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Implementation in Differential Information Economies*

ANDREW POSTLEWAITE

Department of Economics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

AND

DAVID SCHMEIDLER†

Tel Aviv University, Israel

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We consider the problem of implementation of social choice correspondences in differential information economies. We provide necessary conditions for implementation and slightly stronger conditions which are sufficient to guarantee implementation *Journal of Economic Literature* Classification Numbers: 025, 026.

1. INTRODUCTION

Our goal in this paper is to merge several central ideas in economic theory: strategic behavior (incentive compatibility), differential (or incomplete) information, and the Arrow–Debreu model of general equilibrium. By strategic behavior we refer to the literature which models economic institutions as games in strategic form and uses Nash equilibrium as the solution concept. This literature, motivated by informational decentralization questions, deals not with a single economic environment and a single game, but rather considers a class of environments and a strategic outcome function (game form) which is applied uniformly to this class.

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The differential information economy model used here is a simplified version of the model introduced in Postlewaite and Schmeidler [15]. That paper also contains motivation, interpretation, and examples of the model. Myerson [10] contains an exposition of Harsanyi's [4] model of differential information and Harsanyi–Nash (Bayesian) equilibria, and other results including the revelation principle. Several of the results proved in this paper extends to differential information economies results on implementation via Nash equilibria in the complete information framework presented in Maskin [7] and Hurwicz, Maskin, and Postlewaite [6]. For exposition of these and other results for complete information see the surveys by Maskin [8] and Postlewaite [15].

Our model of a pure exchange economy differs from the Arrow–Debreu model in that an agent's characteristics include, in addition to his endowment and preferences, a specification of his information. This is done by making initial endowments and preferences random variables on a set of (Savage) states of nature. An agent's information structure is then given by a partition of the states of nature. When a state of nature occurs, each agent is informed of the event in his partition containing this state. This description of an agent's information is incomplete. Since we are considering strategic behavior, an agent is concerned not only with which states of nature he can distinguish, but also which states can be distinguished by others, since others' behavior, which is a function of their information, affects him. Thus a complete specification of information must include information about others' information, others' information about others' information, and so on. We assume the partitions are common knowledge, thus all relevant information for an agent is captured by the event containing the true state of nature.

If any agent can distinguish two states, all agents' allocations may differ in these two states. An agent who must make a dicision without being able to distinguish the states must trade off his welfare in these two states. Hence we are in a cardinal framework. Thus probability and von Neumann–Morgenstern utilities are included in the description of agents' characteristics as presented in detail in the next section.

The model of a differential information economy is quite flexible in its interpretation. One particular interpretation is of interest. Consider a subset of neoclassical economies with n agents, i.e., n-tuples of preferences and endowments. Each such economy is a trivial differential information economy with a single state of nature; each agent knows the entire vector of characteristics. We can associate with each economy a distinct state of nature and let each agent's information structure be such that all states of nature can be distinguished. The *collection* of trivial differential information economies becomes a single differential information economy itself in this way.

The literature on implementation via Nash equilibria in complete information economies begins with the notion of a social welfare correspondence (or performance correspondence) which associates a non-empty set of allocations with each economy in a given set. In a differential information economy an allocation maps states of the world to the usual (complete information) allocations. A social welfare correspondence, then, is a collection of differential information allocations.

In the complete information framework the problem of implementation is to design a strategic outcome function whose Nash equilibria for any economy coincide with the allocations prescribed by the social welfare correspondence. Here we want to design a strategic outcome function for a differential information economy such that the set of the Bayes equilibrium allocations coincide with the social welfare correspondence.

One of the contributions of this paper is to provide a unified framework in which both differential information and the complete information Nash implementation approach can be accommodated. We described above how a set of complete information economies comprise a single differential information economy. A differential information equilibrium of this differential information economy assigns a Nash equilibrium to each complete information economy. Hence implementation for differential information economies subsumes implementation via Nash equilibria for complete information economies.

The concept of implementation mentioned above is stronger than that used in many economic models. We ask that the *set* of equilibrium (state dependent) allocations coincide with some predetermined set. In other models a weaker condition is sometimes required, namely that a given allocation be an equilibrium of a particular game, with the possibility that there are other equilibria as well. A standard technique is to consider a revelation game and maximize some objective function subject to the constraints that no agent has an incentive to lie. This technique does assure that the resulting outcome will be an equilibrium of some game; however, there may be others as well. This problem is sometimes dismissed with an argument that as long as truthful revelation is an equilibrium, it will somehow be the *salient* equilibrium even if there are other equilibria as well. We will present an example which we believe shows the weakness of this argument.

EXAMPLE. There are three agents 1, 2, and 3 and two states of nature ω_1 and ω_2. All agents believe the two states to be equally likely; agent 1 cannot distinguish the two states while agents 2 and 3 can. There are three alternatives, a, b, and c and the (state dependent) preferences for ω_1 are

$$b\,P_1\,c\,P_1\,a \qquad \text{and} \qquad a\,P_i\,b\,P_i\,c \qquad i=2,3.$$

For ω_2 the preferences are

$$a\,P_1\,c\,P_1\,b \qquad \text{and} \qquad b\,P_i\,a\,P_i\,c \qquad i=2,3.$$

Suppose our objective is to maximize Mr. 1's welfare subject to incentive compatibility constraints. If we consider revelation games we can see that we are able to obtain (b, a) (b in state ω_1 and a in state ω_2) as a state dependent outcome. To see this consider the revelation game which prescribes b when agents 2 and 3 announce ω_1, prescribes a when they announce ω_2, and prescribe c when they announce different states. Truth is trivially an equilibrium; either agent deviating unilaterally can only achieve c, the worst outcome for him. Note that since agent 1 has no information he is a passive player in the revelation game and can be ignored.

There is a problem that arises with the above game. Not only is (b, a) an equilibrium, but so is (a, b); this is the outcome which arises when each agent announces the state opposite of that which he observes. It is an equilibrium for the same reason that truthful revelation is an equilibrium. The difficulty is that not only is there an equilibrium other than (b, a), but that the second equilibrium, (a, b), is preferred by both agents who have any information to reveal. Always telling the truth is an equilibrium and so is always lying. Always lying yields preferred outcomes to all those with nontrivial information. Is the truthful equilibrium the salient equilibrium?

