Elements of Forecasting

in Business, Finance, Economics and Government

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Slides for Projection

Introduction to Forecasting:

Applications, Methods, Books, Journals and Software

1. Forecasting in Action

a. Operations planning and control

b. Marketing

c. Economics

d. Financial speculation

e. Financial risk management

f. Capacity planning

g. Business and government budgeting

i. Demography

j. Crisis management

2. Forecasting Methods: An Overview

[Review of probability, statistics and regression]

Six Considerations Basic to Successful Forecasting

- Forecasts and decisions
- The object to be forecast
- Forecast types
- The forecast horizon
- The information set
- Methods and complexity, the parsimony principle, and the shrinkage principle

Statistical Graphics for Forecasting

Why graphical analysis is important

Simple graphical techniques

Elements of graphical style

Application: graphing four components of real GNP

Modeling and Forecasting Trend

- Modeling trend
- Estimating trend models
- Forecasting trend
- Selecting forecasting models using the Akaike and Schwarz criteria
- Application: forecasting retail sales

Modeling and Forecasting Seasonality

- The nature and sources of seasonality
- Modeling seasonality
- Forecasting seasonal series
- Application: forecasting housing starts

Characterizing Cycles

- Covariance stationary time series
- White noise
- The lag operator
- Wold's theorem, the general linear process, and rational distributed lags
- Estimation and inference for the mean, autocorrelation and partial autocorrelation functions
- Application: characterizing Canadian employment dynamics

Modeling Cycles: MA, AR and ARMA Models

Moving-average (MA) models

Autoregressive (AR) models

Autoregressive moving average (ARMA) models

Application: specifying and estimating models for
forecasting employment

Forecasting Cycles

Optimal forecasts

Forecasting moving average processes

Forecasting infinite-ordered moving averages

Making the forecasts operational

The chain rule of forecasting

Application: forecasting employment

Putting it all Together: A Forecasting Model with Trend, Seasonal and Cyclical Components

Assembling what we've learned

Application: forecasting liquor sales

Recursive estimation procedures for diagnosing and selecting forecasting models

Forecasting with Regression Models

- Conditional forecasting models and scenario analysis
- Accounting for parameter uncertainty in confidence intervals for conditional forecasts
- Unconditional forecasting models
- Distributed lags, polynomial distributed lags, and rational distributed lags
- Regressions with lagged dependent variables, regressions with ARMA disturbances, and transfer function models
- Vector autoregressions
- Predictive causality
- Impulse-response functions and variance decompositions
- Application: housing starts and completions

Evaluating and Combining Forecasts

- Evaluating a single forecast
- Evaluating two or more forecasts: comparing forecast accuracy
- Forecast encompassing and forecast combination
- Application: OverSea shipping volume on the Atlantic East trade lane

Unit Roots, Stochastic Trends, ARIMA Forecasting Models, and Smoothing

Stochastic trends and forecasting

Unit roots: estimation and testing

Application: modeling and forecasting the yen/dollar exchange rate

Smoothing

Exchange rates, continued

Volatility Measurement, Modeling and Forecasting

The basic ARCH process

The GARCH process

Extensions of ARCH and GARCH models

Estimating, forecasting and diagnosing GARCH models

Application: stock market volatility

3. Useful Books, Journals and Software

Books

Statistics review, etc.:

Wonnacott, T.H. and Wonnacott, R.J. (1990), *Introductory Statistics*, Fifth Edition. New York: John Wiley and Sons.

Pindyck, R.S. and Rubinfeld, D.L. (1997), *Econometric Models and Economic Forecasts*, Fourth Edition. New York: McGraw-Hill.

Maddala, G.S. (2001), *Introduction to Econometrics*, Third Edition. New York: Macmillan.

Kennedy, P. (1998), *A Guide to Econometrics*, Fourth Edition. Cambridge, Mass.: MIT Press.

Time series analysis:

Chatfield, C. (1996), *The Analysis of Time Series: An Introduction*, Fifth Edition. London: Chapman and Hall.

Granger, C.W.J. and Newbold, P. (1986), *Forecasting Economic Time Series*, Second Edition. Orlando, Florida: Academic Press.

Harvey, A.C. (1993), *Time Series Models*, Second Edition. Cambridge, Mass.: MIT Press.

Hamilton, J.D. (1994), *Time Series Analysis*. Princeton: Princeton University Press.

Special insights:

Armstrong, J.S. (Ed.) (1999), *The Principles of Forecasting*. Norwell, Mass.: Kluwer Academic Forecasting.

Makridakis, S. and Wheelwright S.C. (1997), *Forecasting: Methods and Applications*, Third Edition. New York: John Wiley.

Bails, D.G. and Peppers, L.C. (1997), *Business Fluctuations*. Englewood Cliffs: Prentice Hall.

Taylor, S. (1996), *Modeling Financial Time Series*, Second Edition. New York: Wiley.

Journals

Journal of Forecasting

International Journal of Forecasting

Journal of Business Forecasting Methods and Systems

Journal of Business and Economic Statistics

Review of Economics and Statistics

Journal of Applied Econometrics

Software

General:

Eviews, S+, Minitab, SAS, etc.

Cross-section: Stata

Open-ended: Matlab

Online Information

Resources for Economists

A Brief Review of Probability, Statistics, and Regression for Forecasting

Discrete Random Variable

Discrete Probability Distribution

Continuous Random Variable

Probability Density Function

Moment

Mean, or Expected Value

Location, or Central Tendency

Variance

Dispersion, or Scale

Standard Deviation

Skewness

Asymmetry

Kurtosis

Leptokurtosis

Normal, or Gaussian, Distribution

Marginal Distribution

Joint Distribution

Covariance

Correlation

Conditional Distribution

Conditional Moment

Conditional Mean

Conditional Variance

Population Distribution

Sample

Estimator

Statistic, or Sample Statistic

Sample Mean

Sample Variance

Sample Standard Deviation

Sample Skewness

Sample Kurtosis

χ^2 Distribution

t Distribution

F Distribution

Jarque-Bera Test

Regression as curve fitting

Least-squares estimation: $$\min_{\beta} \sum_{t=1}^{T} \left[y_t - \beta_0 - \beta_1 x_t\right]^2$$

Fitted values:

$$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 x_t$$

Residuals:

$$e_t = y_t - \hat{y}_t$$

Regression as a probabilistic model

Simple regression:

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

$$\varepsilon_t \overset{iid}{\sim} (0, \sigma^2)$$

Multiple regression:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 z_t + \varepsilon_t$$

$$\varepsilon_t \overset{iid}{\sim} (0, \sigma^2)$$

Mean dependent var   10.23

$$\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$$

S.D. dependent var   1.49

$$SD = \sqrt{\frac{\sum_{t=1}^{T}(y_t - \bar{y})^2}{T-1}}$$

Sum squared resid   43.70

$$SSR = \sum_{t=1}^{T} e_t^2$$

F-statistic   30.89

$$F = \frac{(SSR_{res} - SSR)\ /\ (k-1)}{SSR\ /\ (T-k)}$$

S.E. of regression 0.99

$$s^2 = \frac{\sum_{t=1}^{T} e_t^2}{T-k}$$

$$SER = \sqrt{s^2} = \sqrt{\frac{\sum_{t=1}^{T} e_t^2}{T-k}}$$

R-squared   0.58

$$R^2 = 1 - \frac{\sum_{t=1}^{T} e_t^2}{\sum_{t=1}^{T} (y_t - \bar{y}_t)^2}$$

or

$$R^2 = 1 - \frac{\frac{1}{T}\sum_{t=1}^{T} e_t^2}{\frac{1}{T}\sum_{t=1}^{T} (y_t - \bar{y}_t)^2}$$

Adjusted R-squared   0.56

$$\bar{R}^2 = 1 - \frac{\frac{1}{T-k} \sum_{t=1}^{T} e_t^2}{\frac{1}{T-1} \sum_{t=1}^{T} (y_t - \bar{y}_t)^2}$$

Akaike info criterion   0.03

$$AIC = e^{\left(\frac{2k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$$

Schwarz criterion   0.15

$$SIC = T^{\left(\frac{k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$$

Durbin-Watson stat 1.97

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

$$\varepsilon_t = \varphi \varepsilon_{t-1} + v_t$$

$$v_t \overset{iid}{\sim} N(0, \sigma^2)$$

$$DW = \frac{\sum_{t=2}^{T} (e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2}$$

Regression of y on x and z

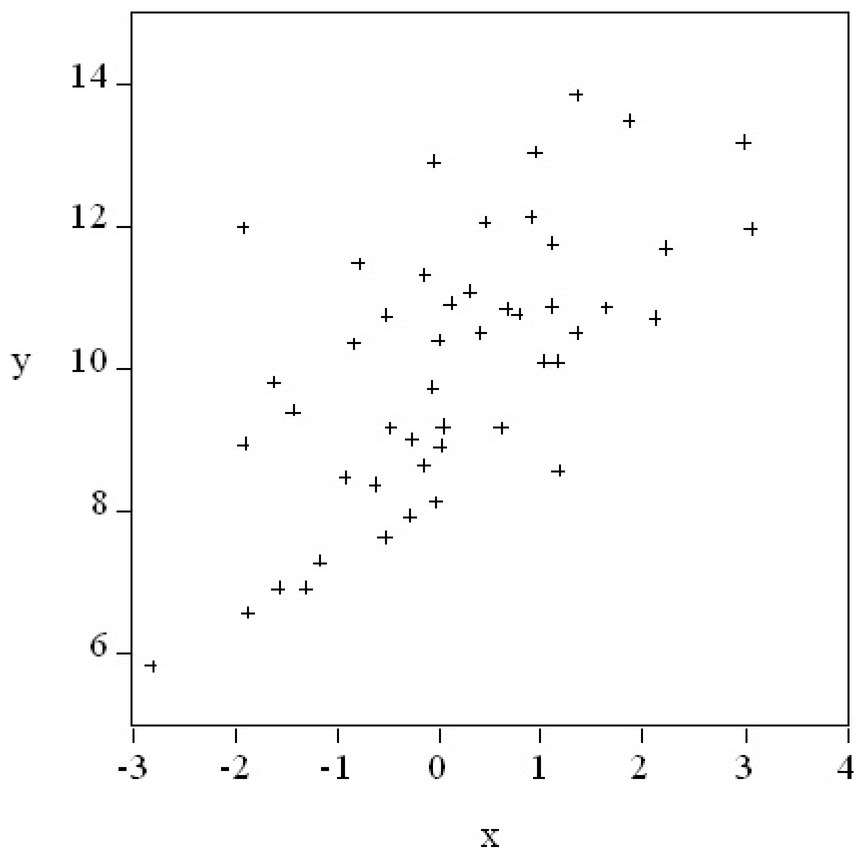
LS // Dependent Variable is Y

Sample: 1960 2007
Included observations: 48

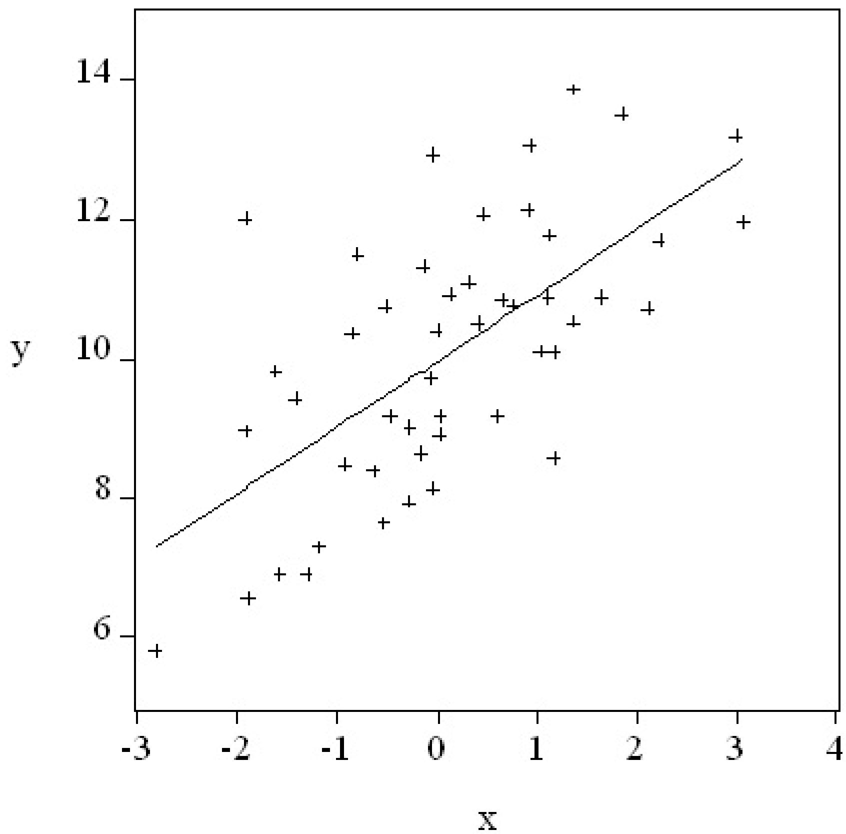
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	9.884732	0.190297	51.94359	0.0000
X	1.073140	0.150341	7.138031	0.0000
Z	-0.638011	0.172499	-3.698642	0.0006

R-squared	0.552928	Mean dependent var	10.08241
Adjusted R-squared	0.533059	S.D. dependent var	1.908842
S.E. of regression	1.304371	Akaike info criterion	3.429780
Sum squared resid	76.56223	Schwarz criterion	3.546730
Log likelihood	-79.31472	F-statistic	27.82752
Durbin-Watson stat	1.506278	Prob(F-statistic)	0.000000

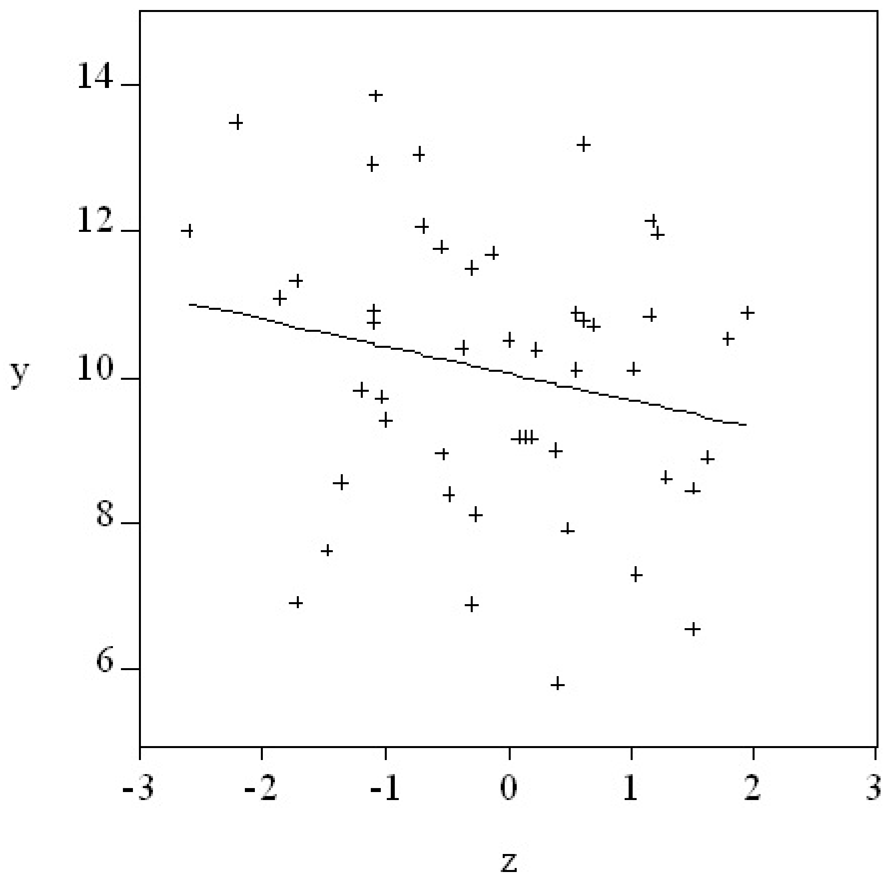
Scatterplot of y versus x



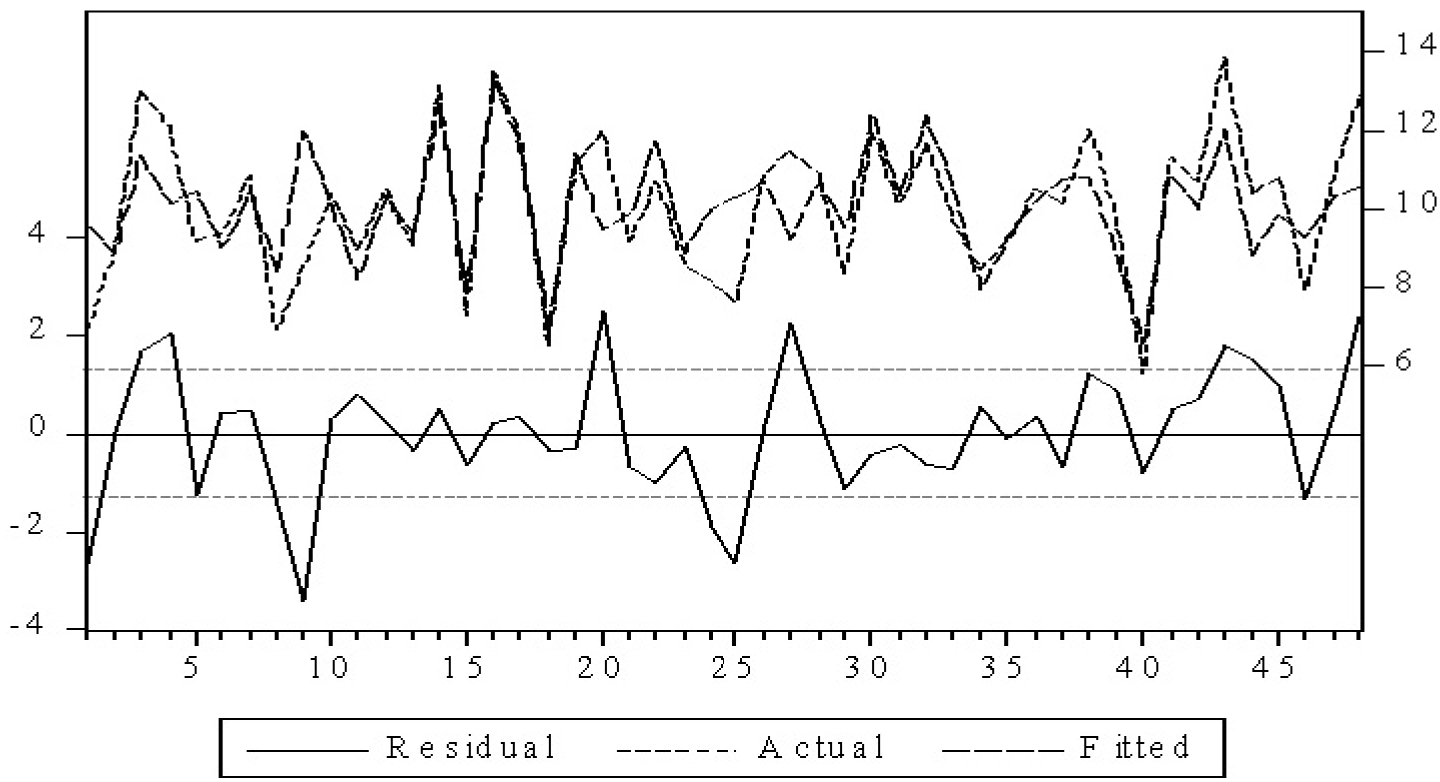
Scatterplot of y versus x
Regression Line Superimposed



Scatterplot of y versus z
Regression Line Superimposed



Residual Plot
Regression of y on x and z



Six Considerations Basic to Successful Forecasting

1. The Decision Environment and Loss Function

$$L(e) = e^2$$

$$L(e) = |e|$$

$$L(y,\hat{y}) = \begin{cases} 0, \text{ if } \operatorname{sign}(\Delta y) = \operatorname{sign}(\Delta \hat{y}) \\ 1, \text{ if } \operatorname{sign}(\Delta y) \neq \operatorname{sign}(\Delta \hat{y}) \end{cases}$$

2. The Forecast Object

Event outcome, event timing, time series.

3. The Forecast Statement

Point forecast, interval forecast, density forecast, probability forecast

4. The Forecast Horizon

h-step ahead forecast

h-step-ahead extrapolation forecast

5. The Information Set

$$\Omega_T^{univariate} = \{y_T, \ y_{T-1}, \ ..., \ y_1\}$$

$$\Omega_T^{multivariate} = \{y_T, \ x_T, \ y_{T-1}, \ x_{T-1}, \ ..., \ y_1, \ x_1\}$$

6. Methods and Complexity, the Parsimony Principle, and the Shrinkage Principle

Signal vs. noise

Smaller is often better

Even incorrect restrictions can help

Decision Making with Symmetric Loss

	Demand High	**Demand Low**
Build Inventory	0	$10,000
Reduce Inventory	$10,000	0

Decision Making with Asymmetric Loss

	Demand High	**Demand Low**
Build Inventory	0	$10,000
Reduce Inventory	$20,000	0

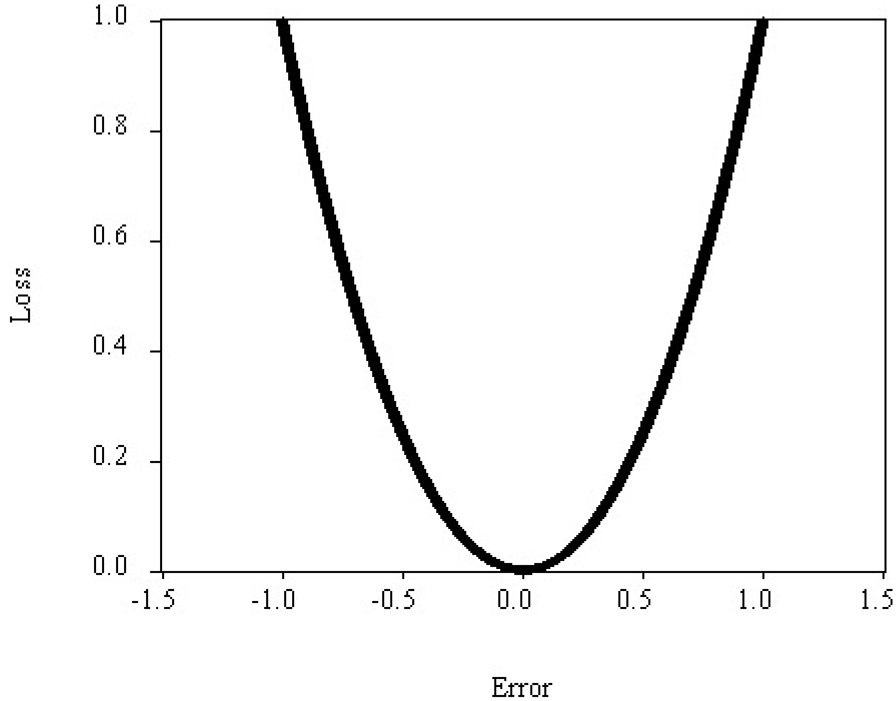
Forecasting with Symmetric Loss

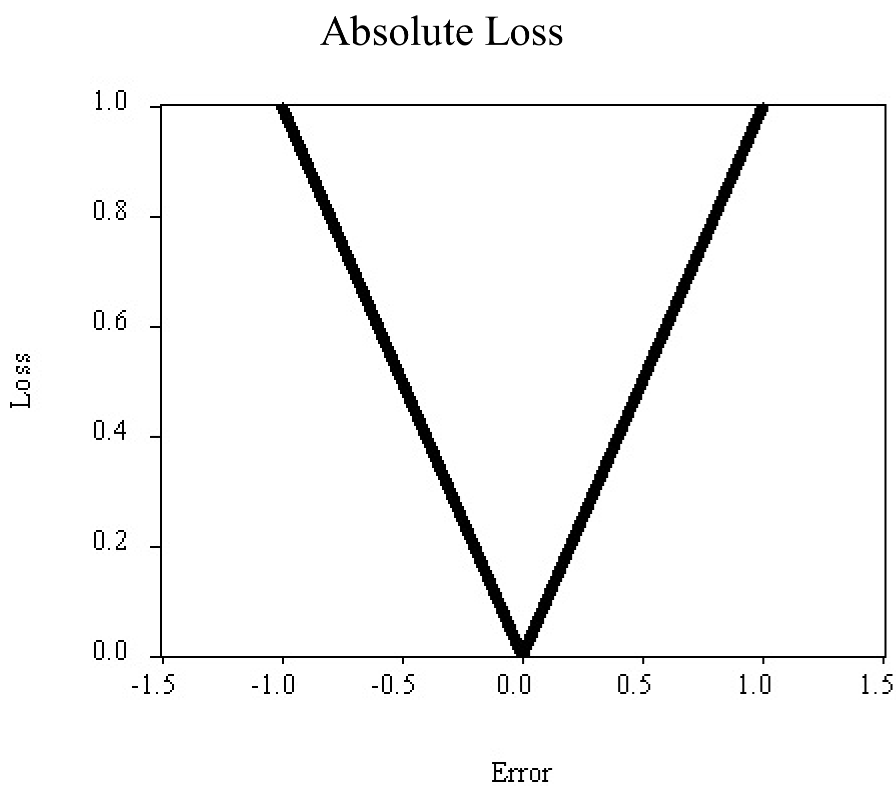
	High Actual Sales	Low Actual Sales
High Forecasted Sales	0	$10,000
Low Forecasted Sales	$10,000	0

Forecasting with Asymmetric Loss

	High Actual Sales	Low Actual Sales
High Forecasted Sales	0	$10,000
Low Forecasted Sales	$20,000	0

Quadratic Loss





Absolute Loss
1.0
0.8
0.6
0.4
0.2
0.0
Loss
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Error

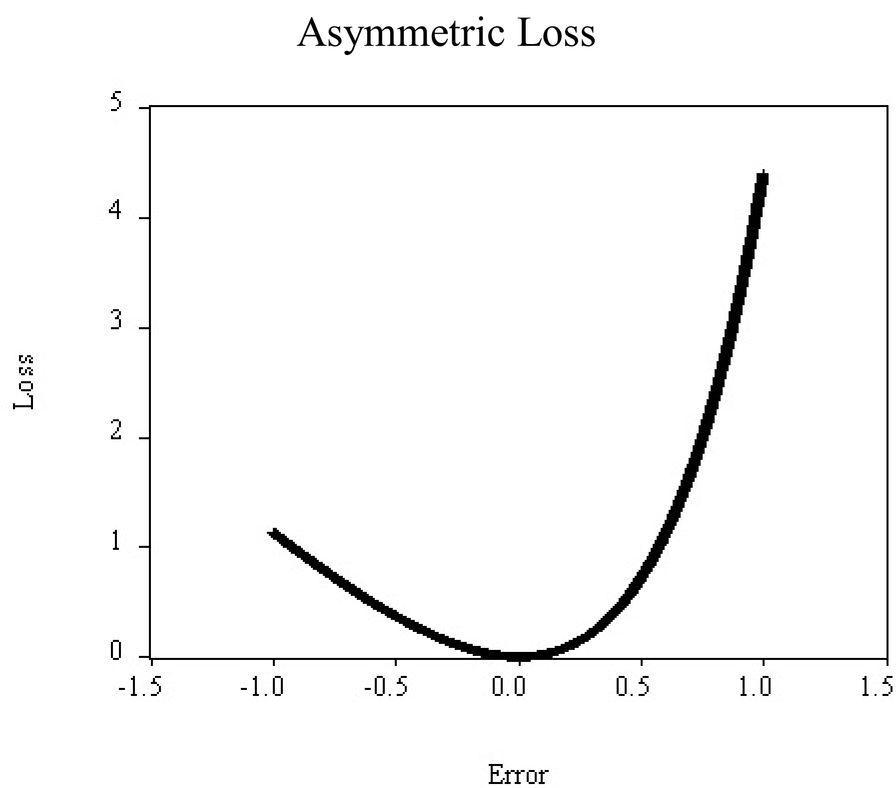

Asymmetric Loss
Loss
Error
0
1
2
3
4
5
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5

FIGURE 2.4 Web Page Growth: Point, Interval, and Density Forecasts

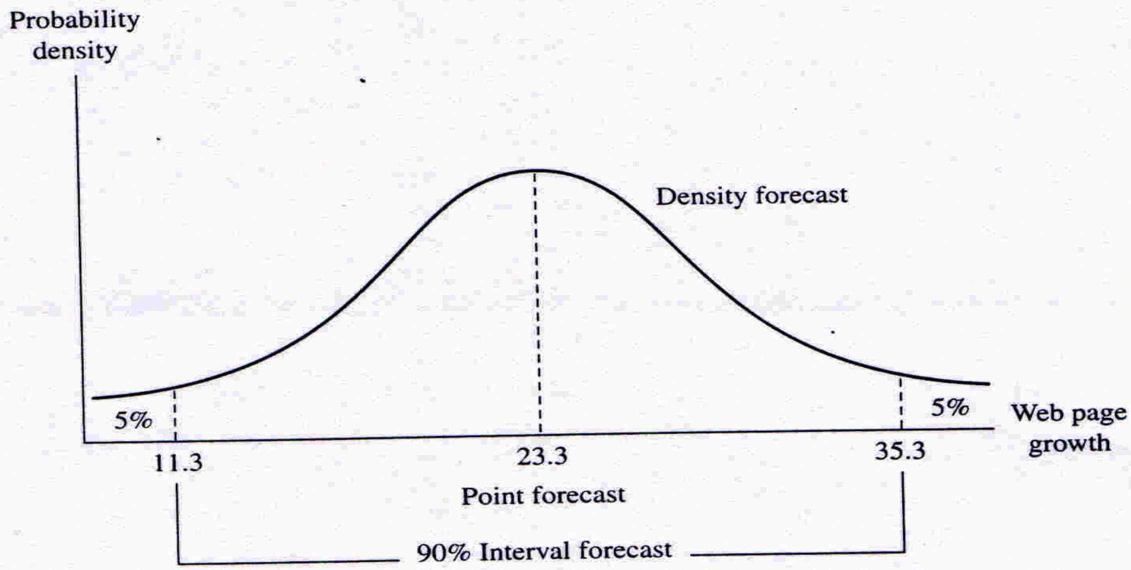


FIGURE 2.5 U.S. Real GDP Growth: Point, Interval, and Density Forecasts

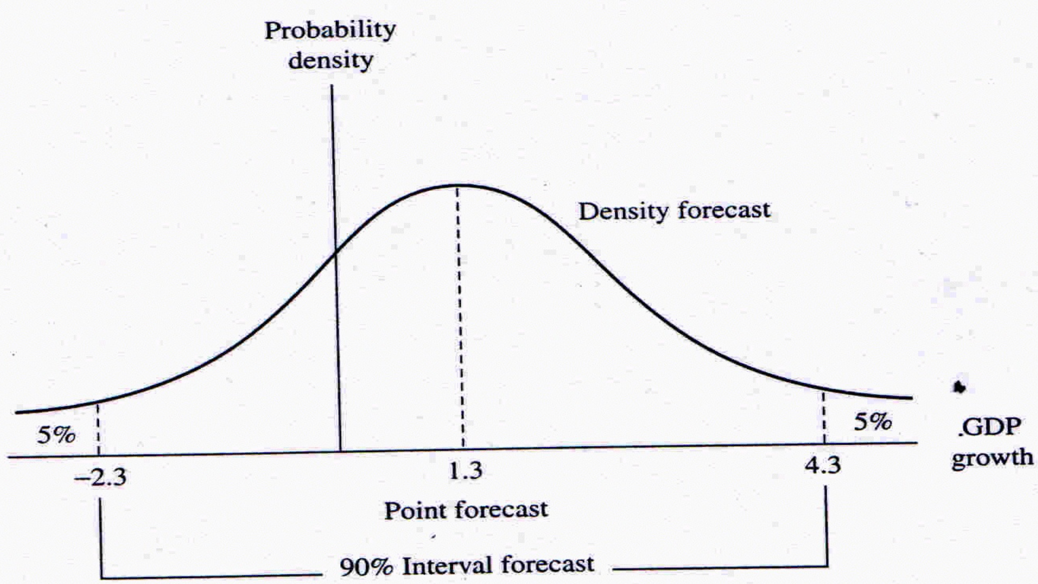


FIGURE 2.6 4-Step-Ahead Point Forecast

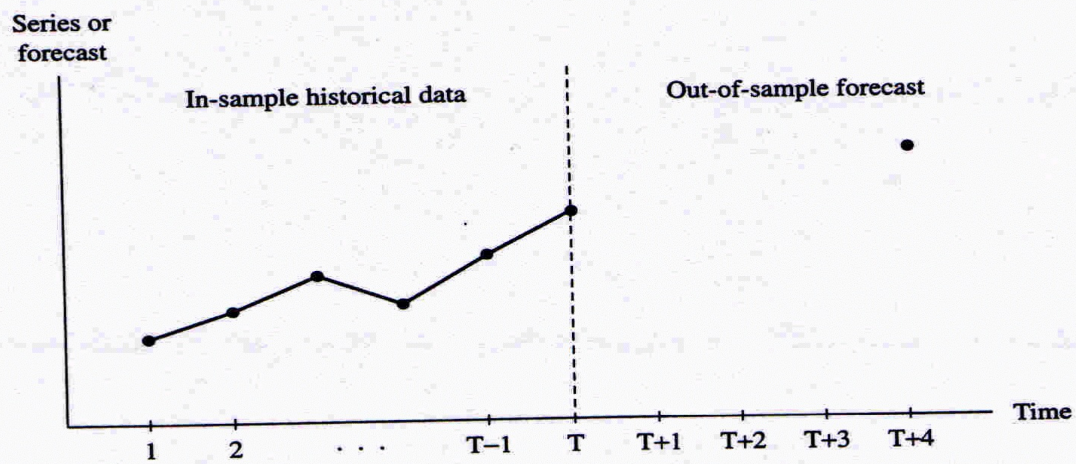
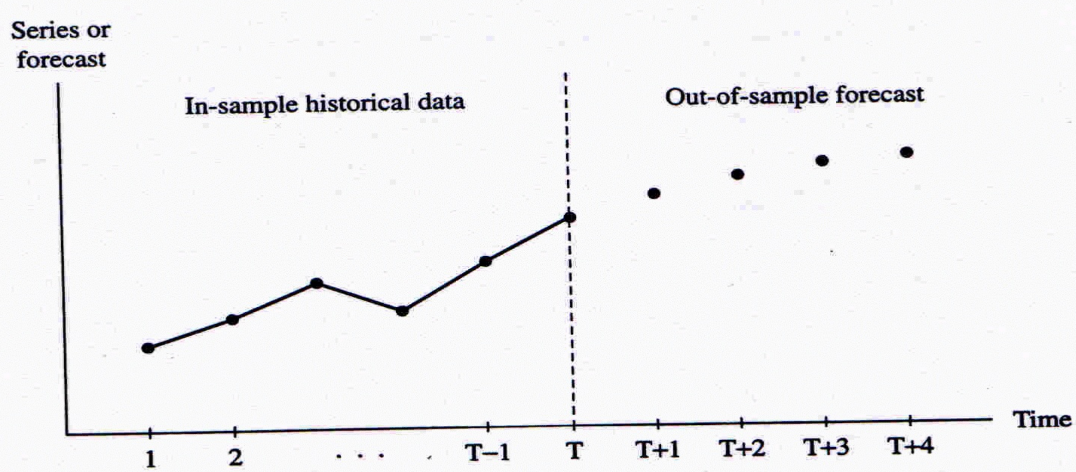


FIGURE 2.7 4-Step-Ahead Extrapolation Point Forecast



Statistical Graphics For Forecasting

1. Why Graphical Analysis is Important

Graphics helps us summarize and reveal patterns in data

Graphics helps us identify anomalies in data

Graphics facilitates and encourages comparison of different pieces of data

Graphics enables us to present a huge amount of data in a small space, and it enables us to make huge datasets coherent

2. Simple Graphical Techniques

Univariate, multivariate

Time series vs. distributional shape

Relational graphics

3. Elements of Graphical Style

Know your audience, and know your goals.

