. These alternatives include foreign currency, bonds like those Aringosa tried to pass in the epigraph, less exotic claims like T-bills or equity shares, and so on. There can also be recognizability differences across this set of alternatives, potentially making, say, some bonds more liquid than others, or making bonds more or less liquid than stocks. As another example, it was often difficult historically for sellers to recognize the quality (i.e., the weight and purity) of gold or silver coins. In modern economies, similar problems can arise with respect to legitimate and counterfeit paper currencies. Most recently, information problems in asset markets have made it increasingly difficult to value complicated bundles of assets like mortgage-backed securities. The general idea is that, in any situation where buyers and sellers are asymmetrically informed about the values of assets, exchange is hindered.

Since recognizability is vital for liquidity, it is desirable to model it endogenously. To this end we allow agents to invest in information – to acquire the knowledge, or perhaps the technology, to distinguish high- and low-quality versions of certain assets. This leads to coordination issues that can generate multiple equilibria. Related results have been discussed previously, but in contrast to that work, multiplicity here is due to explicit general equilibrium asset market effects. For instance, there is a literature studying payment methods, like credit cards, that correctly emphasizes that what sellers accept depends on what buyers carry and vice versa (see Hunt [20] and references therein). While it is not hard to get multiple equilibria by assuming the benefit to using one type of instrument goes up, or the cost goes down, when others also use it, the results here are more subtle. In our model, when more sellers recognize a particular asset, it becomes more liquid, and hence more useful in the exchange process. This makes buyers want more of the asset, and this increases its price. When the asset is more valuable, sellers are more willing to pay to be able to differentiate high- and low-quality versions. This complementarity can lead to multiplicity.

Once liquidity is incorporated into a model, it is apparent that assets generally can be valued for more than their rate of return. The leading example is fiat money, an asset with a perfectly predictable permanent dividend of 0, and hence one that should have a price of 0 according to standard finance theory. In monetary economics, however, agents may value fiat currency, even if it is dominated in return by other assets, because it provides transactions services. The value of fiat money can be interpreted as a liquidity premium. Once this is understood, it must be acknowledged that any asset can bear a liquidity premium, which means that its price can exceed the fundamental price, defined by the present value of its dividend stream. All else equal, if it is harder to trade using asset a_1 than asset a_2 , it seems obvious that the latter will have a higher price and a lower return than the former. The more novel and interesting aspect of the approach taken here is that we endogenize liquidity based on recognizability, and endogenize recognizability by allowing agents to invest in information.

The particular model we use is a multiple-asset version of Lagos and Wright [37]. In this framework, some trades take place in centralized competitive markets, while others take place in decentralized markets with frictions that make either a means of payment or collateral essential. This is a useful setting for studying the relationship between liquidity and asset prices, since the decentralized markets allow one to formalize an asset's role in facilitating transactions, while the centralized markets allow one to price assets competitively using the standard approach of Lucas [44]. Past work exploiting this idea includes Geromichalos et al. [14], Lagos and Rocheteau [36] and Lagos [35], [34], who all study versions of Lagos-Wright with multiple assets. In those papers, however, all assets are equally and perfectly acceptable and hence must have the same return. Lagos [33] takes this a step further by assuming differences in the acceptability of assets, and shows how this can help explain some puzzles in asset-pricing theory, but these differences in acceptability are exogenous.²

²Lagos [33] is silent, however, on *why* the assets have different liquidity properties; in addition to the current paper, see Rocheteau [50] to see how information frictions can generate differential liquidity properties across assets, and how this affects asset prices.

To be clear, we differ from this related work mainly in the way we endogenize differences in liquidity. We do this, again, by having liquidity depend on recognizability, and allowing sellers to invest in information. To do so, however, we must first work out the benefits of being informed, which requires a characterization of the equilibrium for any given level of information. Although this part of the analysis is similar to the papers cited in the previous paragraph, it is necessary to present this material in order to lay the foundation for endogenizing information and liquidity. Moreover, even when one takes information and liquidity as given, there are places where we go beyond and pursue different applications from the above-mentioned work.³

Among the extensions and applications of the basic framework presented below are the following: First, in our baseline model, we specify the environment so that agents who are not informed about a particular asset simply refuse to accept it in exchange. This is technically convenient because it allows us to use standard bargaining theory to determine the terms of trade: since agents only exchange objects that they recognize, they never bargain under asymmetric information. The advantage of modeling information frictions in this way is that it allows us to emphasize liquidity differentials without overly complicating the analysis of the terms of trade. However, we also discuss how one can relax the assumptions in the baseline model, so that assets that are not recognized are still accepted up to a point, based on recent contributions by Rocheteau [49] and Li and Rocheteau [42]. This extension shows that our results are robust in the following sense: if agents have to pay a cost k to be able to produce low-quality assets (e.g., counterfeits), then for $k > 0$ sellers who cannot recognize the quality of a security may accept a limited quantity of it, say $s > 0$, but $s \rightarrow 0$ as $k \rightarrow 0$, recovering our baseline model as a simplified limiting case.

In terms of more substantive applications, a leading case on which we focus, but not the only

³New results, even when we have exogenous information, include, e.g., our analysis in Section 6 of international monetary issues, and our demonstration in Section 3 that even markets that never use money are affected by monetary policy. Also, as a technical matter, the results here are derived using proportional rather than Nash bargaining, which dramatically simplifies the analysis compared to those papers, or compared to the working paper (Lester et al. [40]) where we also used Nash.

one that we consider, concerns two assets, equity and fiat currency. This allows us to highlight some interesting connections between monetary policy and equity prices/returns. Even with exogenous information, inflation makes people want to shift their portfolios out of currency and into alternative assets, increasing the price of, and lowering the return on, these alternatives. We think it is worthwhile to display these results formally in our setup, even if similar results can be found elsewhere and the basic ideas in terms of monetary policy go back a long way.⁴ Much more emerges when we endogenize information. Since the proximate effect of inflation is to decrease the demand for money and increase demand for alternatives, it raises the market value of alternative assets and therefore the incentive to acquire information. Hence inflation increases the liquidity of alternative assets.

One implication is that the share of transactions where cash is apparently required is endogenous: in the case of multiple equilibria, it is not uniquely determined by fundamentals; and even if equilibrium is unique, it is not invariant to policy. This calls into question the practice by many economists of imposing exogenous transactions patterns, as in cash-in-advance models, or, at the opposite extreme, cashless exchange as in New Keynesian Economics. Of course we are not the first to call this into question, but evidently the message has not sunk in. Also, of course, as always the validity of any approach depends on the issues at hand, but it would seem hard to argue that in monetary economics the transactions process should not be endogenous!

⁴A concise statement is contained in Wallace's [54] analysis of OLG (overlapping generations) models:

Of course, in general, fiat money issue is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on the magnitude of the fiat money-financed deficit ... [T]he real rate-of-return distribution faced by individuals in equilibrium is less favorable the greater the fiat money-financed deficit. Many economists seem to ignore this aspect of inflation because of their unfounded attachment to Irving Fisher's theory of nominal interest rates. (According to this theory, (most?) real rates of return do not depend on the magnitude of anticipated inflation.) The attachment to Fisher's theory of nominal interest rates accounts for why economists seem to have a hard time describing the distortions created by anticipated inflation. The models under consideration here imply that the higher the fiat money-financed deficit, the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income. This seems to be what most citizens perceive to be the cost of anticipated inflation.

We think these words ring true, but many questions arise. How can the Fisher Equation not hold? Might it hold for some assets and not others? Why do different assets bear different returns in the first place? In the OLG models Wallace mentions, it is *not* differences in liquidity. This is where more modern monetary theory can help.

As a particular application of this idea, we show how small changes in fundamentals, including the information structure, can generate large responses in transactions patterns, liquidity, asset prices, and welfare, and we suggest, if somewhat tentatively, that this may have something to do with recent financial events.

Other applications include the case of two commodities monies – e.g., gold and silver – and the case of two fiat monies – e.g., dollars and pesos. In the latter application we suppose that, as in many Latin American economies, pesos are more easily recognizable but dollars constitute a better store of value. Consistent with several historical episodes, the theory predicts that when peso inflation is not too high locals in Latin America use mainly pesos as a means of payment, while dollars do not circulate widely, nor are they universally recognized. As peso inflation increases, however, at some point transacting in the local currency becomes very costly, and more agents learn to recognize and use US currency. This is dollarization. Notice, however, that if peso inflation later subsides we should not expect dollars to fall into disuse, because once individuals learn to recognize and use them in transactions they do not quickly forget. This imparts a natural hysteresis effect into dollarization, as has often been discussed but never formalized in this way. Relatedly, we also analyze exchange rates. In particular, we show how to recast some classic results of Kareken and Wallace [24] in a very different model.

The rest of the paper is organized as follows. Section 2 describes the basic assumptions, defines equilibrium and presents a simple example. Section 3 studies the model when information is exogenous as a foundation for what comes next. Section 4, the heart of the paper in terms of novel contributions, endogenizes information and liquidity. Section 5 discusses how to relax the assumption, maintained elsewhere in the paper, that low-quality assets can be produced at no cost. Section 6 discusses international economic issues. Section 7 provides a general discussion of the approach. Section 8 concludes. Proofs of some technical results are relegated to an Appendix.

2 The Model

The general framework is based on Lagos and Wright [37]. In this model, in each period of discrete time, a $[0, 1]$ continuum of infinitely-lived agents participate in two distinct markets: a frictionless centralized market CM, as in standard general equilibrium theory; and a decentralized market DM, where buyers and sellers meet and trade bilaterally, as in search theory. These alternating markets are useful because we can impose interesting frictions in the DM, while the presence of the CM keeps the analysis tractable by helping to reduce the dimensionality of the state space. Also, as we said in the Introduction, the CM allows us to price assets competitively, as in the standard theory of finance, even though we incorporate search, bargaining and information frictions in the DM.

It is assumed that in DM meetings sellers can produce something buyers want, but buyers cannot reciprocate, ruling out direct barter. Buyers could in principle promise to pay sellers in the next meeting of the CM, but standard assumptions imply that they could renege without fear of repercussion. These assumptions, sometimes packaged under the label *anonymity*, are that there is limited commitment, so that promises are not perfectly credible, and a lack of monitoring or record keeping that makes the use of trigger strategies as a punishment device difficult.⁵ Hence, unsecured credit is not available in the DM, and assets have a role in facilitating exchange. Buyers in the DM can either hand over assets directly, or use them as collateral. In the second case, where buyers in the DM use assets as collateral, there is delayed settlement: in the next CM either the loan is repaid or sellers get the collateral, but this is a matter of indifference to both parties in equilibrium. In the first case, where buyers hand over assets directly, there is finality when DM trade occurs. Aside from this detail, the two interpretations (final or deferred settlement) are equivalent.

At each date agents first trade in the DM and then the CM. In the CM there is a consumption good X that all agents can produce one-for-one using labor H , and utility is $U(X) - H$. In

⁵See Kocherlakota [27], Wallace [55], or Aliprantis et al. [3] for formal treatments.

the DM there is another good q that agents value according to $u(q)$ and can be produced at disutility cost $c(q)$. Goods are nonstorable. We assume $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $U'(0) = u'(0) = \infty$. Also, let X^* and q^* solve $U'(X^*) = 1$ and $u'(q^*) = c'(q^*)$. In any bilateral DM meeting, each agent has an equal probability of being a buyer or a seller, so if we normalize the probability of meeting anyone to 2λ , then λ is the probability of being a buyer. Although we do not do so here, one can allow barter or unsecured credit to be available in some meetings without changing the results; it is only necessary that barter or unsecured credit is not available in all meetings.