We would like to point out that it is not simply "bad design" that (a, b) is also an equilibrium. Any revelation game for which (b, a) is an equilibrium must also have (a, b) as an equilibrium. If we are not confident that, of these equilibria, the truthful equilibrium will be played, then we are forced to consider the entire set of equilibria. It is no longer true that we can restrict our attention to revelation games in this case. There are *sets* of allocations which can be precisely the equilibrium set for some game, yet not be the equilibrium set for any revelation game. In fact, we will show that there exists a *non-revelation* game for which in the example above, (b, a) is an equilibrium allocation and (a, b) is *not* an equilibrium.

Proposition 5 will give a sufficient condition for an equilibrium allocation of an arbitrary game to be such that the equilibrium set of the induced revelation game has no additional equilibria. The question of whether a given set of allocations (possibly a singleton) can be precisely the equilibrium set for some game is then addressed. Proposition 1 provides a necessary condition, monotonicity, for such implementation. This condition is essentially an extension of the monotonicity condition used by Maskin [7] and Hurwicz, Maskin, and Postlewaite [6] for complete information environments. Proposition 2 states that a slightly stronger form of monotonicity together with a condition on the collection of agents' information structures is sufficient for a set of allocations to be implemented.

The extension of this result is discussed in Conjecture 4, where we suggest that if a set of allocations satisfy a stronger monotonicity condition and if every allocation in the set satisfies "self selection," the set can be implemented.

Monotonicity and self selection are necessary conditions for implementation; stronger monotonicity and self selection are sufficient conditions.

2. Defferential Information Economy

A pure exchange economy with differential information is a list

$$e = (\Omega, (\Pi_t)_{t \in T}, (\hat{w}_t)_{t \in T}, (\hat{u}_t)_{t \in T}, (P_t(\cdot, \cdot))_{t \in T}).$$

The symbol T stands for an abstract set of n elements that represents the names of the economic agents. There are l Arrow–Debreu commodities in the economy and the consumption set is R^l_+, the nonnegative orthant of euclidean space of dimension l. Both T and l are fixed throughout this paper.

Initial endowments are elements of R^l_+ and an agent's preferences are represented by a von Neumann–Morgenstern (NM) utility function on R^l_+ which is assumed to be real-valued, continuous, quasiconcave, and increasing in the sense that an increase in all l coordinates increases the utility. Furthermore, for every utility function, u, $u(w) = 0$ iff $w = 0$. The set of all such utility functions is denoted by U.

Agents do not have complete information about basic parameters of the economy. Following Savage's (neobayesian) paradigm, the uncertainty is represented by a set of "states of the world" denoted by Ω. A state ω in Ω resolves, by definition, all uncertainties for all agents, with the possible exception of the uncertainties inherent in the definition of Arrow–Debreu contingent commodities. Thus in the differential information (D.I.) framework for all t in T; $\hat{w}_t : \Omega \to R^l_+$ and $\hat{u}_t : \Omega \times R^l_+ \to R$, where $\hat{u}_t(\omega, \cdot) \in U$ for all ω in Ω (or equivalently $\hat{u}_t : \Omega \to U$). A certain measurability condition is imposed in the sequel on $\hat{w}_t$. This condition implies that agent t knows the vector $\hat{w}_t(\omega)$ before the act of exchange.

According to the neobayesian paradigm, every economic agent has a (prior) probability distribution over Ω. Since we are modeling differential information, one can think of a different prior for each t in T. We postulate here, for all t in T, the existence of a partition Π_t of Ω and a conditional (posterior) probability $P_t : \Omega \times \Pi_t \to [0, 1]$.

DI1. Denote by Π the finest partition of Ω which is coarser than each Π_t, t in T. Then every B in Π is finite.

This condition is not necessary but it considerably simplifies the presentation. Without it, appropriate σ-fields have to be defined on Ω, etc.

If ω in Ω occurs, every agent t in T is informed of the element B_t in his partition Π_t which contains ω. The element B of Π which contains ω (and includes each B_t) is *common knowledge at* ω according to the definition of Aumann [1].

For future use we introduce some notation and redefine this concept. For all t in T define $I_t\colon \Omega \to 2^\Omega$ by $I_t(\omega) = B_t \in \Pi_t$ such that $\omega \in B_t$. Extend the definition of I_t to 2^Ω by $I_t(A) = \bigcup_{\omega \in A} I_t(\omega)$. Then $B = I(\omega)$ is *common knowledge at* ω if

$$I(\omega) = I_{t_1}(\omega) \cup I_{t_2}(I_{t_1}(\omega)) \cup I_{t_3}(I_{t_2}(I_{t_1}(\omega))) \cup \cdots$$

where the union is over all finite applications of the operators I_t, $t \in T$ according to some order $t_1, t_2, \ldots$ of agents in T. It is clear that $I(\omega)$ is independent of this order because $I(\omega) \in \Pi$ and is a finite set by **DI1**. On a heuristic level it is assumed that e is common knowledge, i.e., every agent knows e, every agent knows that every agent knows e, every agent knows that every agent knows that every agent knows e, etc.

The following condition is added mainly for the consistency of interpretation.

DI2. For all ω in Ω and all t in T: $P_t(\omega, I_t(\omega)) > 0$.

We now introduce the previously promised condition on the "random" variables $\hat{w}_t(\cdot)$, $t \in T$.

DI3′. For all t in T, $\hat{w}_t\colon \Omega \to R^l_+$ is measurable with respect to Π_t, i.e., $\hat{w}_t$ is constant on each member of Π_t.

Another informational assumption is

DI4. For all ω in Ω: $\bigcap_{t \in T} I_t(\omega) = \{\omega\}$.

The only purpose of this assumption is to simplify presentation. If no one in the economy can distinguish ω and ω' in Ω they constitute one state of nature for all practical purposes. In a more involved model where the informational structure is not exogenously given and information can be acquired, such an assumption might be too restrictive.

