Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
http://www.econ.upenn.edu/pier

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"A Likelihood Analysis of Models with Information Frictions"

by

Leonardo Melosi

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A Likelihood Analysis of Models with Information Frictions

Leonardo Melosi*

University of Pennsylvania

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Abstract

This paper develops a dynamic stochastic general equilibrium model where firms are imperfectly informed. We estimate the model through likelihood-based methods and find that it can explain the highly persistent real effects of monetary disturbances that are documented by a benchmark VAR. The model of imperfect information nests a model of rational inattention where firms optimally choose the variances of signal noise, subject to an information-processing constraint. We present an econometric procedure to evaluate the predictions of this rational inattention model. Implementing this procedure delivers insights on how to improve the fit of rational inattention models.

JEL Classifications: E3, E5, C32, D8.

Keywords: Imperfect common knowledge; rational inattention; Bayesian econometrics; real effects of nominal shocks; VAR identification.

*Corresponding author: Leonardo Melosi, 160 McNeil, 3718 Locust Walk, Philadelphia, PA 19104-6297, USA, email address: lmelosi@econ.upenn.edu, mobile phone: 267-939-4165. I thank Frank Diebold, Jesús Fernández-Villaverde, Dirk Krueger, Bartosz Maćkowiak, Frank Schorfheide, and Mirko Wiederholt for very helpful comments and discussion. I am grateful for comments from participants at the Midwest Macro Meetings 2008, the 23rd Annual Congress of the European Economic Association, Upenn Macro Lunches and Upenn Econometric Lunches.

1 Introduction

This paper develops and estimates a dynamic stochastic general equilibrium (DSGE) model where agents are imperfectly informed, as in Woodford (2002). This type of model is well-suited to explaining highly persistent real effects of money and delayed effects on inflation (Woodford, 2002), which are documented by VAR studies (Christiano *et al.*, 1999, Stock and Watson, 2001, Christiano *et al.*, 2005). Furthermore, this model has another appealing feature as it nests a simple model of rational inattention where firms optimally choose what to pay attention to, subject to an information-processing constraint à la Sims (2003). Whether these models can generate sluggish real effects of nominal shocks hinges upon the parameter values that determine how informed agents are. A shortcoming of the literature is the lack of empirical guidance in selecting these parameter values. We try to counter this shortcoming by estimating these parameters through Bayesian methods.

The paper contributes to the existing literature along three dimensions. First, we show that the estimated model of imperfect information à la Woodford (2002) can account for the strongly persistent real effects of monetary disturbances that characterize the impulse response functions of a benchmark VAR. Second, we present an econometric procedure that evaluates whether the predictions of the rational inattention model are supported by the data. Third, by implementing this procedure, we gain insights into how to improve the fit of rational inattention models.

Following Woodford (2002), we assume that firms do not perfectly observe any realizations of the model variables. There are two state variables in the model: the aggregate technology and the monetary policy stance. Firms observe idiosyncratic noisy signals regarding the state variables and solve a signal extraction problem in order to keep track of the model variables. Since the signal is noisy, firms do not immediately learn the occurrence of monetary disturbances. As a result, the price level fails to adjust enough to entirely neutralize the real effects of nominal shocks (Lucas, 1973). Moreover, because of the idiosyncratic nature of the signals, in the aftermath of a shock firms are also uncertain about what other firms know that other firms know... that other firms know about that shock. This feature of the model is termed imperfect common knowledge. When firms find it optimal to react to changes of endogenous variables (e.g., in the presence of strategic complementarity in price setting), a problem of forecasting the forecast of others of the type envisioned by Townsend (1983b) arises. This feature of the model has been shown to amplify the persistence in economic fluctuations (Townsend, 1983a, 1983b; Hellwig, 2002; Adam, 2008; Angeletos and La'O, 2008; Rondina, 2008; and Lorenzoni, forthcomingA), and in the propagation of monetary disturbances to real variables and prices (Phelps, 1970; Lucas, 1972; Woodford, 2002; Adam, 2007; Gorodnichenko, 2008; Maćkowiak and Wiederholt, 2008; Nimark, 2008; Paciello, 2008; and Lorenzoni, forthcomingB).[1]