Assets are indexed by $j = 1, \dots, n$, and a portfolio is $\mathbf{a} \in \mathbb{R}_+^n$. As in the standard Lucas [44] model, each asset j can be interpreted as a claim to (an equity share in) a tree j , yielding a dividend in terms of fruit δ_j in units of good X each period in the CM. As a special case, asset j may be a fiat object, like outside money. By definition a fiat object is intrinsically worthless (Wallace [54]), which in this context means $\delta_j = 0$. To keep the environment stationary, for any real asset j with $\delta_j > 0$, we fix the supply at A_j . If j is a fiat object, however, we can let the supply change according to $A_j = (1 + \gamma_j)A_j^-$, where A^- is the value of (any variable such as) A in the previous period, without changing real resources because $\delta_j = 0$. Thus, we allow government to issue or retire currency but not to cultivate or cut down fruit-bearing trees. Changes in the supply of fiat objects are accomplished in the CM by lump sum transfers if $\gamma_j > 0$ or taxes if $\gamma_j < 0$. Suppose by way of example that there is exactly one fiat asset j , and let ϕ_j be its price. Stationarity implies $\phi_j A_j = \phi_j^- A_j^-$, which means that γ_j is the inflation rate measured in the price of the fiat asset – a version of the Quantity Theory. We assume $\gamma_j > \beta - 1$, where β is the discount factor, but also consider the limit as $\gamma_j \rightarrow \beta - 1$, which is the Friedman Rule.⁶

We introduce qualitative uncertainty concerning assets, as in Akerlof's [1] lemons model, by assuming that any asset can be of high or low quality. Here, for simplicity, a low-quality asset is completely useless in the sense that it bears no dividend. More generally, the value of

⁶As is standard, there is no equilibrium if $\gamma_j < 1 - \beta$.

any security can be random and agents may have asymmetric information about the probability distribution; the possibility that it may be totally worthless is the special case in which its value may be 0. One interpretation of a worthless asset is that it is *a bad claim to a good tree* – a counterfeit. Another interpretation is that it is *a good claim to a bad tree* – a lemon (tree). For instance, a seller could be offered a bogus equity claim on a profitable company, or he could be offered a legitimate share in a company that was once a going concern but, unbeknownst to him, now has future profit stream of 0. This distinction does not matter for what we do.

In the baseline model, agents can produce worthless assets at any time at cost $k = 0$. This makes worthless assets different from fiat money, even though both have 0 dividends. Fiat money may be valued in equilibrium only if agents cannot costlessly produce passable counterfeit facsimiles of it themselves. So, a bad claim, if recognized as such, will never be accepted, even though agents may accept fiat money when they recognize that it has a 0 dividend, because they cannot produce genuine currency themselves (for details, see Wallace [56]). One reason $k = 0$ simplifies the analysis is that it implies no seller ever accepts assets he cannot recognize, because, if he did, buyers would simply produce and hand over worthless paper.⁷

The assumption $k = 0$ is extreme but also extremely useful: when sellers reject outright assets that they cannot evaluate, we can use simple bargaining theory in the DM. Since unrecognized assets are not even on the table, negotiations always occur under full information. In this way informational frictions help determine liquidity, but we avoid well-known problems with bargaining under asymmetric information. However, in Section 5, we briefly discuss the case $k > 0$ and argue that the main results are robust. When $k > 0$, sellers accept assets that they cannot evaluate, but only up to a point: they will produce $q^k > 0$ for a^k units of the asset,

⁷This is different from work on information-based monetary theory going back to Williamson and Wright [57]. In those models, agents make ex ante choices to bring good or bad assets to the market, and sellers always accept assets with positive probability even if they cannot recognize them. The logic is simple. Suppose there are some informed and uninformed sellers. Informed sellers never accept low-quality assets. If uninformed sellers never accept them, then buyers with such assets cannot trade, and no one brings them to the market. But then uninformed sellers have no reason to reject. The difference here is that buyers can produce worthless assets on the spot. See Lester et al. [40] for more details.

but q^k and a^k can be less than their values under full information. In this case, illiquidity means that only a small amount of an asset is acceptable in DM trades. One can show that $q^k \rightarrow 0$ as $k \rightarrow 0$, so our baseline model is the limiting case where counterfeiting is costless and uninformed agents do not accept unrecognized assets at all.

To be clear, only in the DM is there a problem distinguishing high- and low-quality assets, not in the frictionless CM (one story, stepping outside the formal model, is that there are banks or related institutions freely available only in the CM to certify quality). In any bilateral DM meeting a seller may be informed or uninformed about the quality of any given asset. Subsequently we endogenize the information structure; for now it is taken as given. Index any DM meeting by $S \in \mathcal{P}$ indicating the subset of assets that the seller recognizes, where \mathcal{P} is the power set of $\{1, 2, \dots, n\}$. Let ρ_S be the probability of a type S meeting, or meeting a type S seller. Also, let $\mathcal{P}_j = \{S \in \mathcal{P} : j \in S\}$ be the set of meetings where the seller recognizes asset j . In a type S meeting, a buyer with portfolio \mathbf{a} has liquid or recognizable wealth $y_S(\mathbf{a}) = \sum_{j \in S} (\delta_j + \phi_j) a_j$, and payment to the seller $p_S(\mathbf{a})$ is constrained by $p_S(\mathbf{a}) \leq y_S(\mathbf{a})$. In general, liquid wealth is less than total wealth, $y_S(\mathbf{a}) \leq y(\mathbf{a}) = \sum_{j=1}^n (\delta_j + \phi_j) a_j$.

Let $V(\mathbf{a})$ be the value function for an agent in the DM. In the CM, where all assets are recognized, all that matters for an individual is total wealth $y(\mathbf{a})$, and we write the value function as $W[y(\mathbf{a})]$. Since our linear CM production technology implies the equilibrium real wage is 1, the CM problem is

$$\begin{aligned} W(y) &= \max_{X, H, \hat{\mathbf{a}}} \{U(X) - H + \beta V(\hat{\mathbf{a}})\} \\ \text{s.t. } X &= H + y - \sum_j \phi_j \hat{a}_j + T, \end{aligned} \tag{1}$$

where $\hat{\mathbf{a}} \in \mathbb{R}_+^n$ is the portfolio taken into the next DM, while T is a transfer to accommodate potential changes in the supply of fiat objects. There may be an additional constraint $H \in [0, \bar{H}]$, but assuming it is not binding, we can eliminate H to write

$$W(y) = U(X^*) - X^* + y + T + \max_{\hat{\mathbf{a}}} \left\{ - \sum_j \phi_j \hat{a}_j + \beta V(\hat{\mathbf{a}}) \right\}. \tag{2}$$

It is immediate from (2) that W is linear, $W'(y) = 1$, and $\hat{\mathbf{a}}$ is independent of y and hence \mathbf{a} . This reduces the dimensionality of the state space substantially because we do not have to track the distribution of \mathbf{a} across agents in the DM, since we can restrict attention to the case where they all choose the same $\hat{\mathbf{a}}$. If two assets are perfect substitutes, like a ten dollar bill and two fives, agents may hold different portfolios, but they have the same value. Hence, we focus on a symmetric choice for $\hat{\mathbf{a}}$, satisfying the FOC

$$-\phi_j + \beta \frac{\partial V(\hat{\mathbf{a}})}{\partial \hat{a}_j} \leq 0, = \text{ if } \hat{a}_j > 0 \text{ for } j = 1, \dots, n. \quad (3)$$

These look like conditions one might see in many old, and some not-so-old, models where assets are inserted directly into utility functions. It is important to emphasize, however, that for us $V(\cdot)$ is *not* a primitive – it is the continuation value of participating in the DM.

To characterize the DM terms of trade, a variety of mechanisms can be and have been used in the literature, but for tractability, in this paper we use Kalai’s [23] proportional bargaining solution.⁸ In this class of models, proportional bargaining guarantees that $V(\cdot)$ is concave, and that each agent’s surplus increases monotonically with the match surplus, neither of which is guaranteed with Nash bargaining (Aruoba et al. [5]). A very useful implication of monotonicity here is that agents have no incentive to “hide” some of their asset holdings, as they do with Nash bargaining (Lagos and Rocheteau [36]; Geromichalos et al. [14]). One can deal with these technicalities, but proportional bargaining avoids them, easing the presentation considerably. The theory is robust, however, in the sense that one can derive qualitatively similar results using generalized Nash bargaining (Lester et al. [39]) or Walrasian pricing (Guerrieri [17]).

To apply proportional bargaining, note that the surplus of a buyer who gets q for payment p is $u(q) + W(y - p) - W(y) = u(q) - p$, using the linearity of $W(\cdot)$. Similarly, the surplus of the seller is $p - c(q)$. Consider a type S meeting where the buyer has liquid wealth $y_S(\mathbf{a}) = \sum_{j \in S} (\delta_j + \phi_j) a_j$.

⁸Other options in the literature include Nash bargaining, price taking, price posting, auctions, and pure mechanism design (see the surveys cited in the Introduction for references). Proportional bargaining has several advantages in these models, and for this reason it is being used in many recent applications; see Aruoba et al. [5], Aruoba [4], Rocheteau and Wright [51], and Geromichalos and Simonovska [15].

The proportional solution is given by a payment $p = p_S(\mathbf{a})$ and quantity $q = q_S(\mathbf{a})$ solving

$$\max_{p,q} \{u(q) - p\} \text{ s.t. } u(q) - p = \theta [u(q) - c(q)] \text{ and } p \leq y_S(\mathbf{a}),$$

where $\theta \in [0, 1]$ is the buyer's bargaining power. Notice $p_S(\mathbf{a})$ and $q_S(\mathbf{a})$ depend on the portfolio of the buyer $y_S(\mathbf{a})$ and information of the seller S , but not on the portfolio of the seller or information of the buyer. Define

$$z(q) = \theta c(q) + (1 - \theta)u(q), \tag{4}$$

and let $y^* = z(q^*)$, where $u'(q^*) = c'(q^*)$.

The next result establishes that the buyer pays y^* and gets q^* if his liquid wealth exceeds y^* , and otherwise hands over all of his liquid wealth in exchange for $q < q^*$.

Lemma 1. *If $y_S(\mathbf{a}) \geq y^*$ then $p_S(\mathbf{a}) = y^*$ and $q_S(\mathbf{a}) = q^*$; if $y_S(\mathbf{a}) < y^*$ then $p_S(\mathbf{a}) = y_S(\mathbf{a})$ and $q_S(\mathbf{a}) < q^*$ solves $z(q) = y_S(\mathbf{a})$.*

The proof is omitted as it is basically the same as the result in Lagos and Wright [37], even though they use generalized Nash bargaining.⁹

The value of entering the DM can now be written

$$V(\mathbf{a}) = W[y(\mathbf{a})] + \lambda \sum_{S \in \mathcal{P}} \rho_S \{u[q_S(\mathbf{a})] - p_S(\mathbf{a})\} + K. \tag{5}$$

The first term on the RHS is the value of proceeding to the CM with one's portfolio \mathbf{a} intact. The second term is the probability of being a buyer λ multiplied by the expected trade surplus across types of meetings. The final term K is the expected surplus from being a seller, which

⁹With generalized Nash, Lemma 1 holds as stated if we redefine

$$z(q) = \frac{\theta u'(q)}{\theta u'(q) + (1 - \theta)c'(q)} c(q) + \frac{(1 - \theta)c'(q)}{\theta u'(q) + (1 - \theta)c'(q)} u(q).$$

In both cases $z(q)$ is a convex combination of $u(q)$ and $c(q)$, but with Nash bargaining the weights depend on q . Comparing this with (4), one can perhaps see how proportional bargaining simplifies the algebra, but again, similar results are derived using generalized Nash bargaining in the working paper Lester et al. [39].

as shown above does not depend on \mathbf{a} . Differentiation leads to¹⁰

$$\frac{\partial V}{\partial a_j} = \delta_j + \phi_j + \lambda \sum_{S \in \mathcal{P}_j} \rho_S (\delta_j + \phi_j) \left\{ \frac{u'[q_S(\mathbf{a})]}{z'[q_S(\mathbf{a})]} - 1 \right\}, \quad (6)$$

where we used the fact that, by virtue of Lemma 1:

if $y < y^*$ then $\partial p / \partial a_j = \delta_j + \phi_j$ and $\partial q / \partial a_j = (\delta_j + \phi_j) / z'(q)$;

if $y > y^*$ then $\partial p / \partial a_j = \partial q / \partial a_j = 0$.