EXAMPLE 1. A special case of an economic environment is an environment such that for all t in T and for all ω in Ω: $I_t(\omega) = \{\omega\}$. Consider further the following specifications: $\Omega = R_+^{lT} \times U^T$, and for all t in T, $\hat{w}_t(\omega)$ and $\hat{u}_t(\omega)$ are the corresponding projections of ω. Hence a single incomplete information environment e represents the class of pure exchange economies with cardinal utilities. It is referred to as *common knowledge complete information economy.* The most trivial special case is a *complete information economy* where $\#\Omega = 1$.

The common knowledge complete information economy of Example 1 is a combination of all possible complete information economies. Thus a social welfare correspondence for a differential information economy is, as a special case, a social welfare correspondence for neoclassical economies. In general, the results proved in this model extend to the neoclassical economies case.

The combination property is quite general. Given any collection of D.I. economies, with identical T and l, they can be combined into one D.I. economy. This is done by performing the (disjoint) union of the states of the world (the omegas) and then appropriately redefining the partitions, the random initial endowments, etc.

The condition **DI3**′ suffices for Proposition 1 which presents a necessary condition for implementation. But for the sufficiency result, Proposition 2, every agent has to know the aggregate initial endowment. It is easy to see that this implies that the aggregate initial endowment is constant over common knowledge events. In notation,

DI3″. $\sum_{t \in T} \hat{w}_t(\omega): \Omega \to \mathbb{R}_+^l$ is Π-measurable (i.e., measurable with respect to the algebra generated by Π.)

Instead of **DI3**′ and **DI3**″ we will use the very restrictive condition:

DI3. For all t in T, $\hat{w}_t(\cdot)$ is constant over Ω. Denote it by w_t, and assume $w_t \neq 0$.

This condition is not necessary but it shortens considerably the presentation here, in particular that of strategic outcome functions. It calls for an elaboration.

Suppose first that only **DI3**′ is assumed. Economic institutions of exchange are represented by the concept of a strategic outcome function (SOF). Given a vector w in R_+^l, the sets of acts or moves for an agent whose initial endowment is w is denoted by the set $S(w)$. The domain of a SOF f is $\mathbf{S} = \bigcup_{\mathbf{w} \in R_+^{lT}} X_{t \in T} S(w_t)$. Its range is R_+^{lT}.

Partial informational decentralization is assumed in this work. This means only initial endowments of every agent, given ω, is centrally known. So, on the one hand, an agent cannot choose an act inconsistent with his initial endowment, and on the other hand, for any ω and any list of acts

$\mathbf{s}=(s_t)_{t\in T}$, the resulting (certain) allocation $f(\mathbf{s})=\mathbf{x}$ is feasible, i.e., $\sum_{t\in T} x_t \leqslant \sum_{t\in T} \hat{w}_t(\omega)$. The center is not assumed to know the occurring state ω or to have any private information about ω.

From the agents' point of view the SOF can be described as a function of ω in Ω, since the initial endowments, and through them the available acts depend on ω. One can formulate precisely, through measurability conditions, the information assumed to be centrally known, and impose the feasibility condition on the SOF f. For lack of space we will not do this here. We refer the reader to Postlewaite and Schmeidler [15]. However, after each of the two main results we will point out the adjustments needed if **DI3** is not assumed.

Assuming **DI3** from now on, we have for each t in T a fixed set $S_t=S_t(w_t)$ of acts. We denote $w=\sum_{t\in T} w_t$, $\mathbf{S}=X_{t\in T} S_t$, $A=\{\mathbf{x}\in R_+^{lT} \mid \sum_{t\in T} x_t \leqslant w\}$ and f: $\mathbf{S}\to A$.

The structure of strategic outcome functions is flexible eneough that we can combine several such functions into one without affecting their partial decentralization property. In other words, the situation where the same agents are trading simultaneously on several markets can be analyzed as if those markets were one market.

Given a D.I. economy e and a SOF f a strategy for agent t is a mapping $\sigma_t: \Pi_t \to S_t$. Given a T-list of strategies $\boldsymbol{\sigma}$, agent τ's *best response* is a strategy ρ_τ such that for all s_τ in S_τ and ω in Ω,

$$\sum_{\omega'\in I_\tau(\omega)} P_\tau(\omega', I_\tau(\omega))\, \hat{u}_\tau(\omega', f_\tau((\sigma_t(I_t(\omega')))_{t\neq\tau}, \rho_\tau(I_\tau(\omega))))$$
$$\geqslant \sum_{\omega'\in I_\tau(\omega)} P_\tau(\omega', I_\tau(\omega))\, \hat{u}_\tau(\omega', f_\tau((\sigma_t(I_t(\omega')))_{t\neq\tau}, s_\tau)).$$

A list of strategies $\boldsymbol{\sigma}^*$ is termed a differential information equilibrium (D.I. equilibrium) (or Harsanyi–Nash equilibrium or Bayesian equilibrium) if each agent's strategy is a best response to $\boldsymbol{\sigma}^*$. Our model reduces to Harsanyi's incomplete information game if Ω is finite, the economic structure is deleted and agent's utility given only ω and $\mathbf{s}$ is considered. Then, event B_t in Π_t corresponds to a Harsanyi "type of agent" t. Note that although in our model every agent always plays a pure act, given ω, he conceives the acts of others as mixed. This occurs when $\# I_t(\omega)>1$ and others' strategies are not constant on $I_t(\omega)$.