We evaluate the fit of the model with imperfect common knowledge. For this purpose, we introduce a model that deviates from the one of imperfect common knowledge in only two respects: (1) all agents are perfectly informed, and (2) firms can optimally adjust their prices only at random periods, as in Calvo (1983). The last assumption is common to a very large number of models that have been used as workhorses for monetary policy studies over the last 25 years. We fit both models to a data set that includes U.S. per capita GDP and the U.S. GDP deflator. First, we find that the model with imperfect common knowledge fits the data better than the Calvo model. Second, the model with imperfect information can largely accommodate the persistent real effects of monetary shocks implied by a benchmark VAR. Third, when we replace the mechanism of imperfect common knowledge with that of sticky prices à la Calvo, we observe that such persistence substantially drops.

We modify the model of imperfect common knowledge so as to allow firms to optimally choose the variances of signal noise given an information-processing constraint à la Sims (2003). This model of rational inattention is nested into the model with imperfect common

[1]See Mankiw and Reis (2002a, 2002b, 2006, 2007), and Reis (2006a, 2006b, 2009) for models with information frictions that do not feature imperfect common knowledge but can generate sizeable persistence.

knowledge. The former model makes predictions over the variances of the signal noise. In the latter model these variances are instead structural parameters whose values are learned from the data by estimating the model. We introduce and implement an econometric procedure that allows us to assess to what extent the predictions of this simple model of rational inattention are supported by the data. We find that these predictions are rejected by the data to some extent. Moreover, this exercise delivers interesting insights on how to improve the fit of rational inattention models. In this respect, we observe that capital accumulation would be an important feature to be added to these models.

The procedure to evaluate the predictions of the model of rational inattention can be summarized in four steps. First, we sample with replacement the posterior draws for the parameters of the model with imperfect common knowledge. Second, for each sampled draw, we measure how much information firms acquire per unit of time in the model with imperfect common knowledge. Third, for each sampled draw, we solve the model of rational inattention by using the output of the second step to determine the tightness of firms' information-processing constraint. Fourth, we evaluate whether the variances of signal noise predicted by the two models are similar.

We depart from Woodford (2002) in two respects. First, our empirical strategy is likelihood-based, while Woodford (2002) calibrates the parameters of his model. Second, Woodford's model has one rather than two shocks. Having an additional shock allows us to get around the problem of stochastic singularity when we evaluate the likelihood function. Specifically, we consider a nominal shock and an aggregate technology shock.

This paper is also related to the literature of rational inattention (Sims, 2003, 2006; Luo, 2008; Paciello, 2008; Van Nieuwerburgh and Veldkamp, 2008; Woodford, 2008; and Maćkowiak and Wiederholt, forthcoming). Maćkowiak and Wiederholt (2009) introduce a model where firms optimally decide how much attention to pay to aggregate and idiosyncratic conditions, subject to a constraint on information flows. When they calibrate their model to match the average absolute size of price changes observed in micro data, they find that nominal shocks have sizeable and persistent real effects.

The rest of the paper is organized as follows. Section 2 presents both the model with imperfect common knowledge and the model of rational inattention, as well as the Calvo model. Some features of the first two models are explored in section 3. Section 4 deals with the empirical analysis. In section 5, we conclude.

