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Economics 172
Make – up Quiz 2
Answers

Question 1

1A. Using exactly the same analysis as in Quiz 2, question 1, we find that the ratio of capital per labor in efficiency units that maximizes net output is $\frac{K_{t-1}}{E_t e^{\gamma t}} = 13.33696$. Then we can solve for output in efficiency units as follows:

$$\frac{Q_t}{E_t e^{\gamma t}} = B\left(\frac{K_{t-1}}{E_t e^{\gamma t}}\right)^{1-\beta} = 1.2311 \cdot 13.33696^{0.35} = 3.0484$$

We also know from Quiz 2, question 1 that the capital output ratio α is equal to 4.375. Thus the ratio of capital to labor income is equal to

$$\frac{K_{t-1}}{w_t E_t} = \frac{K_{t-1}}{\beta Q_t} = \frac{1}{\beta} \cdot \alpha = \frac{1}{0.65} \cdot 4.375 = 6.7307$$

1B. We know from Exercise 3 that in this model the capital to labor income ratio is given by

$$\frac{K_{t-1}}{w_t E_t} = \frac{1}{\beta} \cdot \frac{(1 - c_Y \cdot \beta)}{(g + c_A + \delta)}$$

Since we have already solved for the capital to labor income ratio the only unknown in the above equation is c_A and thus we can solve for it, finding it equal to 0.02668.
The saving to labor income ratio is found by using the fact that

$$\frac{S_t}{w_t E_t} = \frac{K_t - K_{t-1}}{w_t E_t} = \frac{g \cdot K_{t-1}}{w_t E_t} = 0.02532 \cdot 6.7307 = 0.17039$$

The interest rate is found by the usual marginal productivity condition

$$(1-\beta)\frac{Q_t}{K_{t-1}} = \rho + \delta \Leftrightarrow \rho = (1-\beta)\frac{Q_t}{K_{t-1}} - \delta = 0.35 \cdot \frac{1}{4.375} - 0.08 = 0.00$$

1C. From Quiz 2, question 1 we know that consumption is equal to

$$C_t = Q_t - (\delta + g')K_{t-1} \Leftrightarrow \frac{C_t}{E_t e^{\gamma t}} = B\left(\frac{K_{t-1}}{E_t e^{\gamma t}}\right)^{1-\beta} - (\delta + g')\frac{K_{t-1}}{E_t e^{\gamma t}} = 3.0484 -$$
$$-(0.08 + 0.02532) \cdot 13.33696 = 1.6439$$

Question 2

2A. In this question the quantity to be maximized is not net output but consumption in labor efficiency units, that is the economy is on the Golden Rule steady state path. From Quiz 2, question 1 we know that consumption per labor in efficiency units is given by

$$\frac{C_t}{E_t e^{\gamma t}} = B\left(\frac{K_{t-1}}{E_t e^{\gamma t}}\right)^{1-\beta} - (\delta + g')\frac{K_{t-1}}{E_t e^{\gamma t}}$$

Maximization requires that the derivative of this expression with respect to capital per efficiency units of labor is set to zero which results in

$$\frac{\partial\left(\frac{C_t}{E\ e^{\gamma t}}\right)}{\partial\left(\frac{K_{t-1}}{E_t e^{\gamma t}}\right)} = (1-\beta)B\left(\frac{K_{t-1}}{E_t e^{\gamma t}}\right)^{-\beta} - (\delta + g') = 0 \Leftrightarrow \frac{K_{t-1}}{E_t e^{\gamma t}} = \left(\frac{(1-\beta)B}{\delta + g}\right)^{\frac{1}{\beta}} = 8.737$$

Following the same procedure as in question 1 we can solve for output in efficiency units of labor

$$\frac{Q_t}{E_t e^{\gamma t}} = B\left(\frac{K_{t-1}}{E_t e^{\gamma t}}\right)^{1-\beta} = 1.2311 \cdot 8.737^{0.35} = 2.629$$

We can find the capital to output ratio by

$$\frac{K_{t-1}}{Q_t} = \frac{\frac{K_{t-1}}{E_t e^{\gamma t}}}{\frac{Q_t}{E_t e^{\gamma t}}} = \frac{8.737}{2.629} = 3.3235$$

and thus the corresponding capital to labor income ratio is

$$\frac{K_{t-1}}{w_t E_t} = \frac{K_{t-1}}{\beta Q_t} = \frac{1}{\beta} \cdot \alpha = \frac{1}{0.65} \cdot 3.3235 = 5.112$$

2B. Using the same procedures and in question 1B we find that

$$c_A = 0.06845, \frac{S_t}{w_t E_t} = 0.12943, \rho = g = 0.02532$$

2C. Using the procedure of question 1C we find that $\frac{C_t}{E_t e^{\gamma t}} = 1.7088$. We thus observe that the steady state consumption per efficiency units is higher that in question 1C. This is to be expected since we specifically set out to maximize consumption in efficiency units and compute the Golden Rule steady state path, on which capital is used as efficiently as possible and thus consumption (in efficiency units) is higher that in any other steady state path.