3. Implementation and Self-Selection

Given a D.I. economy e, a function $\hat{\mathbf{x}}$: $\Omega\to A$ is termed a *D.I. allocation.* We described above how a collection of complete information economies can be considered a single D.I. economy. In this case a D.I. allocation corresponds to a social welfare (choice) function on the collection of complete information economies. A set F of D.I. allocations is termed a *social welfare correspondence*, SWC (given e). A SOF f (faithfully) *implements* F if $F = \{f(\boldsymbol{\sigma}(\cdot)) \mid \boldsymbol{\sigma}$ is a D.I. equilibrium for $(e, f)\}$. We say a SWC F is *implementable* if there exists a SOF f which implements F. Given a T-list of strategies $\boldsymbol{\sigma}$ we denote $\boldsymbol{\sigma}(\omega) \equiv (\sigma_t(I_t(\omega)))_{t \in T}$ for ω in Ω. Also denote by $\sum_t$ agent t's strategies and $\boldsymbol{\Sigma} = X_{t \in T} \sum_t$. Given $\boldsymbol{\sigma}^1$ and $\boldsymbol{\sigma}^2$ in $\boldsymbol{\Sigma}$ we term $\boldsymbol{\sigma}^3$ in $\boldsymbol{\Sigma}$ their *common knowledge concatenation* if for some Π-measurable partition (Ω^1, Ω^2) of Ω, $\sigma^3(\omega) = \sigma^i(\omega)$, $\omega \in \Omega^i$ for $i = 1, 2$. Clearly, if $\boldsymbol{\sigma}^1$ and $\boldsymbol{\sigma}^2$ are D.I. equilibria, so is any of their common knowledge concatenations. Similarly, if F is an implementable D.I. SWC and $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are D.I. allocations in F, any common knowledge concatenation of $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ is in F. From now on we restrict our attention to SWF's, F, which are closed under common knowledge concatenation. Also $\hat{\mathbf{x}}_t(\omega) \neq 0$ for all t, ω and $\hat{\mathbf{x}}$ in F. One might infer, as a result, that we may restrict our theory to cases where Ω is a common knowledge event. But, this is wrong. If a SWC F is implementable separately on each B in Π (i.e., consider $e \mid B$ as a D.I. economy, and $F \mid B$ as a SWC for $e \mid B$) by the same SOF f, it still may not be implementable. This remark will become more obvious after the following definition of monotonicity of a SWF F for a given D.I. economy e.

In the sequel we will write for short $\sum P(\omega, B)\, \hat{u}(\cdots)$ and omit the summation index when the summation is on ω ranging over the event on which $P(\cdot, \cdot)$ has been conditioned.

M (Monotonicity).

Given B and B' in Π with $\alpha: B' \to B$ and given a selection $\hat{\mathbf{x}} \in F \mid B$ one also has $(\hat{\mathbf{x}}(\alpha\omega))_{\omega \in B'} \in F \mid B'$ whenever the following two conditions hold:

(i) For every t in T, α is Π_t-measurable. (We then say that α preserves the information structure.) I.e., for every $t \in T$, and for every $B'_t \in \Pi_t$, $B'_t \subset B$; $\alpha(B'_t) \subset B_t$ for some $B_t \in \Pi_t$, $B_t \subset B$.

(ii) For every t in T, every ω in B', and every $\hat{\mathbf{y}}: B' \to A$ and $\hat{\mathbf{z}}: B \to A$ s.t. $\forall \omega' \in B'$, $\hat{\mathbf{y}}(\omega') = \hat{\mathbf{z}}(\alpha\omega')$; the following holds:

$$\sum P_t(\omega', I_t(\omega))\, \hat{u}_t(\omega', \hat{y}_t(\omega')) > \sum P_t(\omega', I_t(\omega))\, \hat{u}_t(\omega', \hat{x}_t(\alpha\omega'))$$

implies

$$\sum P_t(\omega', I_t(\alpha\omega))\, \hat{u}_t(\omega', \hat{z}_t(\omega')) > \sum P_t(\omega', I_t(\alpha\omega))\, \hat{u}_t(\omega', \hat{x}_t(\omega')).$$

A special but nontrivial case of the above condition occurs when $B = B'$ but α is *not* the identity. The meaning of the condition is easily understood from its use in the proof of the following proposition.

PROPOSITION 1. *A social welfare correspondence F which is implemented on an economy e satisfying* **DI**1 to **DI**4 *is monotonic.*

Proof. Let $\boldsymbol{\sigma}$ be a D.I. equilibrium such that the D.I. allocation $f(\boldsymbol{\sigma}(\cdot))$ belongs to F and for ω in B: $\hat{\mathbf{x}}(\omega) = f(\boldsymbol{\sigma}(\omega))$. We claim that the strategy T-list $\boldsymbol{\sigma}'$ defined by: $\boldsymbol{\sigma}'(\omega) = \boldsymbol{\sigma}(\omega)$ for $\omega \notin B'$ and $\boldsymbol{\sigma}'(\omega) = \sigma(\alpha\omega)$ for $\omega \in B'$, is also a D.I. equilibrium, if (i) and (ii) of M are satisfied. Condition (i) implies that $\boldsymbol{\sigma}'$ is well defined. If $\boldsymbol{\sigma}'$ were not a D.I. equilibrium then some agent, say t, informed of $I_t(\omega)$, $\omega \in B'$, has a better response than $\sigma_t(I_t(\alpha\omega))$ yielding $\hat{\mathbf{y}}(\omega')$, where his share is $\hat{y}_t(\omega')$ for each ω' in $I_t(\omega)$.

However, condition (ii) implies that agent t can successfully defect from σ_t when informed of $I_t(\alpha\omega)$—a contradiction to the assumption that $\boldsymbol{\sigma}$ is a D.I. equilibrium. Hence $\boldsymbol{\sigma}'$ is also a D.I. equilibrium and for all $\omega \in B'$: $f(\boldsymbol{\sigma}'(\omega)) = \hat{\mathbf{x}}(\alpha\omega)$. Q.E.D.

If condition **DI**3′ is assumed instead of **DI**3, then α also has to preserve every agent's endowment, i.e., $\hat{\mathbf{w}}(\omega) = \hat{\mathbf{w}}(\alpha\omega)$ for ω in B'. The feasibility condition for allocations also has to be appropriately adjusted. The following example will help illustrate both the definition of monotonicity and the theorem.

EXAMPLE 2. There are three agents, four equally probable states of nature $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, and one good.

The endowment of each agent is one for each state of nature. The agents' partition are $\{\omega_1, \omega_2\}$ $\{\omega_3, \omega_4\}$ for agent 1, $\{\omega_2, \omega_3\}$ $\{\omega_1, \omega_4\}$ for agent 2, and $\{\omega_1, \omega_3\}$ $\{\omega_2, \omega_4\}$ for agent 3. The utility functions are as given in the table below

	ω_1	ω_2	ω_3	ω_4
1	$2x$	$2x$	x	x
2	x	$2x$	$2x$	x
3	$2x$	x	$2x$	x

Consider the social welfare function $\hat{x}$ given in the next table.