2 The models

In this section we describe three DSGE models. The first model is a model with imperfect common knowledge (henceforth, **ICK model**). In this model, information-processing frictions are modelled by assuming that firms have to solve a signal extraction problem in order to estimate the state of the aggregate technology and that of monetary policy. A feature of this model is that firms take the stochastic process of signals as given. In the second model (henceforth, **rational inattention model**) firms solve the same signal extraction problem as in the ICK model but they are allowed to optimally choose the variances of signal noise, subject to an information-processing constraint of the type used in Sims (2003). In the third model (henceforth, **Calvo model**) all agents have perfect information but they can re-optimize their prices only at random periods, as in Calvo (1983). In the first part of this section we introduce the equations common to all the models. In the remaining part of the section, we analyze the specific features of the three models.

2.1 The common structure

The economy is populated by households, final goods producers (or producers), intermediate goods firms (or firms), a financial intermediary, and a monetary authority (or central bank). Households derive utility from consumption of final goods and disutility from supplying labor to the intermediate goods firms. Furthermore, households face a cash-in-advance (CIA) constraint. The final goods producers are perfectly competitive with a CES production function. The intermediate goods firms operate in a monopolistic competitive environment with a production function that is linear in its unique input, which is labor. Furthermore, there are two shocks: an aggregate productivity shock that affects intermediate goods firms' technology and a monetary policy shock.

At the beginning of period t, the households inherit the entire money stock of the economy, M_t. They decide how much money D_t to deposit at the financial intermediary. These deposits yield interest at rate $R_{H,t}-1$. The financial intermediary receives household deposits and a monetary injection from the monetary authority, which it lends to final goods producers at rate $R_{F,t}-1$. The intermediate goods firms hire labor services from households and produce their output. The firms sell their output to the final goods producers and use the proceeds to pay wages, W_tH_t, where W_t is the nominal hourly wage, and H_t is hours worked, and dividends, Π_t, to households. Households' cash balance increases to $M_t - D_t + W_tH_t + \Pi_t$. The CIA constraint requires that households pay for all consumption purchases with the accumulated cash balances. The producers sell the final goods to households and then pay back their loans. Finally, households receive back their deposits inclusive of interest rate and the net cash inflow of the financial intermediary as dividend Π_t^b.

2.1.1 The representative household

The representative household solves the problem:

$$\max_{\{C_t,H_t,M_{t+1},D_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln C_{t+s} - \alpha H_{t+s}\right]$$

such that

$$P_t C_t \leq M_t - D_t + W_t H_t + \Pi_t \tag{1}$$

$$0 \leq D_t \tag{2}$$

$$M_{t+1} = (M_t - D_t + W_t H_t + \Pi_t - P_t C_t) + R_{H,t} D_t + \Pi_t^b \tag{3}$$

where C_t is the amount of the final good consumed at time t, P_t is the price of the final good at time t, and β is the discount factor.

2.1.2 The technology of the intermediate goods firms

Every intermediate goods firm has the same technology:

$$Y_{i,t} = A_t N_{i,t} \tag{4}$$

where $Y_{i,t}$ is the output produced by the firm i at time t, and $N_{i,t}$ is the labor input demanded by firm i at time t.

We further assume that the aggregate productivity A_t follows a random walk with drift:

$$\ln A_t = \ln a + \ln A_{t-1} + \sigma_a \varepsilon_{a,t} \tag{5}$$

where $\varepsilon_{a,t} \backsim \mathcal{N}(0,1)$. Finally, it turns out to be useful to define:

$$a_t \equiv \ln A_t - \ln a \cdot t \tag{6}$$

2.1.3 The final goods producers

The representative final goods producer combines a continuum of intermediate goods indexed by $i \in [0,1]$ by using the CES technology:

$$Y_t = \left(\int_0^1 (Y_{i,t})^{\frac{\nu-1}{\nu}} \, di \right)^{\frac{\nu}{\nu-1}} \tag{7}$$

where the parameter ν is assumed to be strictly larger than unity.