Show the data, and appeal to the viewer.

Revise and edit, again and again.

4. Application: Graphing Four Components of Real GNP

Anscombe's Quartet

(1)		(2)		(3)		(4)	
x1	y1	x2	y2	x3	y3	x4	y4
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

Anscombe's Quartet
Regressions of yi on xi, i = 1, ..., 4.

LS // Dependent Variable is Y1

Variable	Coefficient	Std. Error	T-Statistic
C	3.00	1.12	2.67
X1	0.50	0.12	4.24

R-squared 0.67 S.E. of regression 1.24

LS // Dependent Variable is Y2

Variable	Coefficient	Std. Error	T-Statistic
C	3.00	1.12	2.67
X2	0.50	0.12	4.24

R-squared 0.67 S.E. of regression 1.24

LS // Dependent Variable is Y3

Variable	Coefficient	Std. Error	T-Statistic
C	3.00	1.12	2.67
X3	0.50	0.12	4.24

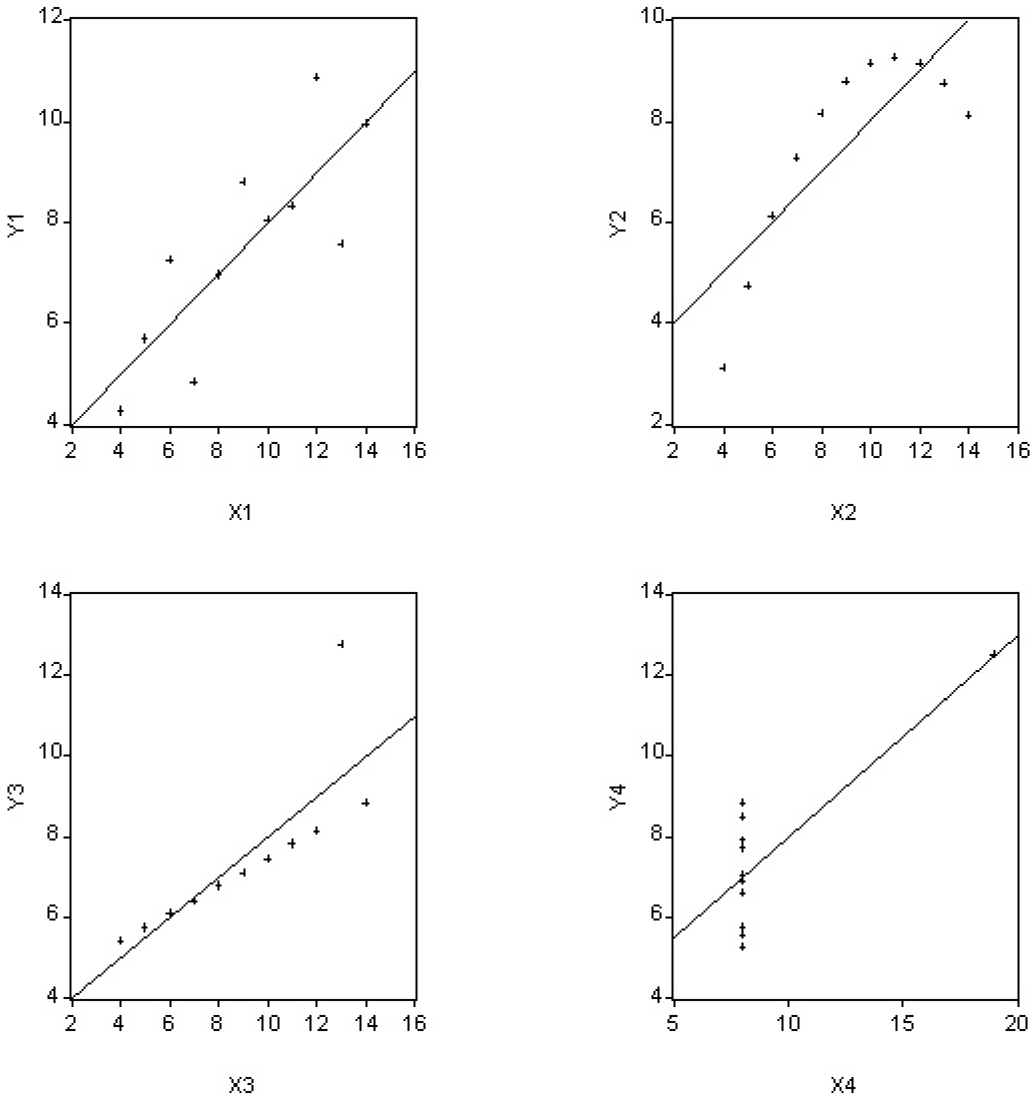
R-squared 0.67 S.E. of regression 1.24

LS // Dependent Variable is Y4

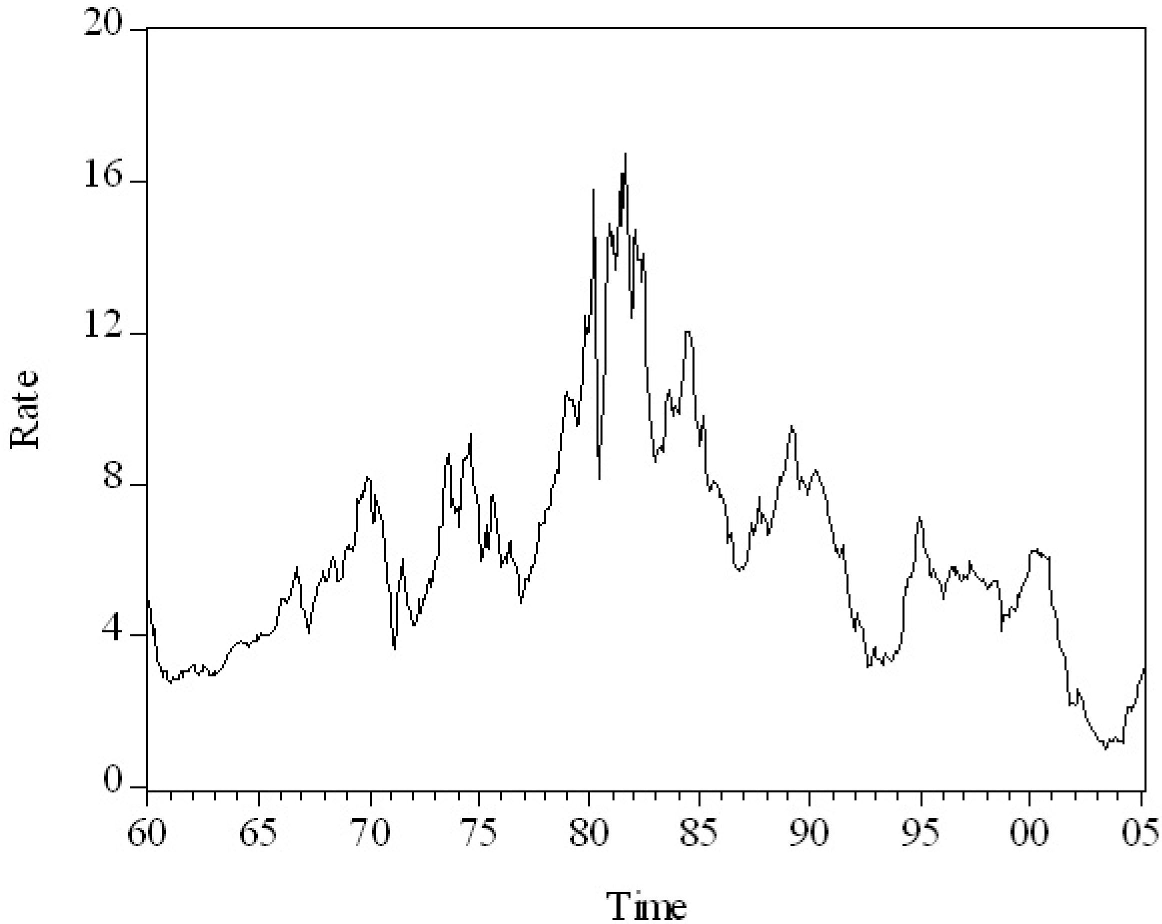
Variable	Coefficient	Std. Error	T-Statistic
C	3.00	1.12	2.67
X4	0.50	0.12	4.24

R-squared 0.67 S.E. of regression 1.24

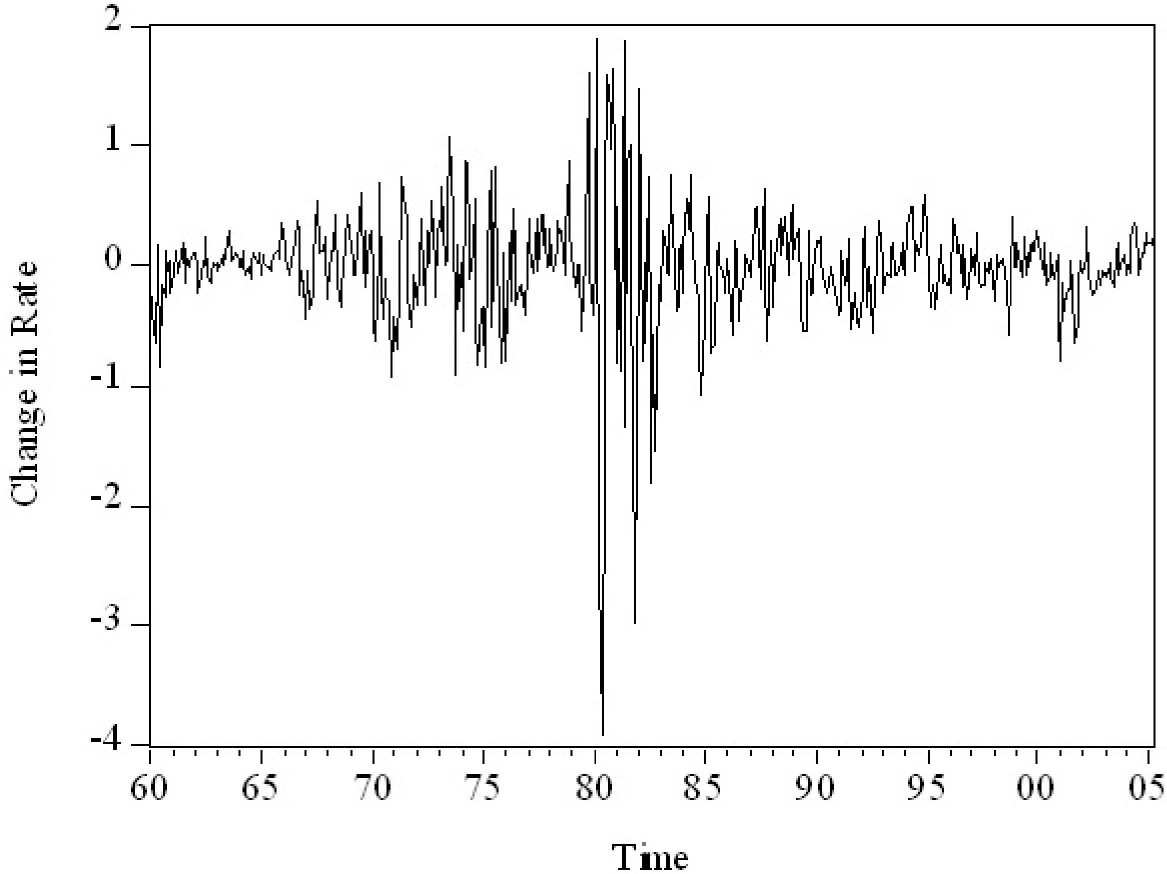
Anscombe's Quartet
Bivariate Scatterplots



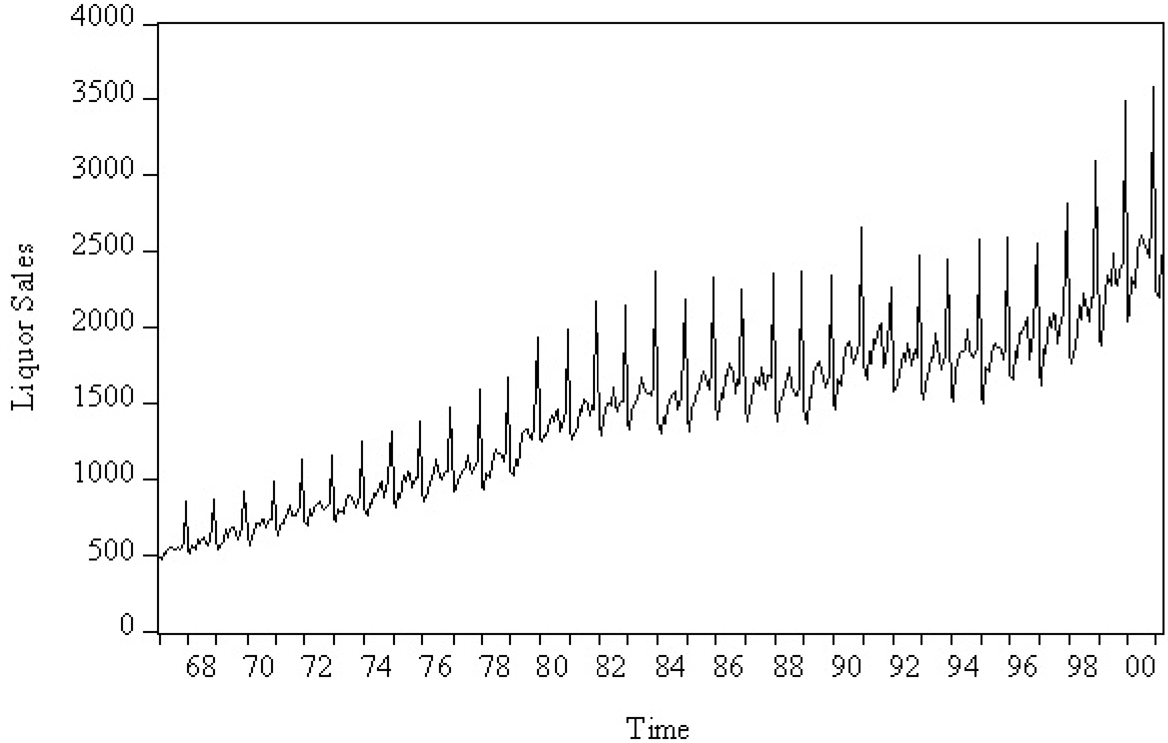
1-Year Treasury Bond Rate



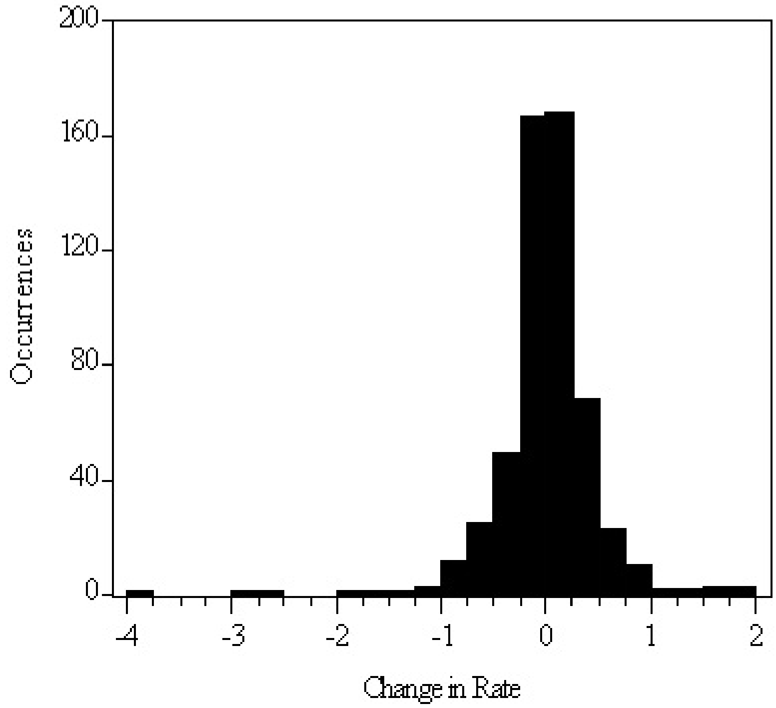
Change in 1-Year Treasury Bond Rate



Liquor Sales



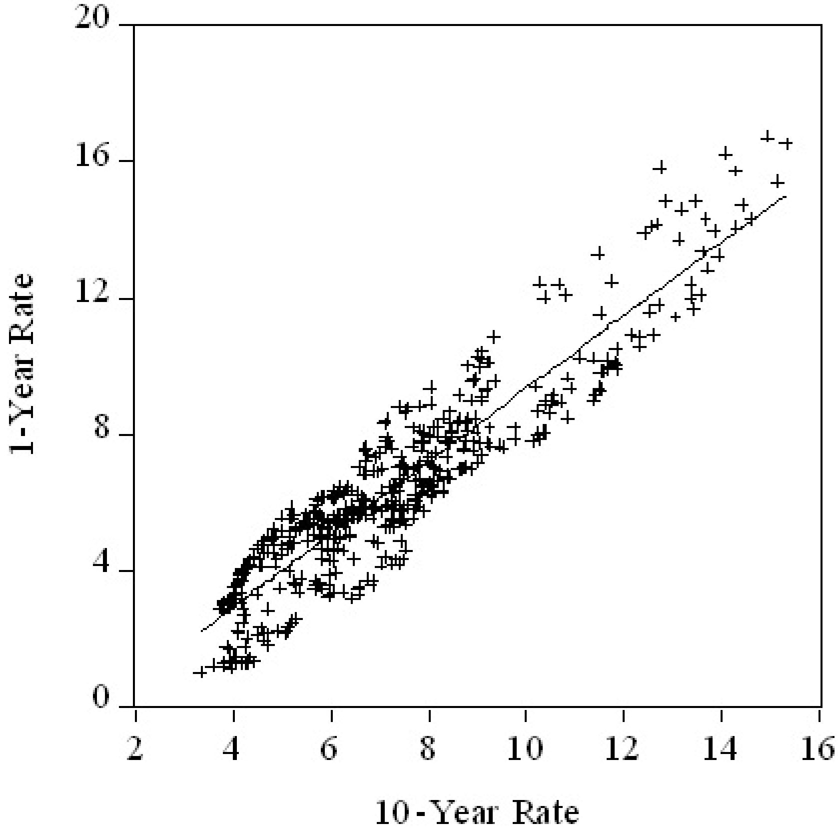
Histogram and Descriptive Statistics
Change in 1-Year Treasury Bond Rate



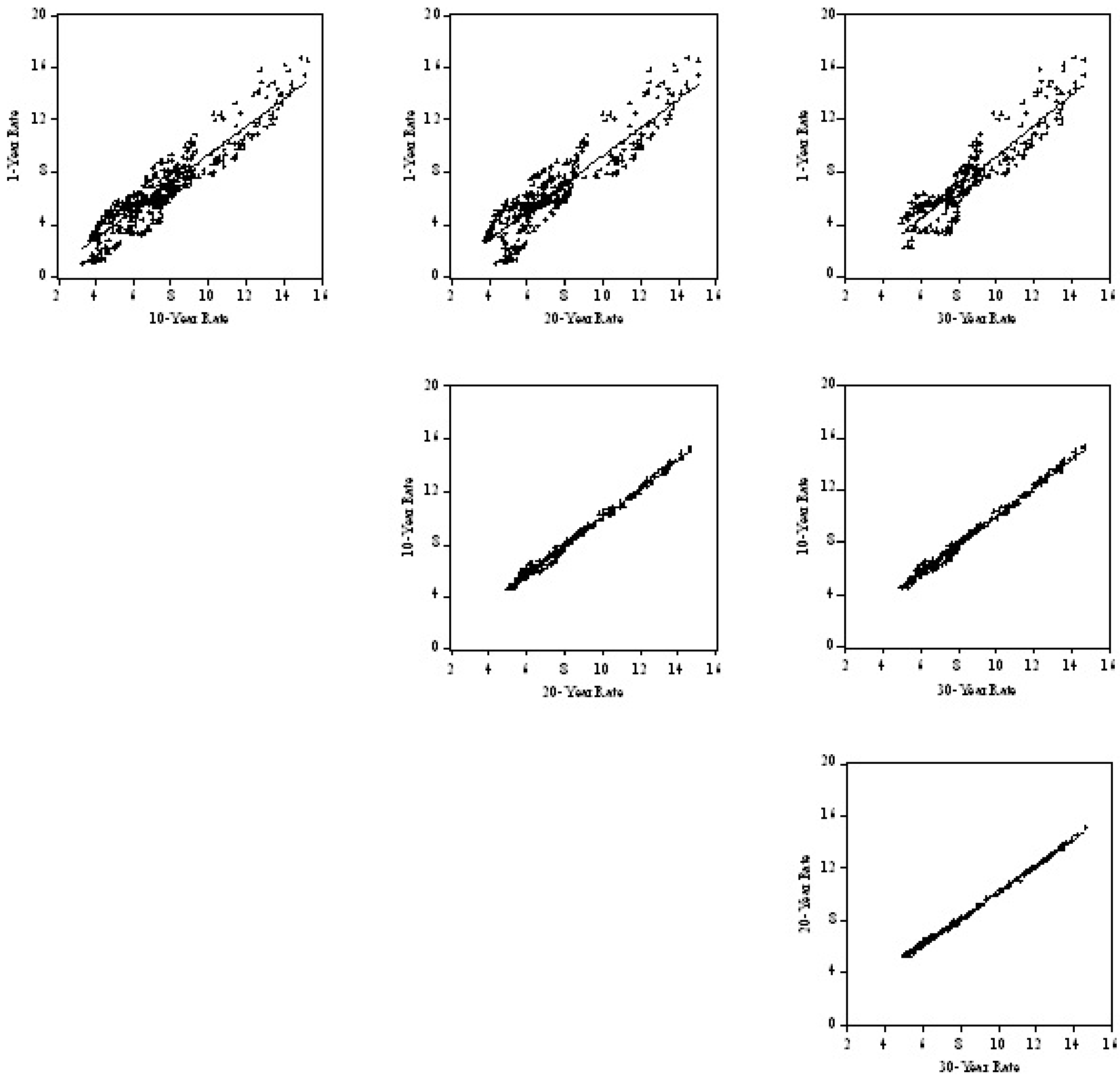
Sample 1960M01 2005M03
Observations 543

Mean	-0.003389
Median	0.000000
Maximum	1.900000
Minimum	-3.910000
Std. Dev.	0.470731
Skewness	-1.380580
Kurtosis	17.44767
Jarque-Bera	4895.124
Probability	0.000000

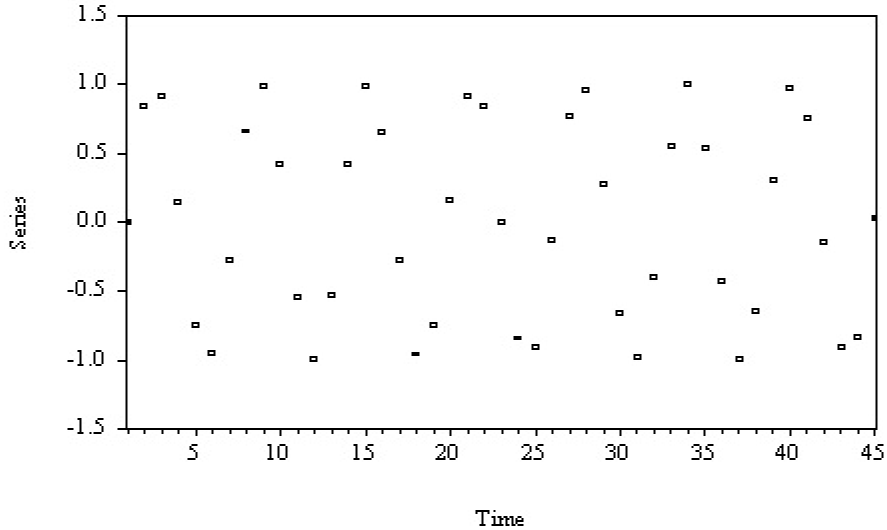
Scatterplot
1-Year versus 10-Year Treasury Bond Rate



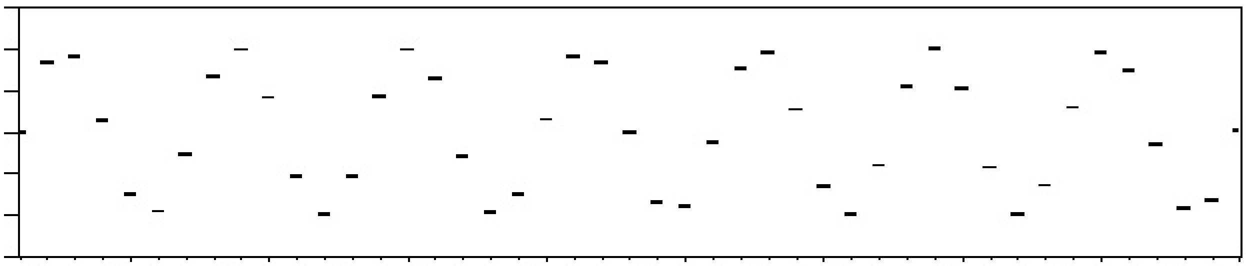
Scatterplot Matrix
1-, 10-, 20-, and 30-Year Treasury Bond Rates