	ω_1	ω_2	ω_3	ω_4
1	1.5	1.5	0	1
2	0	1.5	1.5	1
3	1.5	0	1.5	1

This social welfare function is symmetric and maximizes agents' ex ante expected utility subject to being symmetric. We want to show that this allocation, considered as a social welfare correspondence is not monotonic. Consider $\alpha\colon \Omega \to \Omega$, $\alpha(\omega_i) = \omega_4$, $i = 1,\ldots, 4$. It is clear that α satisfies the measurability condition (i) in the definition of monotonicity. The $\hat{x}(\alpha\omega')$ in part (ii) of the definition is the allocation which arises when $\hat{x}$ is "jumbled" according to α. $\hat{x}(\alpha\omega')$ yields $x(\omega_4)$ for every state. Thus each agent gets one unit independent of the state of nature. Consider agent 1. If he observes the event $\{\omega_1, \omega_2\}$ his utility from $\hat{x}(\alpha\omega')$ is $p(x_1, I_1(\omega_1))\, u(\omega_1, 1) + p(\omega_2, I_1(\omega_1))\, u(\omega_2, 1) = \frac{1}{2}\cdot 2 + \frac{1}{2}\cdot 2 = 2$. Let $\hat{z}(\omega)$ be some allocation and $\hat{y}(\omega') = \hat{z}(\alpha\omega')$; $\hat{y}(\omega) = \hat{z}(\omega_4)$ given our α. If $\hat{y}(\omega')$ is preferred by agent 1 to $\hat{x}(\alpha\omega')$, we must have $\frac{1}{2}u(\omega_1, \hat{y}(\omega_1)) + \frac{1}{2}u(\omega_2, \hat{y}(\omega_2), \hat{y}(\omega_2)) > \alpha$. This implies $\hat{y}(\omega_1) + \hat{y}(\omega_2) > 2$ and thus $\hat{z}(\omega_4) > 1$. This of course implies that $\hat{z}(\omega)$ is preferred to $\hat{x}(\omega)$ in the event that agent 1 sees event $\{\omega_3, \omega_4\}$. Hypothesis (ii) of the definition is thus satisfied. A SWC F which contains the allocation $\hat{x}(\omega)$ must also contain $\hat{x}(\alpha\omega)$ if it is monotonic.

It is easy to see why $\hat{x}(\alpha\omega)$ must be an equilibrium outcome of any SOF for which $\hat{x}(\omega)$ is an equilibrium outcome. If $\hat{x}(\omega)$ is an equilibrium outcome for some SOF, there exist strategies $(\tau_t(\omega))_{t\in T}$ which are equilibrium strategies and yield $\hat{x}(\omega)$. But $\tau_t^1(\omega) = \tau_t(\alpha\omega) = \tau_t(\omega_t)$ yields $x(\hat{\alpha}\omega)$ and any outcome which might prevent $(\tau_t^1(\omega))_{t\in T}$ from being an equilibrium will also prevent $(\tau_t(\omega))_{t\in T}$ from being an equilibrium as well.

For the next proposition a slightly stronger monotonicity condition is assumed. In general, it is not a necessary condition for implementation, but it is necessary for certain kinds of implementation. This result is stated as a corollary to the next proposition.

S^{2}M (Slightly Stronger Monotonicity). Same as M except in (ii) $\hat{\mathbf{z}}(\cdot)$ for $\omega' \in B$, $\omega' \notin \alpha(B')$ is defined: $\hat{\mathbf{z}}(\omega') = \hat{\mathbf{x}}(\omega')$.

Next a condition on the informational structure of the D.I. economy e is introduced, which together with the S^{2}M condition will yield an implementation result.

NEI (Nonexclusivity in Information). For all ω in Ω and t in T: $\bigcap_{t'\neq t} I_{t'}(\omega) = \{\omega\}$. Also $\# T \geq 3$.

No agent has exclusively private information. In the example of Section 2 a much stronger assumption was satisfied; for all ω and t: $I_t(\omega) = \{\omega\}$.

Proposition 2. *Given a D.I. economy e satisfying* **DI**1, **DI**2, **DI**3, *and NEI, a SWC F satisfying S^{2}M is implementable.*

Proof. We construct a SOF which implements F. First, each agent t's acts are defined:

$$S_t^1 = \Pi_t$$
$$S_t^2 = \{\hat{\mathbf{x}}: \Omega \to A\}$$
$$S_t^3 = \{0, 2, 3, ...\} \cup \{\alpha: \Omega \to \Omega\}$$
$$S_t = S_t^1 \times S_t^2 \times S_t^3.$$

We denote $s^3 = \#\{t \in T \mid s_t^3 \neq 0\}$ and define f separately for different subsets of $\mathbf{S}$

$$D_1 = \left\{ \mathbf{s} \in \mathbf{S} \mid \exists \bar{\omega} \in \Omega \text{ s.t. } \bigcap_{t \in T} s_t^1 = \{\bar{\omega}\}, \exists \hat{\mathbf{x}} \in F \text{ s.t. } \forall t \in T, s_t^2 = \hat{\mathbf{x}}, \text{ and } s^3 = 0 \right\}.$$

The acts combinations in D_1 exhibit complete unanimity.

$D_2 = \{\mathbf{s} \in \mathbf{S} \mid s^3 = 1$, $\exists \tau \in T$ s.t. $s_\tau^3 \equiv \alpha$: $\Omega \to \Omega$ and for each $t \in T$, α is Π_t-measurable; $\exists \bar{\omega} \in \Omega$ s.t. $\bigcap_{t \neq \tau} s_t^1 = \{\bar{\omega}\}$; $\exists \hat{\mathbf{x}} \in F$ s.t. $\forall t \neq \tau$, $s_t^2 = \hat{\mathbf{x}}$; Denoting $B \equiv I(\bar{\omega})$ and $B' \equiv I(s_\tau^1)$, $s_\tau^2(\omega) = s_\tau^2(\alpha\omega)$ for $\omega \in B'$ and $s_\tau^2(\omega) = \hat{\mathbf{x}}(\omega)$ for $\omega \in B \setminus B'\}$.