The producer takes input prices P_t^i and output price P_t as given. Furthermore, it has to borrow the cash needed to pay the intermediate goods firms at rate $R_{F,t} - 1$. Hence, its cost function is $\left(\int P_t^i Y_{i,t} di \right) R_{F,t}$. Profit maximization implies that the demand for intermediate goods will be:

$$Y_{i,t} = \left(\frac{P_t^i}{P_t} \right)^{-\nu} Y_t \tag{8}$$

where the competitive price of the final good P_t is given by

$$P_t = \left(\int \left(P_t^i \right)^{1-\nu} di \right)^{\frac{1}{1-\nu}} \tag{9}$$

2.1.4 The financial intermediary

The financial intermediary solves the trivial problem:

$$\max_{\{L_t, D_t\}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \frac{\Pi_{t+s}^b}{Q_{t+s+1}} \right] \tag{10}$$

$$st$$

$$\Pi_t^b = D_t + R_{F,t} L_t - R_{H,t} D_t - L_t + X_t \tag{11}$$

$$L_t \leq X_t + D_t \tag{12}$$

where Q_t is the time 0 value of a unit of the consumption good in period t to the representative household and $X_t = M_{t+1} - M_t$ is the monetary injection.

2.1.5 The monetary authority

The monetary authority sets the growth rate of money so as to ensure that a log-linear combination of output and price level follows an exogenous process of the following type:

$$\Delta \ln \Lambda_t = (1 - \rho_\Lambda) \Lambda^* + \rho_\Lambda \Delta \ln \Lambda_{t-1} + \sigma_\Lambda \varepsilon_{\Lambda,t} \tag{13}$$

with $\varepsilon_{\Lambda,t} \backsim \mathcal{N}(0,1)$ and

$$\ln \Lambda_t = \lambda \ln Y_t + \ln P_t \tag{14}$$

where Δ stands for the first-difference operator, the degree of smoothness in conducting monetary policy ρ_Λ is such that $\rho_\Lambda \in [0,1)$. Λ^* is a parameter that represents the long-run average growth rate of $\ln \Lambda_t$. Moreover, the monetary policy shock $\varepsilon_{\Lambda,t}$ is assumed to be orthogonal to the productivity shock $\varepsilon_{a,t}$. Finally, it is useful to denote:

$$m_t \equiv \ln \Lambda_t - \Lambda^* \cdot t \tag{15}$$

2.2 ICK model

In the ICK model, intermediate goods firms do not face any cost when they adjust their prices. Nonetheless, they cannot observe any realizations of the model variables. Firms observe idiosyncratic noisy signals concerning the state of technology $\ln A_t$ and that of monetary policy $\ln \Lambda_t$. Therefore, they will estimate the model variables by using the history of realizations of their signals. For tractability, it is assumed that the other agents perfectly observe the past and the current realizations of the model variables.

The intermediate goods firms solve:

$$\max_{P_t^i} \; \mathbb{E}\left[\beta^t Q_t \left(P_t^i Y_{i,t} - W_t N_{i,t}\right) | \mathcal{I}_t^i\right], \quad \forall t \in \{1, 2, ...\} \tag{16}$$

$st.$

$$Y_{i,t} = \left(\frac{P_t^i}{P_t}\right)^{-\nu} Y_t, \qquad Y_{i,t} \leq A_t N_{i,t} \tag{17}$$

$$\mathcal{I}_t^i = \left(\{\mathbf{z}_{i,\tau}\}_{\tau=-\infty}^{t}, \Theta_I\right) \tag{18}$$

where Q_t is the time 0 value of a unit of the consumption good in period t to the representative household, which is treated as exogenous by the firm. $\mathcal{I}_t^i$ is the information set available to firm i at time t. This set contains the history of the idiosyncratic signals $\{\mathbf{z}_{i,\tau}\}_{\tau=-\infty}^{t}$ and the vector of model parameters Θ_I, that is

$$\Theta_I \equiv (\nu, \rho_\Lambda, \alpha, \ln a, \Lambda^*, \lambda, \beta, \sigma_\Lambda, \sigma_a, \sigma_{e_1}, \sigma_{e_2}) \tag{19}$$

It is important to emphasize that we assume that at time 0 firms are endowed with an infinite sequence of signals. This assumption simplifies the analysis. Furthermore, the equilibrium laws of motion of all model variables are assumed to be common knowledge among firms.