We can now rewrite (6) as

$$\frac{\partial V}{\partial a_j} = (\delta_j + \phi_j) \left\{ 1 + \lambda \sum_{S \in \mathcal{P}_j} \rho_S \ell [q_S(\mathbf{a})] \right\}, \quad (7)$$

by introducing the *liquidity premium*

$$\ell(q) = \frac{u'(q)}{z'(q)} - 1 = \frac{\theta[u'(q) - c'(q)]}{\theta c'(q) + (1 - \theta)u'(q)}, \quad (8)$$

where the second equality derives $z'(q)$ from (4). This premium is the payoff from a marginal unit of wealth that is liquid, in the sense that it can be used to acquire more q in the DM, as opposed to simply carrying it through to the next CM. From Lemma 1 we have $q \in [0, q^*]$, and from (8) we have $\ell'(q) < 0$ over this range, with $\ell(q) > 0$ if $q < q^*$ and $\ell(q^*) = 0$. For future reference, we say that agents are *satiated in liquidity* at \mathbf{a} when $\ell[q_S(\mathbf{a})] = 0$, or equivalently $q_S(\mathbf{a}) = q^*$, for all S .

Combining (3) and (7), being careful with the timing, we get

$$-\phi_j^- + \beta (\delta_j + \phi_j) \left\{ 1 + \lambda \sum_{S \in \mathcal{P}_j} \rho_S \ell [q_S(\mathbf{a})] \right\} \leq 0, = \text{if } \hat{a}_j > 0, j = 1, \dots, n, \quad (9)$$

where the first term ϕ_j^- is the price of asset j in the previous period's CM while all variables in the second term are in the current period. For any real asset with $\delta_j > 0$, in equilibrium $\phi_j > 0$ and $a_j = A_j > 0$, so (9) holds with equality. For a fiat asset a_j , we can have $\phi_j = 0$, but as

¹⁰Note that in the summation, for any S such that $y_S(\mathbf{a}) \geq y^*$, the term in braces is 0. So the summation is only positive if $y_S(\mathbf{a}) < y^*$ for some S with $\rho_S > 0$. Also, the summation runs only over $S \in \mathcal{P}_j$ since an asset only helps if the seller recognizes it.

long as a_j is valued (9) also holds with equality. Hence, it holds with equality in all relevant situations. Using this and market clearing $\hat{\mathbf{a}} = \mathbf{A}$, we arrive at

$$\phi_j^- = \beta (\delta_j + \phi_j) \left\{ 1 + \lambda \sum_{S \in \mathcal{P}_j} \rho_S \ell [q_S(\mathbf{A})] \right\}, \quad j = 1, \dots, n. \quad (10)$$

Equilibrium asset prices are given by any sequence $\{\phi_t\}_{t=0}^\infty$ satisfying (10) that is nonnegative and satisfies a boundedness condition. The latter condition comes from the natural transversality conditions for this type of model (see Rocheteau and Wright [51]), although this is less relevant here, since we focus on stationary equilibria, where it is satisfied automatically. Given asset prices ϕ_t we can easily determine all of the other endogenous variables. In particular, we can compute $y_S(\mathbf{a})$ in any DM meeting, and then $q_S(\mathbf{a})$ and $p_S(\mathbf{a})$ follow from Lemma 1.

As a benchmark, first suppose that $\lambda = 0$ – i.e., the DM is effectively shut down. Then there are no liquidity considerations, and (10) reduces to $\phi_j^- = \beta (\delta_j + \phi_j)$. The stationary solution to this is $\phi_j = \beta \delta_j / (1 - \beta) = \phi_j^*$, where ϕ_j^* is the *fundamental price* of asset j , defined as the present value of its dividend stream. For any fiat object, therefore, if $\lambda = 0$ then $\phi_j^* = 0$. Now suppose $\lambda > 0$ – i.e., the DM is active. Let $\bar{y}_S(\mathbf{A}) = \sum_{j \in S} A_j \delta_j / (1 - \beta)$ be liquid wealth in meeting S when all assets are priced fundamentally. Agents are satiated in liquidity when $\bar{y}_S(\mathbf{A}) \geq y^*$ with probability 1, and in this case the unique equilibrium also has assets priced fundamentally. But if $\bar{y}_S(\mathbf{A}) < y^*$ for some $S \in \mathcal{P}_j$ with positive probability and some j , then agents are not satiated in liquidity, and ϕ_j can exceed the fundamental price.

Although we are interested mainly in economies with multiple assets, consider an example with $n = 1$ real asset. Dropping the subscript (e.g., writing $a_1 = a$), in steady state (10) becomes

$$\frac{\phi}{\beta (\delta + \phi)} - 1 = \lambda \rho \ell [q(a)], \quad (11)$$

where ρ is the probability a is recognized, and the bargaining solution implies

$$q(a) = \begin{cases} z^{-1}(y) & \text{if } y < y^* \\ q^* & \text{if } y \geq y^* \end{cases}. \quad (12)$$

Inserting $q(a)$ from (12) into (11) implicitly defines the demand for a as a function of the price ϕ . Figure 1 shows the (inverse) demand curve as continuous and decreasing, until it becomes

flat at $A^* = y^*/(\delta + \phi^*)$.¹¹ The market-clearing price corresponds to the intersection of demand with the supply curve, which is vertical at $a = A$.

INSERT FIGURE 1 ABOUT HERE

Suppose a is a T-bill, which is only a slight stretch since one can easily rewrite the model in terms of discount bonds rather than dividend-bearing securities. Then this example constitutes a structural model of the T-bill demand function Krishnamurthy and Vissing-Jorgensen [31] take to the data, where here demand is derived indirectly from the premise that assets are useful in exchange, rather than directly sticking them into the utility function. More generally, our theory delivers an interrelated asset demand system for $\mathbf{a} \in \mathbb{R}_+^n$ as a function of $\phi \in \mathbb{R}_+^n$, analogous to the standard interrelated factor demand systems that are often estimated for firms as functions of input prices. In principle one can estimate this asset demand system jointly – instead of the usual procedure of estimating in isolation the demand for money, the demand for T-bills, etc. In any case, we summarize the results for $n = 1$ as follows:

Proposition 1. *Define $A^* = y^*(1 - \beta)/\delta$. (i) If $A \geq A^*$ then there exists a unique equilibrium with $\phi = \phi^*$, $p = y^* = (\phi^* + \delta)A^*$, and $q = q^*$. (ii) If $A < A^*$, then $\phi > \phi^*$ satisfies (11) with $a = A$, $p = y = (\phi + \delta)A < y^*$, and $q = z^{-1}(y) < q^*$.*

In terms of economics, in case (i) of the Proposition agents are satiated in liquidity and there is no premium, $\phi = \phi^*$. In this case, DM buyers use A^* assets for trade and hold the remaining $A - A^*$ purely as a store of value. In case (ii), however, liquidity is scarce and the asset price bears a premium: $\phi > \phi^*$. To capture fiat money, as a special case, set $\delta = 0$, and let A increase

¹¹To verify this, differentiate to get

$$\frac{d\phi}{da} = \begin{cases} \beta\lambda\rho\ell'(q)(\delta + \phi)^3/z'(q)\delta & \text{if } a < A^* \\ 0 & \text{if } a \geq A^* \end{cases}$$

Since $\ell' < 0$ and $z' > 0$, we have $d\phi/da < 0$ when $a < A^*$. Note that $a < A^*$ iff $q < q^*$ iff $\ell(q) > 0$. From (11), $\ell(q) > 0$ iff $\phi > \phi^*$. Therefore, demand is infinitely elastic at ϕ^* .

at rate γ . In stationary equilibrium the inflation rate is $\phi^-/\phi = 1 + \gamma$, and (10) implies

$$\frac{1 + \gamma - \beta}{\beta} = \lambda\rho\ell(q). \quad (13)$$

Letting $\gamma \rightarrow \beta - 1$, which is the Friedman Rule, we get $\ell(q) = 0$ and $q = q^*$; for any $\gamma > \beta - 1$, there is a liquidity premium, and $q < q^*$.¹²

To say more about the economics, consider for the sake of illustration assets that are totally illiquid, defined here as assets that trade in the CM but cannot be traded in the DM (say, because agents can verify their authenticity with probability 0 in the DM). For instance, let the real and nominal interest rates be denoted r and i for real and nominal illiquid bonds. While we are more interested below in partially liquid assets, for the record, it should be obvious that the returns on completely illiquid bonds satisfy $1 + r = 1/\beta$ and $1 + i = (1 + r)(1 + \gamma)$, the latter being the standard Fisher Equation. Given this, we can always write (13) as $i = \lambda\rho\ell(q)$, which implies $\partial q/\partial i < 0$, and the Friedman Rule can be stated as $i = 0$. In this model $i = 0$ delivers $q = q^*$. The Friedman Rule is of course optimal in many monetary models. Something that is somewhat new here is that, given $i > 0$, the distortion is greater when information problems are more severe, i.e., q is smaller when ρ is smaller.

3 Inflation and Asset Prices

Consider $n = 2$ assets: $a_1 = m$ is fiat currency with $\delta_1 = 0$ and $A_1 = M$; $a_2 = a$ is a real asset with $\delta_2 = \delta > 0$ and $A_2 = A$. We write their prices as $\phi_1 = \phi$ and $\phi_2 = \psi$, and focus on stationary monetary equilibrium where $\phi^-/\phi = M/M^- = 1 + \gamma$. For this exercise we abstract from counterfeit currency considerations and assume agents always recognize money: $m \in S$ with probability 1. However, we assume only a fraction of agents recognize real assets: $a \in S$ with probability $\rho \in (0, 1)$. Call the event $S_1 = \{m\}$, in which case a seller in the DM does not recognize a , a *type 1 meeting*. The buyer's liquid wealth in such a meeting is $y_{S_1}(m, a) = y_1 = \phi m$

¹²This is a difference between proportional and generalized Nash bargaining: the latter implies $q \rightarrow q^*$ as $\gamma \rightarrow \beta - 1$ if and only if $\theta = 1$; the former $q \rightarrow q^*$ as $\gamma \rightarrow \beta - 1$ for all θ .

and the terms of trade are (p_1, q_1) , as given in Lemma 1. Similarly, call $S_2 = \{m, a\}$ a *type 2 meeting*. Liquid wealth in such a meeting is $y_{S_2}(m, a) = y_2 = \phi m + (\psi + \delta)a$ and the terms of trade are (p_2, q_2) .

Taking ρ as given for now, the DM value function is the special case of (5) given by

$$V(m, a) = W(y_2) + \lambda_1 [u(q_1) - p_1] + \lambda_2 [u(q_2) - p_2] + K, \quad (14)$$

where $\lambda_1 = \lambda(1 - \rho)$ and $\lambda_2 = \lambda\rho$, while (10) reduces to

$$\phi^- = \beta\phi [1 + \lambda_1\ell(q_1) + \lambda_2\ell(q_2)] \quad (15)$$

$$\psi^- = \beta(\psi + \delta) [1 + \lambda_2\ell(q_2)]. \quad (16)$$

The bargaining solution and market clearing imply $z(q_1) = \phi M$ and $z(q_2) = \phi M + (\psi + \delta)A$ as long as these yield $q_j < q^*$; otherwise $q_j = q^*$. Using this to eliminate (q_1, q_2) , (15)-(16) becomes a system of equations in asset prices, as in the general case. It is easier in this application, however, to work with quantities rather than prices.

Using $(\psi + \delta)A = z(q_2) - z(q_1)$, $1 + i = (1 + \gamma)/\beta$ and $\beta = 1/(1 + r)$, we reduce (15)-(16) to

$$i = \lambda_1\ell(q_1) + \lambda_2\ell(q_2) \quad (17)$$

$$(1 + r)A\delta = [z(q_2) - z(q_1)][r - \lambda_2\ell(q_2)]. \quad (18)$$

A stationary monetary equilibrium is summarized by a positive solution (q_1, q_2) to (17)-(18), as long as $q_j < q^*$. Notice that $q_1 < q_2$ and $q_j = q^*$ if and only if $\ell(q_j) = 0$ if and only if $y_j \geq y^*$. Also, if $i > 0$ then $y_1 < y^*$ and $q_1 < q^*$ (because a small reduction in q near q^* has a negligible effect, by the envelope theorem, while there is a first-order cost to carrying money when $i > 0$). It is possible to have $y_2 \geq y^*$, whence liquidity in a type 2 meeting suffices to purchase q^* . In this case, $q_2 = q^*$, $\ell(q_2) = 0$, and $q_1 = \tilde{q} < q^*$ solves $\lambda_1\ell(\tilde{q}) = i$. But it is also possible to have $y_2 < y^*$ and $q_2 < q^*$. Either case may obtain, depending on parameters, and in particular depending on whether the real asset is relatively abundant or scarce.