For $\mathbf{s}$ in D_2 all the agents except one, τ, act as they act in D_1. Agent τ objects. He implicitly claims that the true state is in s_τ^1, and the others colluded via α. For convenience we will denote $\hat{\mathbf{y}}(\omega) = s_\tau^2(\omega)$ for $\omega \in B'$ and $\hat{\mathbf{z}}(\omega) = s_\tau^2(\omega)$ for $\omega \in B$. The conditions on α, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are those in the statements of the conditions M and $\mathrm{S}^2\mathrm{M}$. Define

$$D_a = \left\{ \mathbf{s} \in D_2 \,\middle|\, \sum P_\tau(\omega, I_\tau(\bar{\omega}))\, \hat{u}_\tau(\omega, \hat{\mathbf{z}}(\omega)) \leqslant \sum P_\tau(\omega, I_\tau(\bar{\omega}))\, \hat{u}_\tau(\omega, \hat{\mathbf{x}}(\omega)) \right\}$$

$$D_b = \left\{ \mathbf{s} \in D_2 \,\middle|\, \sum P_\tau(\omega, I_\tau(\bar{\omega}))\, \hat{u}_\tau(\omega, \hat{\mathbf{z}}(\omega)) > \sum P_\tau(\omega, I_\tau(\omega))\, \hat{u}_\tau(\boldsymbol{\omega}, \hat{\mathbf{x}}(\omega)) \right\}.$$

The set of strategies combinations D_2 is the disjoint union of D_a and D_b. We define

$$D_3 = \{\mathbf{s} \in \mathbf{S} \mid s^3 = 0, \mathbf{s} \notin D_1 \text{ but there are } t' \text{ and } s'_{t'}, \text{ s.t. } (\mathbf{s} \mid s'_{t'}) \in D_1\} \cup \{\mathbf{s} \in \mathbf{S} \mid s^3 = 1 \text{ but } \mathbf{s} \notin D_2\}$$

$$D_4 = \mathbf{S} \setminus (D_1 \cup D_2 \cup D_3).$$

Define

$$\bar{k} = \max\{s_t^3 \mid t \in T \text{ and } s_t^3 : \Omega \to \Omega \text{ counts as } 1\},$$

$$K = \{t \in T \mid s_t^3 = \bar{k}\} \text{ and } k = \# K$$

$$\begin{aligned} f(\mathbf{s}) &= \hat{\mathbf{x}}(\bar{\omega}) && \mathbf{s} \in D_1 \\ &= \hat{\mathbf{z}}(\bar{\omega}) && \mathbf{s} \in D_a \\ &= \hat{\mathbf{x}}(\bar{\omega}) && \mathbf{s} \in D_b \\ &= 0 && \mathbf{s} \in D_3 \\ &= w/k \text{ to } t \in K, 0 \text{ to } t \notin K, && \mathbf{s} \in D_4. \end{aligned}$$

On D_4 those that announce highest s_t^3 divide equally w where announcing α counts 1. As a result only s in D_1 can be a D.I. equilibrium act. So if σ is a D.I. equilibrium σ_t^1: $\Pi_t \to \Pi_t$ for all t so that there exists $\alpha: \Omega \to \Omega$ which is Π_t-measurable for all t and such that $\alpha(I_t(\omega)) = \sigma_t^1(I_t(\omega))$ for all $\omega \in \Omega$.

Hence "truth" is a D.I. equilibrium. More precisely defining for an arbitrary $\hat{\mathbf{x}}$ in F: $\sigma_t^2(I_t(\omega)) = \hat{\mathbf{x}}$, $\sigma_t^1(I_t(\omega)) = I_t(\omega)$, and $\sigma_t^3 = 0$ for all t and a we get a D.I. equilibrium whose D.I. allocation is $\hat{\mathbf{x}}$. An agent can affect the outcome by defecting from $\boldsymbol{\sigma}$ only if he is not better off. He can play an act which will move s from D_1 to D_a or D_3.

However there may be an equilibrium $\boldsymbol{\sigma}$ where σ_t^1 is not always truthful i.e., the α induced by $\boldsymbol{\sigma}$ is not the identity. In this case the induced D.I allocation is in F by S^2M condition. Otherwise there will be an agent τ who will act as described by D_a. Q.E.D

If the function α of D_2 or $M(i)$, $\alpha: B' \to B$ is onto (for all $B' \in \Pi$), then the conditions M and S^2M coincide. The proof of Proposition 2 also implies:

COROLLARY 1. *Suppose that for a given D.I. economy and SWC F, a SOF f as described in the proof of Proposition 2 implements F. Then F satisfies the S^2M condition.*

If the initial endowments are not constant over Ω it has to be assured that they are constant on every B in Π. In addition to the restriction on S as that described after the proof of Proposition 1, S_t^1 must be restricted to those B_t's in Π_t where t's endowment is his true one.

Remark. Propositions 1 and 2 extend Maskin's [7] implementation results as applied to economic environments (see Hurwicz, Maskin, and Postlewaite [6])).

Next we wish to discuss the relationship between our implementation results and the literature on partial equilibrium models using Baye equilibrium as the solution concept, e.g. the literature on implicit contracts, the principal-agent problem, auctions, optimal taxation, etc. In these latter models some agents typically have private information; at least one agent can distinguish two states which cannot be distinguished by any other agents. Thus these informational structures do not satisfy our non-exclusivity of information assumption. Hence our theorem giving sufficient conditions for implementation does not apply. These models rely on self selection for implementation. If the strategic outcome function gives an agent the proper incentive to "reveal" his information, it is not necessary that any other agent possess this information. Our sufficiency results focus on the informational structure. Note, however, that monotonicity remains a necesssary condition independent of the informational structure and the number of agents.

Consider a differential information economy e satisfying **DI1–DI4** and a social welfare correspondence F. We will now define a concept of self selection for an agent τ.

SS* (SELF-SELECTION). A SWC satisfies self-selection on $B \in \Pi$ if for any $\tau \in T$, $\omega_1, \omega_2 \in B$, and any $\hat{x} \in F$:

$$\sum P(\omega, I_\tau(\omega_1))\, u_\tau(\hat{x}_\tau(\omega), \omega) \geqslant \sum P(\omega, I_\tau(\omega_1))\, u(\hat{y}_\tau(\omega), \omega)$$

where $\hat{y}$ is defined as follows,

if $\emptyset \neq I_\tau(\omega_2) \cap (\bigcap_{t \neq \tau} I_t(\omega)) \equiv \omega'$ then $\hat{y}(\omega) = \hat{x}(\omega')$ and
if $\emptyset = I_\tau(\omega_2) \cap (\bigcap_{t \neq \tau} I_t(\omega))$ then $\hat{y}(\omega) = 0$.