Firm i's signal model is

$$\begin{bmatrix} z_{1,i,t} \\ z_{2,i,t} \end{bmatrix} = \begin{bmatrix} m_t \\ a_t \end{bmatrix} + \begin{bmatrix} e_{1,i,t} \\ e_{2,i,t} \end{bmatrix} \tag{20}$$

where $\mathbf{z}_{i,t} \equiv [z_{1,i,t}, z_{2,i,t}]'$, $\mathbf{e}_{i,t} \equiv [e_{1,i,t}, e_{2,i,t}]'$ and

$$\mathbf{e}_{i,t} \overset{iid}{\backsim} \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}_e\right), \; \mathbf{\Sigma}_e = \begin{bmatrix} \sigma_{e_1}^2 & 0 \\ 0 & \sigma_{e_2}^2 \end{bmatrix} \tag{21}$$

Note that a_t and m_t are the state variables of the model and the signal noises $e_{1,i,t}$ and $e_{2,i,t}$

are assumed to be *iid* across firms and time.

Assuming that the two signals are orthogonal may be considered a strong assumption. After all, firms might learn about a given state variable by processing signals concerning the other state variable. We find, however, that relaxing this assumption of orthogonality of signals does not substantially affect the main predictions of the estimated model.

Finally, one should notice that, as in Woodford (2002), firms are assumed to perfectly observe neither the amount of labor hired $N_{i,t}$ nor the quantity sold $Y_{i,t}$. They are able to get information about these variables indirectly through their estimates of the state variables.

2.3 The rational inattention model

The model of rational inattention relies on three fundamental assumptions. First, information about all model variables is freely available to decision makers. Second, information needs to be processed before being used for decision-making. Third, intermediate goods firms face limitations on the amount of information they can process per unit of time. As a result, firms will optimally decide how much information they want to acquire about each variable that matters for their price-setting decisions. For tractability, it is assumed that the other agents do not face any information-processing constraints.

In full-fledged models of rational inattention (e.g., Maćkowiak and Wiederholt, forthcoming), agents optimally choose the stochastic process of signals, subject to an information-processing constraint à la Sims (2003). Unlike these models, we parametrically restrict the set of signal processes that firms can select. Specifically, we assume that firms optimally choose among signals that follow a "true state plus white noise Gaussian error" process. Hence, what firms are allowed to choose are the variances of signals in equations (20)-(21). Nevertheless, one can show that the signal process (20)-(21) is not optimal if profit function is not quadratic or λ is not equal to unity (Maćkowiak and Wiederholt, forthcoming, sections 6 and 7). We introduce these parametric restrictions for tractability. Moreover, we assume that firms can choose the stochastic process of signals at time 0 but they cannot reconsider their decision thereafter. In section 3.2, we will show that this last assumption is not critical for our results.

At period zero, firms allocate their attention by solving:

$$\max_{\sigma_{e_1,i},\ \sigma_{e_2,i}} \mathbb{E}\left[\sum_{t=1}^{\infty} \beta^t Q_t \left(P^*_{i,t} Y_{i,t} - W_t N_{i,t}\right) |\mathcal{I}^i_t\right], \tag{22}$$

st

$$P^*_{i,t} = \arg\max_{P^i_t} \mathbb{E}\left[\beta^t Q_t \left(P^i_t Y_{i,t} - W_t N_{i,t}\right) |\mathcal{I}^i_t\right] \tag{23}$$

$$Y_{i,t} = \left(\frac{P^*_{i,t}}{P_t}\right)^{-\nu} Y_t, \qquad Y_{i,t} \leq A_t N_{i,t} \tag{24}$$