Proposition 2. Define A^* by

$$\frac{A^*\delta}{r} = \frac{z(q^*) - z(\tilde{q})}{1+r} > 0.$$

(i) If $A \geq A^*$ there exists a unique stationary monetary equilibrium, with $(q_1, q_2) = (\tilde{q}, q^*)$, $\phi = z(\tilde{q})/M$ and $\psi = \delta/r$. (ii) If $A < A^*$ there exists a unique stationary monetary equilibrium, where (q_1, q_2) solves (17)-(18), $\phi = z(q_1)/M$ and $\psi = [z(q_2) - z(q_1)]/A - \delta > \delta/r$.

The proof is in the Appendix, but the results should be clear from Figure 2, which displays functions $q_2 = \mu(q_1)$ and $q_2 = \alpha(q_1)$ defined by (17) and (18), respectively. It is easy to show $\mu(\cdot)$ is decreasing and $\alpha(\cdot)$ increasing, and they intersect for some $q_1 \in [0, q^*]$. For $A \leq A^*$, this intersection determines the equilibrium $(q_1, q_2) \in [0, q^*]^2$. For $A > A^*$, the intersection occurs at $q_2 > q^*$, but since $q_2 > q^*$ is not possible, equilibrium is $(q_1, q_2) = (\tilde{q}, q^*)$. Thus, the unique (stationary monetary) equilibrium is given by the intersection of $\mu(q_1)$ and $\bar{\alpha}(q_1) = \min\{\alpha(q_1), q^*\}$.

INSERT FIGURE 2 ABOUT HERE

When $A < A^*$, the asset bears a liquidity premium. Table 1 shows the effects of changing the rate of monetary expansion γ and the recognizability parameter ρ in this case (derivations are in the Appendix). Increasing γ increases inflation and lowers the return on perfectly liquid money, $R_m = \phi/\phi^-$. It has no effect on the return of illiquid real bonds, $R_r = 1+r = 1/\beta$, while the return on illiquid nominal bonds $R_n = 1+i = (1+\gamma)/\beta$ increases one-for-one with inflation (Fisher's theory). For our *partially* liquid asset a , an increase in γ shifts the μ curve southwest and leaves α unchanged, reducing q_1, q_2 and the return $R_a = (\psi + \delta)/\psi$. Intuitively, as inflation increases, agents try to economize on cash, reducing the CM price ϕ and DM value of money $q_1 = z^{-1}(\phi M)$. As agents desire fewer money balances, they endeavor to shift into real assets, which are (imperfect) substitutes for cash. With a fixed supply A , this raises the price ψ , lowers the dividend-price ratio, and hence lowers the return R_a .¹³

¹³This sounds like some discussions in monetary policy circles, claiming that easing monetary policy reduces

ζ	$\frac{\partial q_1}{\partial \zeta}$	$\frac{\partial q_2}{\partial \zeta}$	$\frac{\partial \phi}{\partial \zeta}$	$\frac{\partial \psi}{\partial \zeta}$	$\frac{\partial R_m}{\partial \zeta}$	$\frac{\partial R_a}{\partial \zeta}$
γ	-	-	-	+	-	-
ρ	-	?	-	+	0	-

Table 1: Effects of parameters when $A < A^*$.

Figure 3 plots ψ against i for an example, for different values of ρ . As ρ increases, so does $\partial\psi/\partial i$. Intuitively, as a becomes more liquid, the effect on ψ of i becomes stronger, because a is a better substitute for m . In terms of changes in ρ , more generally, Table 1 indicates that when the recognizability of a goes up, at the margin agents desire a reallocation of their portfolios out of m and into a . This drives ϕ down and ψ up, with ambiguous effects on q_2 . All these results are for the case $A \leq A^*$. When $A > A^*$, $q_2 = q^*$ and $q_1 = \tilde{q}$, where $\ell(\tilde{q}) = i/\lambda_1$. In this case, an increase in γ or ρ reduces q_1 and ϕ , with no effect on q_2 or ψ , which are pinned down by fundamentals. Again, it clearly matters whether A is above or below A^* , as we discuss further in Section 7.

INSERT FIGURE 3 ABOUT HERE

Before endogenizing information, we mention one more application with ρ fixed. This shows how monetary policy affects not only agents or markets that use money, but also those that do not. Consider two distinct decentralized markets, call them DM_1 and DM_2 . After each meeting of the CM, some agents go to DM_1 and others to DM_2 , say because different goods are traded there. In DM_2 all sellers accept both a and m , which means no one brings m to DM_2 . Meanwhile, in DM_1 only a fraction ρ of sellers accept a , so agents bring a portfolio (m, a) as in the benchmark model. One can show (Lester et al. [39]) that an increase in γ reduces consumption and welfare in *both* markets, even though there is no money in DM_2 . Intuitively,

interest rates, which is hard to understand in terms of Fisher's theory. It also helps us make sense of the remarks by Wallace [54] in footnote 3: assets bear different returns because of liquidity differentials, and Fisher's theory may fail for liquid assets because they are partial substitutes for cash. See Geromichalos et al. [14] and references therein for related results. See Li and Li [41] for a model that delivers the opposite result: increases in inflation decrease the price and increase the return on real assets (in that model, assets are complements rather than substitutes for cash, because they are used to collateralize monetary rather than real loans).

as γ increases, agents going to DM_1 shift out of m and into a , driving up ψ . This lowers the return on a , as well as the amount held by agents in DM_2 . This lowers consumption and utility for agents in DM_2 even though they never use money.¹⁴

4 Endogenous Information

We now use the above results to endogenize ρ in a model with two assets. For the sake of illustration, suppose one is a real asset a , and the other a fiat object m , although nothing much depends on this (one can similarly consider two fiat objects, like dollars and pesos, or two real assets, like stocks and bonds, or gold and silver). Assume that agent $h \in [0, 1]$ has an ex ante choice whether to acquire at cost $\kappa(h)$ the information, or perhaps the technology, that allows him to recognize the quality of a , maintaining the assumption that everyone can recognize m at zero cost. Without loss of generality, label agents so that $\kappa(h)$ is weakly increasing. Agent h accepts a in the DM if and only if he pays $\kappa(h)$, since this is the only way to distinguish asset quality, and if an uninformed seller were to accept a , buyers would hand over worthless facsimiles. The fraction of agents that incur the cost of becoming informed therefore determines ρ , the fraction that accept a in the DM. Depending on the application at hand, one can either assume $\kappa(h)$ is a one-time cost or a flow cost that h must pay each period; for now we assume it is a flow cost.

One can imagine several interpretations of κ . It is typically thought to be costly to learn to use a new medium of exchange, for a variety of reasons (some of which some are discussed in Lotz and Rocheteau [43]). Historically, it was difficult to distinguish the weight and fineness of coins without devices like scales and touchstones. It would have been even harder in those days to distinguish real from bogus paper claims, especially for the many who could not read. Although literacy has improved since then, distinguishing low- from high-quality assets remains a costly endeavor – a contemporary financial institution that wants to trade pools of asset-backed

¹⁴This extension was motivated by those who say monetary policy is irrelevant, to them, because they never use cash. The logic here shows that this position is, in addition to egocentric, incorrect.

securities, say, must set up a department with analysts to ascertain their values. Other costs may be technological, as in the case of debit cards, where sellers must buy a machine to transfer funds from one account to another. As these all seem potentially relevant, we are agnostic about the exact nature of κ .

Conditional on a fraction $\rho \in [0, 1]$ of others being informed, let $\Pi(\rho)$ denote the benefit to an individual agent from becoming informed. To determine this, let $\Sigma_1(\rho) = z[q_1(\rho)] - c[q_1(\rho)]$ denote a seller's surplus in a type 1 meeting, where $z(q)$ defined in (4) gives the real value of the payment to the seller as a function of q , and $q_1(\rho)$ is the equilibrium outcome characterized in Proposition 2. Similarly define $\Sigma_2(\rho) = z[q_2(\rho)] - c[q_2(\rho)]$ for a type 2 meeting. Then

$$\Pi(\rho) = \beta\lambda [\Sigma_2(\rho) - \Sigma_1(\rho)].$$

After inserting $z(q) = \theta u(q) + (1 - \theta)c(q)$, this reduces to

$$\Pi(\rho) = \beta\lambda (1 - \theta) \{u[q_2(\rho)] - c[q_2(\rho)] - u[q_1(\rho)] + c[q_1(\rho)]\}. \quad (19)$$

The following useful result, proved in the Appendix, says that the benefit of being informed is increasing in the measure of informed agents.

Lemma 2. $\Pi(\rho)$ is increasing.

Equilibrium with endogenous information is defined as a triple (q_1, q_2, ρ) such that: (i) given ρ , (q_1, q_2) solve (17)-(18) from the model with ρ fixed; and (ii) given (q_1, q_2) , the measure of informed agents ρ satisfies one of the following configurations: $\rho = 0$ and $\Pi(0) \leq \kappa(0)$; $\rho = 1$ and $\Pi(1) \geq \kappa(1)$; or $\rho \in (0, 1)$ and $\Pi(\rho) = \kappa(\rho)$. To state this last condition equivalently but slightly differently, in a way that will be useful later, let $\mathcal{M}(C)$ be the measure of agents satisfying criterion C . Then define the aggregate best response correspondence $T : [0, 1] \rightarrow [0, 1]$ by

$$T(\rho) = \{\rho' \in [0, 1] : \rho' = \mathcal{M}[\kappa < \Pi(\rho)] + \Phi \mathcal{M}[\kappa = \Pi(\rho)] \text{ for some } \Phi \in [0, 1]\}. \quad (20)$$

Thus, all agents with $\kappa < \Pi(\rho)$ must invest, since the benefit exceeds the cost; while any agent with $\kappa = \Pi(\rho)$ can invest in information with an arbitrary probability Φ since they are

indifferent. Equilibrium condition (ii) is then satisfied by a fixed point of this correspondence, $\rho^* = T(\rho^*)$.

We will use $T(\rho)$ later, when we consider the general case, but we begin with the simpler special case where the cost is degenerate: $\kappa = \bar{\kappa}$ for all agents. First, note that for a given ρ , the equilibrium is independent of the information cost (distribution). Therefore, even though $\Pi(\rho)$ depends on the equilibrium objects $q_1(\rho)$ and $q_2(\rho)$, $\Pi(\rho)$ does not depend on $\bar{\kappa}$, so we know that we can find $\bar{\kappa} \geq 0$ such that $\bar{\kappa} \leq \Pi(0)$. This means the cost of information is less than the benefit for $\rho = 0$, and *a fortiori* for all ρ . Hence, $\bar{\kappa} \leq \Pi(0)$ implies there is a unique equilibrium and it entails $\rho^* = 1$. Similarly, we can set $\bar{\kappa} \geq \Pi(1)$, which implies there is a unique equilibrium and it entails $\rho^* = 0$. And, since typically $\Pi(1) > \Pi(0)$, except for extreme parameter values like $A = 0$, we can set $\bar{\kappa}$ such that $\Pi(0) < \bar{\kappa} < \Pi(1)$ which implies there will be exactly three equilibria: $\rho^* = 0$; $\rho^* = 1$; and the unique $\rho^* \in (0, 1)$ solving $\bar{\kappa} = \Pi(\rho^*)$. In this case, if no one else is informed then it is not worthwhile to invest in information, so m is the only liquid asset; but if everyone else is informed then it is worthwhile for all to invest, and a is perfectly liquid; and there is a mixed strategy equilibrium where a is partially liquid.