The intersection in the previous line is at most a single point by our assumption **DI4**. This definition is essentially the same as used in many other models except that we include the possibility that the intersection is empty.

To understand better the meaning of self-selection, consider a situation where the allocation $\hat{x}$ from the definition above is obtained in an equilibrium of a SOF. By the revelation principle (see, e.g., Myerson [10] or Postlewaite and Schmeidler [14] for a statement in the present model) there is a revelation outcome function for which truth is an equilibrium yielding $\hat{x}$. For a state of nature ω_1, agent τ could announce some event other than the true event, $I_\tau(\omega')$. The false statement may or may not be detectable, i.e., the intersection of his announced (false) event, $I_\tau(\omega_2)$ with the true events announced by others, $\bigcap_{\tau \neq t} I_t(\omega')$ may be empty or a singleton, ω'. (Detection of **a** deviation from truth is in general weaker than detection of **the** deviator.) Assuming that the worst outcome for τ for those states in which a lie has been detected is that he consumes the bundle 0, then the revelation outcome function must result in an outcome no worse for τ than $\hat{y}$ for this announcement. Hence, if the inequality in the definition is not satisfied, $\hat{x}$ cannot be an equilibrium outcome for the revelation game. So a fortiori, $\hat{x}$ cannot be an equilibrium for any SOF.

Formally, we have just proved the following:

PROPOSITION 3. *Given a D.I. economy e satisfying* **DI**1 *to* **DI**4, *a D.I. allocation* $\hat{x}$ *is an equilibrium outcome for some SOF only if it satisfies self-selection.*

The D.I. allocation $\hat{x}$ above is considered a SWC which is a singleton.

COROLLARY 2. *A social welfare correspondence F which is implemented on an economy e satisfying* **DI**1 *to* **DI**4 *satisfies self-selection.*

Proof. By definition of implementation, there must be an SOF f such that every allocation in the SWC is an equilibrium outcome for f. But by the proposition, each allocation must then satisfy self-selection. Hence the correspondence F satisfies self-selection. Q.E.D.

In the definition of self-selection it is implicitly assumed that the worst outcome for an agent is to receive zero consumption. This comes from the definition of $\hat{y}(\omega)=0$ in the case that a lie has been detected, i.e., $I_\tau(\omega_2)\cap(\bigcap_{t\neq\tau} I_t(\omega_2))$. One might consider outcomes which include the possibility of "punishments" in addition to the confiscation of initial endowment. Self-selection could then be redefined with the utility of punishment replacing the utility of zero consumption.

The problem we are dealing with here arises in the case that the deviation of a single agent from truth telling in the revelation game may be detectable. The statement above regarding the possibility of punishment has to do with the case in which there is a positive probability that such deviations may be detected. There is also the possibility that for some information structures such deviations are *never* detectable. This is the case when for a common knowledge event B the individuals' partitions $(\Pi_t \mid B)_{t\in T}$ are qualitatively independent on B. (I.e., $\bigcap_{t\in T} B_t \neq \varnothing$ if for all $t\in T$, $B_t\in \Pi_t \mid B$.). Clearly any deviation from truth, when restricted to B, is undetectable. This is the information structure for the standard private values auction models, for example. A special case of qualitative independent is where one agent, say τ, has complete information on B, i.e., for each $\omega\in B$, $I_\tau(\omega)=\{\omega\}$, and for any other agent $t\neq\tau$, $I_t(\omega)=B$. This is the information structure for the prototypical principal-agent problem.

If qualitative independence describes an information structure in which no deviation from truth in the revelation game will be detected, the other extreme, i.e., that *any* such deviation will be detected is implied by the condition of non-exclusive information. In this case, $\hat{y}(\omega)$ in the definition of self-selection equals 0 for every ω, and hence self-selection is trivially satisfied for any allocation $\hat{x}$.

We will now discuss an extension of Proposition 2 by substituting NEI by S.S.

CONJECTURE 1. *Given a* **DI** *economy e with* $\# T > 2$ *satisfy* **DI**1 *to* **DI**4, *a SWC F satisfying SS and a stronger monotonicity condition is implementable.*

Instead of defining precisely what we mean by a stronger monotonicity condition, let us return to the proof of Proposition 2. The strategic outcome function used in the proof of this proposition is not well defined if NEI is not satisfied. Specifically, the set $\bigcap_{t \neq \tau} s_t^{\prime}$ in the definition of D_2 will not always be a singleton. This difficulty can be remedied if we define in D_2

$$\{\bar{\omega}\} = \bigcap_{t \neq \tau} s_t^1 \cap \alpha(s_\tau^1).$$

With this redefined SOF, any $\hat{x} \in F$ will be a DI equilibrium allocation. This part of the proof of Proposition 2 carries over when NEI is replaced by SS. The problem arises in that there may be equilibria for this SOF which are not in F. A strengthening of monotonicity which suffices for the conjecture to be true is to assume *a fortiori* that these equilibria are contained in the SWC. This condition could be stated directly in terms of the SWC F, but it would be even more cumbersome than the conditions M and S^2M.

To the extent that monotonicity (M) and the stronger monotonicity condition suggested above differ, we have not provided necessary and sufficient conditions for implementation. This remains an open problem.

4. EQUILIBRIA OF REVELATION OUTCOME FUNCTIONS

Given a D.I. economy e and a SOF f denote by Σ^* the set of D.I. equilibria for (e, f). Given an equilibrium $\boldsymbol{\sigma}^*$ in Σ^* one can construct, by the revelation principle, an outcome function $g = g(e, f, \boldsymbol{\sigma}^*)$, where truth is a D.I. equilibrium of g resulting in the same outcome as $\boldsymbol{\sigma}^*$. One can also construct a SOF $h = h(e, f, \Sigma^*)$ associated with e, f and the set of all D.I. equilibria of (e, f). It will be referred to as the extended revelation outcome function. The agents are asked to reveal their private information and the equilibrium strategy $\boldsymbol{\sigma}^* \in \Sigma^*$ they agreed upon. In our interpretation the agents discuss and reach a nonbinding agreement about the equilibrium strategy to be used in the revelation game. Thus "extended truth" means the true private information and the equilibrium strategy are agreed upon unanimously.