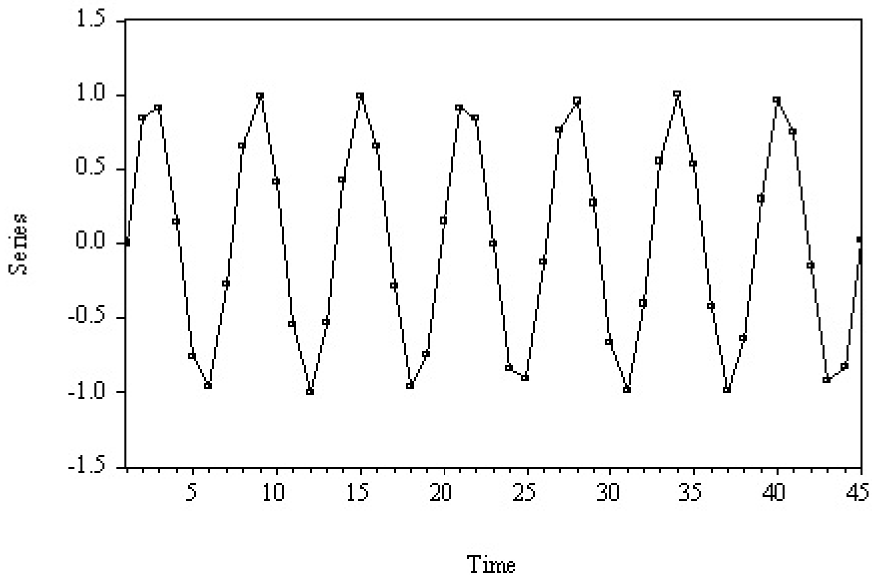
Time Series Plot
Aspect Ratio 1:1.6

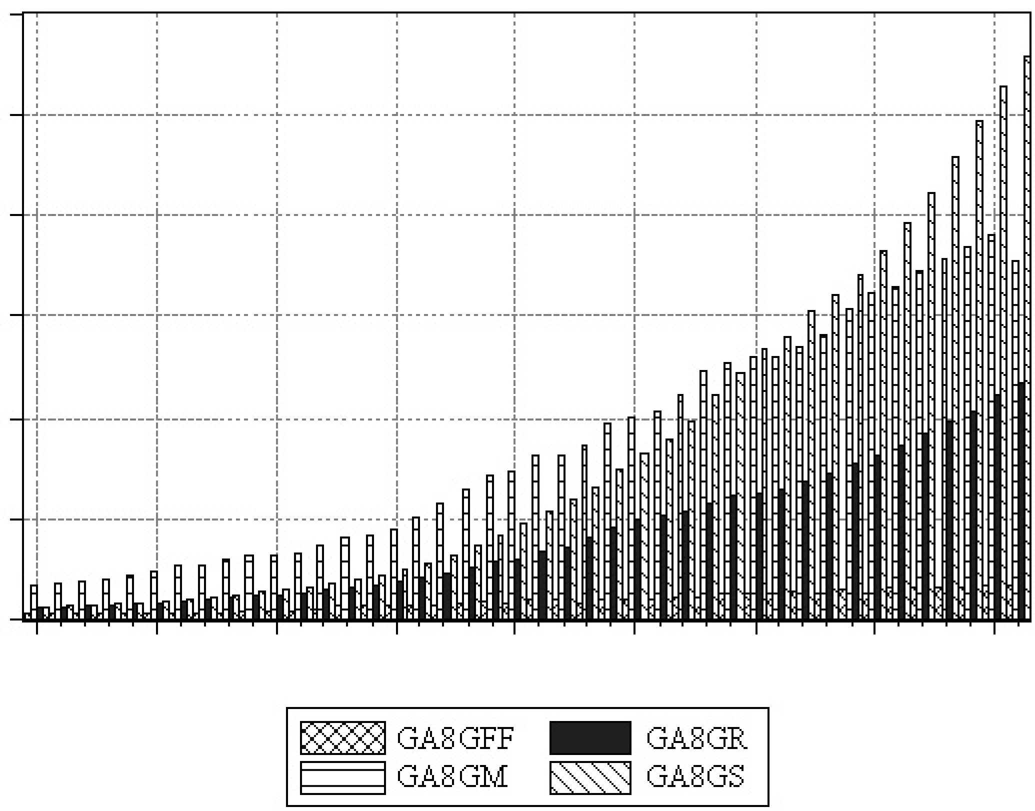


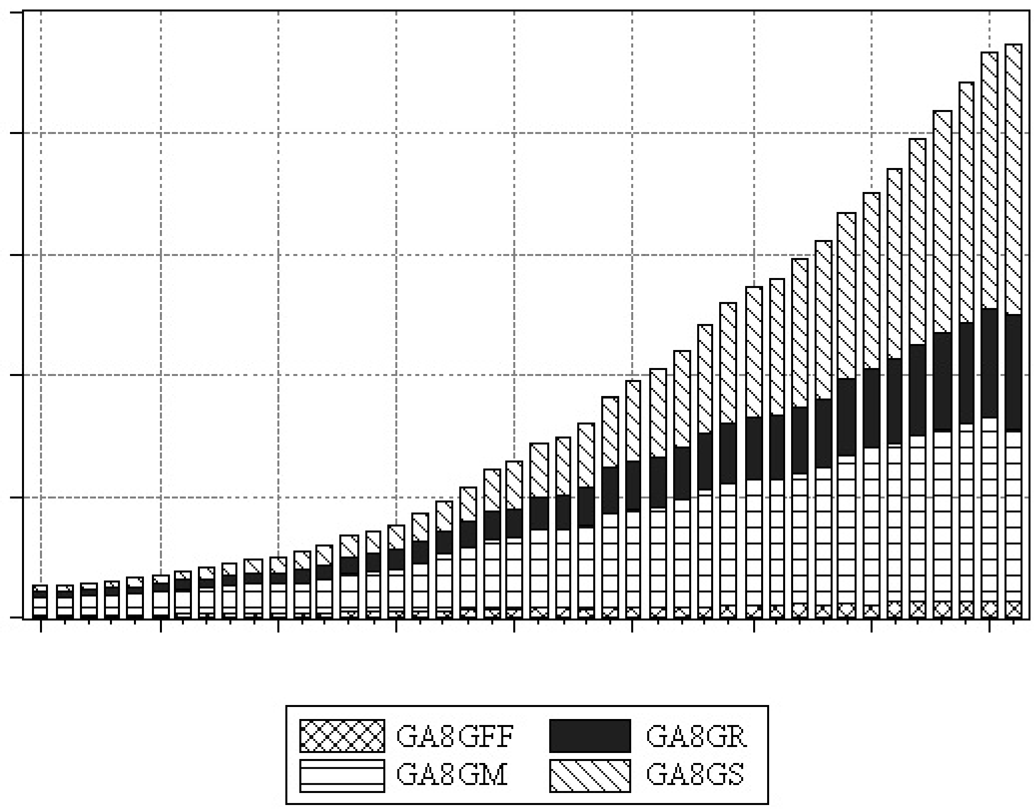
Time Series Plot
Banked to 45 Degrees



Time Series Plot
Aspect Ratio 1:1.6




GA8GFF
GA8GR
GA8GM
GA8GS


GA8GFF
GA8GR
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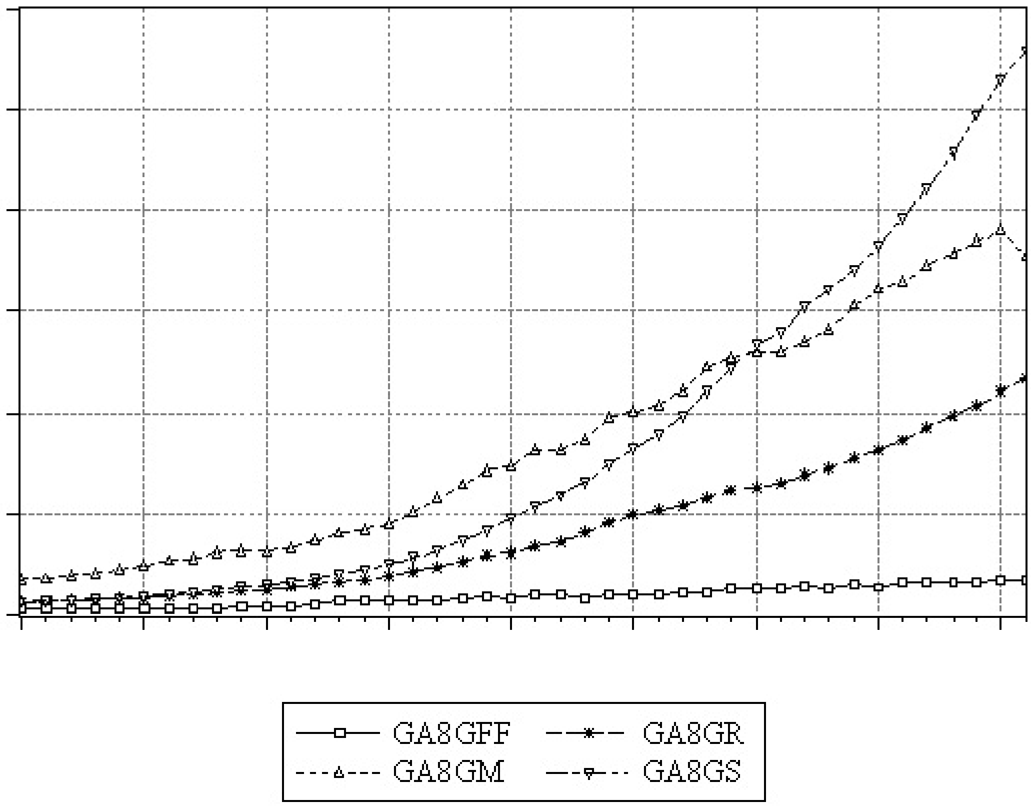
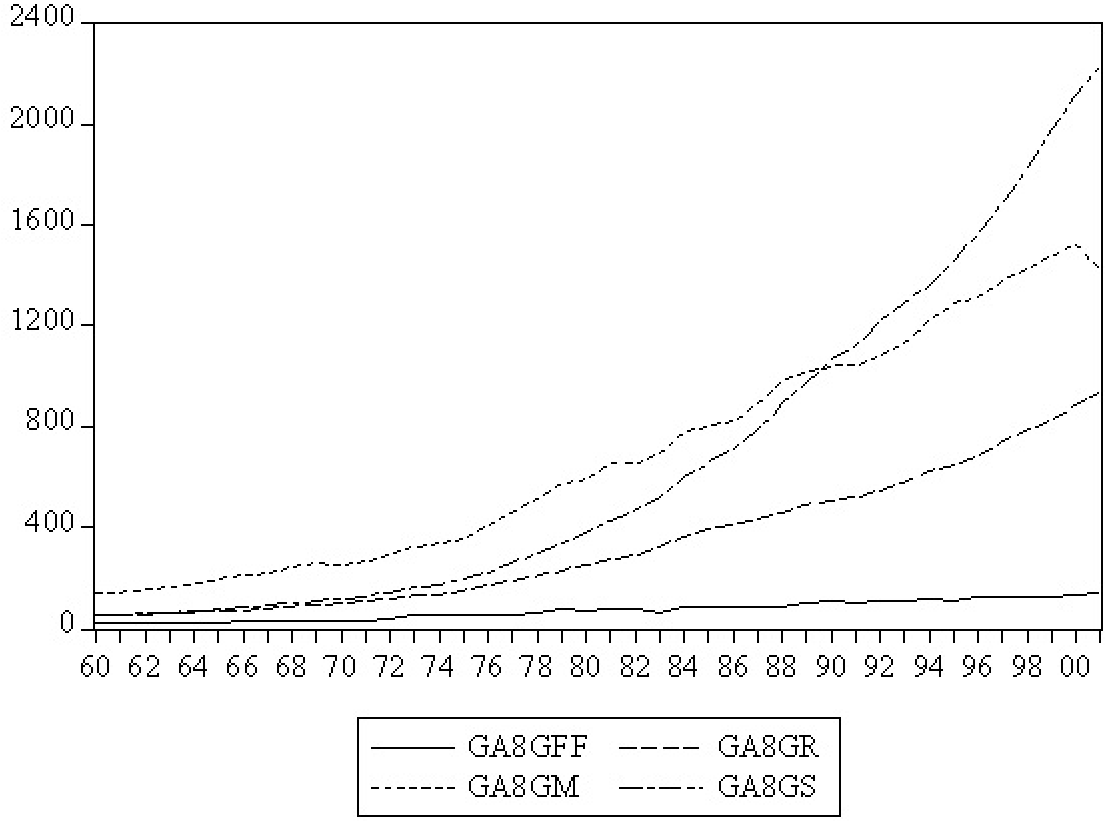
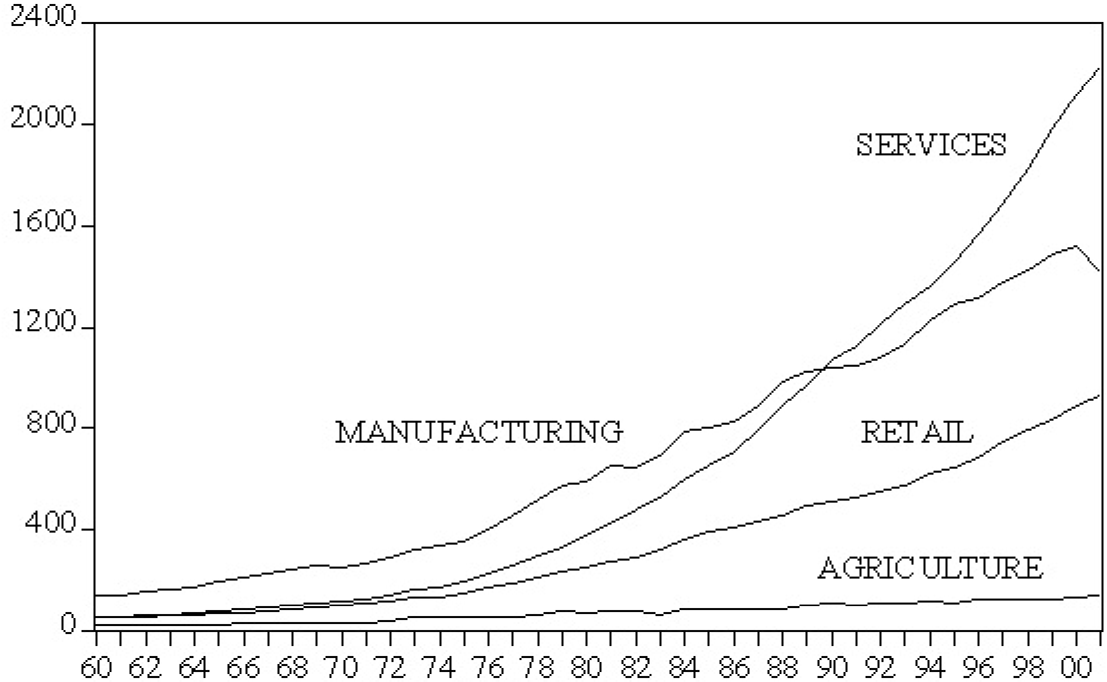
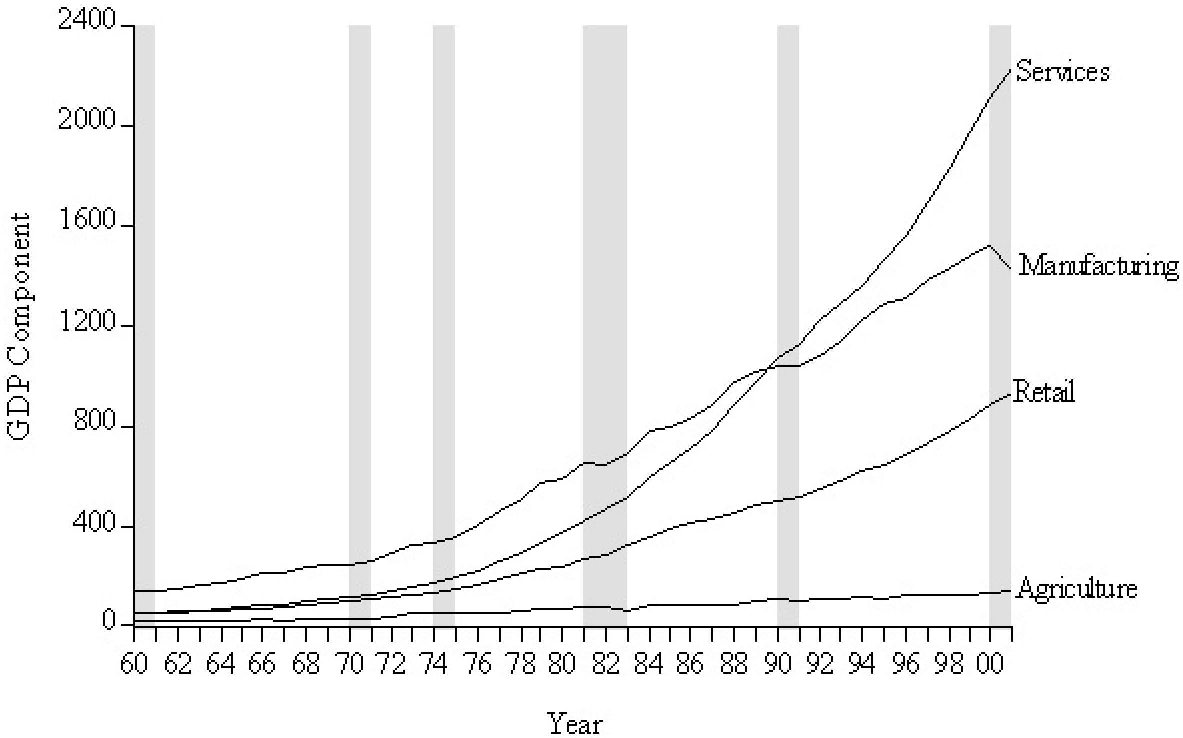
GA8GS


GA8GFF
GA8GR
GA8GM
GA8GS


2400
2000
1600
1200
800
400
0
60 62 64 66 68 70 72 74 76 78 80 82 84 86 88 90 92 94 96 98 00
GA8GFF
GA8GR
GA8GM
GA8GS


2400
2000
1600
1200
800
400
0
SERVICES
MANUFACTURING
RETAIL
AGRICULTURE
60 62 64 66 68 70 72 74 76 78 80 82 84 86 88 90 92 94 96 98 00

Components of Real GDP (Millions of Current Dollars, Annual)



Modeling and Forecasting Trend

1. Modeling Trend

$$T_t = \beta_0 + \beta_1 TIME_t$$

$$T_t = \beta_0 + \beta_1 TIME_t + \beta_2 TIME_t^2$$

$$T_t = \beta_0 e^{\beta_1 TIME_t}$$

$$\ln(T_t) = \ln(\beta_0) + \beta_1 TIME_t$$

2. Estimating Trend Models

$$(\hat{\beta}_0,\ \hat{\beta}_1) = \underset{\beta_0,\beta_1}{\text{argmin}} \sum_{t=1}^{T} \left[y_t - \beta_0 - \beta_1 TIME_t\right]^2$$

$$(\hat{\beta}_0,\ \hat{\beta}_1,\ \hat{\beta}_2) = \underset{\beta_0,\beta_1,\beta_2}{\text{argmin}} \sum_{t=1}^{T} \left[y_t - \beta_0 - \beta_1 TIME_t - \beta_2 TIME2_t\right]^2$$

$$(\hat{\beta}_0,\ \hat{\beta}_1) = \underset{\beta_0,\beta_1}{\text{argmin}} \sum_{t=1}^{T} \left[y_t - \beta_0 e^{\beta_1 TIME_t}\right]^2$$

$$(\hat{\beta}_0,\ \hat{\beta}_1) = \underset{\beta_0,\beta_1}{\text{argmin}} \sum_{t=1}^{T} \left[\ln y_t - \ln\beta_0 - \beta_1 TIME_t\right]^2$$

3. Forecasting Trend

$$y_t = \beta_0 + \beta_1 TIME_t + \varepsilon_t$$

$$y_{T+h} = \beta_0 + \beta_1 TIME_{T+h} + \varepsilon_{T+h}$$

$$y_{T+h,T} = \beta_0 + \beta_1 TIME_{T+h}$$

$$\hat{y}_{T+h,T} = \hat{\beta}_0 + \hat{\beta}_1 TIME_{T+h}$$

$$y_{T+h,T} \pm 1.96\sigma$$

$$\hat{y}_{T+h,T} \pm 1.96\hat{\sigma}$$

$$N(y_{T+h,T}, \sigma^2)$$

$$N(\hat{y}_{T+h,T}, \hat{\sigma}^2)$$

4. Selecting Forecasting Models

$$\text{MSE} = \frac{\sum_{t=1}^{T} e_t^2}{T}$$

$$R^2 = 1 - \frac{\sum_{t=1}^{T} e_t^2}{\sum_{t=1}^{T} (y_t - \bar{y})^2}$$

$$s^2 = \frac{\sum_{t=1}^{T} e_t^2}{T-k}$$

$$s^2 = \left(\frac{T}{T-k}\right) \frac{\sum_{t=1}^{T} e_t^2}{T}$$

$$\bar{R}^2 = 1 - \frac{\sum_{t=1}^{T} e_t^2 \;/\; T-k}{\sum_{t=1}^{T} (y_t - \bar{y})^2 \;/\; T-1} = 1 - \frac{s^2}{\sum_{t=1}^{T} (y_t - \bar{y})^2 \;/\; T-1}$$

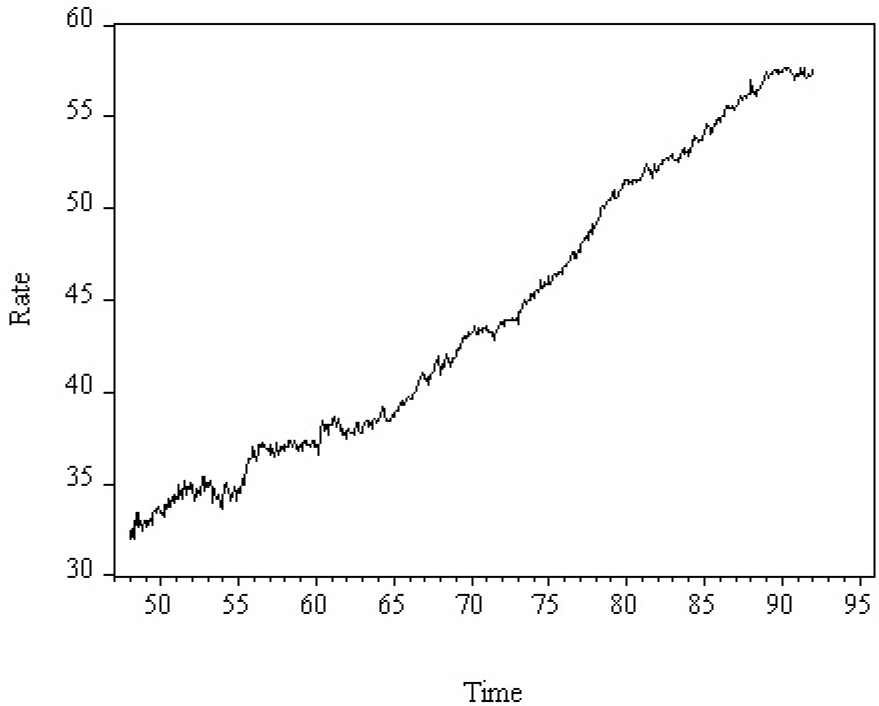
$$AIC = e^{\left(\frac{2k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$$

$$SIC = T^{\left(\frac{k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$$

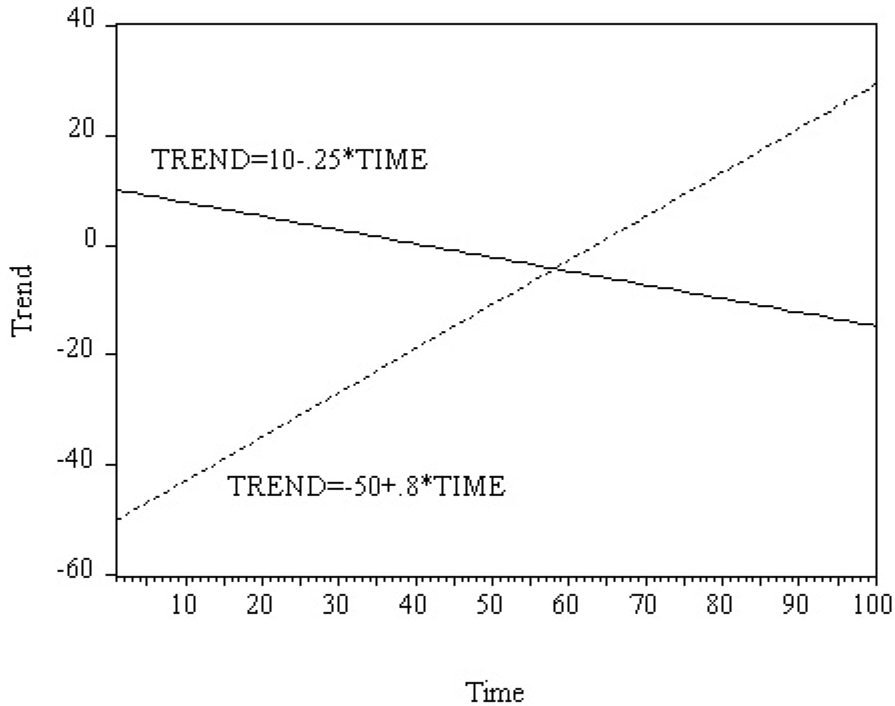
Consistency

Efficiency

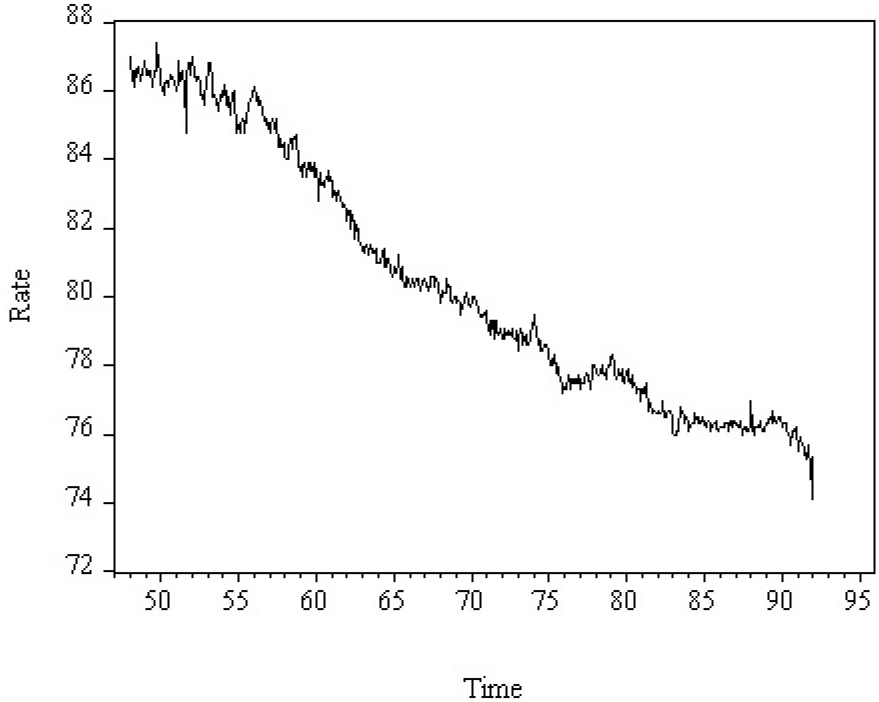
Labor Force Participation Rate
Females



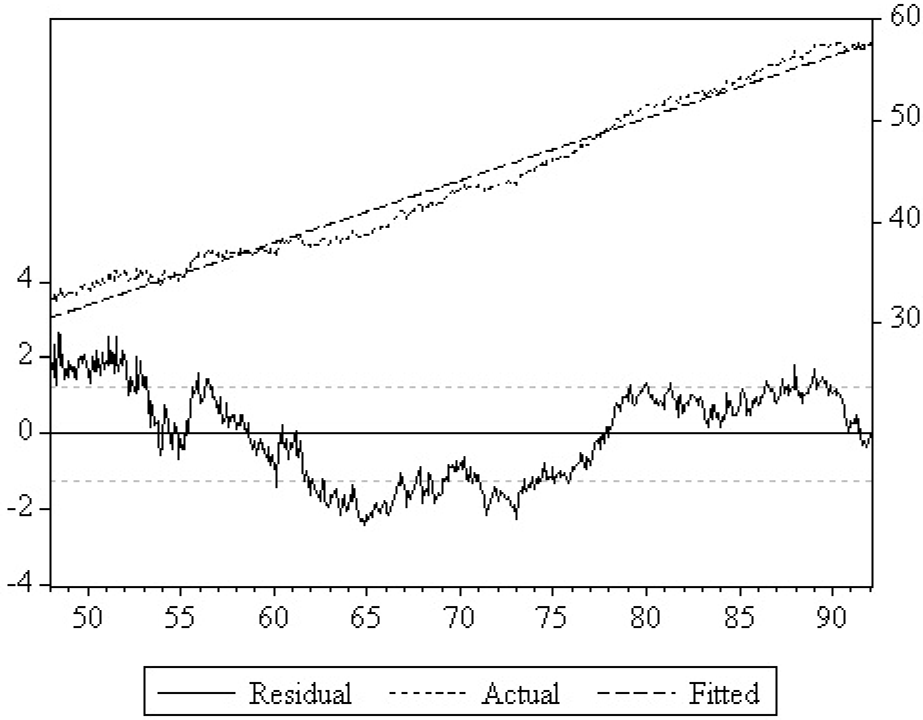
Increasing and Decreasing Linear Trends



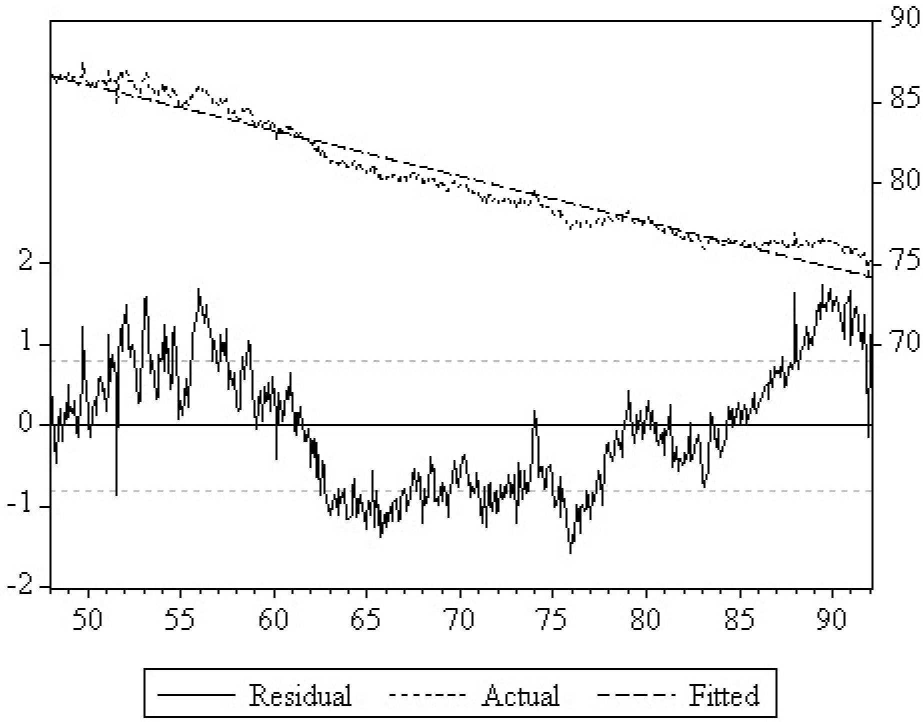
Labor Force Participation Rate
Males



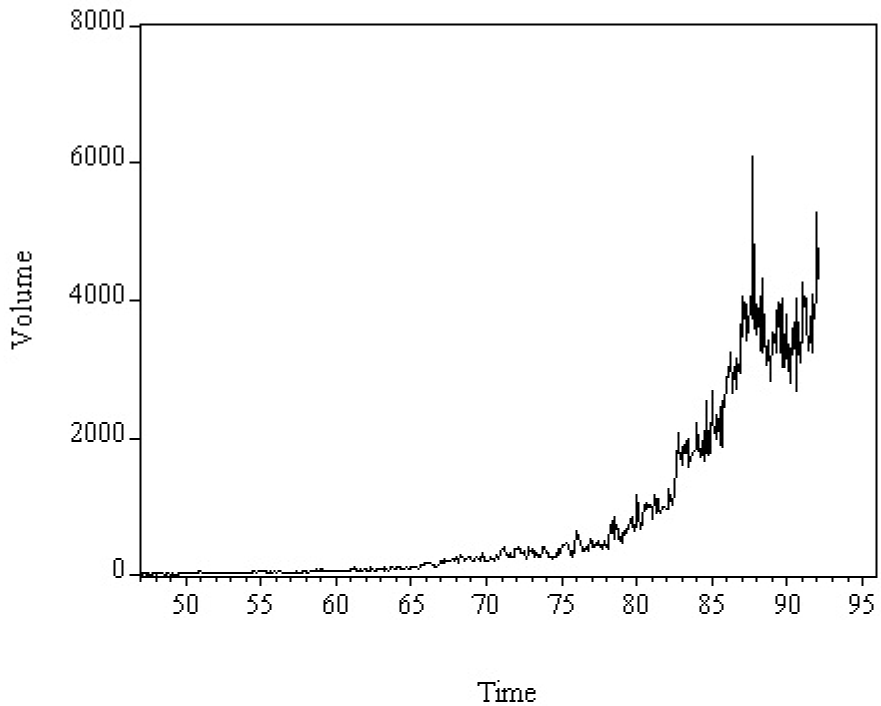
Linear Trend
Female Labor Force Participation Rate



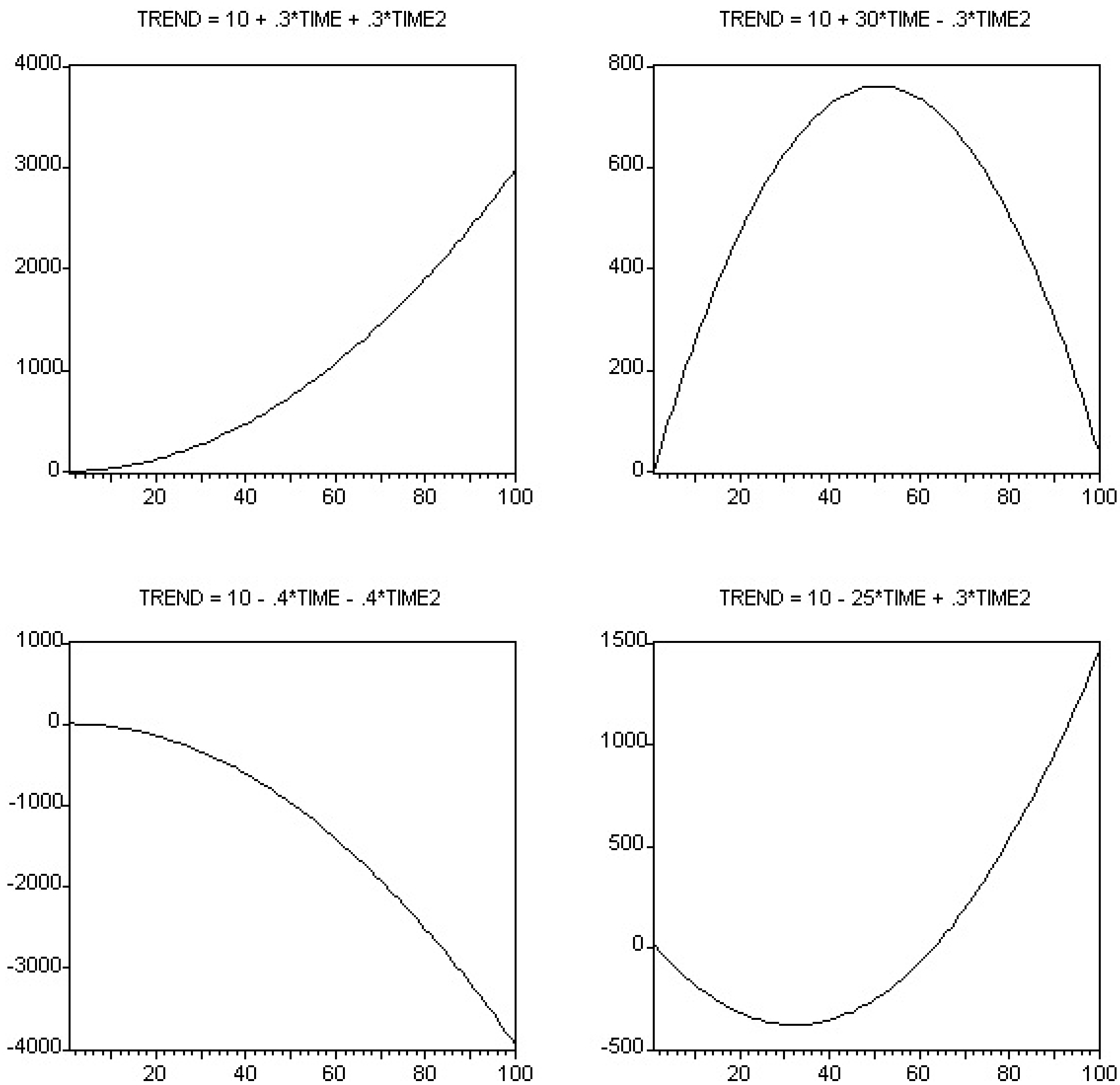
Linear Trend
Male Labor Force Participation Rate