In terms of the likelihood of multiplicity, it is obvious from the above observations that we need $\bar{\kappa}$ to be neither too big nor too small. One can also characterize the policy parameters that deliver uniqueness or multiplicity, using Figure 2. For instance, as we approach the Friedman rule, $i \rightarrow 0$, it is clear that q_1 and q_2 both go to q^* , so $\Pi(\rho)$ gets small for all ρ and hence there is a unique equilibrium $\rho^* = 0$. Intuitively, there is no sense making a costly investment in information about the alternative asset a when m delivers q^* in all meetings. As i gets bigger, q_1 and q_2 both decrease, but as $i \rightarrow \infty$ we have $q_1 \rightarrow 0$ while $q_2 \rightarrow q_0 > 0$. In this case there will be an equilibrium with $\rho > 0$ as long as $\bar{\kappa}$ is not too high, and there could still be multiple equilibria. One can similarly analyze how the outcome depends on other parameters.¹⁵

¹⁵Consider, e.g., the effective stock of real assets $A\delta$. Since an increase in $A\delta$ shifts up the α curve in Figure 2 without affecting the μ curve, this decreases q_1 and increases q_2 . This makes it more likely that there will be an equilibrium with $\rho > 0$, and possibly multiple equilibria. If $A\delta$ is small, naturally, the likely outcome is a unique equilibrium with $\rho^* = 0$ – why invest in costly information about something that is not abundant?

The intuition for multiplicity here is straightforward, but nonetheless interesting, because of the way it works through general equilibrium asset market effects. For small values of ρ , when the asset is illiquid, the CM demand for the asset is low. As a result, its price ψ is relatively low, and few agents find it worthwhile investing resources so that they can recognize it. As ρ increases, the asset becomes more useful as a medium of exchange, causing demand in the CM to increase for a and decrease for m . As a result, ψ rises and ϕ falls (see Table 1), so that the payoff to being able to accept both a and m increases relative to the payoff from being able to accept only m . Therefore, as ρ increases more agents are inclined to invest in information. The bottom line is that there exists a strategic complementarity between agents who acquire information, manifesting itself through the equilibrium prices of the assets.

We also want to consider the more general case, where we do not assume $\kappa = \bar{\kappa}$ for all agents. One reason is that the equilibrium with $\rho^* \in (0, 1)$, which is interesting because a is partially liquid, is not *stable* according to a simplistic but nevertheless standard criterion: if we change ρ^* to $\rho^* + \varepsilon$ for arbitrarily small $\varepsilon > 0$ ($\varepsilon < 0$), then it is a best response for everyone (no one) to invest, leading to the corner equilibrium with $\rho = 1$ ($\rho = 0$). Consider a distribution of costs with CDF $F : \mathbb{R} \rightarrow [0, 1]$, where $F(\bar{\kappa}) = \mathcal{M}(\kappa \leq \bar{\kappa})$. For an arbitrary distribution, the correspondence defined in (2) is non-decreasing in the following sense:

Lemma 3. *For any ρ' and $\rho'' > \rho'$, $\inf T(\rho') \leq \inf T(\rho'')$.*

Lemma 3 follows directly from Lemma 2; intuitively, since the value of information $\Pi(\rho)$ is increasing in ρ , it cannot be a best response for an agent with cost κ to invest in information when a fraction ρ' of other agents invest and not when a $\rho'' > \rho'$ invest. In general, $T(\rho)$ is horizontal in any interval (ρ', ρ'') such that $\mathcal{M}[\Pi(\rho') < \kappa < \Pi(\rho'')] = 0$ (increasing ρ over such an interval does not increase the measure agents who invest), and vertical at any ρ such that $\mathcal{M}[\kappa = \Pi(\rho)] > 0$ (at any such ρ there is a mass of agents who are indifferent to investing). Two examples are shown in Figure 4. The one on the left is for a discrete distribution: $\kappa = \kappa_1$ for a measure p_1 of agents; $\kappa = \kappa_2$ for a measure $p_2 - p_1$ of agents; and $\kappa = \kappa_3$ for the rest. The

case on the right is for a continuous distribution. The upper graphs show $\Pi(\rho)$ and $\kappa(\rho)$, and the lower graphs use these to construct the correspondence $T(\rho)$.

INSERT FIGURE 4 ABOUT HERE

Lemma 3 implies immediately that an equilibrium – i.e., a fixed point of $T(\rho)$ – always exists, without even worrying about whether T is continuous. The logic is the same as that behind Tarski’s fixed point theorem, which assumes monotonicity but not continuity. To see how it works, consider first the case where $T : [0, 1] \rightarrow [0, 1]$ is a (single-valued) function. Suppose it does not have a fixed point. Then, in particular, $T(0) > 0$. As we increase ρ from 0 to 1, $T(\rho)$ is pinched between the 45° line and 1 – it cannot jump over the 45° line, because it is increasing. Hence, it has a fixed point $\rho^* = T(\rho^*)$. Clearly, the same logic applies if T is a correspondence satisfying Lemma 3. So there must be some $\rho^* \in [0, 1]$ such that $\rho^* \in T(\rho^*)$.

For an arbitrary $F(\kappa)$, $T(\rho)$ can have any number of fixed points in $[0, 1]$. Consider the example on the left in Figure 4. If $\rho = 0$ the benefit of information is not worth the cost for anyone, $\Pi(0) < \kappa_1 < \kappa_2 < \kappa_3$; so there is an equilibrium with $\rho = 0$. If we increase ρ to ρ_1 then those with the lowest cost are just indifferent about investing in information, $\Pi(\rho_1) = \kappa_1$; so there is an equilibrium where a fraction of the agents with κ_1 invest, and the rest of these agents do not, nor do any of those with κ_2 or κ_3 . And if we increase ρ to $\rho_2 = p_1$ there is an equilibrium where all of the low cost agents and none of the other agents invest. And so on. The example on the right is similar, although in this case $\rho = 0$ is not an equilibrium, since $\Pi(0)$ exceeds κ for some agents, and the lowest equilibrium ρ is ρ_1 , where the marginal investor has cost $\kappa(\rho_1) = \Pi(\rho_1)$. There is another equilibrium at $\rho = \rho_2$ and, in this case, another at $\rho = 1$, since $\Pi(1)$ exceeds κ for even the highest-cost agent. In either example, there are some equilibria where T cuts the 45° line from below, which are unstable in the sense discussed above, and some where T cuts the 45° line from above, which are stable.¹⁶

¹⁶In the general $F(\kappa)$ case, to give simple conditions that guarantee an interior equilibrium, simply assume that $\kappa(h) = 0$ for a positive measure of agents, so they always invest in information, and that $\kappa(h)$ is arbitrarily high

We summarize the key results below – mainly, equilibrium always exists, since as discussed above there is always a solution to $\rho = T(\rho)$, and once we have ρ , (q_1, q_2) and the other endogenous variables, including assets prices (ψ, ϕ) , follow from the analysis in previous sections. Also, it is easy to generate multiple equilibria, with different solutions to $\rho = T(\rho)$, and these different equilibria imply different values for the other equilibrium variables as discussed above – e.g., higher ρ is associated with higher ψ and lower ϕ .

Proposition 3. *Equilibrium with endogenous information exists. If $\kappa(h)$ is small for a positive measure of agents and $\kappa(h)$ is big for a positive measure, any equilibrium satisfies $\rho^* \in (0, 1)$. Multiple equilibria can exist, although the Friedman rule necessarily delivers a unique (and efficient) outcome.*

In terms of economic content, several points are worth mentioning. First, when $\rho^* = 0$, a is illiquid and the DM looks like a cash-in-advance market as in, e.g., Lucas and Stokey [45]. When $\rho^* = 1$, a is perfectly liquid and circulates concurrently with m , at least as long as $(1+r)\delta A < rz(q^*)$ (see footnote 17). If this inequality is reversed, then $\rho^* = 1$ implies m is not valued, and the DM looks like a cashless economy such as the one studied by, e.g., Woodford [60]. Our point, however, is *not* to provide microfoundations for any particular *ad hoc* assumption on transaction patterns, like money is always used or money is never used. To the contrary, as soon as one endogenizes the set of transactions where a particular instrument is used for payments, or equivalently, as collateral, it becomes all too evident that in general this set is neither uniquely determined nor structurally invariant. There need not be a unique solution to $\rho^* = T(\rho^*)$, and

for a positive measure of agents, so they never invest. To give more detail, first impose $(1+r)\delta A < rz(q^*)$, which guarantees there is a monetary equilibrium even if $\rho = 1$ (we need this here because if $\rho = 1$ and A is big money will not be valued, which did not come up before because we had $\rho < 1$). Consider the payoff to an individual from being informed when $\rho = 0$. Since $\rho = 0$ implies $q_1 = \hat{q} = \ell^{-1}(i/\lambda)$ and $\psi = \delta/r$, if a single individual were to be informed, instead of accepting only m in exchange for q_1 , he could accept m and a for $q_2 = \min\{q^*, \bar{q}\}$, where \bar{q} solves $(1+r)A\delta = [z(\bar{q}) - z(\hat{q})]r$. Since $\Pi(\rho)$ is increasing, $\Pi(0)$ is the minimum possible benefit to information. Now consider the benefit when $\rho = 1$. Since $(1+r)\delta A < rz(q^*)$, equilibrium exists with $q_2 = \hat{q} > q_1 = \underline{q} > 0$, where $(1+r)A\delta = [z(\hat{q}) - z(\underline{q})](r-i)$. Given this, $\Pi(1)$ is the maximum possible benefit to being informed. Equilibrium with $\rho^* = 0$ exists if $\kappa(0) \geq \Pi(0)$, and it is unique if $\kappa(0) > \Pi(1)$. Similarly, equilibrium with $\rho^* = 1$ exists if $\Pi(1) \geq \kappa(1)$, and it is unique when $\Pi(0) > \kappa(1)$. For an interior equilibrium $\rho^* \in (0, 1)$, sufficient conditions are either $\kappa(0) < \Pi(0)$ and $\kappa(1) > \Pi(1)$, or $\kappa(0) > \Pi(0)$ and $\kappa(1) < \Pi(1)$.

even if there is, ρ^* will very obviously depend on policy and other parameters in general.¹⁷

The theory implies that small changes in the environment, including the cost of information, can have large effects. Consider changing from $F_1(\kappa)$ to $F_2(\kappa)$, where the latter first-order stochastically dominates the former. This shifts $T(\rho)$ down for all ρ . In the case of a unique interior equilibrium, this unambiguously reduces ρ^* . More dramatically, in Figure 5 the shift changes the cardinality of the equilibrium set: starting in equilibrium with the highest ρ , a small increase in the cost of verifying asset quality leads to a drastic change in liquidity, with commensurate changes in prices, allocations and payoffs. Although we would not push this too far, here, one can view recent financial crises through the lens of this theory. Even if a change in the cost of information does not alter the structure of the equilibrium set, the model generates internal multiplier effects. Suppose there is a unique equilibrium $\rho^* \in (0, 1)$. When the cost of information goes up, the number of informed agents ρ^* falls, holding other variables constant. If we may be permitted to indulge in pseudo-dynamics, for a moment, the fall in ρ^* leads to a decrease in the value of the asset, so even fewer agents are willing to bear the cost of information. This continues as the initial increase in cost leads to further and further declines in asset prices.¹⁸

INSERT FIGURE 5 ABOUT HERE

A similar multiplier effect holds for an increase in inflation. Initially, higher i lowers the value of money ϕ , which increases ρ^* , which further decreases ϕ , and so on. Notice, however, that while increasing inflation may increase the liquidity of some assets, this is not desirable from a welfare perspective. As mentioned earlier, the efficient outcome is achieved when inflation is

¹⁷In models that impose a cash-in-advance constraint on some goods and not others, changes in parameters can generate adjustment on the intensive margin (i.e., on quantities consumed of so-called cash goods and so-called credit goods), but, with few exceptions, there is no adjustment along the extensive margin determining which trades use which instruments.