$$\mathcal{I}^i_t = \left(\{\mathbf{z}_{i,\tau}\}^t_{\tau=-\infty}, \Theta_R\right) \tag{25}$$

$$\begin{bmatrix} z_{1,i,t} \\ z_{2,i,t} \end{bmatrix} = \begin{bmatrix} m_t \\ a_t \end{bmatrix} + \begin{bmatrix} e_{1,i,t} \\ e_{2,i,t} \end{bmatrix} \tag{26}$$

$$\mathbf{e}_{i,t} \overset{iid}{\backsim} \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}_e\right), \ \mathbf{\Sigma}_e = \begin{bmatrix} \sigma^2_{e_1,i} & 0 \\ 0 & \sigma^2_{e_2,i} \end{bmatrix} \tag{27}$$

$$\kappa_{m,i,t} + \kappa_{a,i,t} \leq \kappa, \qquad \text{any } t > 0 \tag{28}$$

where Θ_R is a vector including all the parameters of the model,

$$\Theta_R \equiv (\nu, \rho_\Lambda, \alpha, \ln a, \Lambda^*, \lambda, \beta, \sigma_\Lambda, \sigma_a, \kappa) \tag{29}$$

The variables $\kappa_{m,i,t}$ and $\kappa_{a,i,t}$ denote the information flow from signal $z_{1,i,t}$ to the state of monetary policy, m_t, and that from signal $z_{2,i,t}$ to the state of technology, a_t, respectively. Moreover, the parameter κ quantifies the overall amount of information firms can process in each period. Finally, we define the vector $\mathbf{z}_{i,t} \equiv [z_{1,i,t}, z_{2,i,t}]'$.

Notice that firms have to solve two problems: a price-setting problem and a problem of how to allocate their attention between the two state variables. In the problem of allocating the attention, firms optimally choose the variances of signal noise. Notice that when firms decide how to allocate their attention, they are aware that this choice will affect the objective function (23) and in turn the optimal price-setting policy. Moreover, conditional to these variances of signal noise, rationally inattentive firms face the same price-setting problem as that in the ICK model.

The information set (25) is of the same type as that in the ICK model. Equations (26)-(27) restrict the set of signal processes that can be chosen by firms to be "true state plus white noise Gaussian error" processes. The information-processing constraint (28) sets an upper bound $\kappa \in \mathbb{R}_+$ on the overall amount of information firms can gather at any time t. We define the information flows $\kappa_{m,i,t}$ and $\kappa_{a,i,t}$ in this constraint as follows:

$$\kappa_{m,i,t} \equiv H\left(m_t | z_{1,i}^{t-1}\right) - H\left(m_t | z_{1,i}^{t}\right) \tag{30}$$

$$\kappa_{a,i,t} \equiv H\left(a_t | z_{2,i}^{t-1}\right) - H\left(a_t | z_{2,i}^{t}\right) \tag{31}$$

where $H\left(m_t | z_{1,i}^{\tau}\right)$ and $H\left(a_t | z_{2,i}^{\tau}\right)$ are the conditional entropies of the state variable m_t and a_t, given the history of signals up to time τ, $\mathbf{z}_i^{\tau}$. In information theory (Shannon, 1948), entropy is an axiomatic measure of conditional uncertainty about random variables (Ash, 1990). For instance, the entropy of m_t conditional to the sequence of signals $z_{1,i}^t$ is given by $\int_{-\infty}^{\infty} \log_2 \left[p\left(m_t | z_{1,i}^t\right)\right] p\left(m_t | z_{1,i}^t\right) dm_t$, where $p\left(m_t | z_{1,i}^t\right)$ is the conditional probability density function of m_t. Since all shocks and noise in the model are Gaussian, one can show that the following results hold:

$$H\left(m_t | z_{1,i}^{\tau}\right) \equiv \frac{1}{2} \log_2 \left[2\pi e \cdot VAR\left(m_t | z_{1,i}^{\tau}\right)\right] \tag{32}$$

$$H\left(a_t|z_{2,i}^{\tau}\right) \equiv \frac{1}{2}\log_2\left[2\pi e \cdot VAR\left(a_t|z_{2,i}^{\tau}\right)\right] \tag{33}$$