Formally we define for all t in T and all ω in Ω: $S_t^h(\omega) = \Pi_t \Sigma^*$. If for all t in T, agent t plays (B_t, σ_t^*) then $h(((B_t, \sigma_t^*))_{t \in T}) = f((\sigma_t^*(B_t))_{t \in T})$. Clearly, "extended truth" is D.I. equilibrium for (e, h).

There are several drawbacks to the revelation principle. First, for each D.I. equilibrium $\boldsymbol{\sigma}^*$ of (e, f) a separate SOF g is constructed. This is taken care of by the concept of extended revelation. Second, given such a SOF it may have D.I. equilibria in adition to the truth. Furthermore, these aditional equilibria may yield outcomes which are not equilibrium outcomes of the original SOF f. Such examples are presented by (among others) Postlewaite and Schmeidler [15] and Repullo [18]. Another drawback not dealt with here is the possibility that the revelation outcome function is much more complicated than the original SOF.

PROPOSITION 5. *Let g be the revelation SOF associated with a SOF f, a D.I. economy e, and a D.I. equilibrium* $\boldsymbol{\sigma}^*$ of (e, f). *Suppose that for all t in T: $\{\sigma_t^*(B_t) \mid B_t \in \Pi_t \equiv S_t^g\} = S_t \equiv S_t^f$. Then every D.I. equilibrium outcome of (e.g.) is also an equilibrium outcome of (e, f).*

Proof. Let $\boldsymbol{\sigma}^g$ be a D.I. equilibrium of (e, g) and let $\boldsymbol{\sigma}'$ be the induced strategies list for (e, f). That is, for all t and ω: $\sigma_t'(B_t) = \sigma_t^*(B_t')$, where $B_t' = \sigma_t^g(B_t)$. By definition $\boldsymbol{\sigma}'$ yields the same outcome as $\boldsymbol{\sigma}^g$.

If, by way of negation, $\boldsymbol{\sigma}'$ is not an equilibrium strategy for (e, f) then for some t, σ_t' is not a best response to $\boldsymbol{\sigma}'$. Let then $\theta_t(\cdot)$ be a better response. But by the condition of the proposition, for each B_t in Π_t there exists B_t' in S_t^g such that $\sigma_t^*(B_t') = \theta_t(B_t)$. Hence σ_t^g is not a best response to $\boldsymbol{\sigma}^g$, a contradiction. Q.E.D.

This result has been independently derived by Repullo [18]; a variant for complete information economies was obtained by Hurwicz [5]. We can also state the condition analogous to that of Proposition 3 that prevents equilibria outcomes additional to those of the extended truthful revelation. Indeed the condition in Proposition 6, which follows, is weaker than that of Proposition 5 since the set of all equilibria strategies may use more acts than used by one specific equilibrium in this set.

PROPOSITION 6. *If for all t in T every act in $S_t \equiv S_t^f$ in (e, f), of t is used by some equilibrium strategy of t at some ω, then the D.I. equilibria* *allocations of the extended revelation SOF h (and e) coincide with those of* (e, f).

The proof of Proposition 6 is a repetition of the proof of Proposition 5.

As promised we return to Example 1 in the Introduction. Recall that the example had the property that a revelation outcome function was designed so that the desired outcome (b, a) was an equilibrium outcome, but that for this SOF, (a, b) was also an equlibrium outcome. We will now demonstrate a non-revelation SOF for which (b, a) is an equilibrium outcome and (a, b) is not. Let the set of actions available to agents 2 and 3 remain the same. For them the game remains a "revelation game." Let agent 1 have the set of actions consisting of two elements, $\{A, I\}$. The SOF will be as follows: If agent 1 chooses A, the outcome is as in the earlier SOF: if agents 2 and 3 disagree in their announcements, the outcome is c. If they both announce ω_1, the outcome is b and if they announce ω_2, the outcome is a. If agent 1 chooses I and 2 and 3 disagree in their announcements, c is again the outcome. The outcomes in the case that 1 chooses I and 2 and 3 agree are reversed, however, agreement on ω_1 yields a and agreement of ω_2 yields b. As before (b, a) is an equilibrium outcome (arising from the strategies A for 1 and truthful revelation for 2 and 3). Now (a, b) is *not* an equilibrium outcome. For any strategy triple giving rise to (a, b), agent 1 can change his strategy from I to A (or the reverse) and change the outcome from (a, b) to (b, a). Since (b, a) is preferred by 1, (a, b) is not an equilibrium outcome. Thus, as we promised, we have constructed an SOF such that (b, a) is an equilibrium outcome but (a, b) is not.

A few comments are in order. First, the strategy set for agent 1, $\{A, I\}$ can be thought of as accepting the announcements of agents 2 and 3 or inverting them. They correspond roughtly to the α in S^3 of the strategy definition of the set of acts in the proof of Proposition 2. Agent 1 is given a strategy set here which allows him to "unjumble" the "jumbled announcements of agents 2 and 3.

The second point to be made concerns Proposition 5. That proposition states that given an equilibrium allocation no new equilibrium allocations would be added in the induced revelation zone if the strategies of the agents were onto their act spaces. Note that for an equilibrium which gives rise to (b, a), agent 1 is playing either A or I, but not both. (There are two equilibria which yield (b, a)). Thus, agent I's strategy is *not* onto his act space, and indeed, the induced revelation game has an additional equilibrium outcome (a, b) as we saw in the Introduction.

The last point is that it may seem strange that giving agent 1, who has no information, a choice can eliminate the equilibrium outcome which is bad from his point of view, while preserving the good one. The point of the example is simply to demonstrate that the set of equilibrium outcomes can be shrunk if we extend our attention from revelation SOF's to non-revelation SOF's. It is worthwhile to observe that there are still several equilibria in the non-revelation SOF that give rise to the same outcome. The manner in which the one equilibrium is eliminated is somewhat disturbing.

Bibliographic Note. Since this paper was written, there has been additional related work by Blume and Easley [2] and Palfrey and Srivastava [11, 12].

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