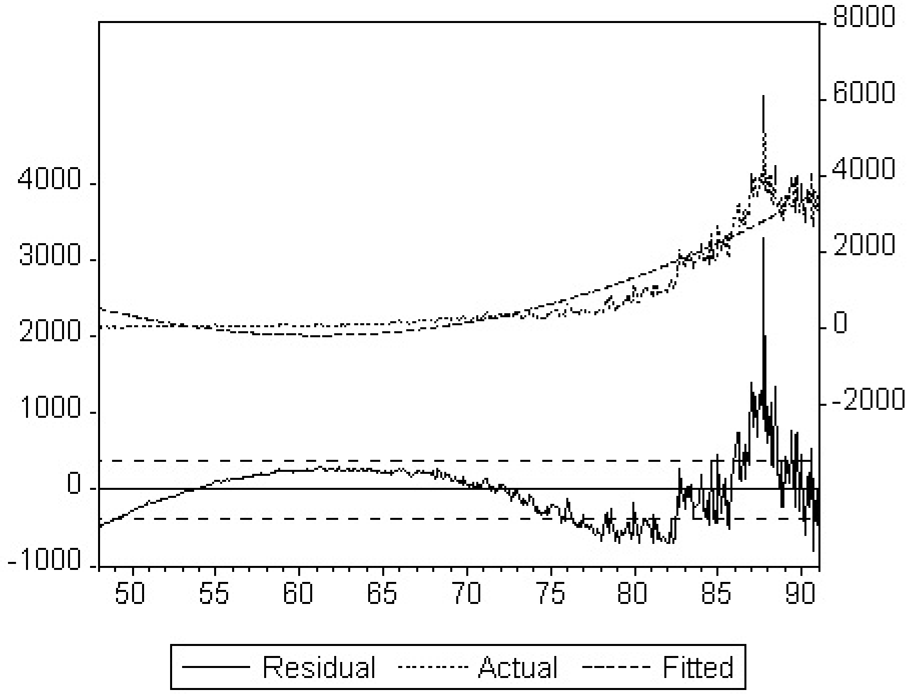
Volume on the New York Stock Exchange



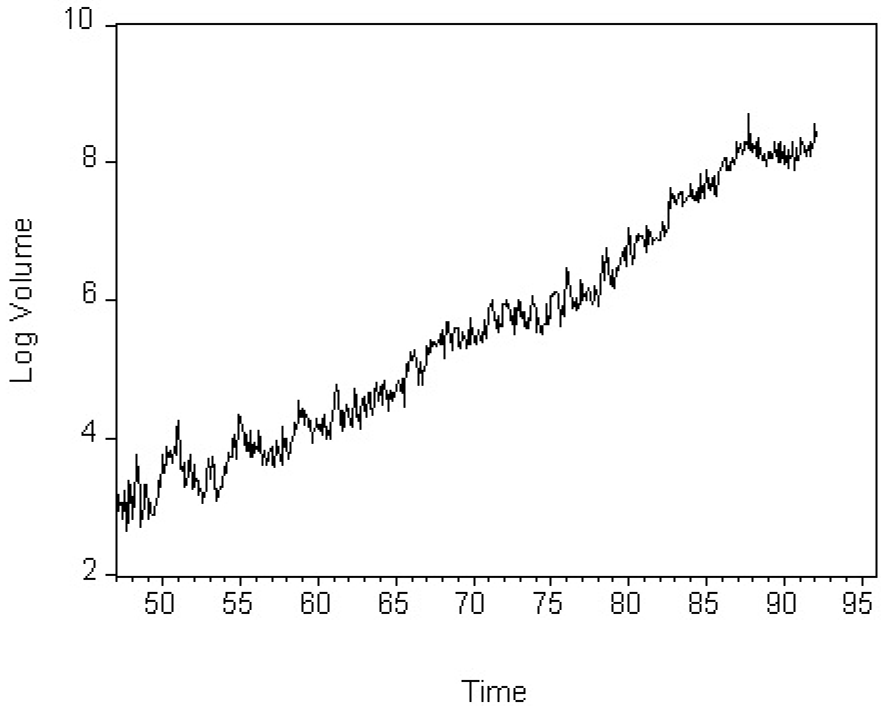
Various Shapes of Quadratic Trends



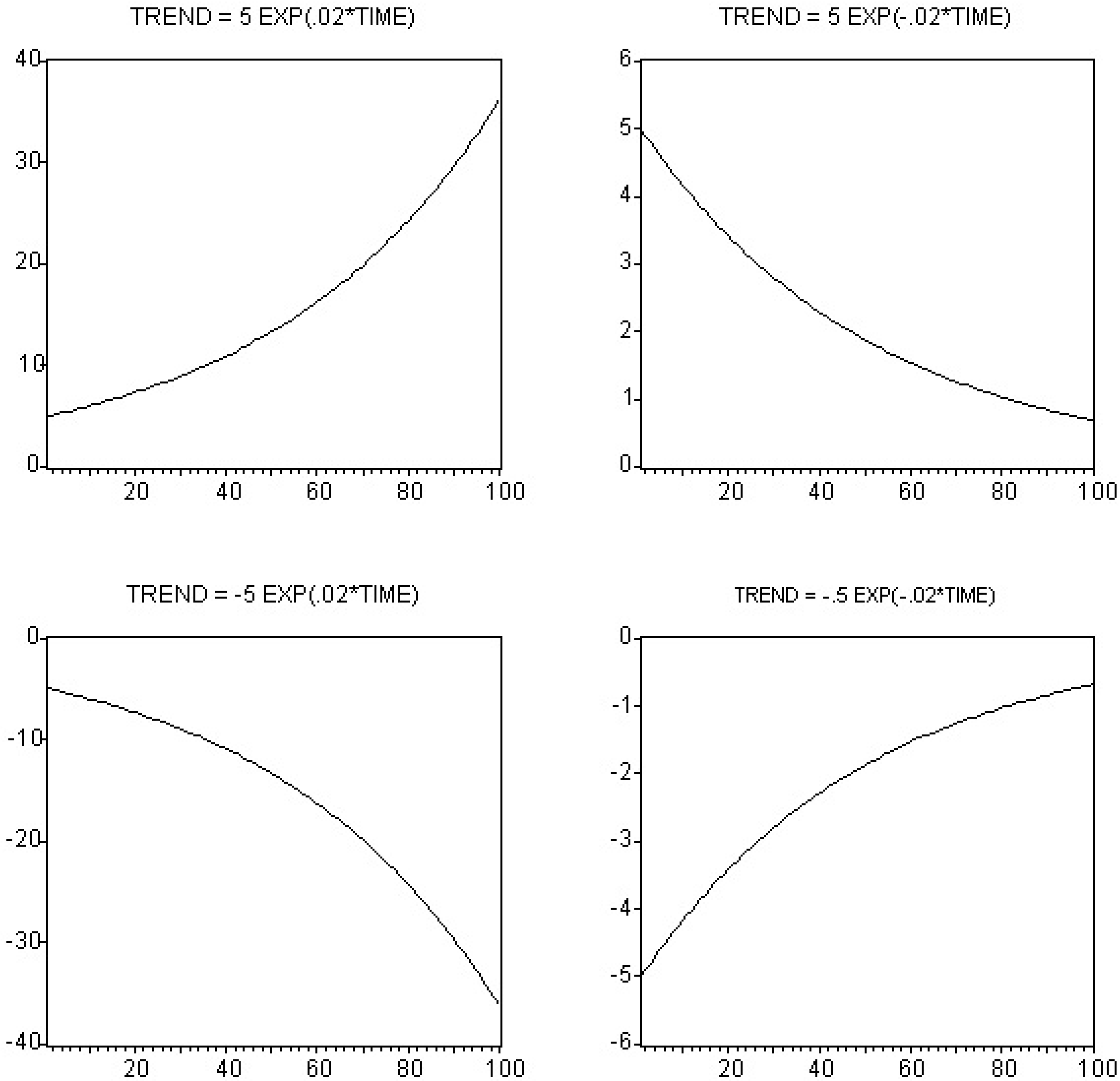
Quadratic Trend
Volume on the New York Stock Exchange



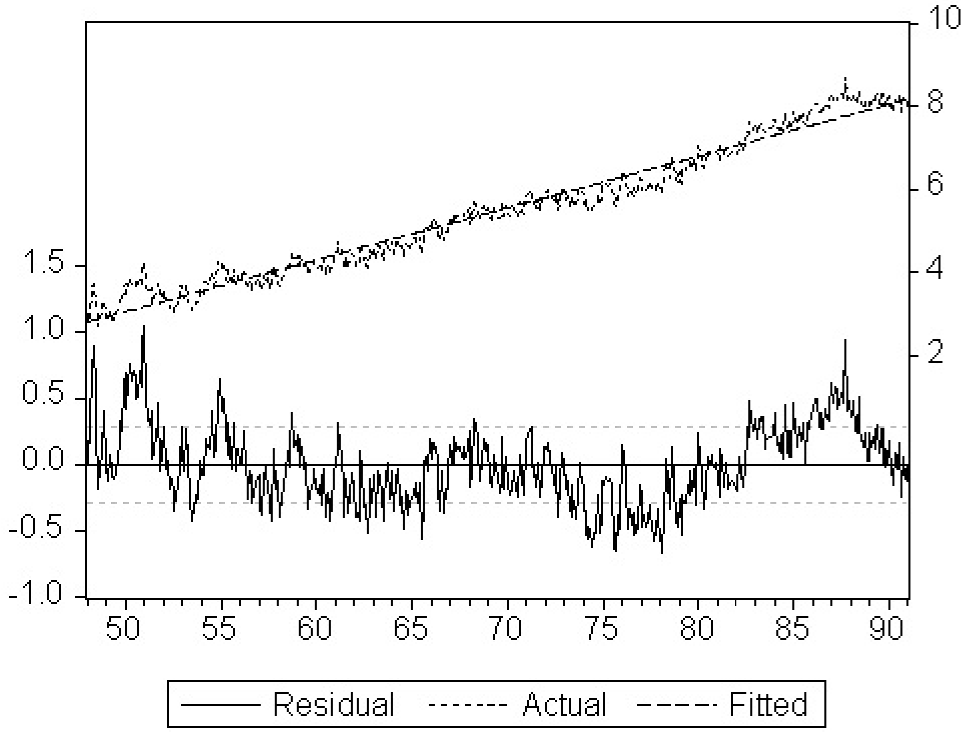
Log Volume on the New York Stock Exchange



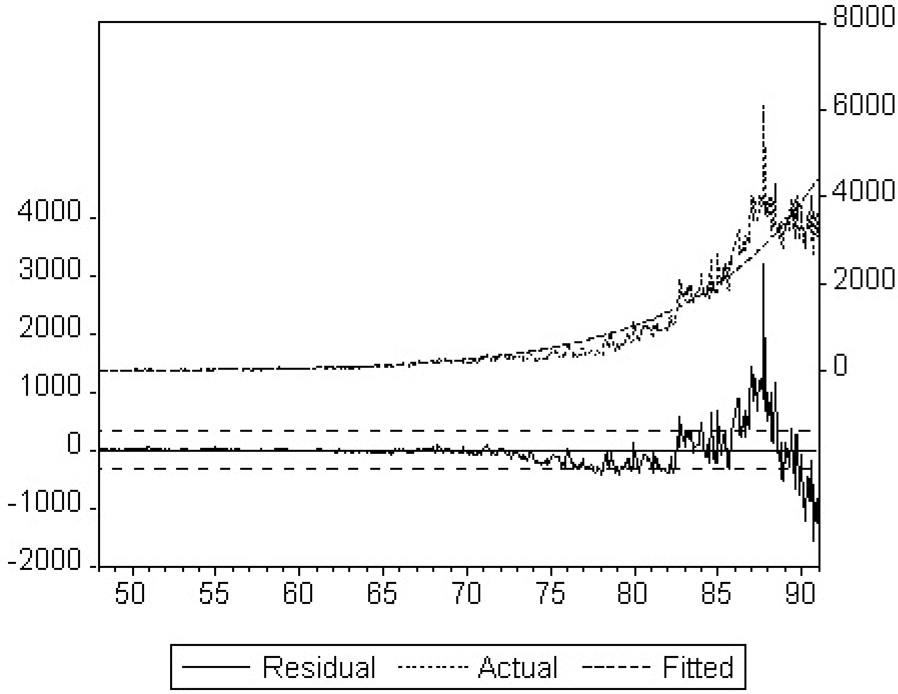
Various Shapes of Exponential Trends



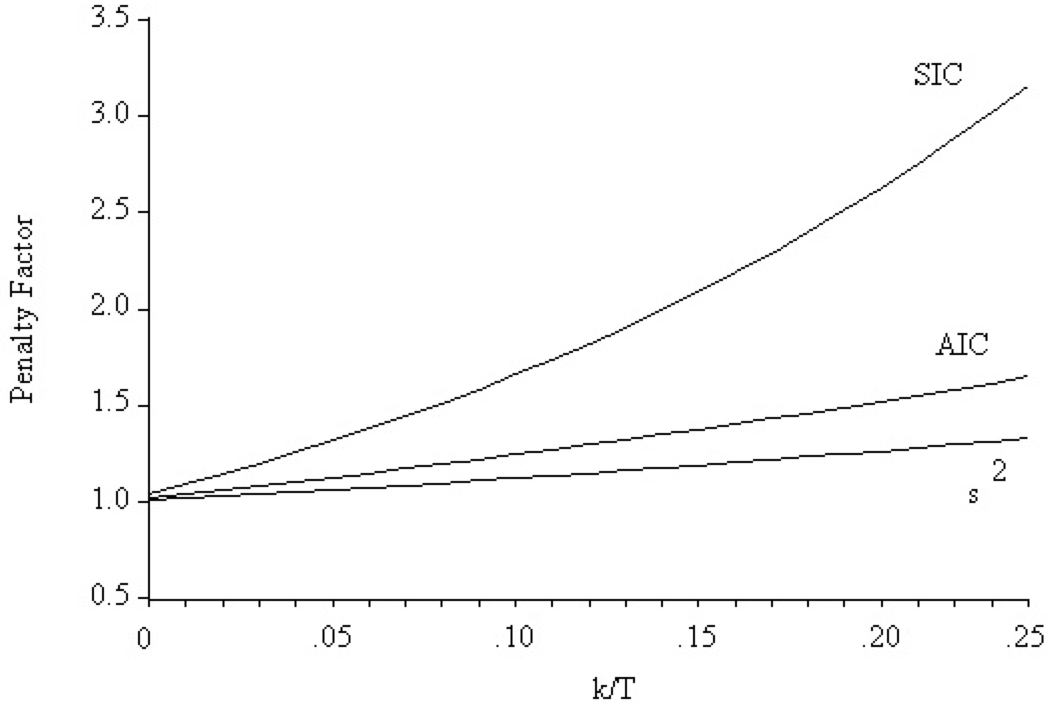
Linear Trend
Log Volume on the New York Stock Exchange

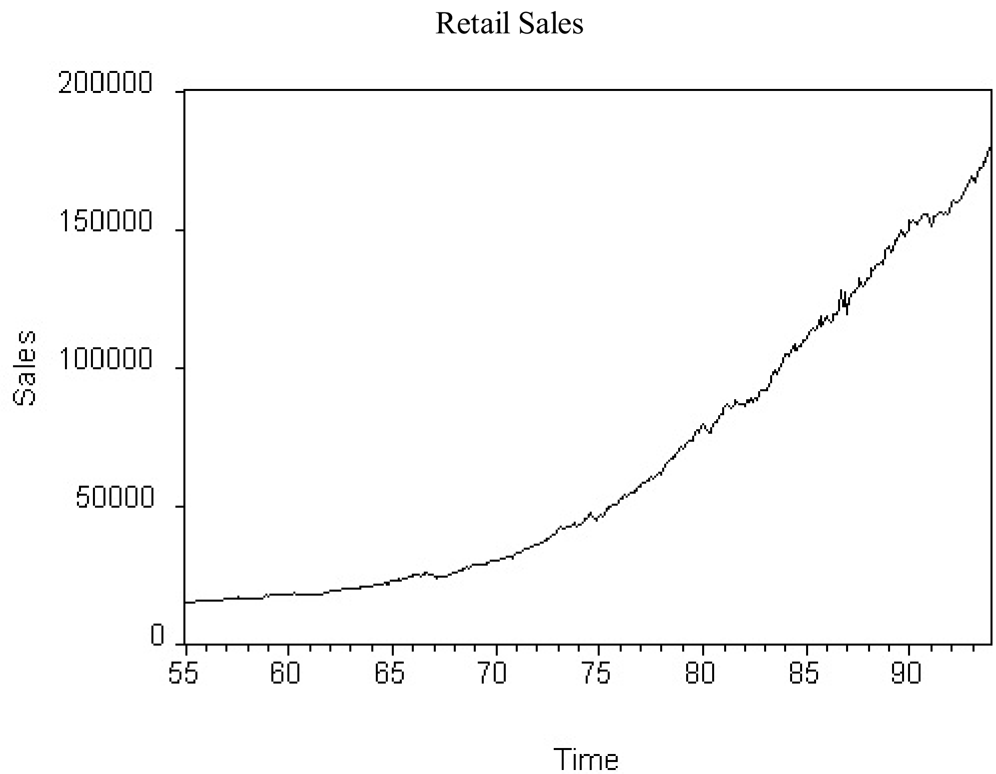


Exponential Trend
Volume on the New York Stock Exchange



Degrees-of-Freedom Penalties
Various Model Selection Criteria




Retail Sales
Sales
Time
200000
150000
100000
50000
0
55
60
65
70
75
80
85
90

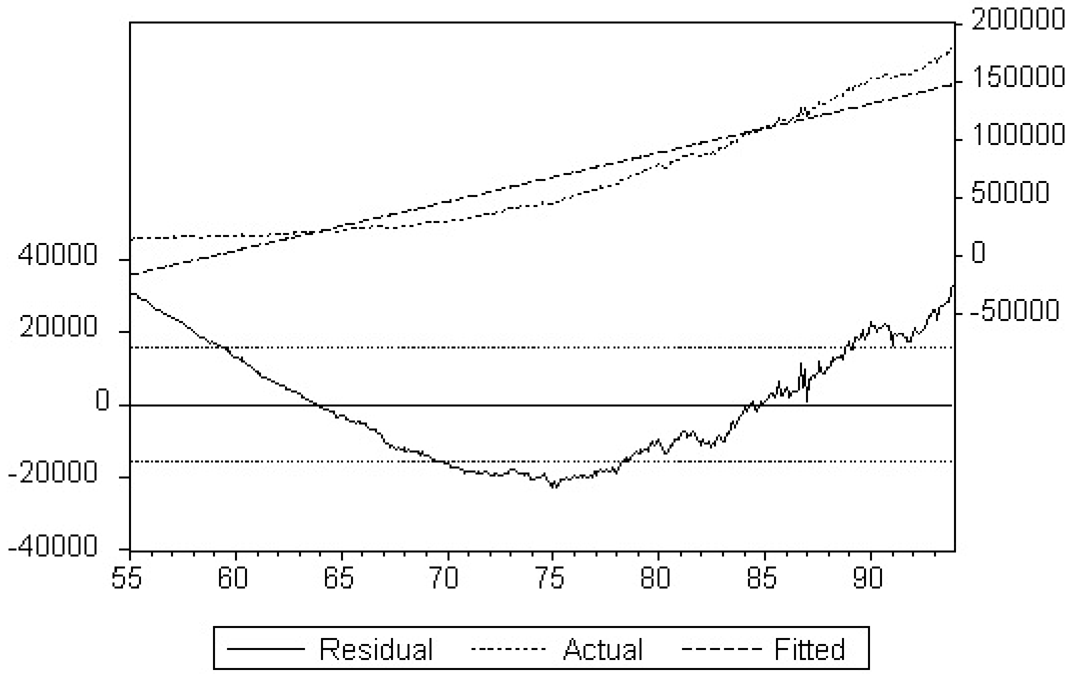
Retail Sales
Linear Trend Regression

Dependent Variable is RTRR
Sample: 1955:01 1993:12
Included observations: 468

Variable	Coefficient	Std. Error	T-Statistic	Prob.
C	-16391.25	1469.177	-11.15676	0.0000
TIME	349.7731	5.428670	64.43073	0.0000

R-squared	0.899076	Mean dependent var	65630.56
Adjusted R-squared	0.898859	S.D. dependent var	49889.26
S.E. of regression	15866.12	Akaike info criterion	19.34815
Sum squared resid	1.17E+11	Schwarz criterion	19.36587
Log likelihood	-5189.529	F-statistic	4151.319
Durbin-Watson stat	0.004682	Prob(F-statistic)	0.000000

Retail Sales
Linear Trend Residual Plot



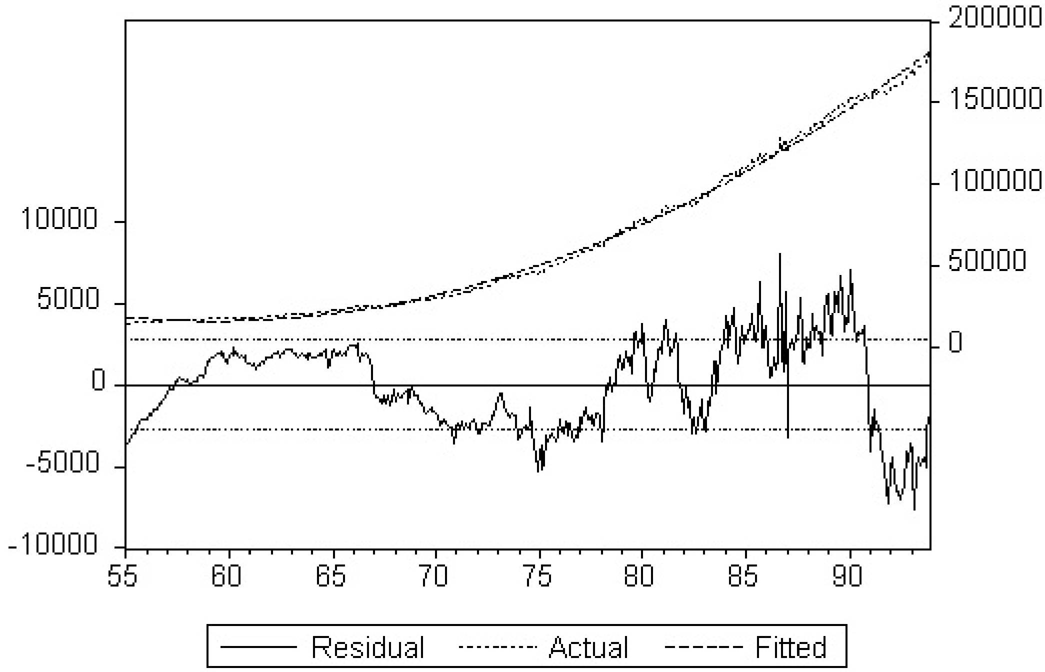
Retail Sales
Quadratic Trend Regression

Dependent Variable is RTRR
Sample: 1955:01 1993:12
Included observations: 468

Variable	Coefficient	Std. Error	T-Statistic	Prob.
C	18708.70	379.9566	49.23905	0.0000
TIME	-98.31130	3.741388	-26.27669	0.0000
TIME2	0.955404	0.007725	123.6754	0.0000

R-squared	0.997022	Mean dependent var	65630.56
Adjusted R-squared	0.997010	S.D. dependent var	49889.26
S.E. of regression	2728.205	Akaike info criterion	15.82919
Sum squared resid	3.46E+09	Schwarz criterion	15.85578
Log likelihood	-4365.093	F-statistic	77848.80
Durbin-Watson stat	0.151089	Prob(F-statistic)	0.000000

Retail Sales
Quadratic Trend Residual Plot



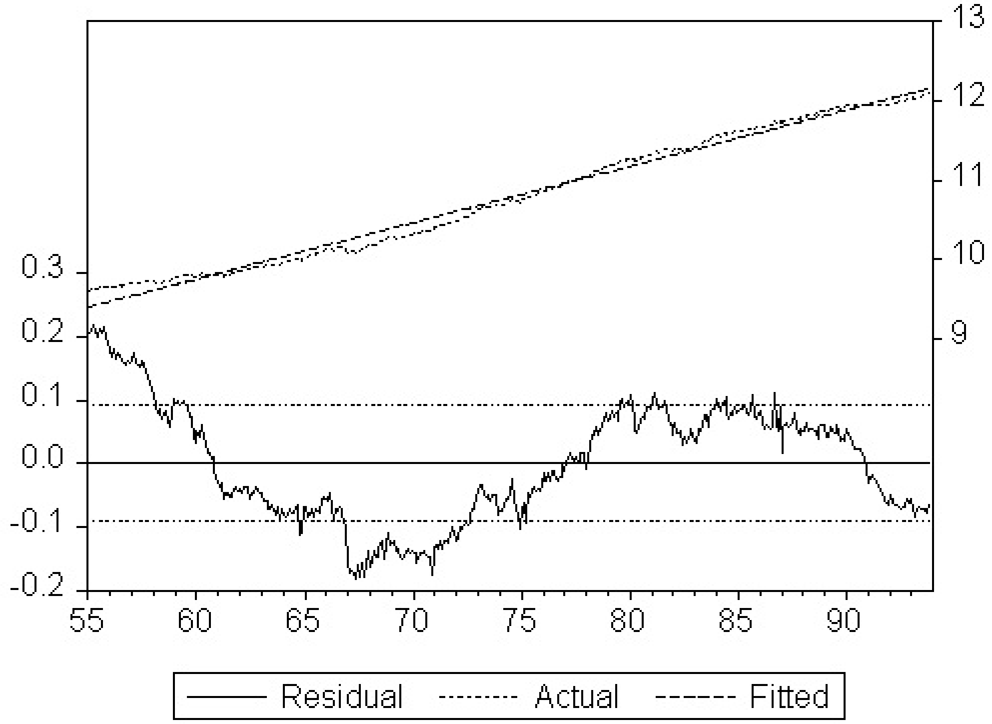
Retail Sales
Log Linear Trend Regression

Dependent Variable is LRTRR
Sample: 1955:01 1993:12
Included observations: 468

Variable	Coefficient	Std. Error	T-Statistic	Prob.
C	9.389975	0.008508	1103.684	0.0000
TIME	0.005931	3.14E-05	188.6541	0.0000

R-squared	0.987076	Mean dependent var	10.78072
Adjusted R-squared	0.987048	S.D. dependent var	0.807325
S.E. of regression	0.091879	Akaike info criterion	-4.770302
Sum squared resid	3.933853	Schwarz criterion	-4.752573
Log likelihood	454.1874	F-statistic	35590.36
Durbin-Watson stat	0.019949	Prob(F-statistic)	0.000000

Retail Sales
Log Linear Trend Residual Plot



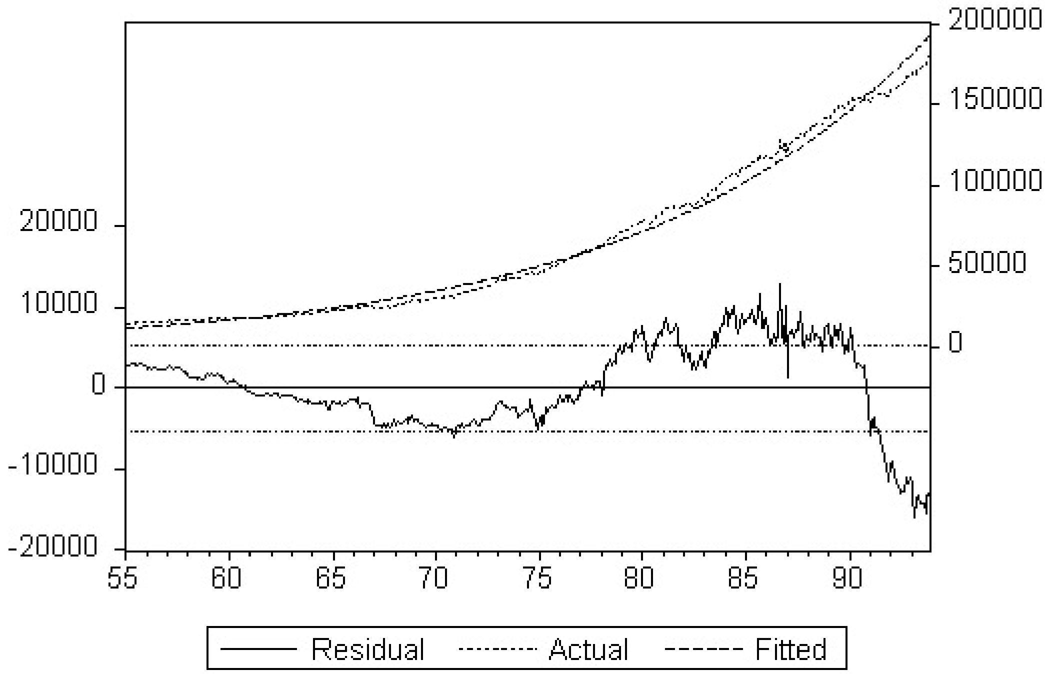
Retail Sales
Exponential Trend Regression

Dependent Variable is RTRR
Sample: 1955:01 1993:12
Included observations: 468
Convergence achieved after 1 iterations
RTRR=C(1)*EXP(C(2)*TIME)

	Coefficient	Std. Error	T-Statistic	Prob.
C(1)	11967.80	177.9598	67.25003	0.0000
C(2)	0.005944	3.77E-05	157.7469	0.0000

R-squared	0.988796	Mean dependent var	65630.56
Adjusted R-squared	0.988772	S.D. dependent var	49889.26
S.E. of regression	5286.406	Akaike info criterion	17.15005
Sum squared resid	1.30E+10	Schwarz criterion	17.16778
Log likelihood	-4675.175	F-statistic	41126.02
Durbin-Watson stat	0.040527	Prob(F-statistic)	0.000000

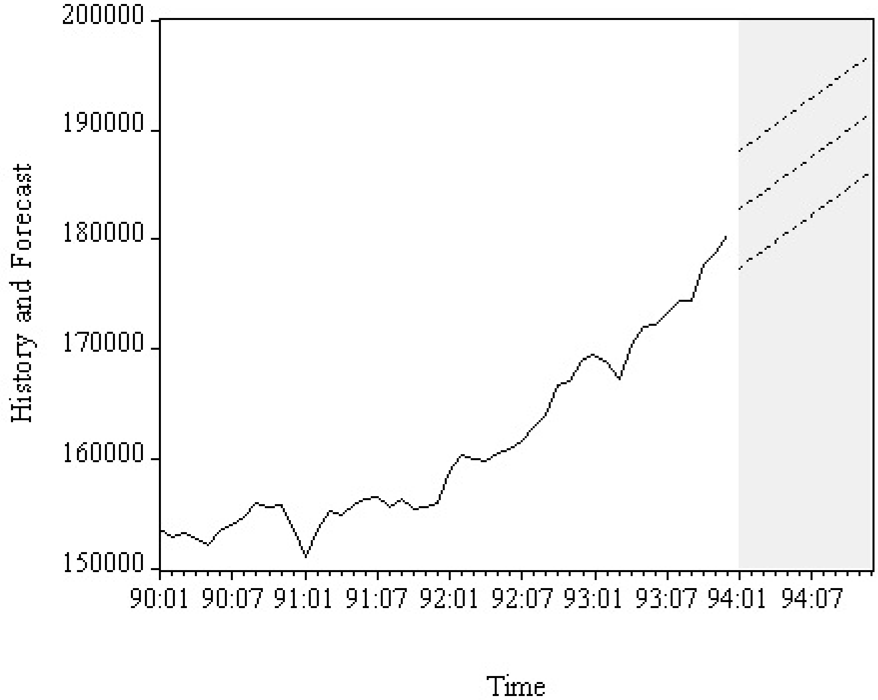
Retail Sales
Exponential Trend Residual Plot



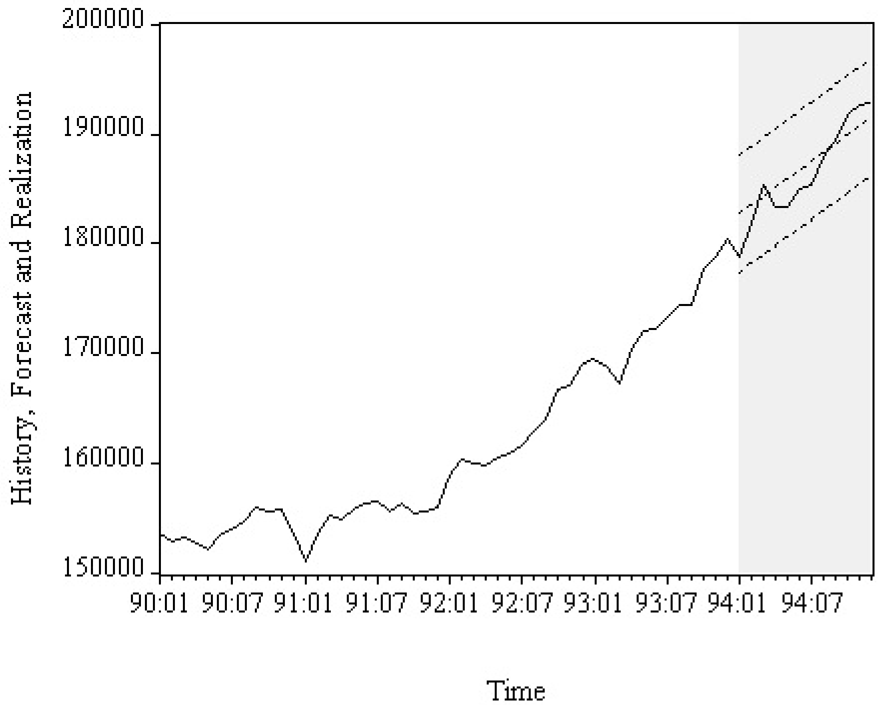
Model Selection Criteria
Linear, Quadratic and Exponential Trend Models

	Linear Trend	Quadratic Trend	Exponential Trend
AIC	19.35	15.83	17.15
SIC	19.37	15.86	17.17