¹⁸In terms of observations, in the case of mortgage-backed securities, e.g., when housing prices fell the value of the assets became difficult to discern, and counterparties became increasingly unwilling to accept them. The haircut on agency mortgage-backed securities, e.g., went from 2.5 % in the spring of 2007 to 8.5 % in the fall of 2008 (Krishnamurthy [30]).

minimized, at the Friedman Rule. This policy not only satiates agents in liquidity, resulting in $q = q^*$ in all trades, it additionally allows us to save on the cost of information acquisition, given the maintained assumption that all agents recognize cash without having to invest (although it could also be interesting to proceed without this assumption).

On a related point, to close this section, we sketch how to use the framework to think about choices over alternative commodity monies. Consider the case of two real assets, say a_1 is gold and a_2 is silver, to capture in a stylized way the long and contentious debate over bimetallism in US history (see Friedman [12] for a synopsis and references). Assume without loss in generality that the two metals have the same dividend $\delta > 0$ by appropriately choosing units. Also, assume $\max\{A_1, A_2\} < A^* = ry^*/(1+r)\delta$, so that neither is individually plentiful enough to support q^* . To ease the presentation, let the cost of recognizing them be the same, $\kappa_1(h) = \kappa_2(h) = \kappa(h)$. This means household have to pay $2\kappa(h)$ to be able to evaluate both gold and silver, which may not be the most realistic scenario (say, if they only need to buy one scale), but is fine for the purpose of illustration.

Consider a candidate equilibrium in which everyone becomes informed about gold but not silver. Let the CM prices be ϕ_1^G and ϕ_2^G and the DM quantity $q^G = z^{-1}[(\phi_1^G + \delta)A_1]$ in this scenario, where gold is the unique medium of exchange, since it is uniquely recognizable. By the usual reasoning,

$$\frac{\phi_1^G}{\beta(\phi_1^G + \delta)} - 1 = \lambda\ell(q^G) \text{ and } \phi_2^G = \delta/r,$$

so that gold bears a liquidity premium while silver is priced fundamentally. Suppose an individual deviates and becomes informed about silver *in addition to* gold, and let \tilde{q}^G be the amount he produces in a DM meeting. If $(\phi_1^G + \delta)A_1 + (\phi_2^G + \delta)A_2 > y^*$ then $\tilde{q}^G = q^*$; otherwise $\tilde{q}^G < q^*$ solves $z(\tilde{q}^G) = (\phi_1^G + \delta)A_1 + (\phi_2^G + \delta)A_2$. Also, let $\hat{q}^G = z^{-1}[(\phi_2^G + \delta)A_2]$ be the quantity he produces if he deviates by becoming informed about silver *instead of* gold. To support this equilibrium we need: (i) no agent wants to learn to accept silver as well as gold; (ii) no agent wants to learn to accept silver instead of gold; and (iii) all agents want to learn to accept gold.

These conditions all hold when

$$\kappa(0) > \beta\lambda [z(\tilde{q}^G) - c(\tilde{q}^G) - z(q^G) + c(q^G)]; q^G \geq \tilde{q}^G; \kappa(1) < \beta\lambda [z(q^G) - c(q^G)]. \quad (21)$$

Symmetrically, an equilibrium where silver is the unique medium of exchange, with CM prices ϕ_1^S and ϕ_2^S and DM quantity q^S , exists when (21) holds after changing the superscript from G to S . Both equilibria exist for some parameters (e.g., when A_1 and A_2 are not too different), but one may dominate the other. If $A_1 < A_2 < A^*$, so that gold is more scarce than silver, e.g., a gold equilibrium can exist even though a silver equilibrium would support more DM trade. It may be better still to have both gold and silver used as money, of course, but it is important not to neglect the cost $\kappa(h)$ in this calculation. We leave further analysis to future work. The point here is to show that once exchange patterns and transaction instruments are endogenous, one can study a variety of questions in historical and contemporary economics.

5 Costly Counterfeiting

In the model presented above, tractability is enhanced by the property that sellers who do not recognize an asset reject it outright, since this avoids complications associated with bargaining under private information. This property follows from the assumption that the cost of producing worthless facsimiles of an asset is $k = 0$. We now relax this assumption by considering $k > 0$. In this case, sellers may accept an asset that they do not recognize up to a point: they only produce q^k for a^k units of the asset, where generally $a^k < a$ when k is small. Such restrictions on asset transferability are often imposed exogenously – with Kiyotaki and Moore [25], [26] and Holmstrom and Tirole [19] providing some of the best examples – but here the restriction is an equilibrium outcome. This means it is not generally invariant to parameter changes, of course, and in particular, $a^k \rightarrow 0$ and $q^k \rightarrow 0$ as $k \rightarrow 0$, which implies that our baseline model can be considered a useful limiting case.

We go into more detail in the working paper, while here we only sketch the ideas, since the approach is a direct application of recent results by Rocheteau [49] and Li and Rocheteau [42].

To be clear about the assumptions, we assume that buyers in DM meetings make take-it-or-leave-it offers, as Li-Rocheteau assume in their baseline analysis, although they also show how to relax this. It is important to relax this if one wants to endogenize sellers' information, of course, since when buyers make take-it-or-leave-it offers, sellers get no return and hence will never invest in information. However, to illustrate the basic method we stick to take-it-or-leave-it offers here and refer readers to the above-mentioned papers.

Consider the case of two assets, m and a , where again everybody recognizes m and no one recognizes a , so that $\rho = 0$; this is without much loss in generality, since if any seller were to recognize a , bargaining could proceed under full information. Now agents in the DM can produce any number of worthless facsimiles of a , at zero marginal cost if they pay a fixed cost $k > 0$ each period in the CM. Also, following the related literature, assume bad assets fully depreciate after one period (e.g., imagine that in the CM they are identified, confiscated and destroyed, the way counterfeit currency is usually handled).

Let χ denote the probability that an agent pays cost k to acquire the counterfeiting technology. After making this decision, agents in the CM choose a portfolio $[m(\chi), a(\chi)]$ and then proceed to the DM. With probability λ , each agent is a buyer in a match and makes a take-it-or-leave-it offer (q, d, s) , asking for q units of the good in exchange for d dollars and s shares of the asset, with no stipulation that these shares are real or counterfeit, since the seller cannot tell the difference. Any offer must be feasible, in the sense that a buyer must be able to come up with the dollars and (genuine or counterfeit) shares offered. In response, the seller chooses a probability of accepting, denoted ξ . Unfortunately, as is common in models with bargaining under asymmetric information, there are many equilibria in this game. Specifically, if the choice of χ occurs before the offer (q, d, s) , the seller has to formulate beliefs about χ based on the offer. Since sequential equilibrium imposes little discipline on beliefs, many equilibria are possible.

Rocheteau and Li-Rocheteau, following work by In and Wright [21], consider the *reverse-order game* in which buyers first choose (q, d, s) and then χ . Changing the timing in this way

does not affect the payoffs or information of the seller; indeed, the two games have the same reduced normal form.¹⁹ With the new timing, however, the game described above has proper subgames, and thus sequential equilibrium imposes strict discipline: following any offer, the seller's belief about χ must be consistent with equilibrium strategies. Li-Rocheteau show that there is a unique equilibrium of this game, which is also an equilibrium of the original game for specific reasonable beliefs (and so the approach can be thought of as a refinement on the original game).

In fact, these authors show that the unique equilibrium can be found simply by maximizing the payoff of the buyer subject to two constraints: the usual condition that makes the seller willing to accept in the presence of no private information, $c(q) \leq \phi d + (\psi + \delta)s$; and the so-called no-counterfeiting condition, $k \geq s[\psi - \beta(1 - \rho\lambda)(\psi + \delta)]$. The latter condition ensures $\chi = 0$, since it costs more to pay the fixed cost of being able to counterfeit than the buyers gets in exchange for his assets. This obviously holds if and only if s is not too big, which makes it reasonable to believe the buyer is not a counterfeiter. This is the bound on asset transferability mentioned earlier. Related to our comments on cash-in-advance and other *ad hoc* transaction patterns, it is apparent that this constraint ought to be endogenous, and is not generally invariant to policy interventions.

Moreover, and most relevant for our purposes, it is immediate from the no-counterfeiting condition that $s \rightarrow 0$ as $k \rightarrow 0$. Hence, our baseline model can be regarded as a special case where counterfeiting is costless, $k = 0$. But the results are robust in the sense that similar economic outcomes emerge when $k > 0$. In the working paper we characterize equilibrium for all $k \geq 0$. The outcomes are economically interesting, but since the algebra is tedious, and since the methods follow Rocheteau and Li-Rocheteau closely, we do not include the results here.

¹⁹This approach is in the spirit of the invariance condition required for strategic stability in Kohlberg and Mertens [29], which requires that a solution to a game should also be a solution to any other game with the same reduced normal form.

6 International Economics

Consider the case of an economy with two fiat objects, as in a Latin American country with both dollars and pesos. Let M_j denote the stock of fiat currency $j \in \{1, 2\}$, which grows at rate γ_j . Let ρ_1 be the probability sellers recognize only M_1 , ρ_2 the probability they only recognize M_2 , and ρ_{12} the probability they recognize both. With a slight abuse of notation, let $\lambda_1 = \lambda\rho_1$, $\lambda_2 = \lambda\rho_2$, and $\lambda_{12} = \lambda\rho_{12}$ denote the probabilities that an agent is a buyer in the respective meetings. We begin with an exogenous information structure, where λ_1 , λ_2 and λ_{12} are fixed. In any equilibrium in which both currencies are valued, the quantities traded in DM meetings are $q_1 = z^{-1}(\phi_1 M_1)$, $q_2 = z^{-1}(\phi_2 M_2)$ and $q_{12} = z^{-1}(\phi_1 M_1 + \phi_2 M_2)$. For each currency, $1 + \pi_j = \phi_j^- / \phi_j$ is the inflation rate, which may or may not equal $1 + \gamma_j$ in this application, as we will see. Using the Fisher Equation, as before, $1 + i_j = \phi_j^- / (\phi_j \beta)$ is the interest rate on an illiquid nominal bond denominated in currency j .

If both currencies are valued, the equilibrium conditions are

$$i_1 = \lambda_1 \ell(q_1) + \lambda_{12} \ell(q_{12}) \quad (22)$$

$$i_2 = \lambda_2 \ell(q_2) + \lambda_{12} \ell(q_{12}). \quad (23)$$

There are several cases of interest. The simplest is when some sellers recognize currency 1 and others recognize currency 2, but no one recognizes both: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_{12} = 0$. In this case we can solve independently for q_1 and q_2 from $\ell(q_j) = i_j / \lambda_j$, and here $\pi_j = \gamma_j$. Both currencies are valued, even if one is being issued at a higher rate and hence has a higher inflation and nominal interest rate, since M_1 and M_2 are each essential for some meetings. It is immediate that $\partial q_j / \partial \lambda_j > 0$ and $\partial q_j / \partial i_j < 0$.²⁰

For the next case suppose $\rho_1 = 0$, $\rho_2 > 0$, and $\rho_{12} > 0$, which captures the scenario where

²⁰Notice that λ_1 and i_1 do not affect q_2 , and vice versa, but this result is not especially robust. If there were an asset a_3 , agents would hold all three. Then an increase in i_1 makes them want to shift out of m_1 and into a_3 , driving up ϕ_3 . This makes them want to shift out of a_3 and into m_2 . This increased demand for m_2 affects ϕ_2 and q_2 . So, even if i_1 does not affect m_2 directly, there are equilibrium effects via a_3 .

all sellers accept pesos, and some also accept dollars. The equilibrium conditions are

$$\begin{aligned} i_1 &= \lambda_{12}\ell(q_{12}) \\ i_2 &= \lambda_2\ell(q_2) + \lambda_{12}\ell(q_{12}), \end{aligned}$$

where $q_2 = z^{-1}(\phi_2 M_2)$, $q_{12} = z^{-1}(\phi_1 M_1 + \phi_2 M_2)$, and again $\pi_j = \gamma_j$. Combining these yields $i_2 - i_1 = \lambda_2\ell(q_2)$, which implies that both currencies are valued only if $i_2 > i_1$. Since dollars are strictly dominated by pesos in terms of liquidity (i.e., anyone who accepts dollars also accepts pesos), the former will only be held if it promises a larger return. Among other results, Table 2 shows that increasing the rate at which we issue dollars γ_1 increases π_1 and i_1 , which decreases ϕ_1 , increases ϕ_2 , and obviously increases the exchange rate $e = \phi_2/\phi_1$.