See Cover and Thomas (1991). The unit of measure of these conditional entropies and consequently that of information flows $\kappa_{m,i,t}$ and $\kappa_{a,i,t}$ is 1 bit.[2] Moreover, as in the ICK model, we assume that the equilibrium laws of motion of all variables are common knowledge.

2.4 A sticky price model à la Calvo (1983)

In the Calvo model all agents perfectly observe the past and current realizations of the model variables. Moreover, the prices charged by each firm are re-optimized only at random periods. The key (simplifying) assumption is that the probability that a given firm will adjust its price within a particular period is independent of the state of the model, the current price charged, and how long ago it was last re-optimized. Firms that do not re-optimize index their prices at the balance-growth-path inflation rate.

We assume that only a fraction $(1-\theta_p)$ of firms re-optimize their prices, while the remaining θ_p fraction does not reset them. The problem of the intermediate goods firms that are allowed to adjust their prices in period t is:

$$\max_{P_t^i} \mathbb{E}_t \sum_{s=0}^{\infty} \theta_p^s \beta^{t+s} Q_{t+s}\left(P_t^i - MC_{t+s}\right)\frac{Y_{i,t+s}}{P_{t+s}} \tag{34}$$

$st.$

$$MC_{t+s} = \frac{W_{t+s}}{A_{t+s}},\ Y_{i,t+s} = \left(\frac{P_t^i}{P_{t+s}}\right)^{-\nu} Y_{t+s} \tag{35}$$

where Q_{t+s} is the marginal utility of a unit of consumption at time $t+s$ in terms of the utility of the representative household at time t, and MC_{t+s} stands for the nominal marginal costs in period $t+s$. We consider only the symmetric equilibrium at which all firms will

[2] If we had used the natural logarithm instead of the logarithm of base two in equation (32)-(33), these quantities would have been measured in nats.

choose the same optimal price $P_t^i = P_t^*$. On aggregate, we have

$$P_t^{1-\nu} = \left[(1-\theta_p) P_t^{*(1-\nu)} + \theta_p \ (\pi_* P_{t-1})^{1-\nu} \right] \tag{36}$$

where π_* is the balance-growth-path (gross) inflation rate. We denote Θ_C as the set of parameters of the Calvo model:

$$\Theta_C \equiv (\nu, \rho_\Lambda, \alpha, \ln a, \Lambda^*, \lambda, \beta, \theta_p, \sigma_\Lambda, \sigma_a) \tag{37}$$

3 Log-linearization and features of the models

All the models presented in the previous section are log-linearized before being solved. The exogenous processes (5) and (13) induce both a deterministic and a stochastic trend to all endogenous variables, except labor. We will detrend the non-stationary variables before log-linearizing the models. It is useful to define the stationary variables as follows:

$$y_t \equiv \frac{Y_t}{A_t}, \quad p_t \equiv \frac{A_t^\lambda P_t}{\Lambda_t}, \quad p_t^i = \frac{A_t^\lambda P_t^i}{\Lambda_t} \tag{38}$$

In order to log-linearize the models with information frictions,[3] we take the following steps. First, we derive the price-setting equation by solving the intermediate goods firms' problem in both models with information frictions. Second, we transform the variables according to the definitions (38). Third, we log-linearize the resulting price-setting equation around the perfect-information symmetric steady state. Henceforth, when we refer to the three models we mean their log-linear approximations.

[3] How to log-linearize and solve the Calvo model is standard and hence omitted. We use the routine *gensys* developed by Sims (2002) to numerically solve this model.