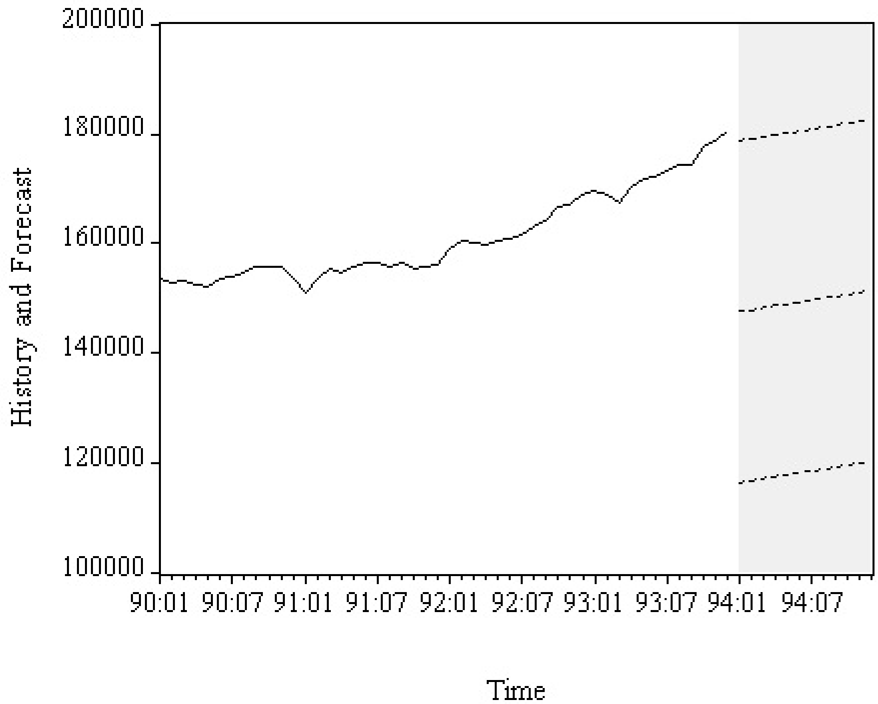
Retail Sales
History, 1990.01 - 1993.12
Quadratic Trend Forecast, 1994.01-1994.12



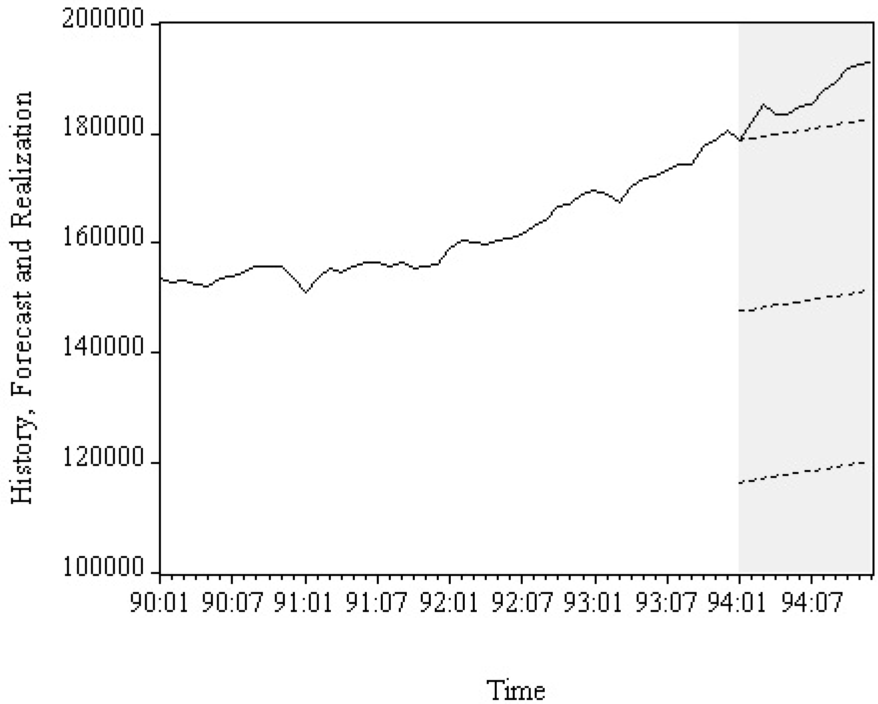
Retail Sales
History, 1990.01 - 1993.12
Quadratic Trend Forecast and Realization, 1994.01-1994.12



Retail Sales
History, 1990.01 - 1993.12
Linear Trend Forecast, 1994.01-1994.12



Retail Sales
History, 1990.01 - 1993.12
Linear Trend Forecast and Realization, 1994.01-1994.12



Modeling and Forecasting Seasonality

1. The Nature and Sources of Seasonality

2. Modeling Seasonality

$$D_1 = (1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, ...)$$
$$D_2 = (0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, ...)$$
$$D_3 = (0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, ...)$$
$$D_4 = (0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, ...)$$

$$y_t = \sum_{i=1}^{s} \gamma_i D_{it} + \varepsilon_t$$

$$y_t = \beta_1 TIME_t + \sum_{i=1}^{s} \gamma_i D_{it} + \varepsilon_t$$

$$y_t = \beta_1 TIME_t + \sum_{i=1}^{s} \gamma_i D_{it} + \sum_{i=1}^{v_1} \delta_i^{HD} HDV_{it} + \sum_{i=1}^{v_2} \delta_i^{TD} TDV_{it} + \varepsilon_t$$

3. Forecasting Seasonal Series

$$y_t = \beta_1 TIME_t + \sum_{i=1}^{s} \gamma_i D_{it} + \sum_{i=1}^{v_1} \delta_i^{HD} HDV_{it} + \sum_{i=1}^{v_2} \delta_i^{TD} TDV_{it} + \varepsilon_t$$

$$y_{T+h} = \beta_1 TIME_{T+h} + \sum_{i=1}^{s} \gamma_i D_{i,T+h} + \sum_{i=1}^{v_1} \delta_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{v_2} \delta_i^{TD} TDV_{i,T+h} + \varepsilon_{T+h}$$

$$y_{T+h,T} = \beta_1 TIME_{T+h} + \sum_{i=1}^{s} \gamma_i D_{i,T+h} + \sum_{i=1}^{v_1} \delta_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{v_2} \delta_i^{TD} TDV_{i,T+h}$$

$$\hat{y}_{T+h,T} = \hat{\beta}_1 TIME_{T+h} + \sum_{i=1}^{s} \hat{\gamma}_i D_{i,T+h} + \sum_{i=1}^{v_1} \hat{\delta}_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{v_2} \hat{\delta}_i^{TD} TDV_{i,T+h}$$

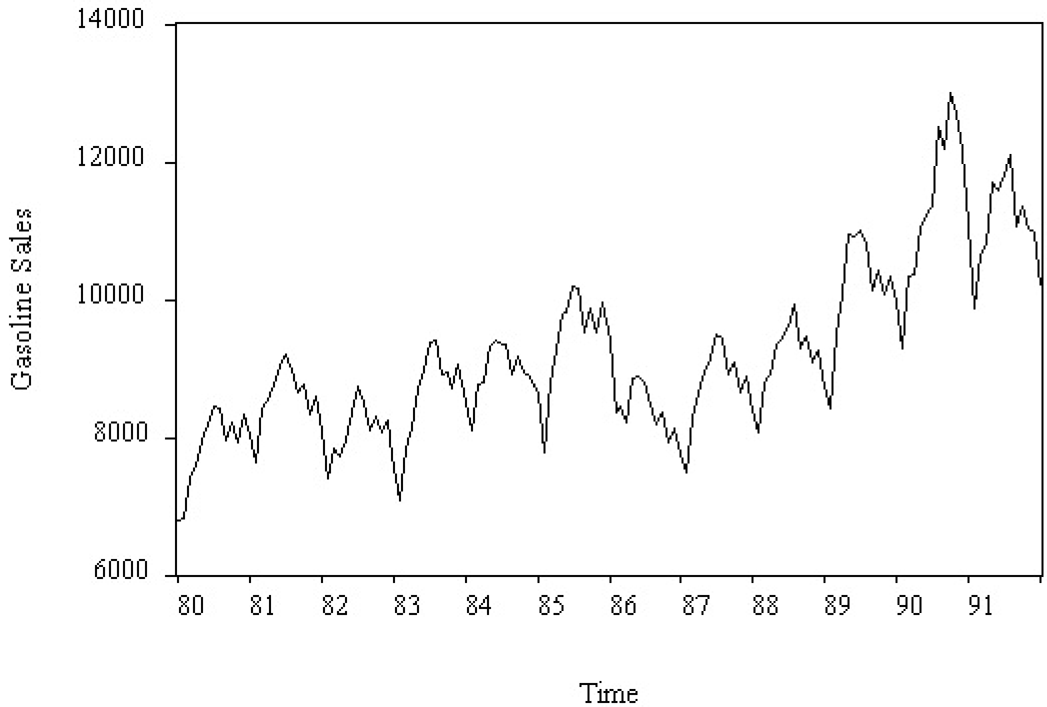
$$y_{T+h,T} \pm 1.96\sigma$$

$$\hat{y}_{T+h,T} \pm 1.96\hat{\sigma}$$

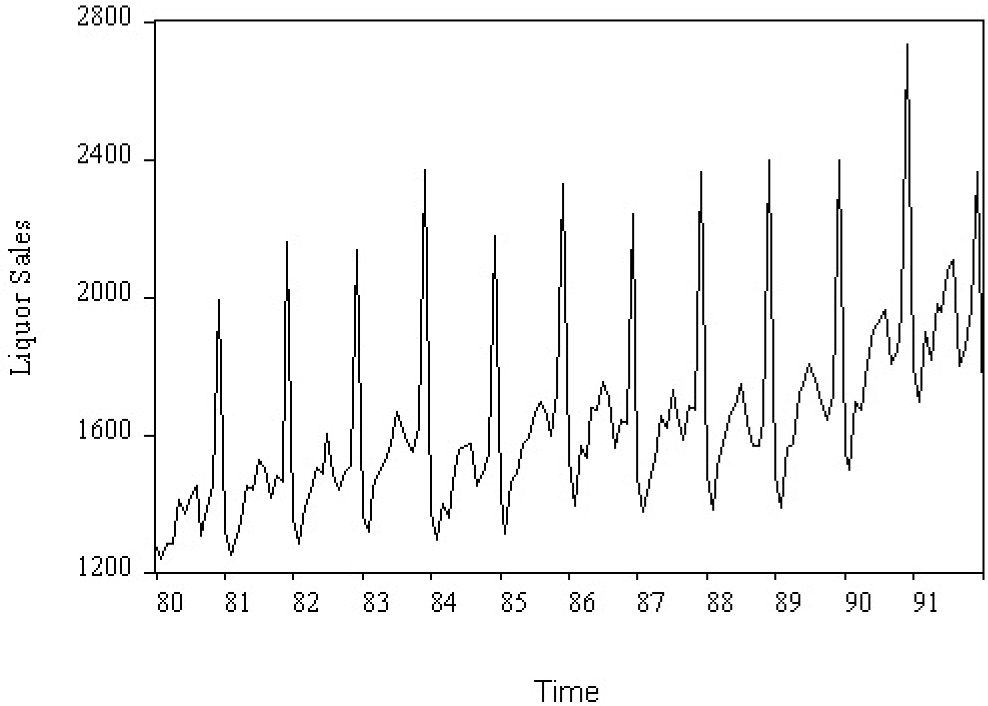
$$N(y_{T+h,T},\ \sigma^2)$$

$$N(\hat{y}_{T+h,T},\ \hat{\sigma}^2)$$

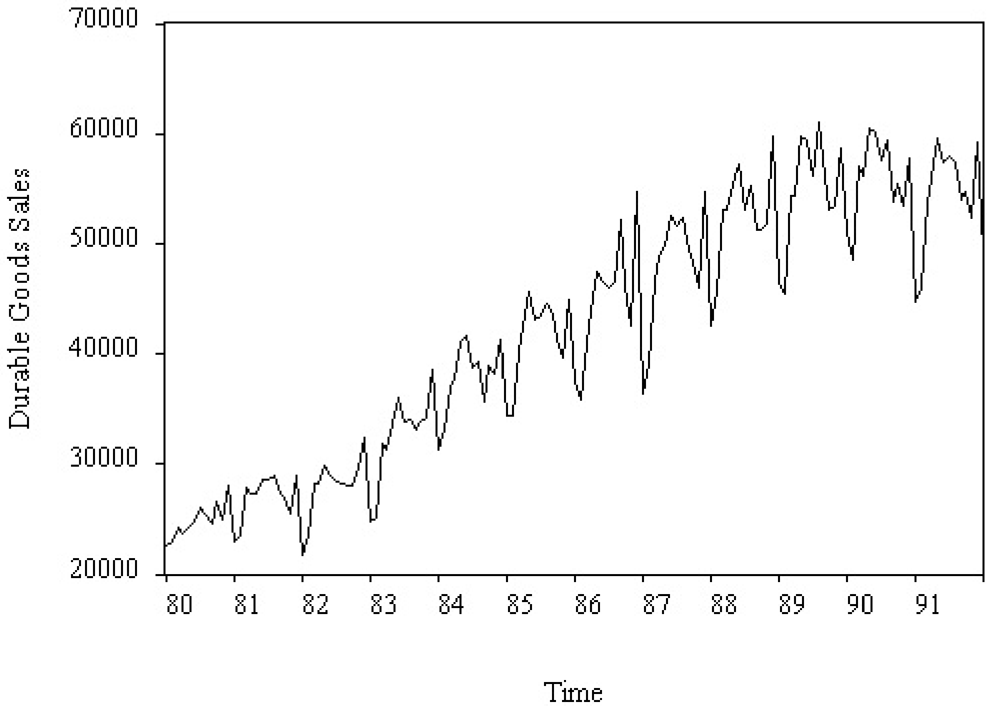
Gasoline Sales



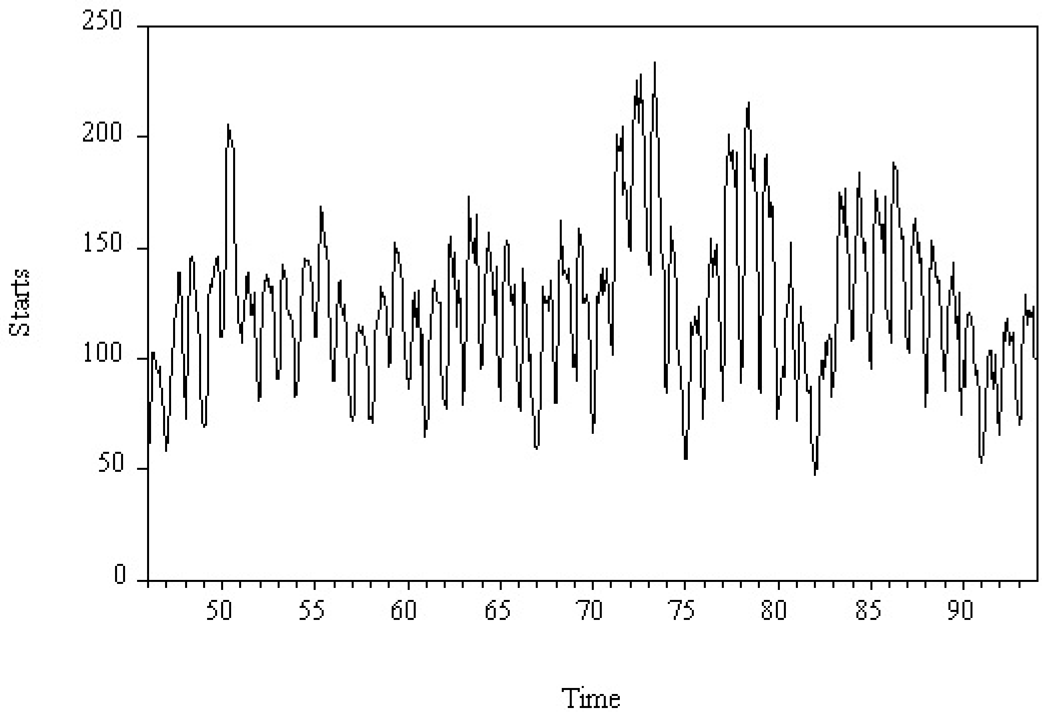
Liquor Sales

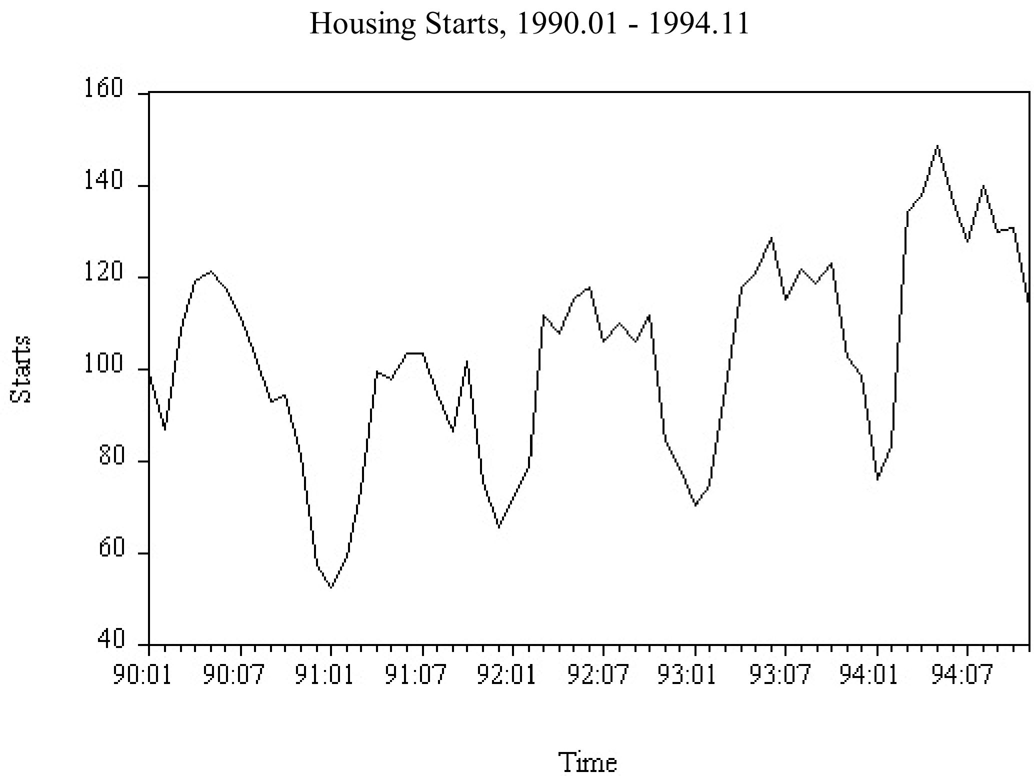


Durable Goods Sales



Housing Starts, 1946.01 - 1994.11




Housing Starts, 1990.01 - 1994.11
160
140
120
100
80
60
40
Starts
90:01
90:07
91:01
91:07
92:01
92:07
93:01
93:07
94:01
94:07
Time

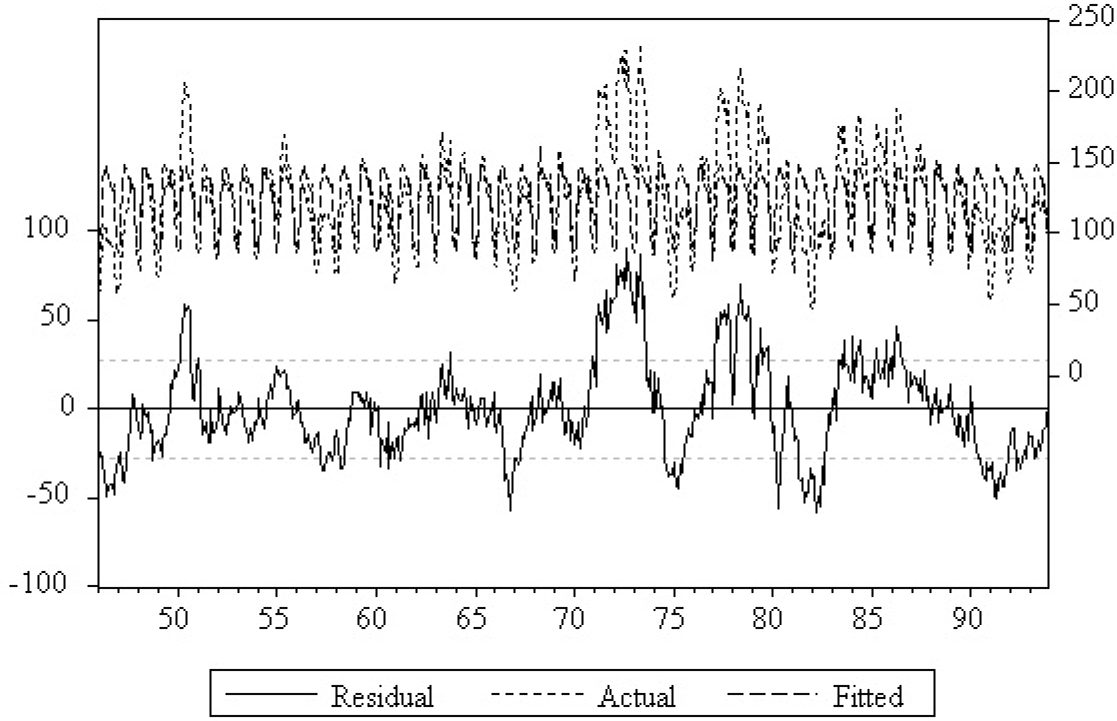
Regression Results
Seasonal Dummy Variable Model
Housing Starts

LS // Dependent Variable is STARTS
Sample: 1946:01 1993:12
Included observations: 576

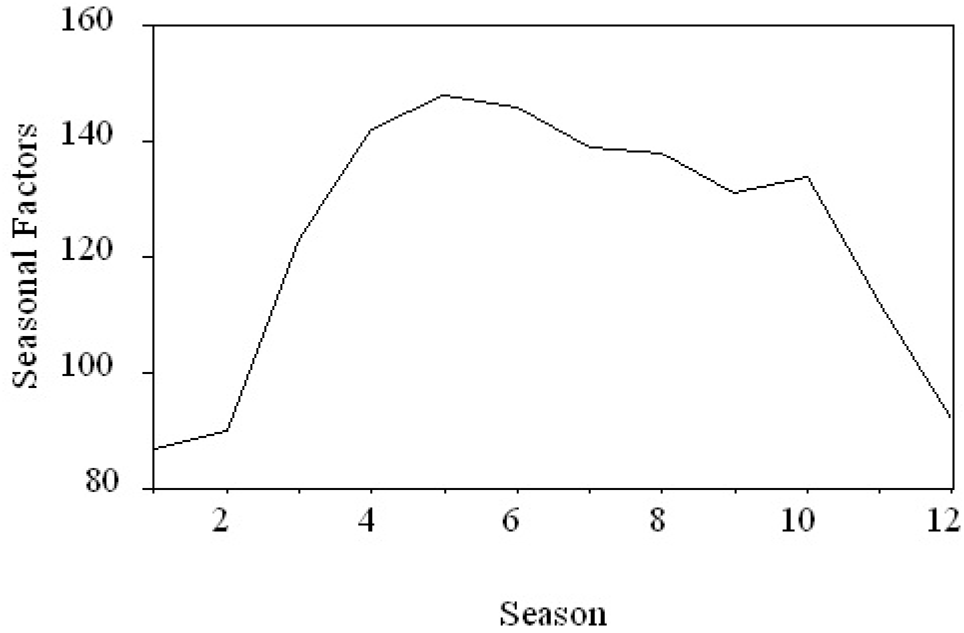
Variable	Coefficient	Std. Error	t-Statistic	Prob.
D1	86.50417	4.029055	21.47009	0.0000
D2	89.50417	4.029055	22.21468	0.0000
D3	122.8833	4.029055	30.49929	0.0000
D4	142.1687	4.029055	35.28588	0.0000
D5	147.5000	4.029055	36.60908	0.0000
D6	145.9979	4.029055	36.23627	0.0000
D7	139.1125	4.029055	34.52733	0.0000
D8	138.4167	4.029055	34.35462	0.0000
D9	130.5625	4.029055	32.40524	0.0000
D10	134.0917	4.029055	33.28117	0.0000
D11	111.8333	4.029055	27.75671	0.0000
D12	92.15833	4.029055	22.87344	0.0000

R-squared	0.383780	Mean dependent var	123.3944
Adjusted R-squared	0.371762	S.D. dependent var	35.21775
S.E. of regression	27.91411	Akaike info criterion	6.678878
Sum squared resid	439467.5	Schwarz criterion	6.769630
Log likelihood	-2728.825	F-statistic	31.93250
Durbin-Watson stat	0.154140	Prob(F-statistic)	0.000000

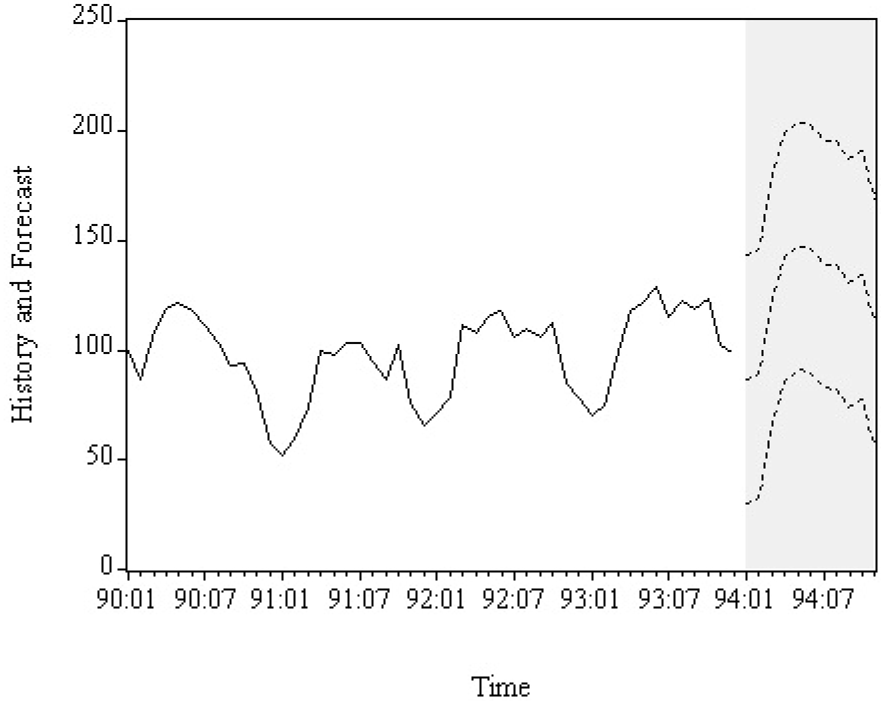
Residual Plot



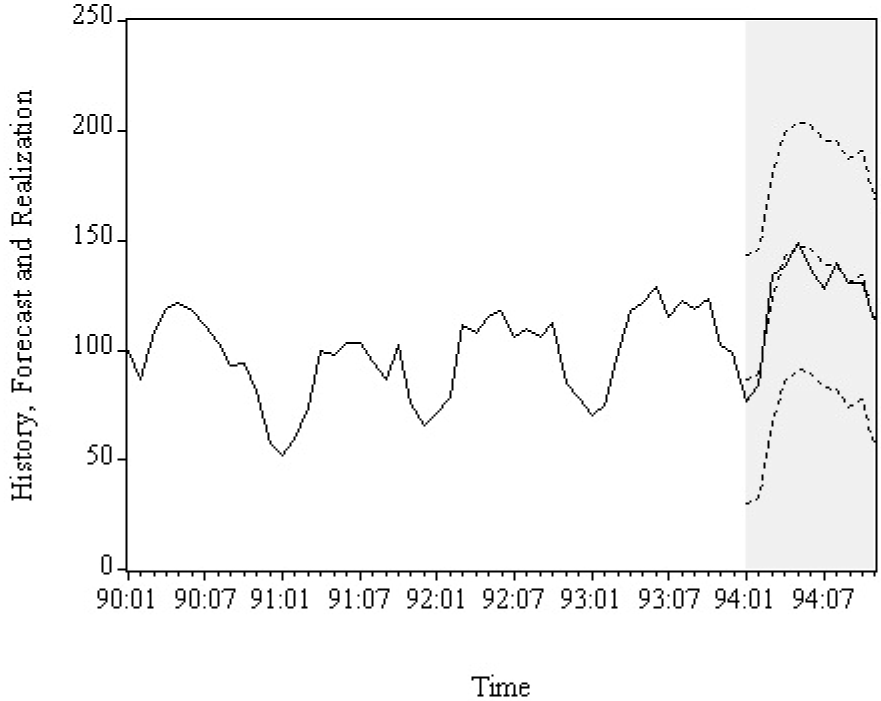
Estimated Seasonal Factors
Housing Starts



Housing Starts
History, 1990.01-1993.12
Forecast, 1994.01-1994.11



Housing Starts
History, 1990.01-1993.12
Forecast and Realization, 1994.01-1994.11



Characterizing Cycles

1. Covariance Stationary Time Series

Realization

Sample path

Covariance stationarity

$$Ey_t = \mu_t$$

$$Ey_t = \mu$$

$$\gamma(t,\tau) = cov(y_t, y_{t-\tau}) = E(y_t - \mu)(y_{t-\tau} - \mu)$$

$$\gamma(t,\tau) = \gamma(\tau)$$

$$\gamma(0) = cov(y_t, y_t) = var(y_t) < \infty$$

$$\text{corr}(x,y) = \frac{\text{cov}(x,y)}{\sigma_x \ \sigma_y}$$

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)}, \ \tau = 0, \ 1, \ 2, \ \ldots.$$

$$\rho(\tau) = \frac{\text{cov}(y_t, \ y_{t-\tau})}{\sqrt{\text{var}(y_t)} \ \sqrt{\text{var}(y_{t-\tau})}} = \frac{\gamma(\tau)}{\sqrt{\gamma(0)}\sqrt{\gamma(0)}} = \frac{\gamma(\tau)}{\gamma(0)}$$

$p(\tau)$ regression of y_t on $y_{t-1}, \ldots, y_{t-\tau}$

2. White Noise

$$y_t \sim WN(0, \sigma^2)$$

$$y_t \overset{iid}{\sim} (0, \sigma^2)$$

$$y_t \overset{iid}{\sim} N(0, \sigma^2)$$

$$E(y_t) = 0$$

$$\text{var}(y_t) = \sigma^2$$

$$\gamma(\tau) = \begin{cases} \sigma^2, & \tau = 0 \\ 0, & \tau \geq 1 \end{cases}$$

$$\rho(\tau) = \begin{cases} 1, & \tau = 0 \\ 0, & \tau \geq 1 \end{cases}$$

$$p(\tau) = \begin{cases} 1, & \tau = 0 \\ 0, & \forall\tau \geq 1 \end{cases}$$

$$E(y_t|\Omega_{t-1}) = 0$$

$$\text{var}(y_t|\Omega_{t-1}) = E[(y_t - E(y_t|\Omega_{t-1}))^2|\Omega_{t-1}] = \sigma^2$$

3. The Lag Operator

$$L y_t = y_{t-1}$$

$$L^2 y_t = L(L(y_t)) = L(y_{t-1}) = y_{t-2}$$

$$B(L) = b_0 + b_1 L + b_2 L^2 + \dots b_m L^m$$

$$L^m y_t = y_{t-m}$$

$$\Delta y_t = (1-L) y_t = y_t - y_{t-1}$$

$$(1 + .9L + .6L^2) y_t = y_t + .9 y_{t-1} + .6 y_{t-2}$$

$$B(L) = b_0 + b_1 L + b_2 L^2 + \dots = \sum_{i=0}^{\infty} b_i L^i$$

$$B(L)\,\varepsilon_t = b_0\,\varepsilon_t + b_1\,\varepsilon_{t-1} + b_2\,\varepsilon_{t-2} + \dots = \sum_{i=0}^{\infty} b_i \varepsilon_{t-i}$$

4. Wold's Theorem, the General Linear Process, and Rational Distributed Lags

Wold's Theorem

Let $\{y_t\}$ be any zero-mean covariance-stationary process. Then:

$$y_t = B(L)\varepsilon_t = \sum_{i=0}^{\infty} b_i \varepsilon_{t-i}$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

where $b_0 = 1$ and $\sum_{i=0}^{\infty} b_i^2 < \infty$.