ζ	$\frac{\partial q_{12}}{\partial \zeta}$	$\frac{\partial q_2}{\partial \zeta}$	$\frac{\partial \phi_1}{\partial \zeta}$	$\frac{\partial \phi_2}{\partial \zeta}$	$\frac{\partial e}{\partial \zeta}$
i_1	-	+	-	+	+
i_2	0	-	+	-	-

Table 2: Effects of policy changes.

Instead of taking recognizability to be exogenous, to discuss dollarization, we now allow locals to invest in order to recognize foreign currency. For this application we change the interpretation of $\kappa(h)$ to a one-time, not a per-period, cost.²¹ Now the decision to acquire the information needed to recognize US dollars is based on a comparison of the individual cost $\kappa(h)$ with the expected benefit of accepting both pesos and dollars forever. This is given by

$$\frac{\Pi(\rho_{12})}{1 - \beta} = \frac{\beta\lambda(1 - \theta)}{1 - \beta} \{[u(q_{12}) - c(q_{12})] - [u(q_2) - c(q_2)]\},$$

where it is understood that q_{12} and q_2 depend on ρ_{12} . One can show that $\Pi(\rho_{12})$ is increasing as $\Pi(\rho)$ was in Section 4. Intuitively, when the measure of locals who accept dollars increases, ϕ_1 goes up and this gives other locals more incentive to learn to trade in dollars. As before, this can generate multiple equilibria, some where dollars circulate widely and are highly valued, and others where this is not the case.

²¹Discussion of why it might be costly to learn to use a new medium of exchange, with a particular focus on dollarization, is contained in Uribe [53], Guidotti and Rodriguez [18], and Dornbusch et al. [9].

Moreover, $\Pi(\rho_{12})$ shifts up when we raise γ_2 . Assuming there is a unique equilibrium $\rho_{12} \in (0, 1)$, this leads to an increase in i_2 and hence dollarization. This effect can be dramatic in practice. As Guidotti and Rodriguez [18] report, dollarization in Bolivia went from close to 0 in 1985 to nearly 50% in 1987. Note the asymmetry here when $\kappa(h)$ is a one-time cost: although an increase in peso inflation leads to dollarization, a subsequent decrease does *not* reverse the process. This is because once locals learn how to trade using dollars, they do not forget when peso inflation comes down. This captures the notion of hysteresis in dollarization that has been discussed in the literature but not formalized in this way. The theory also predicts ρ_{12} increases if we lower i_1 , instead of raising i_2 . Thus, dollars are more likely to circulate in Latin America when either peso inflation is high or dollar inflation is low, consistent with conventional wisdom.²²

The next case we consider is $\rho_1 = \rho_2 = 0$ and $\rho_{12} > 0$, so that the two currencies are perfect substitutes. To reduce notation, assume $\rho_{12} = 1$. Then, in any equilibrium in which both currencies are valued, we have

$$i_1 = \lambda \ell(q) \text{ and } i_2 = \lambda \ell(q),$$

where $q = q_{12}$. A necessary condition for this type of equilibrium is that $i_1 = i_2$, but note that, in this case, this does *not* necessarily mean the growth rates of M_1 and M_2 have to be the same (see below). This case is studied by Kareken and Wallace [24] in an OLG model, where they prove the exchange rate is indeterminate, and argue that this is natural since $e = \phi_2/\phi_1$ is the relative price of two fiat objects. We think this is worth re-examining in our model, not just to show yet another application, but because their result seems to be under-appreciated.²³

There are two versions of Kareken-Wallace indeterminacy: one in a stationary and the other in a non-stationary context. To begin with the stationary result, which is much easier, suppose

²²We are obviously being a little loose in this paragraph about expectations of policy changes, which matter for one-time investment decisions. For the sake of discussion, assume that both increases and decreases in inflation come as complete surprises.

²³Perhaps this is because, as they themselves put it, “some economists are more than a little doubtful about all OLG models.” Again, this is where modern monetary theory may help.

$\gamma_1 = \gamma_2 = \gamma$, and consider an equilibrium where $i_1 = i_2 = i$ and q solves $i = \lambda\ell(q)$. This implies $z(q) = \phi_1 M_1 + \phi_2 M_2$ is constant as well. Now consider any arbitrary constant exchange rate $e > 0$, and let $M = M_1 + eM_2$ denote the aggregate supply of money measured in units of currency 1, which is growing at rate γ . Since $\phi_1 M = z(q)$ is constant, ϕ_1 decreases at rate γ , as does ϕ_2 . An equilibrium is comprised of time-invariant values for q and e , plus paths for ϕ_1 and ϕ_2 , such that q satisfies $i = \lambda\ell(q)$, ϕ_1 and ϕ_2 decrease at rate γ , and $z(q) = \phi_1 [M_1 + eM_2]$. Consider this last condition at date $t = 0$. There are two free variables – the exchange rate e and the initial price ϕ_1 – that must satisfy a single equality. This makes e indeterminate.

This initially surprising result has an intuitive interpretation. Suppose there are two fiat objects, blue notes and red notes, both of which are being issued at the same rate and are equally liquid. If $e = 2$, we could call blue and red notes 5 and 10 dollar bills, say, while if $e = 4$ we could call them 5 and 20 dollar bills. Prices at time $t = 0$ will adjust, which can affect the total amount of money measured in units of blue notes, but this does not affect q . Still, due to fiscal considerations, the choice of e is not neutral. Suppose naturally that lump sum transfers of new notes go to different subsets of agents: Americans get dollars and Mexicans get pesos. For group 1, the value of the transfer in terms of CM numeraire is $T_1 = \gamma M_1 \phi_1$, while for group 2 it is $T_2 = \gamma M_2 \phi_1 e$. When we increase e , ϕ_1 and ϕ_2 both fall, while T_1 falls and T_2 rises. Since the transfers are lump sum they do not affect q , but they do affect CM hours worked and hence welfare for the two groups. Therefore e matters.

To briefly sketch the nonstationary Kareken-Wallace result, suppose the two currencies are being issued at different rates γ_1 and γ_2 (both still constant over time). Consider an arbitrary $e > 0$. Given the stocks of each currency at $t = 0$, $M_{1,0}$ and $M_{2,0}$, for all future t we have $M_{j,t} = (1 + \gamma_j)^t M_{j,0}$. Letting $M_t = M_{1,t} + eM_{2,t}$, the growth rate of this aggregate $1 + \bar{\gamma}_t = M_t/M_{t-1}$ satisfies

$$1 + \bar{\gamma}_t = (1 + \gamma_1) \left(\frac{1}{1 + \omega_t} \right) + (1 + \gamma_2) \left(\frac{1}{1 + 1/\omega_t} \right),$$

where $\omega_t = e(1 + \gamma_2)^{t-1} M_{2,0} / (1 + \gamma_1)^{t-1} M_{1,0}$, and $\bar{\gamma}_t \rightarrow \max\{1 + \gamma_1, 1 + \gamma_2\}$ as $t \rightarrow \infty$. In

equilibrium with both currencies valued,

$$(1+r)\phi_{j,t-1}/\phi_{j,t} = 1 + \lambda\ell(q_t). \quad (24)$$

Then, using $z(q_t) = \phi_{1,t}M_t$ for all t , we have

$$\frac{\phi_{j,t-1}}{\phi_{j,t}} = \bar{\gamma}_t \frac{z(q_{t-1})}{z(q_t)}. \quad (25)$$

Combining (24) and (25) yields a first order difference equation,

$$(1+r)\bar{\gamma}_t \frac{z(q_{t-1})}{z(q_t)} = 1 + \lambda\ell(q_t). \quad (26)$$

As in Kareken-Wallace, for any e there exists a unique solution to this difference equation satisfying the nonnegativity and boundedness requirements. This equilibrium has a time-varying supply of real balances, which makes it hard to characterize, but again nothing pins down e . Note that deriving this result within the context of our model potentially allows us to move beyond the original Kareken-Wallace result: while they assumed that both currencies are perfectly liquid, in our search-based model one could allow dollars and pesos to have different probabilities of acceptance, either exogenously or endogenously. More work on these issues would be interesting, but is beyond the scope of the current paper.

7 Discussion

The basic ideas here are not new. Samuelson [52], for one, put it this way:

It is true that in a world involving no transaction friction and no uncertainty, there would be no reason for a spread between the yield on any two assets, and hence there would be no difference in the yield on money and on securities... In fact, in such a world securities themselves would circulate as money and be acceptable in transactions... Of course, the above does not happen in real life, precisely because uncertainty, contingency needs, non-synchronization of revenues and outlay, transactions frictions, etc., etc. are all with us.

Samuelson and his contemporaries, however, did not write down explicit models incorporating the relevant frictions. Nor did they say much about why these frictions, if they were in a model, would be expected to generate a role for money or liquid securities, as opposed to credit.

On informational frictions, in particular, consider Jevons [22], who defined a quality called *cognizability* as follows:

By this name we may denote the capability of a substance for being easily recognized and distinguished from all other substances. As a medium of exchange, money has to be continually handed about, and it will occasion great trouble if every person receiving currency has to scrutinize, weigh, and test it. If it requires any skill to discriminate good money from bad, poor ignorant people are sure to be imposed upon... Precious stones, even if in other respects good as money, could not be so used, because only a skilled lapidary can surely distinguish between true and imitation gems.

A century later, Alchian [2] argued “Any exchange proposed between two parties with two goods will be hindered... the less fully informed are the two parties about the true characteristics of the proffered goods.” Like us, he assumed “interpersonal differences exist in degrees of knowledge about different goods – either by fortuitous circumstance or by deliberate development of such knowledge,” and that assets or goods “differ in the costs of determining or conveying to others their true qualities and attributes.” Alchian concluded, “It is not the absence of a double coincidence of wants, nor of the costs of searching out the market of potential buyers and sellers of various goods, nor of record keeping, but the costliness of information about the attributes of goods available for exchange that induces the use of money in an exchange economy.” Although these are interesting ideas, modern theory tries to be explicit and precise about how it all might work. To do so, one has to make certain assumptions and abstractions, and here we discuss some issues that come up in the process.

First, given our setup, agents in the model can (sometimes) use assets to purchase goods directly, which may not be so typical in actual markets. As noted earlier, however, an alternative interpretation that is more realistic is that agents use assets as collateral. Also, as Ravikumar and Shao [48] point out, sometimes one actually can write checks on mutual funds. Although we do not literally have checkable mutual funds in the formal model, the analysis still captures the idea that assets can help consumers making payments. For investors or firms, this may be even more relevant. In OTC (over-the-counter) markets trades often boil down to an exchange of securities, such as swaps between fixed- and variable-rate assets. The general message we are trying to convey is that assets may help individual agents in the exchange process, and this means they can command a liquidity premium.

One might also object to our real assets directly generating utility via dividends. Although this is standard, if one prefers, in our CM dividends can equivalently be paid in cash. One might also question the DM specification more broadly, featuring as it does search and bargaining. In OTC markets, however, these features are quite realistic, as sophisticated products like derivatives or complex securitized loans are traded by dealers who must search for counterparties and bargain over the terms of trade.²⁴ We do not claim that our DM constitutes the definitive model of such activity; only that search and bargaining are not unnatural for thinking about finance generally. Moreover, random matching is a convenient way to model agents meeting and trading with each other, rather than merely trading against budget equations. By modeling trade in this way, one can start to ask whether they use barter, credit or certain assets in exchange. But one does not have to take random matching literally, and can replace it with directed search, or replace search entirely with preference and technology shocks.²⁵

²⁴As Duffie et al. [10] put it, “Many assets, such as mortgage-backed securities, corporate bonds, government bonds, US federal funds, emerging-market debt, bank loans, swaps and many other derivatives, private equity, and real estate, are traded in [OTC] markets. Traders in these markets search for counterparties, incurring opportunity or other costs. When counterparties meet, their bilateral relationship is strategic; prices are set through a bargaining process that reflects each investor’s alternatives to immediate trade.”