3.1 Quantifying the size of information frictions in the ICK model

The following definitions turn out to be useful for evaluating the size of the information frictions in the log-linear ICK model.

Definition: *Firms' overall level of attention $\varkappa$ is the amount of information that firms process about both state variables in the unit of time.*

Definition: *Firms' allocation of attention to a given state variable is the ratio of the amount of processed information about that state variable to the overall level of attention.*

The *overall level of attention* $\varkappa$ is defined as $\varkappa \equiv \kappa_m + \kappa_a$, where κ_m and κ_a are computed exactly as the information flows in equations (30)-(33). The quantities $\varkappa$, κ_m and κ_a turn out not to vary across periods and firms[4] and are all measured in bits. Moreover, the *allocation of attention* to the state of technology Υ_a can be computed as follows: $\Upsilon_a \equiv \frac{\kappa_a}{\varkappa}$.

Characterizing the parameter $\varkappa$ and Υ_a for the log-linearized ICK model requires computing the conditional variances of m_t and a_t in equations (32)-(33) for a given set of parameters Θ_I. In order to numerically pin down these variances, one has to apply the Kalman filter to the state-space model whose transition equations are given by equations (5) and (13) and the measurement equations are defined by equations (20)-(21). We can concisely represent this result through the mapping ϕ_I:

$$(\varkappa, \kappa_m, \kappa_a)' = \phi_I(\Theta_I) \tag{39}$$

We denote the pair of information flows (κ_m, κ_a) as firms' allocation of attention in the ICK model.

[4]Since firms are assumed to receive infinitely many signals at time $t = 0$, the conditional variances $VAR(m_t|z_{1,i}^{\tau})$ and $VAR(a_t|z_{1,i}^{\tau})$, $\tau \in \{t, t-1\}$ any $t > 0$, do not change over time. Moreover, in the ICK model, these conditional variances are the same across firms because firms face the same variances of signal noise and all shocks are Gaussian. If these variances do not change across periods and firms, neither do information flows κ_m and κ_a. See equations (32)-(33).

3.2 Some property of the rational inattention model

In the log-linear rational inattention model, firms' profit function is log-quadratic. It can be shown that when the profit function is quadratic, the optimal signal is Gaussian (Sims, 2003). This implies that the assumption we made in section 2.3 that signals follow a Gaussian process is not critical.

In section 2.3, we also assumed that firms decide their allocation of attention at time 0. They are not allowed to reconsider the allocation of attention in any subsequent periods. If firms' profit function is quadratic, this assumption does not give rise to a problem of time inconsistency of firms' policies. The reason behind this result is as follows.[5] When firms' profit function is quadratic, it can be shown that the allocation-of-attention problem (22)-(28) turns out to be that of choosing the variances of signal noise so as to minimize the conditional variance of the profit-maximizing price under perfect information (i.e., when $\kappa \to \infty$). This conditional variance does not change over time in periods $t > 0$ because firms receive an infinite sequence of signals at time $t = 0$ and the rational inattention model is linear and Gaussian. Therefore, the objective function of the allocation-of-attention problem does not change over time, and hence, firms do not have any incentives to reconsider their allocation of attention in periods $t > 0$.

Moreover, if their profit function is quadratic, the optimal variances of signal noise can be shown to be the same across firms. Since all shocks are Gaussian and firms receive an infinite sequence of signals at time $t = 0$, the conditional variance of the profit-maximizing price under perfect information is the same for all firms. Therefore, in a quadratic-Gaussian framework, the objective function of the allocation-of-attention problem is the same across firms. Thus, every firm will find it optimal to choose the same allocation of attention. The optimal variances of signal noise will be denoted $\left(\sigma^*_{e_1}\right)^2$ and $\left(\sigma^*_{e_2}\right)^2$.

[5] A more detailed proof of this result is provided in Maćkowiak and Wiederholt (2009).