The General Linear Process

$$y_t = B(L)\varepsilon_t = \sum_{i=0}^{\infty} b_i \varepsilon_{t-i}$$

$$\varepsilon_t \overset{iid}{\sim} WN(0, \sigma^2),$$

where $b_0=1$ and $\sum_{i=0}^{\infty} b_i^2 < \infty$.

$$E(y_t) = E(\sum_{i=0}^{\infty} b_i \varepsilon_{t-i}) = \sum_{i=0}^{\infty} b_i E\varepsilon_{t-i} = \sum_{i=0}^{\infty} b_i \bullet 0 = 0$$

$$\text{var}(y_t) = \text{var}(\sum_{i=0}^{\infty} b_i \varepsilon_{t-i}) = \sum_{i=0}^{\infty} b_i^2 \text{var}(\varepsilon_{t-i}) = \sum_{i=0}^{\infty} b_i^2 \sigma^2 = \sigma^2 \sum_{i=0}^{\infty} b_i^2$$

$$E(y_t|\Omega_{t-1}) = E(\varepsilon_t|\Omega_{t-1}) + b_1 E(\varepsilon_{t-1}|\Omega_{t-1}) + b_2 E(\varepsilon_{t-2}|\Omega_{t-1}) + \ldots = 0 + b_1\varepsilon_{t-1} + b_2\varepsilon_{t-2} + \ldots = \sum_{i=1}^{\infty} b_i \varepsilon_{t-i}$$

$$\mathrm{var}(y_t|\Omega_{t-1}) = E[(y_t - E(y_t|\Omega_{t-1}))^2|\Omega_{t-1}] = E(\varepsilon_t^2|\Omega_{t-1}) = E(\varepsilon_t^2) = \sigma^2$$

Rational Distributed Lags

$$B(L) = \frac{\Theta(L)}{\Phi(L)}$$

$$\Theta(L) = \sum_{i=0}^{q} \theta_i L^i$$

$$\Phi(L) = \sum_{i=0}^{p} \varphi_i L^i$$

$$B(L) \approx \frac{\Theta(L)}{\Phi(L)}$$

5. Estimation and Inference for the Mean, Autocorrelation and Partial Autocorrelation Functions

$$\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$$

$$\rho(\tau) = \frac{E\left[(y_t - \mu)(y_{t-\tau} - \mu)\right]}{E[(y_t - \mu)^2]}$$

$$\hat{\rho}(\tau) = \frac{\frac{1}{T}\sum_{t=\tau+1}^{T}\left[(y_t - \bar{y})(y_{t-\tau} - \bar{y})\right]}{\frac{1}{T}\sum_{t=1}^{T}(y_t - \bar{y})^2} = \frac{\sum_{t=\tau+1}^{T}\left[(y_t - \bar{y})(y_{t-\tau} - \bar{y})\right]}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

$$\hat{y}_t = \hat{c} + \hat{\beta}_1 y_{t-1} + \ldots + \hat{\beta}_\tau y_{t-\tau}$$

$$\hat{p}(\tau) \equiv \hat{\beta}_\tau$$

$$\hat{\rho}(\tau),\ \hat{p}(\tau) \sim N\left(0, \frac{1}{T}\right)$$

$$\hat{\rho}(\tau) \sim N(0, \frac{1}{T})$$

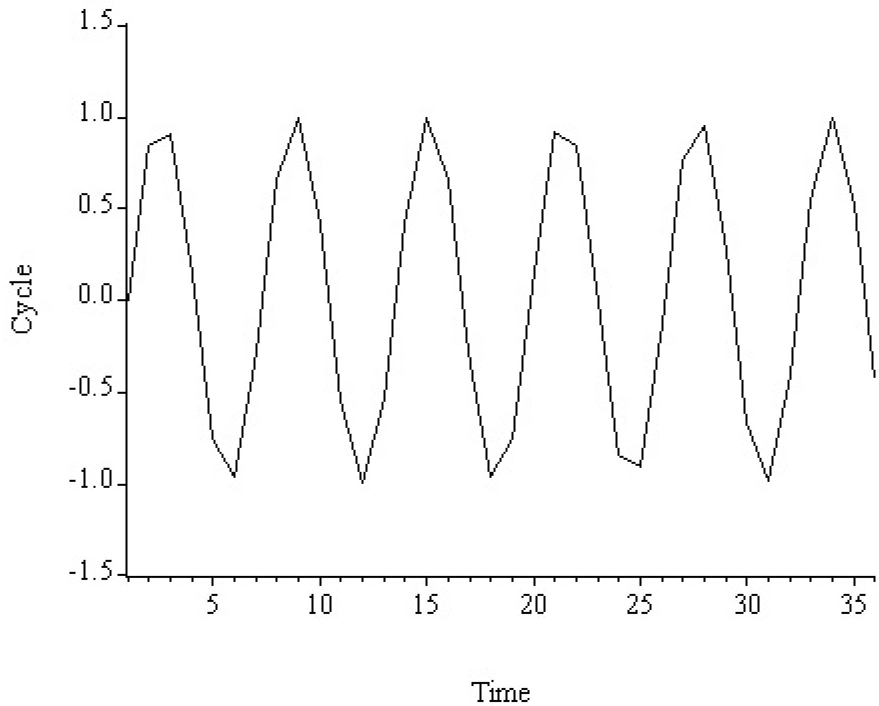
$$\sqrt{T}\hat{\rho}(\tau) \sim N(0, 1)$$

$$T \hat{\rho}^2(\tau) \sim \chi^2_1$$

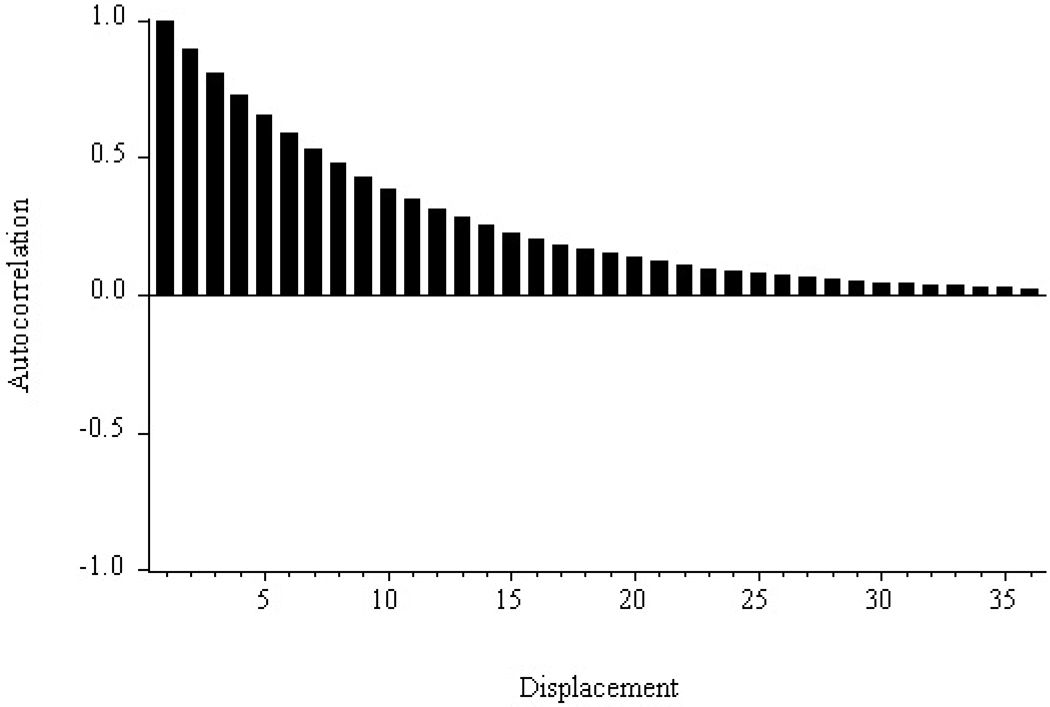
$$Q_{BP} = T \sum_{\tau=1}^{m} \hat{\rho}^2(\tau)$$

$$Q_{LB} = T(T+2) \sum_{\tau=1}^{m} \left(\frac{1}{T-\tau} \right) \hat{\rho}^2(\tau)$$

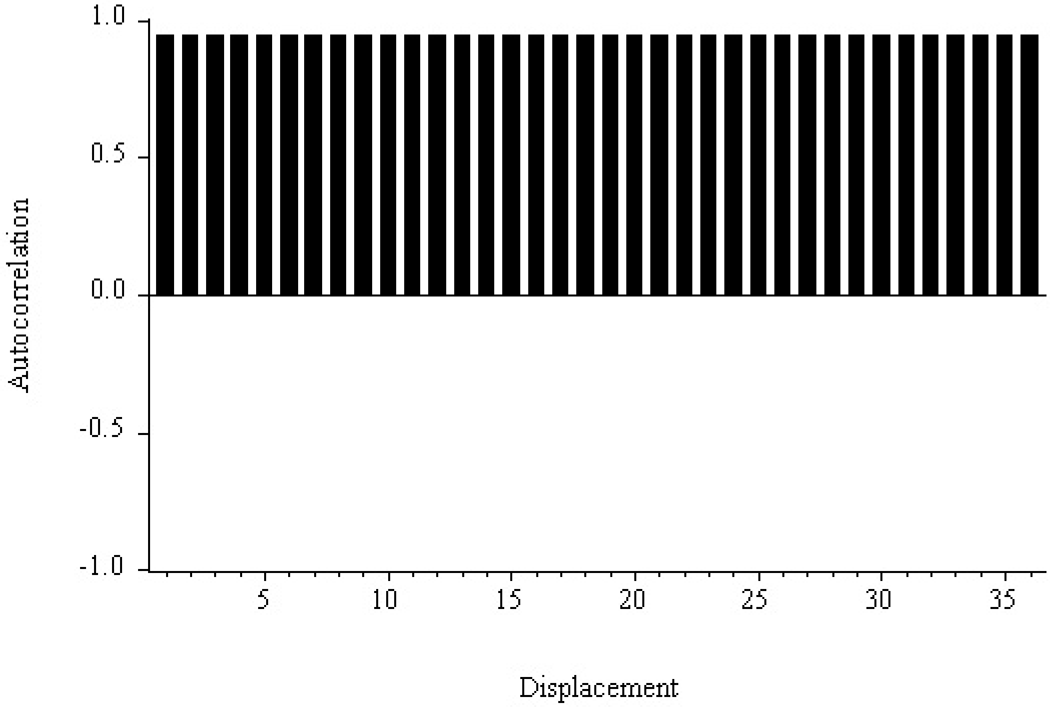
A Rigid Cyclical Pattern



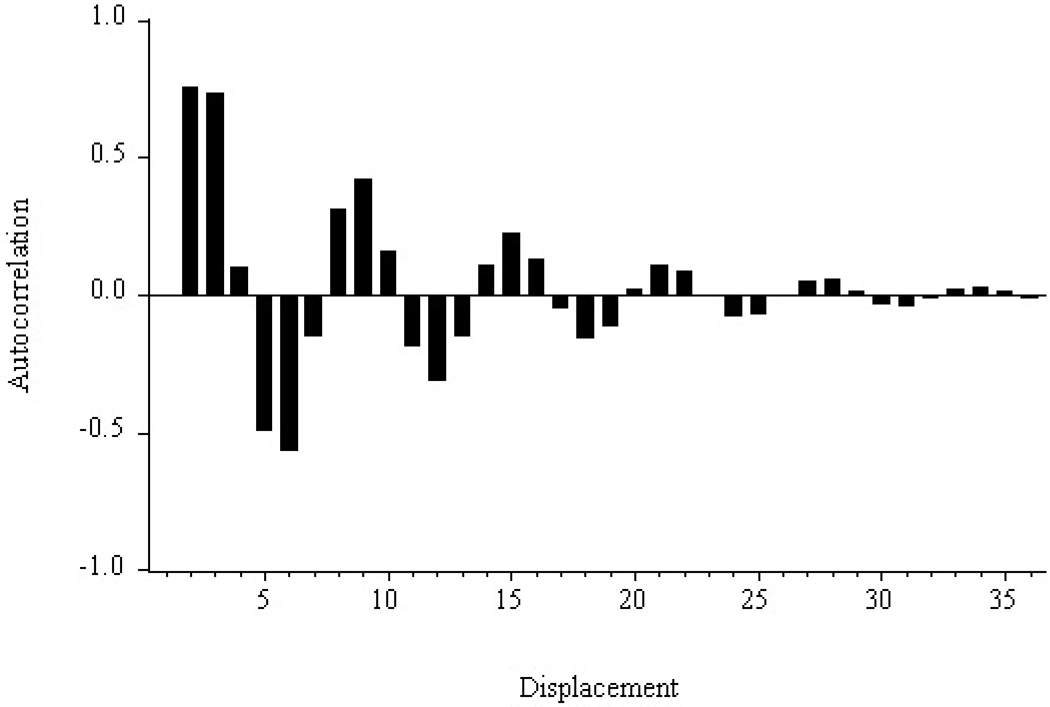
Autocorrelation Function, One-Sided Gradual Damping



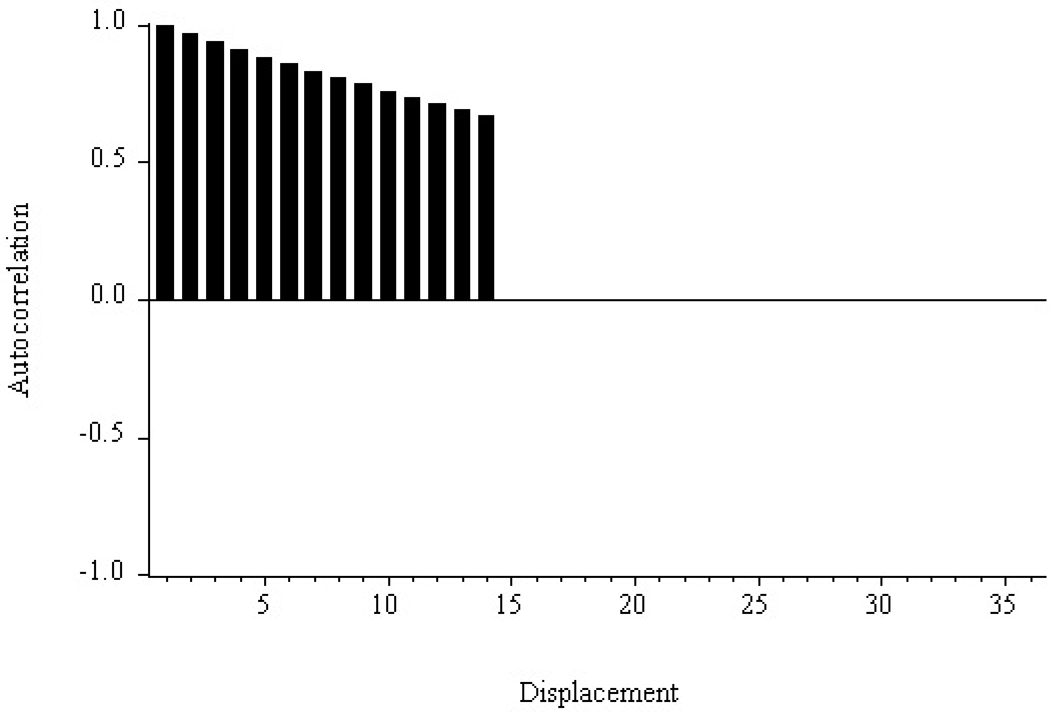
Autocorrelation Function, Non-Damping

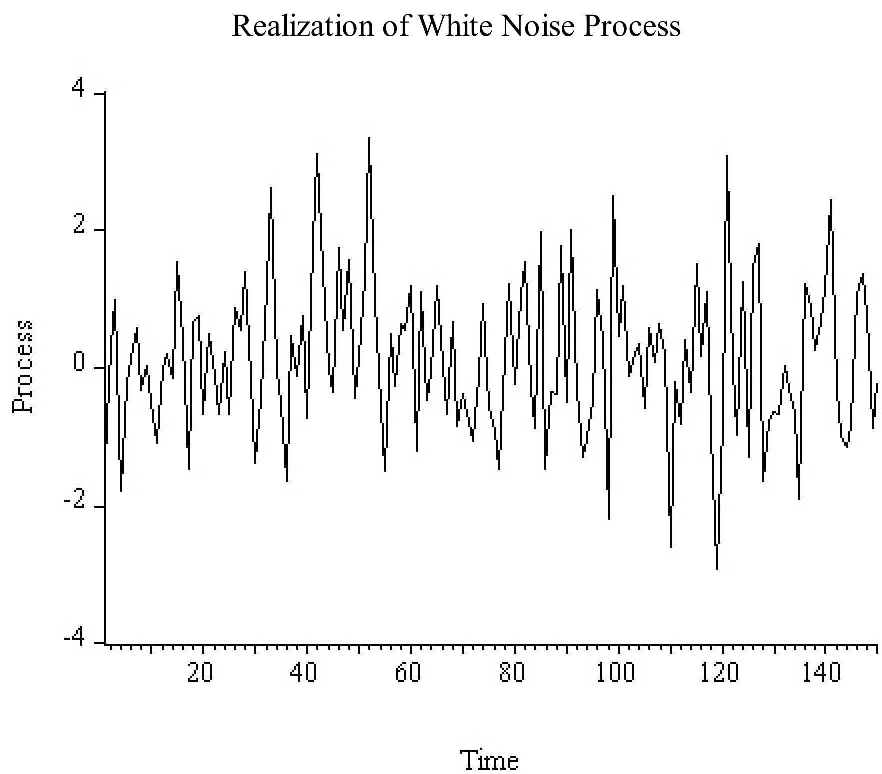


Autocorrelation Function, Gradual Damped Oscillation



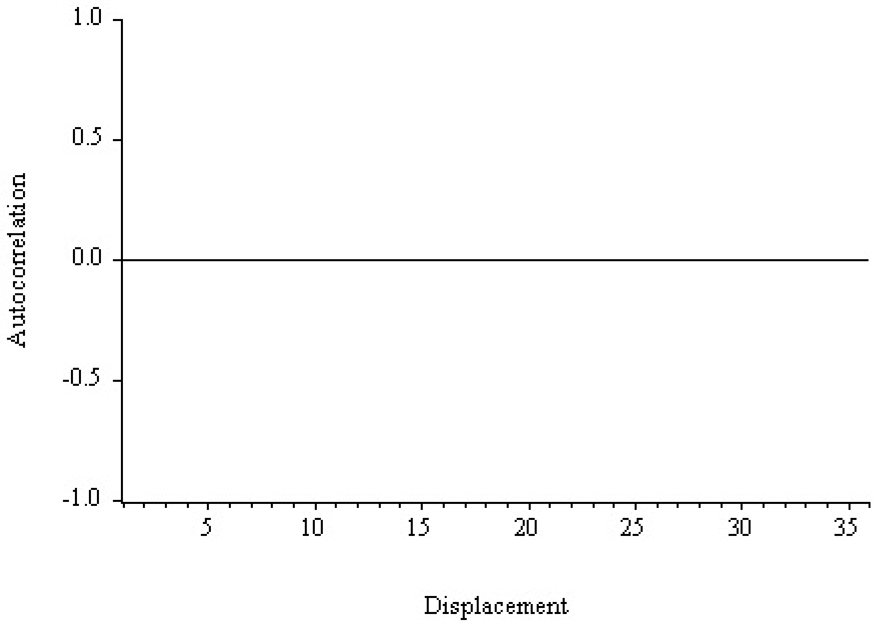
Autocorrelation Function, Sharp Cutoff

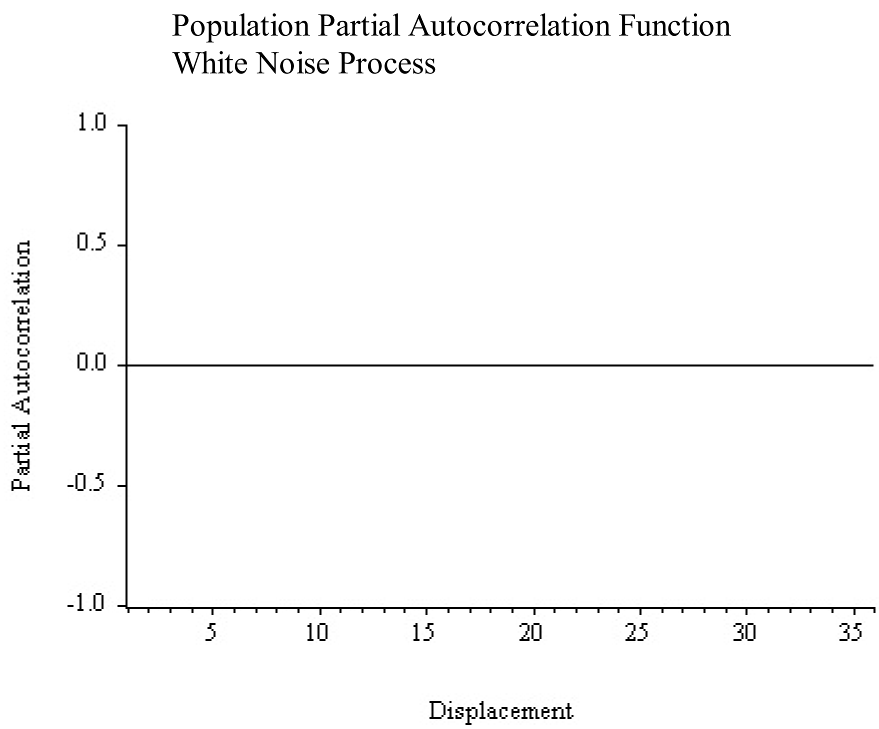


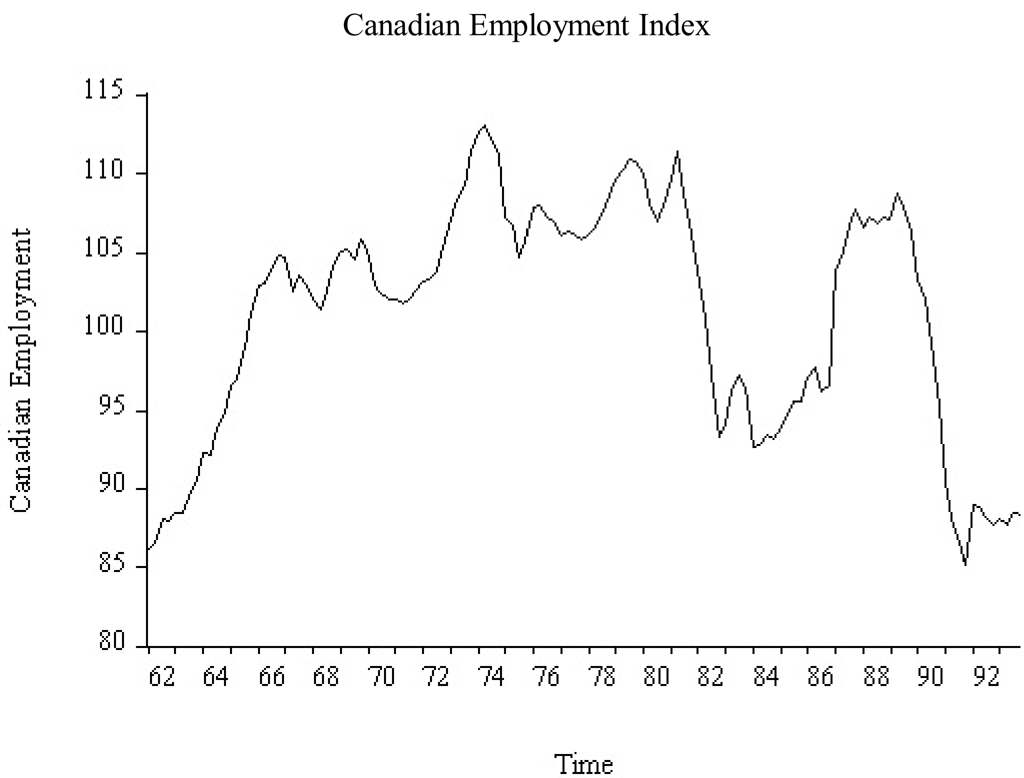


Realization of White Noise Process
4
2
0
-2
-4
Process
20
40
60
80
100
120
140
Time

Population Autocorrelation Function
White Noise Process




Population Partial Autocorrelation Function
White Noise Process
Partial Autocorrelation
1.0
0.5
0.0
-0.5
-1.0
5
10
15
20
25
30
35
Displacement

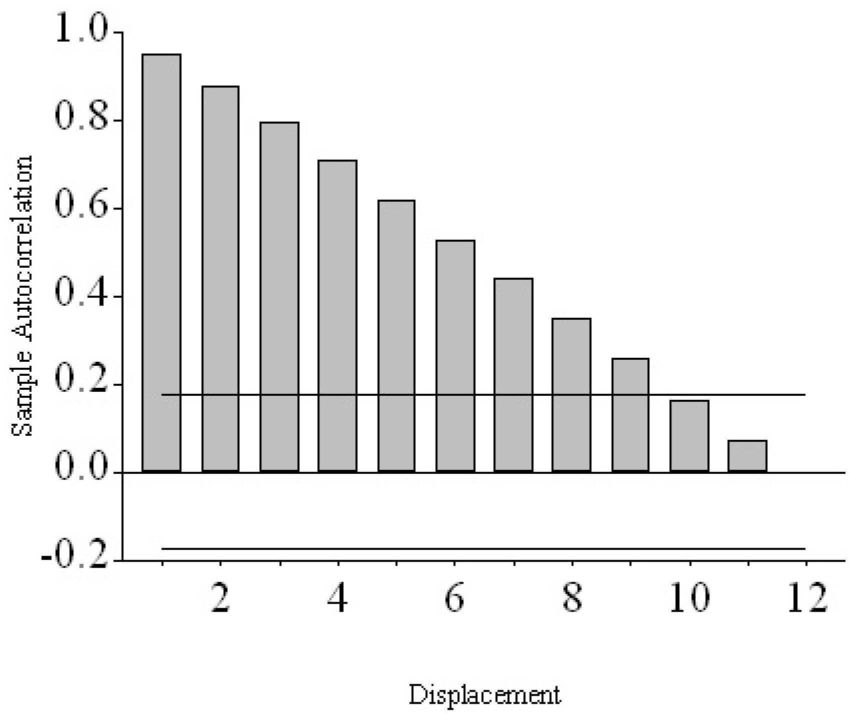

Canadian Employment Index
Canadian Employment
115
110
105
100
95
90
85
80
62 64 66 68 70 72 74 76 78 80 82 84 86 88 90 92
Time

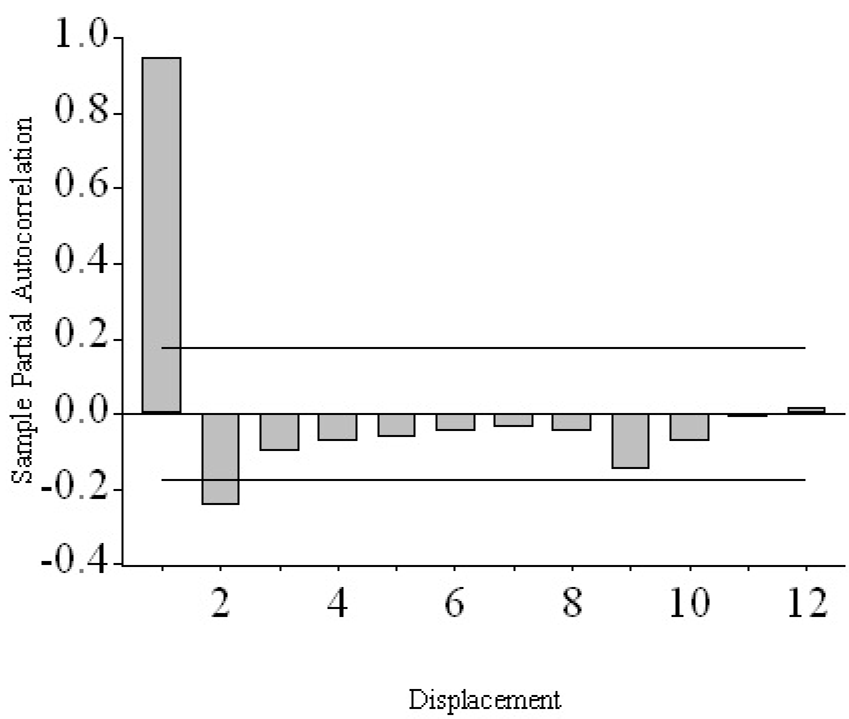
Canadian Employment Index
Correlogram

Sample: 1962:1 1993:4
Included observations: 128

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.949	0.949	.088	118.07	0.000
2	0.877	-0.244	.088	219.66	0.000
3	0.795	-0.101	.088	303.72	0.000
4	0.707	-0.070	.088	370.82	0.000
5	0.617	-0.063	.088	422.27	0.000
6	0.526	-0.048	.088	460.00	0.000
7	0.438	-0.033	.088	486.32	0.000
8	0.351	-0.049	.088	503.41	0.000
9	0.258	-0.149	.088	512.70	0.000
10	0.163	-0.070	.088	516.43	0.000
11	0.073	-0.011	.088	517.20	0.000
12	-0.005	0.016	.088	517.21	0.000

Canadian Employment Index
Sample Autocorrelation and Partial Autocorrelation Functions,
With Plus or Minus Two Standard Error Bands





Modeling Cycles: MA, AR, and ARMA Models

The MA(1) Process

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1} = (1 + \theta L)\varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

If invertible:

$$y_t = \varepsilon_t + \theta y_{t-1} - \theta^2 y_{t-2} + \theta^3 y_{t-3} - \ldots$$

$$Ey_t = E(\varepsilon_t) + \theta E(\varepsilon_{t-1}) = 0$$

$$\mathrm{var}(y_t) = \mathrm{var}(\varepsilon_t) + \theta^2 \mathrm{var}(\varepsilon_{t-1}) = \sigma^2 + \theta^2\sigma^2 = \sigma^2(1 + \theta^2)$$

$$E(y_t|\Omega_{t-1}) = E((\varepsilon_t + \theta\varepsilon_{t-1})|\Omega_{t-1}) = E(\varepsilon_t|\Omega_{t-1}) + \theta E(\varepsilon_{t-1}|\Omega_{t-1}) = \theta\varepsilon_{t-1}$$

$$\mathrm{var}(y_t|\Omega_{t-1}) = E[(y_t - E(y_t|\Omega_{t-1}))^2|\Omega_{t-1}] = E(\varepsilon_t^2|\Omega_{t-1}) = E(\varepsilon_t^2) = \sigma^2$$

$$\gamma(\tau) = E(y_t y_{t-\tau}) = E((\varepsilon_t+\theta\varepsilon_{t-1})(\varepsilon_{t-\tau}+\theta\varepsilon_{t-\tau-1})) = \begin{cases} \theta\sigma^2, & \tau=1 \\ 0, & \text{otherwise} \end{cases}$$

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)} = \begin{cases} \dfrac{\theta}{1+\theta^2}, & \tau=1 \\ 0, & \text{otherwise} \end{cases}$$

The MA(q) Process

$$y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} = \Theta(L)\varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

where

$$\Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$$

The AR(1) Process

$$y_t = \varphi y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

If covariance stationary:

$$y_t = \varepsilon_t + \varphi\varepsilon_{t-1} + \varphi^2\varepsilon_{t-2} + \ldots$$

Moment structure:

$$E(y_t) = E(\varepsilon_t + \varphi\varepsilon_{t-1} + \varphi^2\varepsilon_{t-2} + ...)$$

$$= E(\varepsilon_t) + \varphi E(\varepsilon_{t-1}) + \varphi^2 E(\varepsilon_{t-2}) + ...$$

$$= 0$$

$$\mathrm{var}(y_t) = \mathrm{var}(\varepsilon_t + \varphi\varepsilon_{t-1} + \varphi^2\varepsilon_{t-2} + ...)$$

$$= \sigma^2 + \varphi^2\sigma^2 + \varphi^4\sigma^2 + ...$$

$$= \sigma^2 \sum_{i=0}^{\infty} \varphi^{2i}$$

$$= \frac{\sigma^2}{1-\varphi^2}$$

$$E(y_t | y_{t-1}) = E((\varphi y_{t-1} + \varepsilon_t) \mid y_{t-1})$$

$$= \varphi E(y_{t-1} | y_{t-1}) + E(\varepsilon_t | y_{t-1})$$

$$= \varphi y_{t-1} + 0$$

$$= \varphi y_{t-1}$$

$$\text{var}(y_t | y_{t-1}) = \text{var}((\varphi y_{t-1} + \varepsilon_t) \mid y_{t-1})$$

$$= \varphi^2 \text{var}(y_{t-1} | y_{t-1}) + \text{var}(\varepsilon_t | y_{t-1})$$

$$= 0 + \sigma^2$$

$$= \sigma^2$$

Autocovariances and autocorrelations:

$$y_t = \varphi y_{t-1} + \varepsilon_t$$

$$y_t y_{t-\tau} = \varphi y_{t-1} y_{t-\tau} + \varepsilon_t y_{t-\tau}$$

For $\tau \geq 1$,

$$\gamma(\tau) = \varphi \gamma(\tau - 1).$$

(Yule-Walker equation) But $\gamma(0) = \frac{\sigma^2}{1-\varphi^2}$. Thus

$$\gamma(\tau) = \varphi^\tau \frac{\sigma^2}{1-\varphi^2}, \ \tau = 0, \ 1, \ 2, \ ....$$

and

$$\rho(\tau) = \varphi^\tau, \ \tau = 0, \ 1, \ 2, \ ....$$

Partial autocorrelations:

$$p(\tau) = \begin{cases} \varphi, & \tau = 1 \\ 0, & \tau > 1 \end{cases}$$

The AR(p) Process

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \dots + \varphi_p y_{t-p} + \varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

The ARMA(1,1) Process

$$y_t = \varphi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

MA representation if invertible:

$$y_t = \frac{(1 + \theta L)}{(1 - \varphi L)} \varepsilon_t$$

AR representation of covariance stationary:

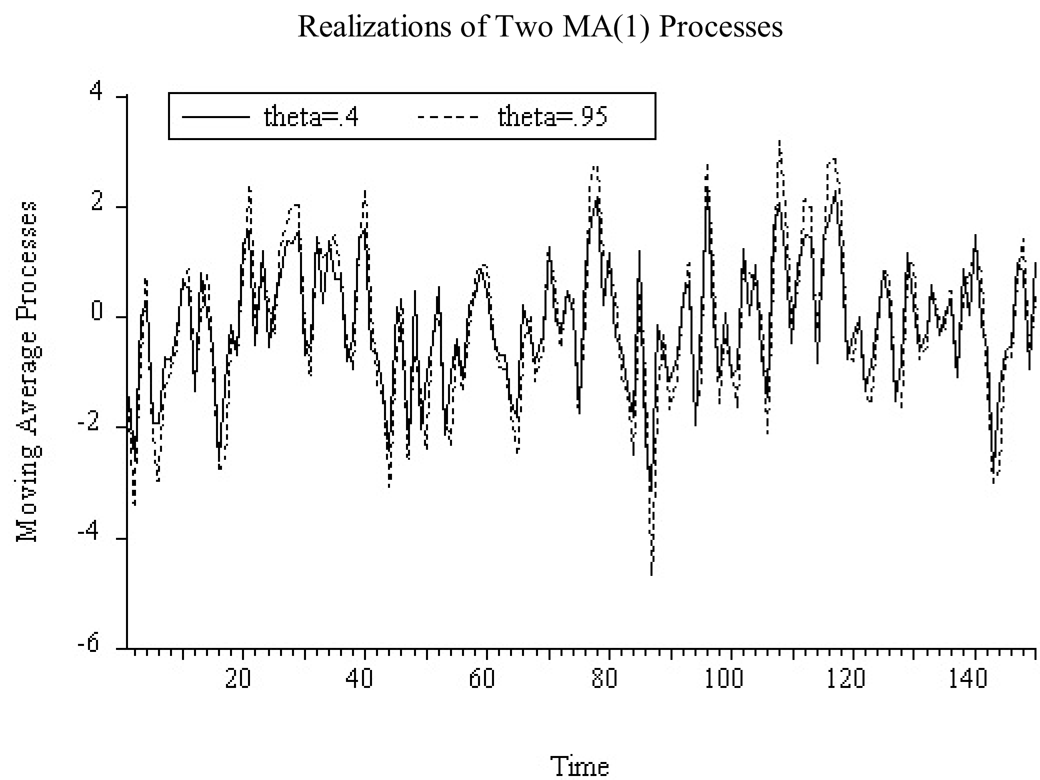
$$\frac{(1 - \varphi L)}{(1 + \theta L)} y_t = \varepsilon_t$$

The ARMA(p,q) Process

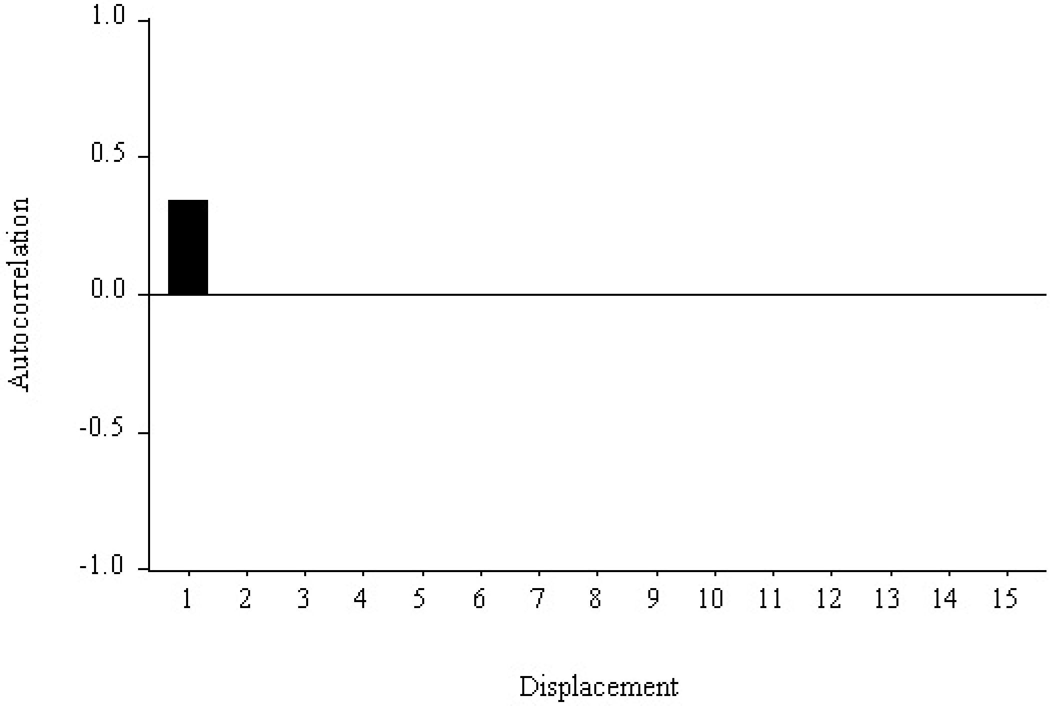
$$y_t = \varphi_1 y_{t-1} + \ldots + \varphi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

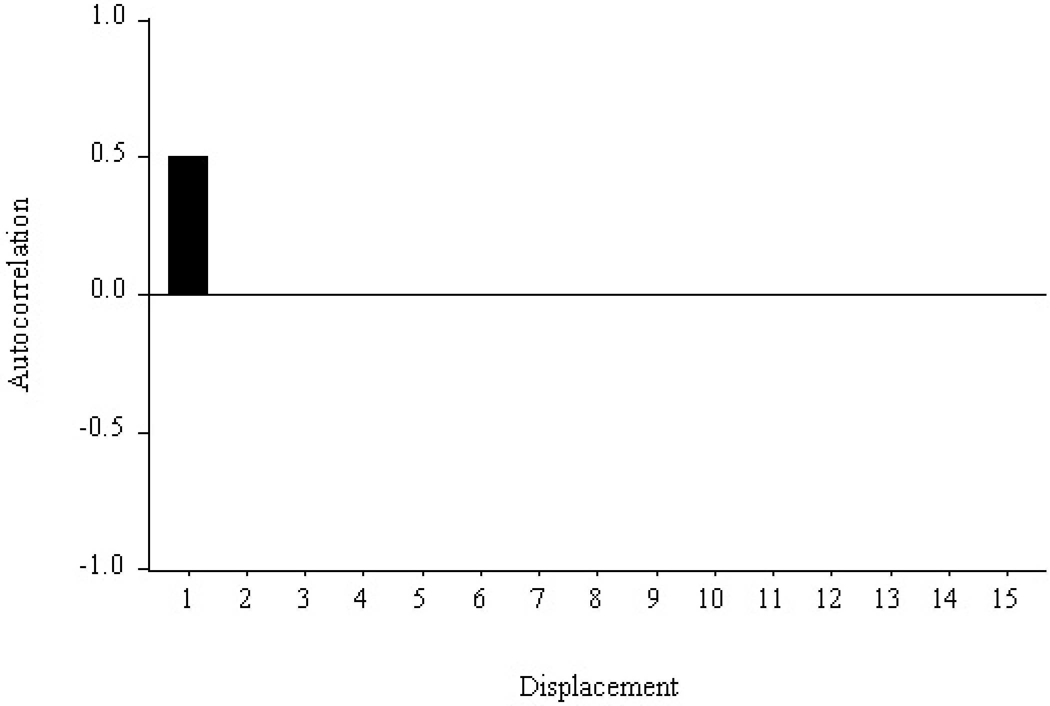
$$\Phi(L) y_t = \Theta(L) \varepsilon_t$$

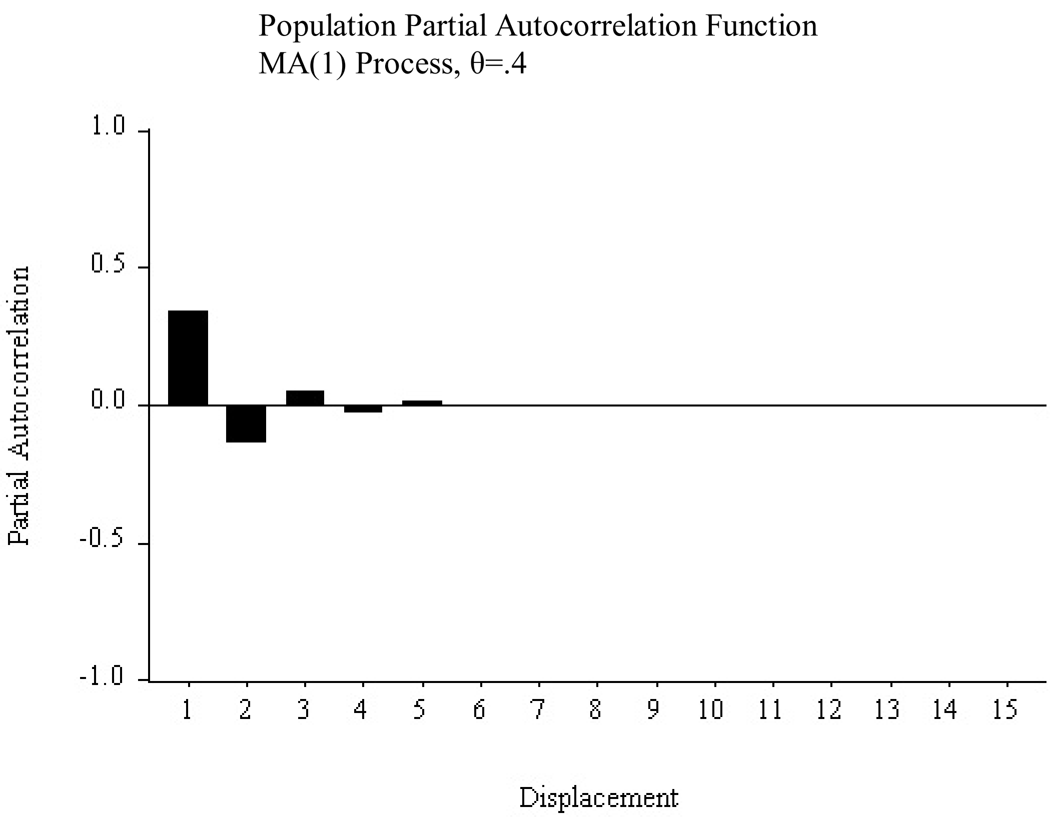

Realizations of Two MA(1) Processes
theta=.4
theta=.95
Moving Average Processes
4
2
0
-2
-4
-6
20
40
60
80
100
120
140
Time

Population Autocorrelation Function
MA(1) Process, θ=.4

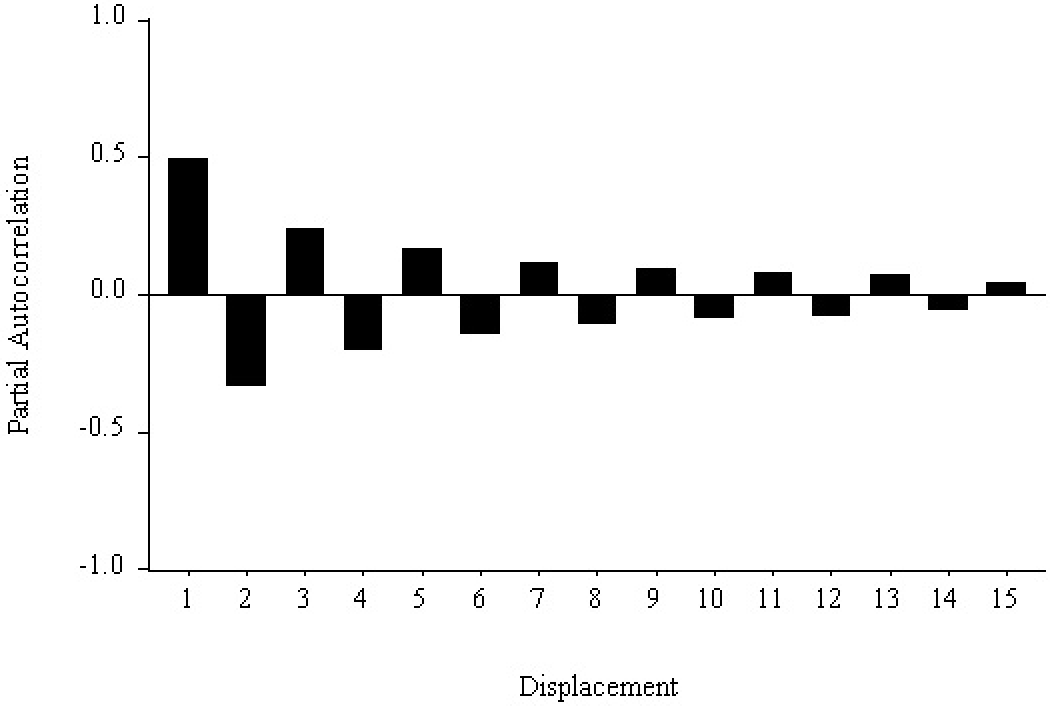


Population Autocorrelation Function
MA(1) Process, θ=.95

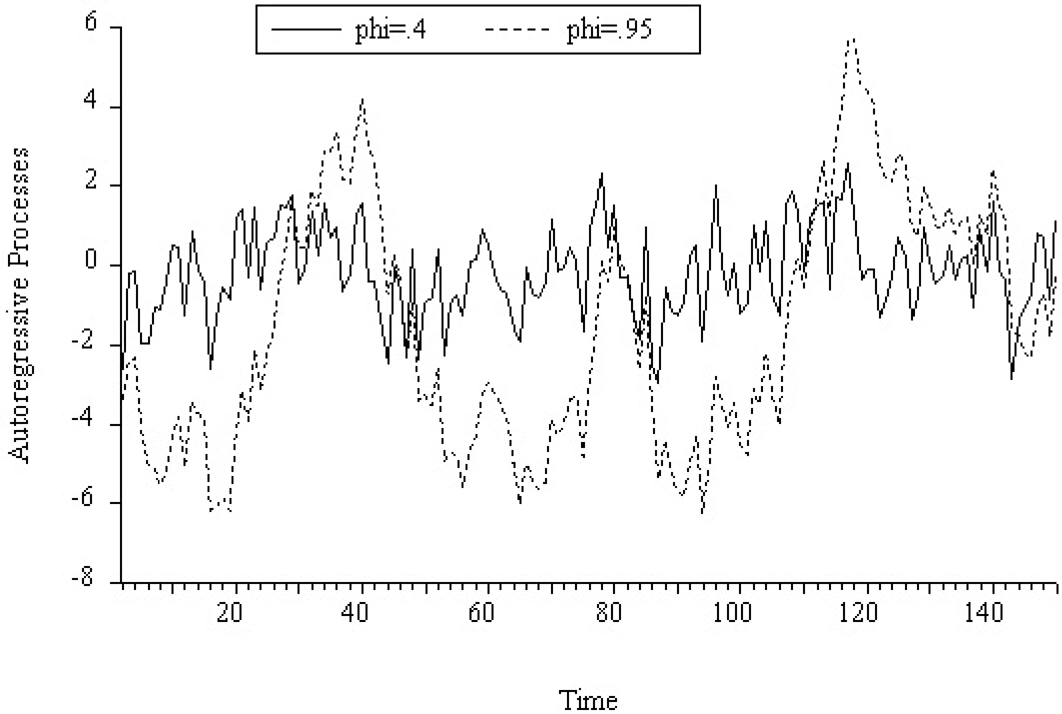



Population Partial Autocorrelation Function
MA(1) Process, θ=.4
Partial Autocorrelation
1.0
0.5
0.0
-0.5
-1.0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Displacement

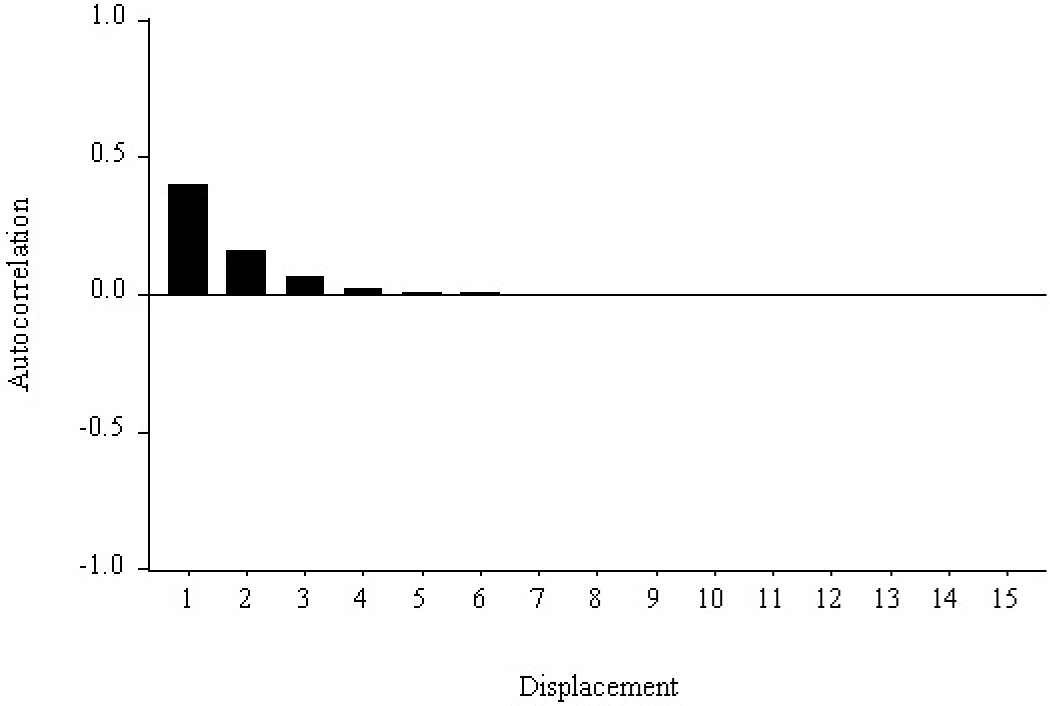
Population Partial Autocorrelation Function
MA(1) Process, θ=.95



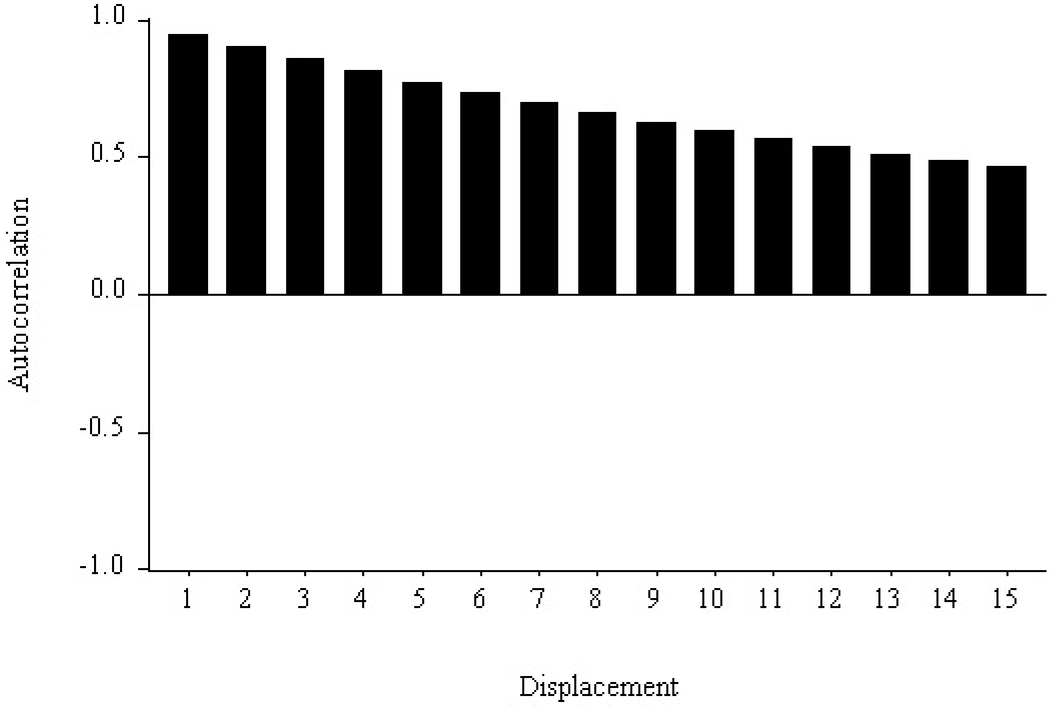
Realizations of Two AR(1) Processes



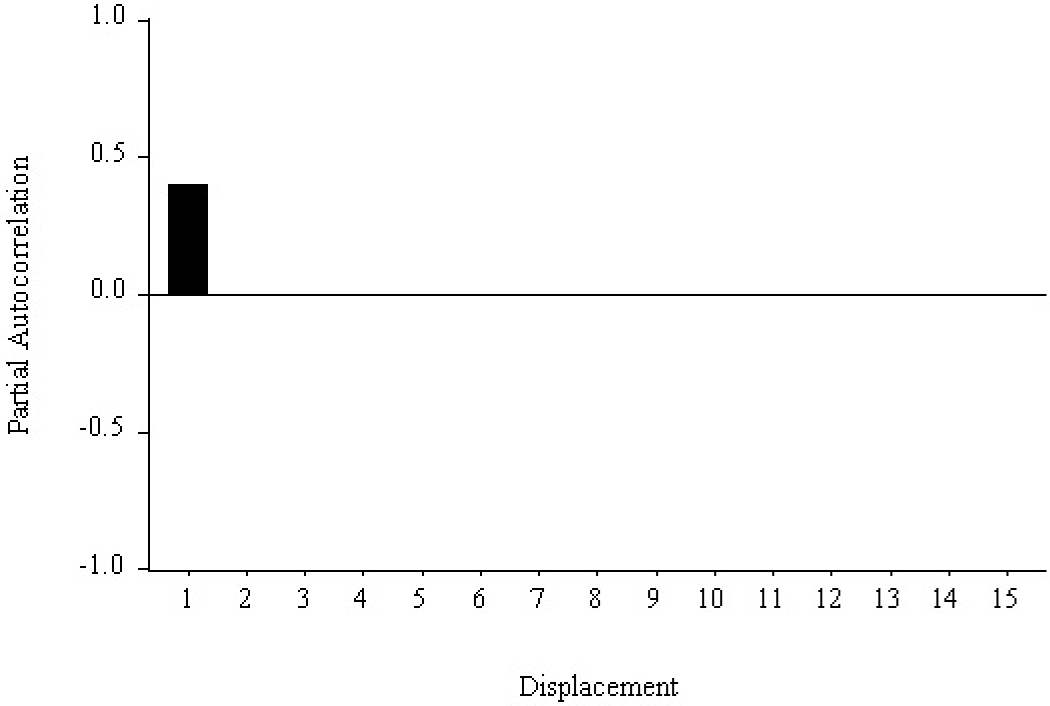
Population Autocorrelation Function
AR(1) Process, φ=.4



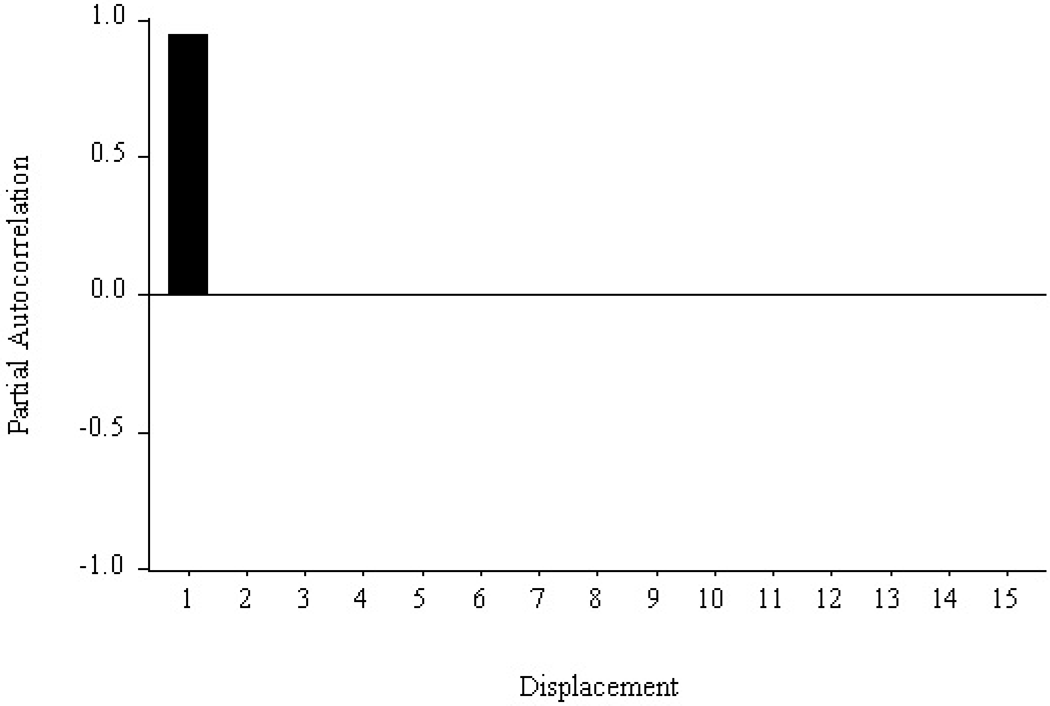
Population Autocorrelation Function
AR(1) Process, φ=.95

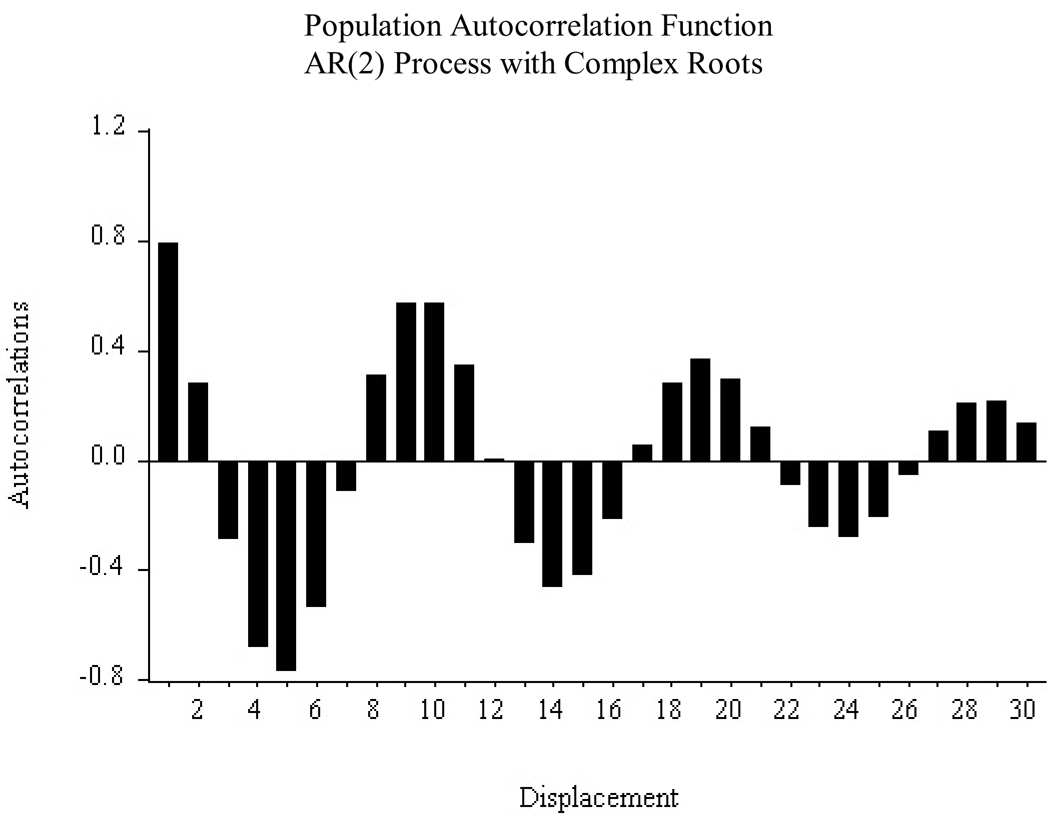


Population Partial Autocorrelation Function
AR(1) Process, φ=.4



Population Partial Autocorrelation Function
AR(1) Process, φ=.95




Population Autocorrelation Function
AR(2) Process with Complex Roots
Autocorrelations
1.2
0.8
0.4
0.0
-0.4
-0.8
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
Displacement

Employment
MA(4) Model

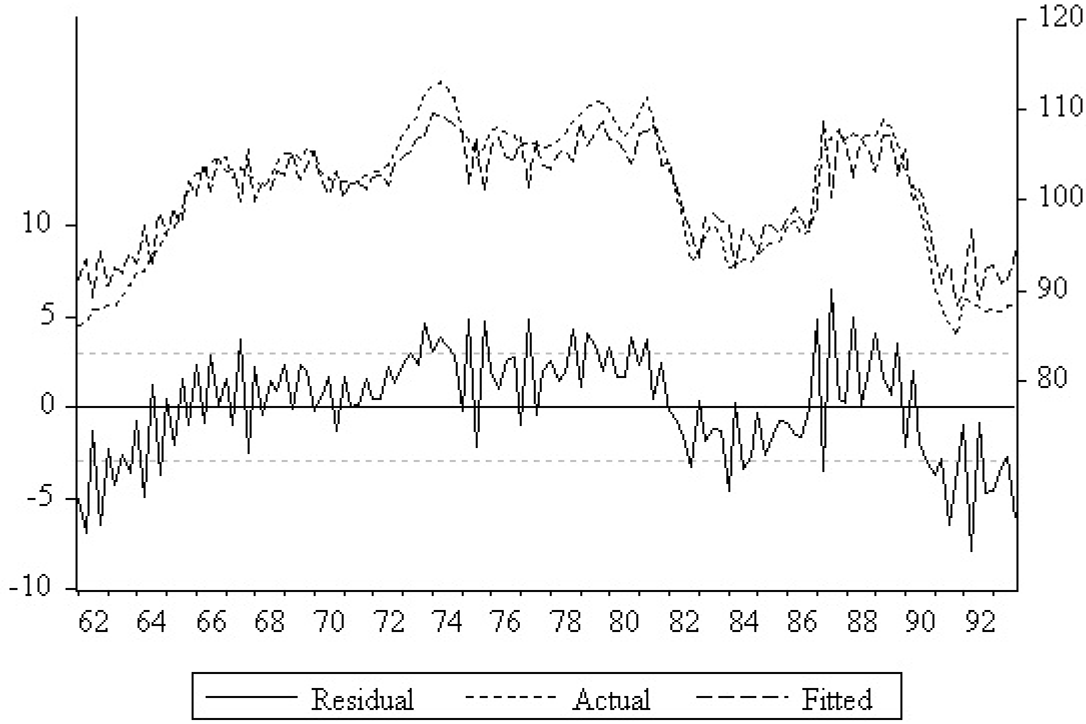
LS // Dependent Variable is CANEMP
Sample: 1962:1 1993:4
Included observations: 128
Convergence achieved after 49 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	100.5438	0.843322	119.2234	0.0000
MA(1)	1.587641	0.063908	24.84246	0.0000
MA(2)	0.994369	0.089995	11.04917	0.0000
MA(3)	-0.020305	0.046550	-0.436189	0.6635
MA(4)	-0.298387	0.020489	-14.56311	0.0000

R-squared	0.849951	Mean dependent var	101.0176
Adjusted R-squared	0.845071	S.D. dependent var	7.499163
S.E. of regression	2.951747	Akaike info criterion	2.203073
Sum squared resid	1071.676	Schwarz criterion	2.314481
Log likelihood	-317.6208	F-statistic	174.1826
Durbin-Watson stat	1.246600	Prob(F-statistic)	0.000000