²⁵Again, see the surveys cited in the Introduction, for explicit descriptions of these alternative settings.

Also, many results hinge here on whether assets are scarce or plentiful, $A < A^*$ or $A > A^*$. Only in the former case do we get liquidity premia, the possible failure of the Fisher Equation, the result that easy money can reduce real returns, etc. Which is the empirically relevant case? Caballero [8] advocates the position that there is indeed a dearth of financial assets in the world. The reason is that the demand for such assets is huge. As Geanakoplos and Zame [13] put it, the total [value] of collateralized lending is enormous: the value of U.S. residential mortgages alone exceeds \$9.7 trillion (only slightly less than the \$10.15 trillion total capitalization of S&P 500 firms).” To understand why there is so much demand for liquidity relative to GDP, note that many payments must be made on transactions that do not add to net output. An example is clearing and settlement among banks and related institutions. The average daily values of transactions on the two big settlement systems in the US, Fedwire and CHIPS, are currently around \$2.3 trillion and \$1.4 trillion, meaning the value of annual output flows throughout the system used every 4 days.²⁶

Although we use clearing to motivate the demand for liquidity, in the model taken at face value, assets are used to finance consumption. Although there is more to be done, we mention that models in this general class can be and have been used to formally study payments among financial institutions (Koepl, Monnet and Temzilides [28]; Lester [38]). Also, regarding whether $A < A^*$ or $A > A^*$, we note that mitigating the demand for liquidity is the fact that one can sometimes use the same assets as collateral multiple times, plus the fact that some intermediaries with illiquid assets issue liquid liabilities. But, in any case, it is important to keep in mind recognizability and related properties of assets when thinking about these issues. The total value of assets may well be large, especially if one includes various types of intangible, or human, capital. But to the extent that these assets are difficult to use in transactions or as collateral, liquidity may be scarce. The issue is not whether there is a shortage of real wealth; the issue is whether it is easy to pull together enough assets with desirable properties – recognizability,

²⁶For more details, see www.frbservices.org/operations/fedwire/fedwire_funds_services_statistics.html and www.chips.org/docs/000652.pdf.

transferability, etc. – to satisfy the need.

8 Conclusion

There is a tradition arguing that informational frictions are central for understanding money. We are convinced that they are central for understanding liquidity generally. In our framework, real and fiat assets can differ in terms of recognizability, and this can give rise to differential liquidity premia. We analyzed implications for monetary policy. We endogenized recognizability and liquidity by letting agents invest in information. We proved existence with endogenous information, and discussed the possibility of multiplicity. One lesson is that small changes in fundamentals may have big effects. Another is that models with exogenous transactions patterns, like those that impose cash-in-advance restrictions, or those that only consider cashless limiting cases, or those that simply assume constraints on asset transferability, are problematic because generally transaction patterns are neither uniquely determined nor invariant to interventions. This is one (more) reason to prefer models where the exchange process and hence the role of assets in this process are modeled relatively explicitly.

Several assumptions kept the analysis tractable. One is that bad assets can be produced at cost $k = 0$, which implies that agents who cannot verify an asset's authenticity never accept it, and this avoids technical problems with bargaining under private information. However, we sketched the case $k > 0$, and as $k \rightarrow 0$ the outcomes converge to the those in our benchmark economy. Another ingredient that imparts tractability is proportional bargaining, but similar results can be derived with other solution concepts. Still, more could be done in terms of studying different mechanisms. Another outstanding issue is to bringing intermediation more clearly into the picture. It might also be interesting to add a secondary market – one convening, say, between the CM and DM – where agents swap assets before trading for goods. One could also pursue empirical implications, as discussed in the context of our interrelated asset demand system. We leave all this to future work.

Appendix

Proof of Proposition 2: Suppose $A < A^*$. We claim there exists a unique (q_1, q_2) with $q_1 > 0$ and $q_2 < q^*$ that satisfies the equilibrium conditions (17)-(18). Let μ and α denote the implicit functions (17) and (18), respectively, that map q_1 into q_2 . We have

$$\begin{aligned}\mu'(q_1) &= -\frac{\lambda_1 \ell'(q_1)}{\lambda_2 \ell'(q_2)} < 0 \\ \alpha'(q_1) &= \frac{z'(q_1) [r - \lambda_2 \ell(q_2)]}{z'(q_2) [r - \lambda_2 \ell(q_2)] - [z(q_2) - z(q_1)] \lambda_2 \ell'(q_2)} > 0,\end{aligned}$$

where $r - \lambda_2 \ell(q_2) \geq 0$ follows from $q_2 \geq q_1$. Let \check{q} satisfy $\lambda_1 \ell(\check{q}) = i + \lambda_2$, with $0 < \check{q} < \tilde{q} \leq q^*$. Since $\ell'(q) < 0$ and $\lim_{q \rightarrow \infty} \ell(q) = -1$, it is easy to see that $\lim_{q_1 \rightarrow \check{q}^+} \mu(q_1) = \infty$. Moreover, we claim $\lim_{q_1 \rightarrow \check{q}^+} \alpha(q_1) < \infty$. Suppose not, so that $\lim_{q_1 \rightarrow \check{q}^+} \alpha(q_1) = \infty$. Since $z' \geq 0$, this implies $(1+r)A\delta \geq [z(q^*) - z(\check{q})](r + \lambda_2) > [z(q^*) - z(\tilde{q})]r$, which implies $A > A^*$, a contradiction. Therefore $\lim_{q_1 \rightarrow \check{q}^+} \mu(q_1) > \lim_{q_1 \rightarrow \check{q}^+} \alpha(q_1)$. Moreover, it is straightforward to establish that $\mu(q^*) < q^* < \alpha(q^*)$ when $A < A^*$. Therefore, since μ and α are continuous, $\mu' < 0$ and $\alpha' > 0$, $\mu(q') > \alpha(q')$ for some q^* , and $\alpha(q^*) > \mu(q^*)$, we conclude that there exists a unique pair (q_1, q_2) with $q_1 > 0$ and $q_2 < q^*$ that satisfies (17)-(18).

Now suppose $A \geq A^*$. We claim that there is no pair (q_1, q_2) with $q_2 < q^*$ satisfying (17)-(18). Suppose, towards a contradiction, that such a pair exists. Since $q_2 < q^*$, it must be that $q_1 > \tilde{q}$ by (17). This implies, $(1+r)A\delta < [z(q^*) - z(\tilde{q})]r$, or $A < A^*$, a contradiction. Therefore, it must be that $q_2 = q^*$. Then $q_1 = \tilde{q}$, and the rest follows immediately. ■

Results in Table 1: Let Δ denote the determinant of the following matrix:

$$\begin{bmatrix} \lambda_1 \ell'(q_1) & \lambda_2 \ell'(q_2) \\ [\lambda_2 \ell(q_2) - r] z'(q_1) & [r - \lambda_2 \ell(q_2)] z'(q_2) - [z(q_2) - z(q_1)] \lambda_2 \ell'(q_2) \end{bmatrix}$$

Again, since $r \geq \lambda_2 \ell(q_2)$, we have $\Delta < 0$. Then it is easy to compute:

$$\begin{aligned}\frac{\partial q_1}{\partial i} &= \frac{[r - \lambda_2 \ell(q_2)] z'(q_2) - [z(q_2) - z(q_1)] \lambda_2 \ell'(q_2)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial i} &= \frac{[r - \lambda_2 \ell(q_2)] z'(q_1)}{\Delta} < 0 \\ \frac{\partial q_1}{\partial \rho} &= \frac{\lambda[\ell(q_1) - \ell(q_2)][r - \lambda_2 \ell(q_2)] z'(q_2) - [z(q_2) - z(q_1)] \lambda_2 \ell'(q_2) \lambda \ell(q_1)}{\Delta} < 0 \\ \frac{\partial q_2}{\partial \rho} &= \frac{\lambda[\ell(q_1) - \ell(q_2)][r - \lambda_2 \ell(q_2)] z'(q_1) + [z(q_2) - z(q_1)] \lambda_1 \ell'(q_1) \lambda \ell(q_2)}{\Delta}.\end{aligned}$$

Given $z(q_1) = \phi M$ and $z(q_2) - z(q_1) = (\psi + \delta)A$, we have

$$\begin{aligned}\frac{\partial \psi}{\partial i} &= \frac{[z(q_2) - z(q_1)]\lambda_2 \ell'(q_2) z'(q_1)}{A\Delta} > 0 \\ \frac{\partial \phi}{\partial i} &= \frac{z'(q_1)}{M} \frac{\partial q_1}{\partial i} < 0 \\ \frac{\partial \psi}{\partial \rho} &= \frac{[z(q_2) - z(q_1)][\lambda_1 \ell'(q_1)\lambda \ell(q_2) z'(q_2) + \lambda_2 \ell'(q_2)\lambda \ell(q_1) z'(q_1)]}{\Delta} > 0 \\ \frac{\partial \phi}{\partial \rho} &= \frac{z'(q_1)}{M} \frac{\partial q_1}{\partial \rho} < 0.\end{aligned}$$

To see that $\lim_{\rho \rightarrow 0} \partial \psi / \partial i = 0$, note that $q_1 \rightarrow \hat{q} = \ell^{-1}(i/\lambda) \in (0, q^*)$ as $\rho \rightarrow 0$, while $q_2 \rightarrow \min\{q^*, \bar{q}\}$, where \bar{q} satisfies $(1+r)\delta A = r[z(\bar{q}) - z(\hat{q})]$. Therefore, $\lim_{\rho \rightarrow 0} [z(q_2) - z(q_1)]\ell'(q_2)z'(q_1)$ is strictly negative and bounded, as is $\lim_{\rho \rightarrow 0} \Delta = \lambda \ell'(q_1) r z'(q_2)$. Since $\lim_{\rho \rightarrow 0} \lambda_2 = 0$, it follows that $\partial \psi / \partial i \rightarrow 0$ as $\rho \rightarrow 0$. ■

Proof of Lemma 2: Substituting $z(q)$ into $\Pi(\rho) = \beta \lambda [\Sigma_2(\rho) - \Sigma_1(\rho)]$, we have

$$\Pi(\rho) = \beta \lambda (1 - \theta) [(u_2 - c_2) - (u_1 - c_1)],$$

where $u_1 = u[q_1(\rho)]$ and so on. Therefore,

$$\Pi' \approx (u'_2 - c'_2) \partial q_2 / \partial \rho - (u'_1 - c'_1) \partial q_1 / \partial \rho,$$

where \approx means the two expressions have the same sign. Inserting $\partial q_j / \partial \rho$, we get

$$\begin{aligned}\Pi' &\approx (u'_2 - c'_2) [-\lambda(1-\rho)\ell'_1 \ell_2 (z_2 - z_1) + (\ell_2 - \ell_1)(r - \lambda \rho \ell_2) z'_1] \\ &\quad - (u'_1 - c'_1) [(\ell_2 - \ell_1)(r - \lambda \rho \ell_2) z'_2 + \lambda \rho \ell'_2 \ell_1 (z_2 - z_1)] \\ &= (\ell_2 - \ell_1)(r - \lambda \rho \ell_2) [z'_1 (u'_2 - c'_2) - z'_2 (u'_1 - c'_1)] \\ &\quad - \lambda (z_2 - z_1) [\rho \ell_1 \ell'_2 (u'_1 - c'_1) + (1-\rho)\ell'_1 \ell_2 (u'_2 - c'_2)].\end{aligned}$$

Since $z_2 > z_1$ and $u'_i \geq c'_i$ for all $q_j \leq q^*$, the second term is positive. Since $\ell_2 < \ell_1$ and $r > \lambda \rho \ell_2$, a sufficient condition for $\Pi' > 0$ is therefore

$$\begin{aligned}0 &\geq z'_1 (u'_2 - c'_2) - z'_2 (u'_1 - c'_1) \\ &= [\theta c'_1 + (1-\theta) u'_1] (u'_2 - c'_2) - [\theta c'_2 + (1-\theta) u'_2] (u'_1 - c'_1) \\ &= c'_1 u'_2 - u'_1 c'_2.\end{aligned}$$

Since c is convex and u concave, the proof is complete. ■

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