Inverted MA Roots	.41	-.56+.72i	-.56 -.72i	-.87

Employment
MA(4) Model
Residual Plot

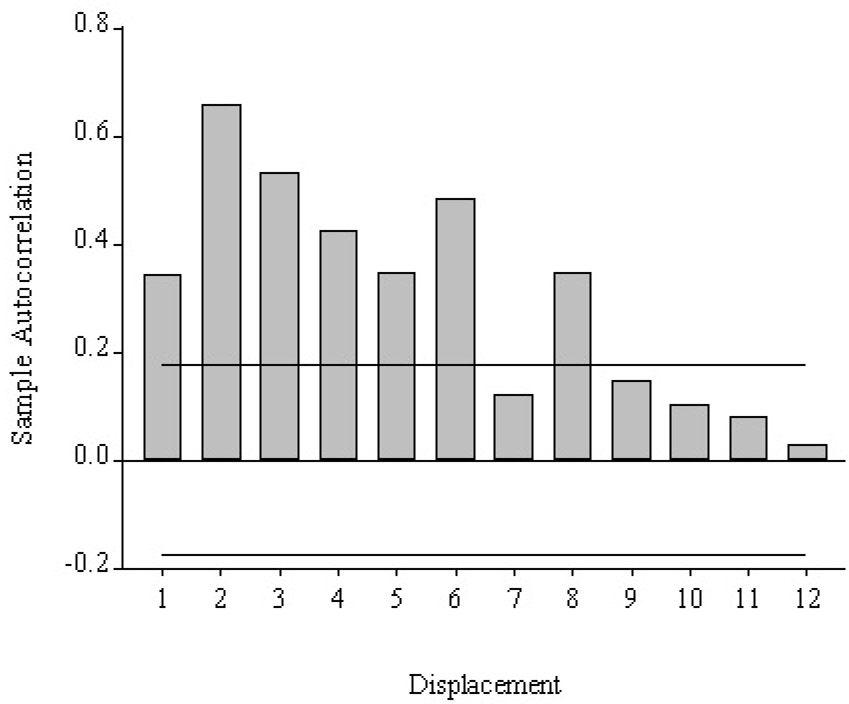


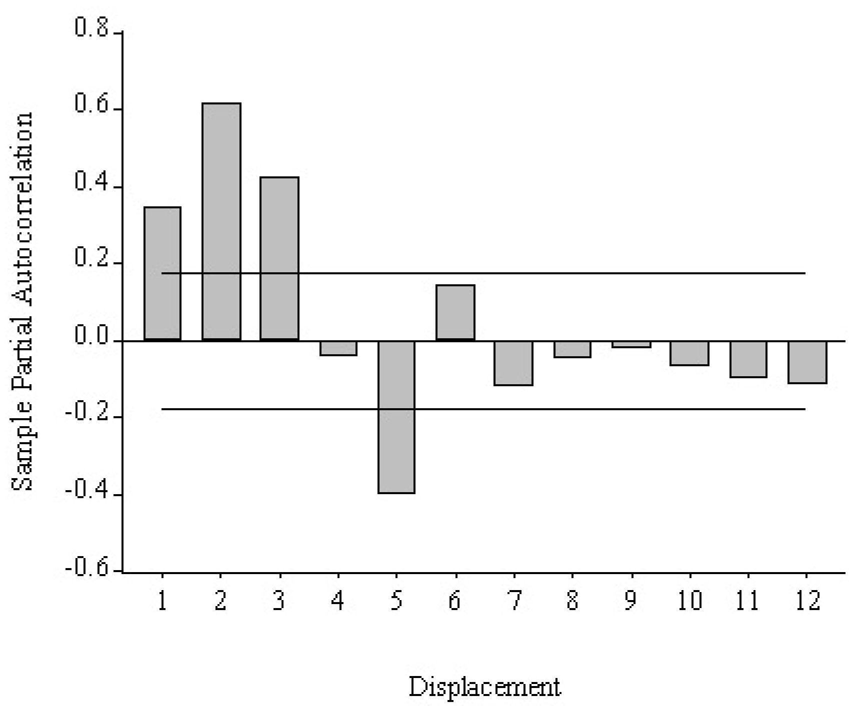
Employment
MA(4) Model
Residual Correlogram

Sample: 1962:1 1993:4
Included observations: 128
Q-statistic probabilities adjusted for 4 ARMA term(s)

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.345	0.345	.088	15.614	
2	0.660	0.614	.088	73.089	
3	0.534	0.426	.088	111.01	
4	0.427	-0.042	.088	135.49	
5	0.347	-0.398	.088	151.79	0.000
6	0.484	0.145	.088	183.70	0.000
7	0.121	-0.118	.088	185.71	0.000
8	0.348	-0.048	.088	202.46	0.000
9	0.148	-0.019	.088	205.50	0.000
10	0.102	-0.066	.088	206.96	0.000
11	0.081	-0.098	.088	207.89	0.000
12	0.029	-0.113	.088	208.01	0.000

Employment
MA(4) Model
Residual Sample Autocorrelation and Partial Autocorrelation Functions,
With Plus or Minus Two Standard Error Bands





Employment
AR(2) Model

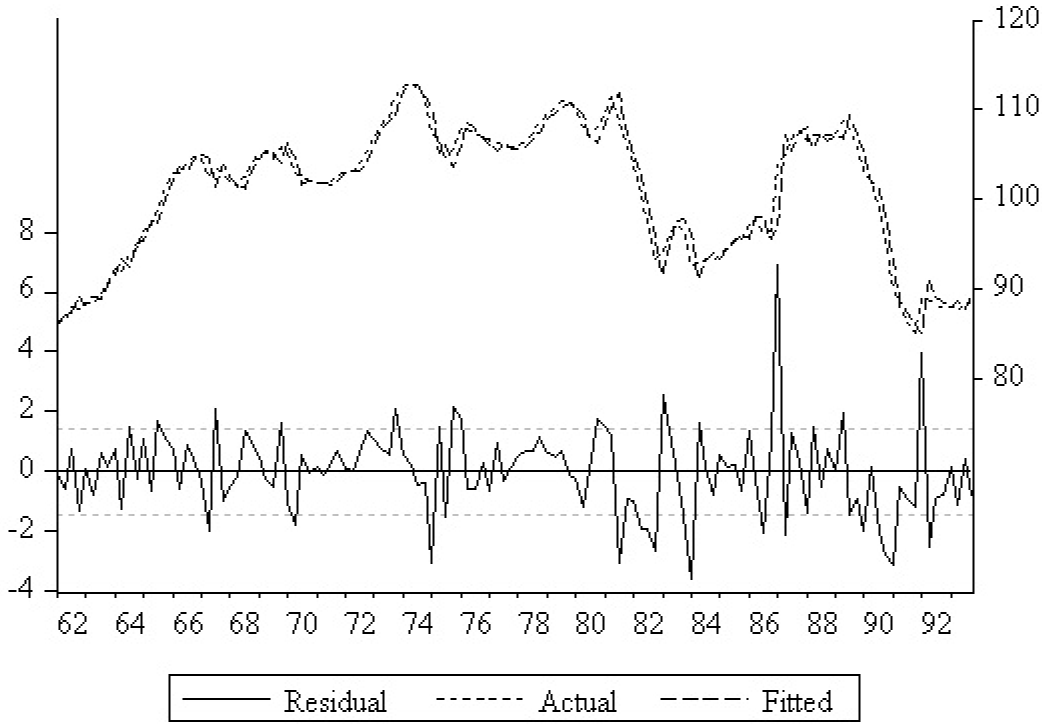
LS // Dependent Variable is CANEMP
Sample: 1962:1 1993:4
Included observations: 128
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	101.2413	3.399620	29.78017	0.0000
AR(1)	1.438810	0.078487	18.33188	0.0000
AR(2)	-0.476451	0.077902	-6.116042	0.0000

R-squared	0.963372	Mean dependent var	101.0176
Adjusted R-squared	0.962786	S.D. dependent var	7.499163
S.E. of regression	1.446663	Akaike info criterion	0.761677
Sum squared resid	261.6041	Schwarz criterion	0.828522
Log likelihood	-227.3715	F-statistic	1643.837
Durbin-Watson stat	2.067024	Prob(F-statistic)	0.000000

Inverted AR Roots	.92	.52

Employment
AR(2) Model
Residual Plot

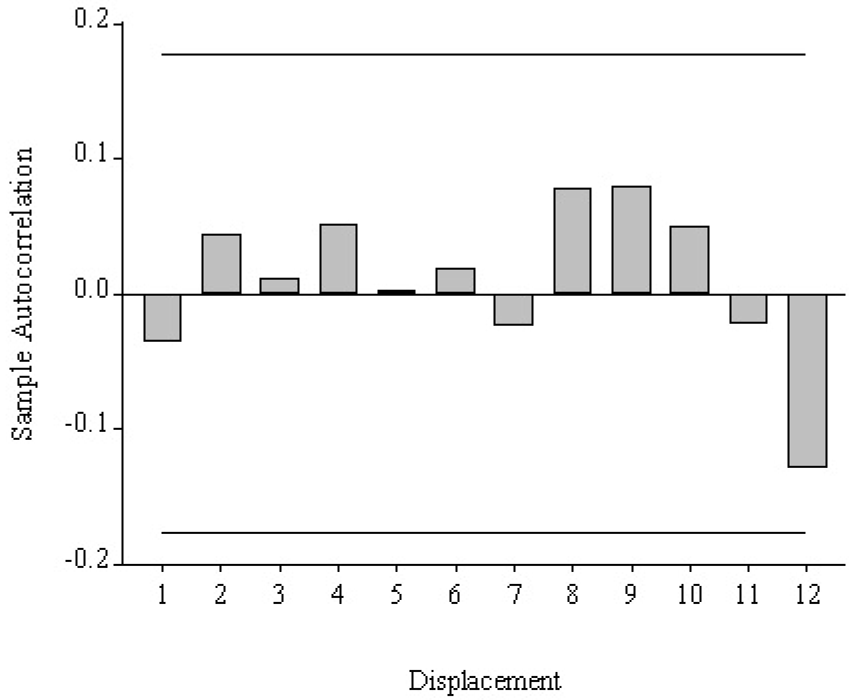


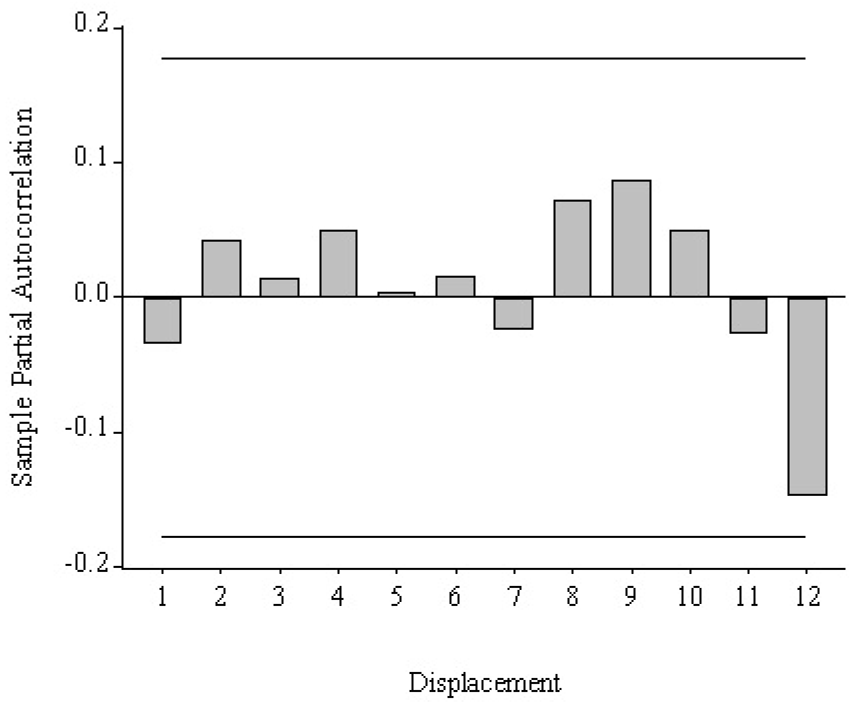
Employment
AR(2) Model
Residual Correlogram

Sample: 1962:1 1993:4
Included observations: 128
Q-statistic probabilities adjusted for 2 ARMA term(s)

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	-0.035	-0.035	.088	0.1606	
2	0.044	0.042	.088	0.4115	
3	0.011	0.014	.088	0.4291	0.512
4	0.051	0.050	.088	0.7786	0.678
5	0.002	0.004	.088	0.7790	0.854
6	0.019	0.015	.088	0.8272	0.935
7	-0.024	-0.024	.088	0.9036	0.970
8	0.078	0.072	.088	1.7382	0.942
9	0.080	0.087	.088	2.6236	0.918
10	0.050	0.050	.088	2.9727	0.936
11	-0.023	-0.027	.088	3.0504	0.962
12	-0.129	-0.148	.088	5.4385	0.860

Employment
AR(2) Model
Residual Sample Autocorrelation and Partial Autocorrelation Functions,
With Plus or Minus Two Standard Error Bands





Employment
AIC Values
Various ARMA Models

				MA Order		
		0	1	2	3	4
	0		2.86	2.32	2.47	2.20
	1	1.01	.83	.79	.80	.81
AR Order	2	.762	.77	.78	.80	.80
	3	.77	.761	.77	.78	.79
	4	.79	.79	.77	.79	.80

Employment
SIC Values
Various ARMA Models

				MA Order		
		0	1	2	3	4
	0		2.91	2.38	2.56	2.31
	1	1.05	.90	.88	.91	.94
AR Order	2	.83	.86	.89	.92	.96
	3	.86	.87	.90	.94	.96
	4	.90	.92	.93	.97	1.00

Employment
ARMA(3,1) Model

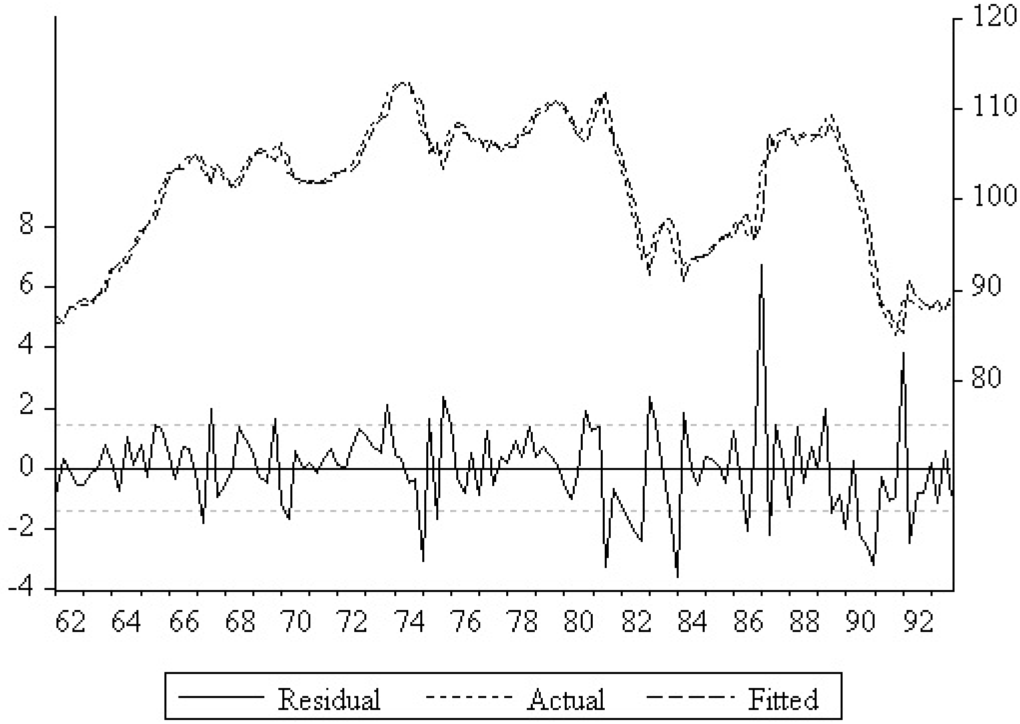
LS // Dependent Variable is CANEMP
Sample: 1962:1 1993:4
Included observations: 128
Convergence achieved after 17 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	101.1378	3.538602	28.58130	0.0000
AR(1)	0.500493	0.087503	5.719732	0.0000
AR(2)	0.872194	0.067096	12.99917	0.0000
AR(3)	-0.443355	0.080970	-5.475560	0.0000
MA(1)	0.970952	0.035015	27.72924	0.0000

R-squared	0.964535	Mean dependent var	101.0176
Adjusted R-squared	0.963381	S.D. dependent var	7.499163
S.E. of regression	1.435043	Akaike info criterion	0.760668
Sum squared resid	253.2997	Schwarz criterion	0.872076
Log likelihood	-225.3069	F-statistic	836.2912
Durbin-Watson stat	2.057302	Prob(F-statistic)	0.000000

Inverted AR Roots	.93	.51	-.94
Inverted MA Roots	-.97		

Employment
ARMA(3,1) Model
Residual Plot

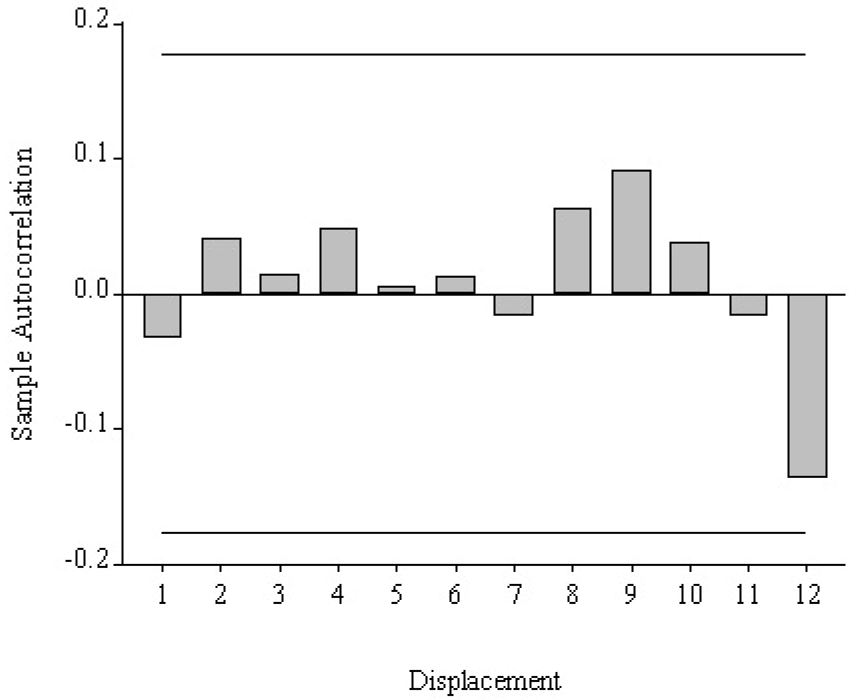


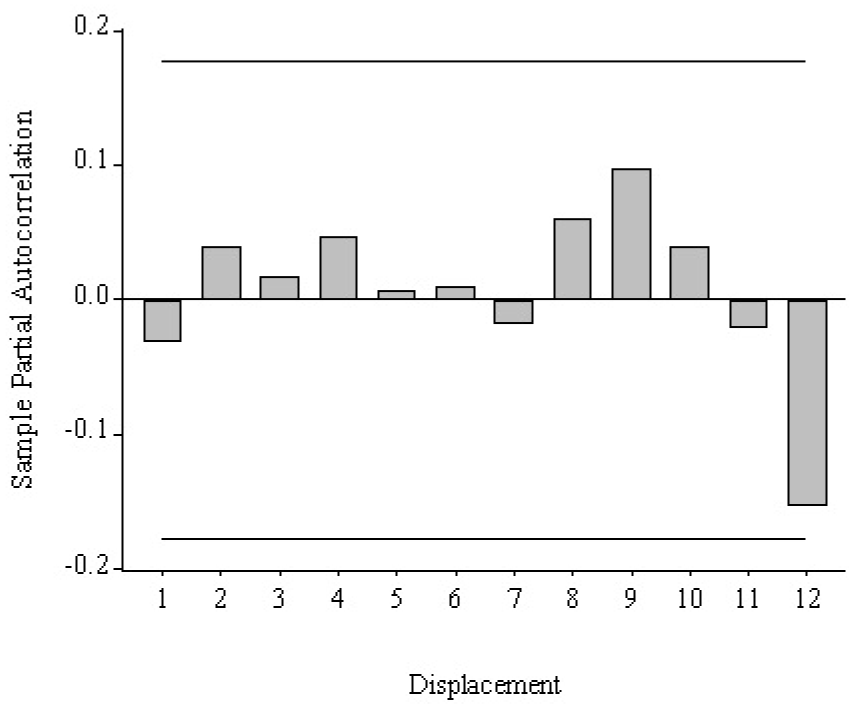
Employment
ARMA(3,1) Model
Residual Correlogram

Sample: 1962:1 1993:4
Included observations: 128
Q-statistic probabilities adjusted for 4 ARMA term(s)

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	-0.032	-0.032	.09	0.1376	
2	0.041	0.040	.09	0.3643	
3	0.014	0.017	.09	0.3904	
4	0.048	0.047	.09	0.6970	
5	0.006	0.007	.09	0.7013	0.402
6	0.013	0.009	.09	0.7246	0.696
7	-0.017	-0.019	.09	0.7650	0.858
8	0.064	0.060	.09	1.3384	0.855
9	0.092	0.097	.09	2.5182	0.774
10	0.039	0.040	.09	2.7276	0.842
11	-0.016	-0.022	.09	2.7659	0.906
12	-0.137	-0.153	.09	5.4415	0.710

Employment
ARMA(3,1) Model
Residual Sample Autocorrelation and Partial Autocorrelation Functions,
With Plus or Minus Two Standard Error Bands





Forecasting Cycles

$$\Omega_T = \{y_T, y_{T-1}, y_{T-2}, ...\},$$

$$\Omega_T = \{\varepsilon_T, \varepsilon_{T-1}, \varepsilon_{T-2}, ...\}.$$

Optimal Point Forecasts for Infinite-Order Moving Averages

$$y_t = \sum_{i=0}^{\infty} b_i \varepsilon_{t-i},$$

where $\varepsilon_t \sim WN(0, \sigma^2)$, $b_0 = 1$, and $\sigma^2 \sum_{i=0}^{\infty} b_i^2 < \infty$.

$$y_{T+h} = \varepsilon_{T+h} + b_1 \varepsilon_{T+h-1} + ... + b_h \varepsilon_T + b_{h+1} \varepsilon_{T-1} + ...$$

$$y_{T+h,T} = b_h \varepsilon_T + b_{h+1} \varepsilon_{T-1} + ...$$

$$e_{T+h,T} = (y_{T+h} - y_{T+h,T}) = \sum_{i=0}^{h-1} b_i \varepsilon_{T+h-i},$$

$$\sigma_h^2 = \sigma^2 \sum_{i=0}^{h-1} b_i^2.$$

Interval and Density Forecasts

$$y_{T+h} = y_{T+h,T} + e_{T+h,T}.$$

95% h-step-ahead interval forecast:

$$y_{T+h,T} \pm 1.96\sigma_h .$$

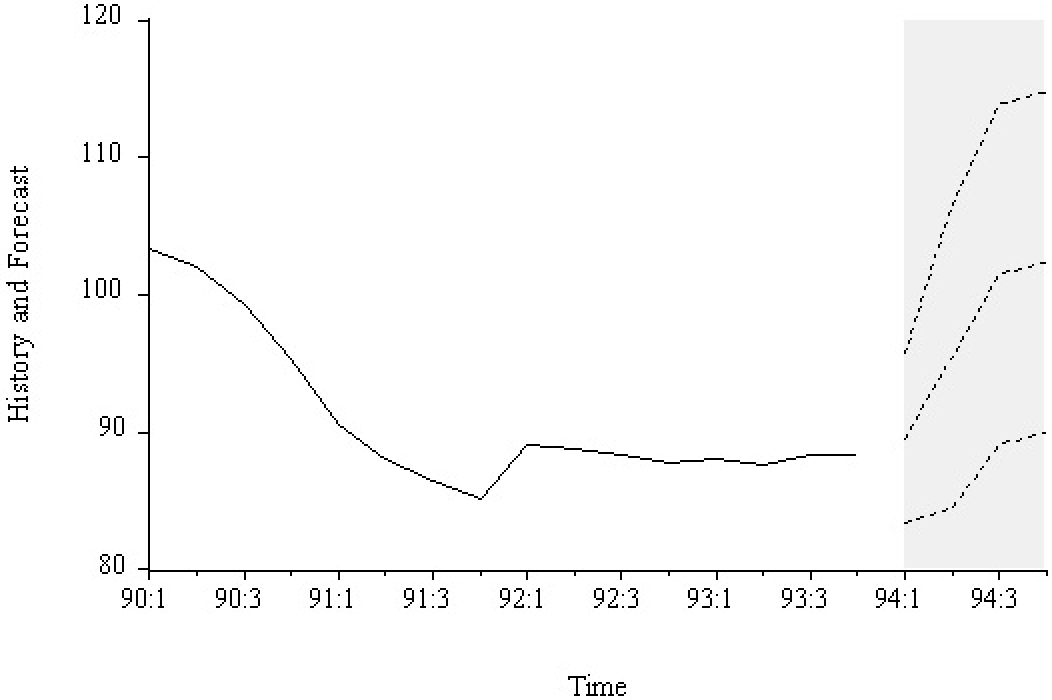
h-step-ahead density forecast:

$$N(y_{T+h,T}, \sigma_h^2)$$

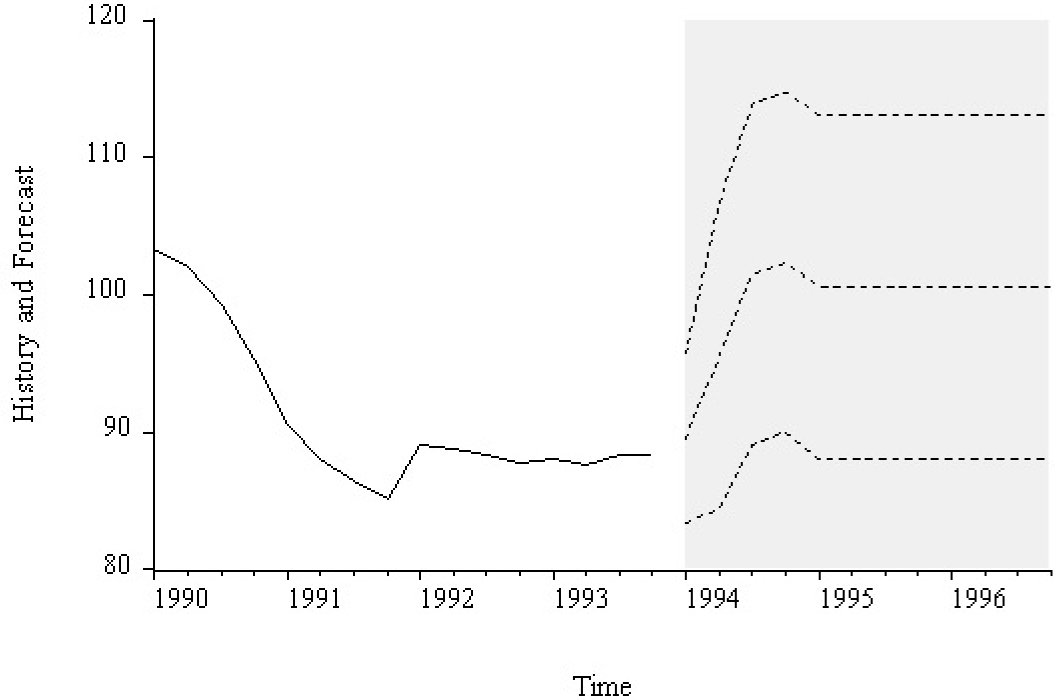
Making the Forecasts Operational

The Chain Rule of Forecasting

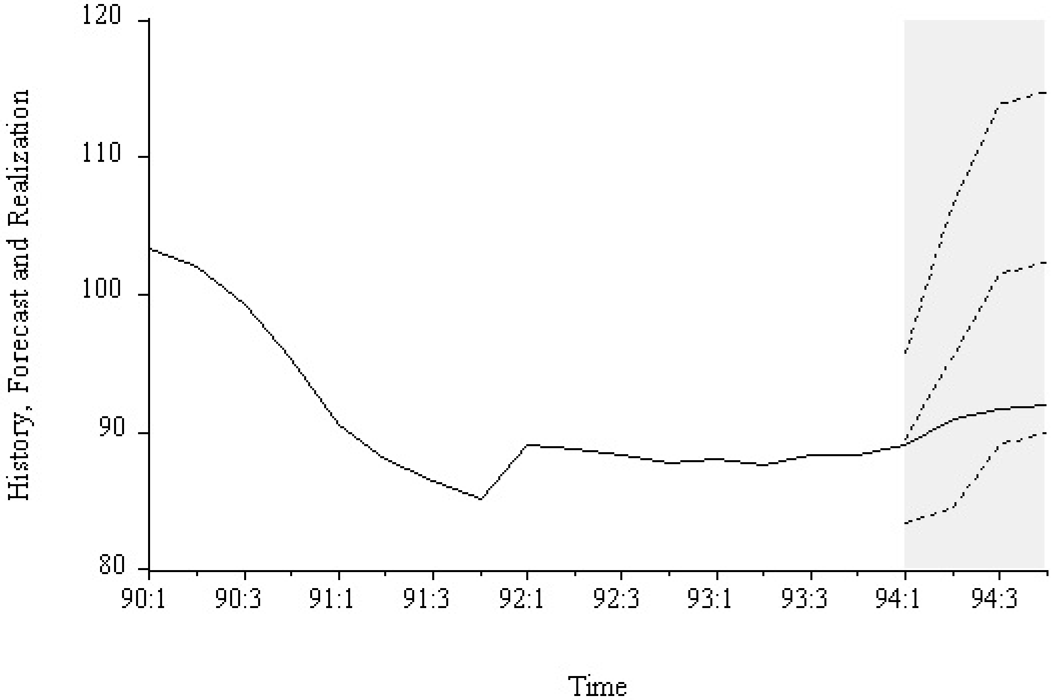
Employment History and Forecast
MA(4) Model



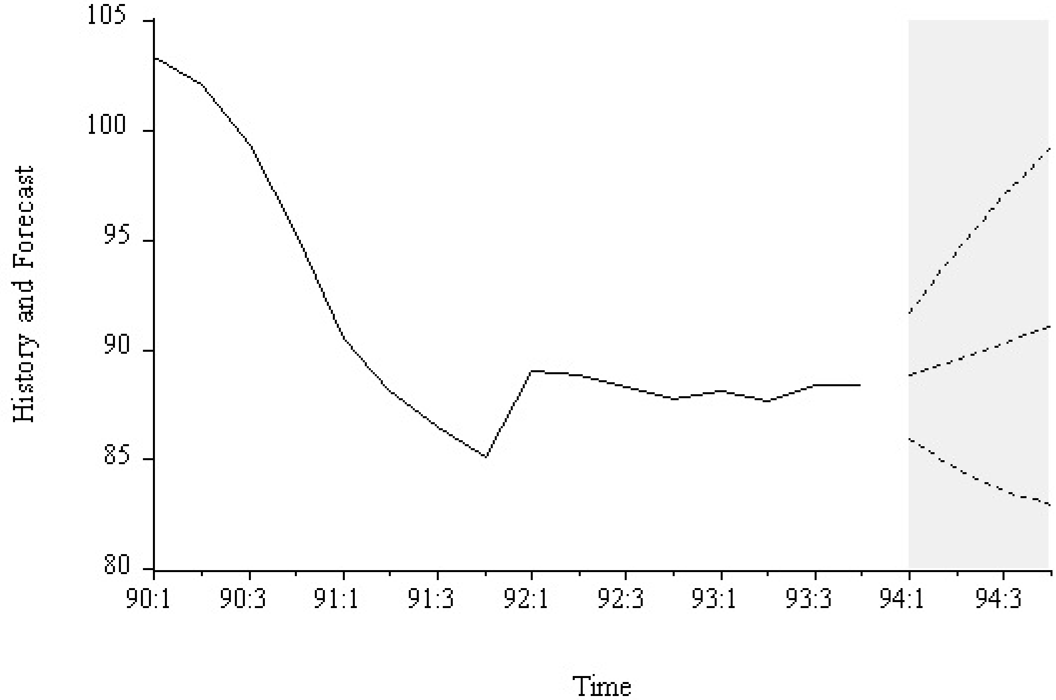
Employment History and Long-Horizon Forecast
MA(4) Model

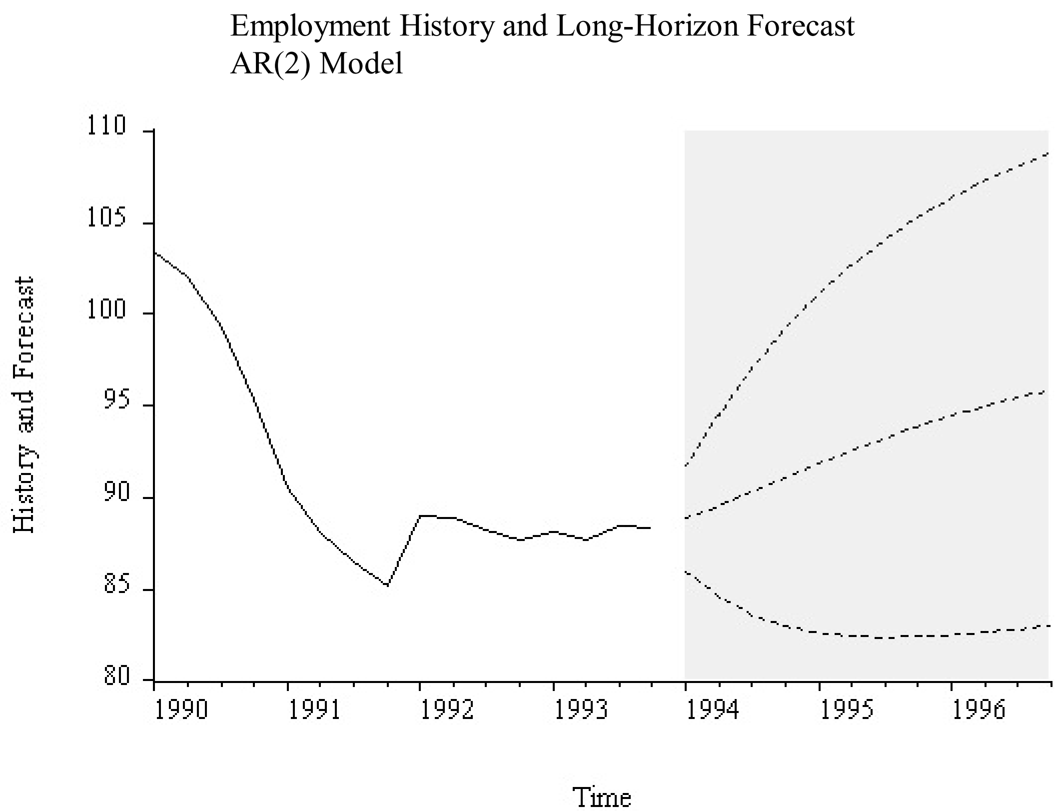


Employment History, Forecast and Realization
MA(4) Model



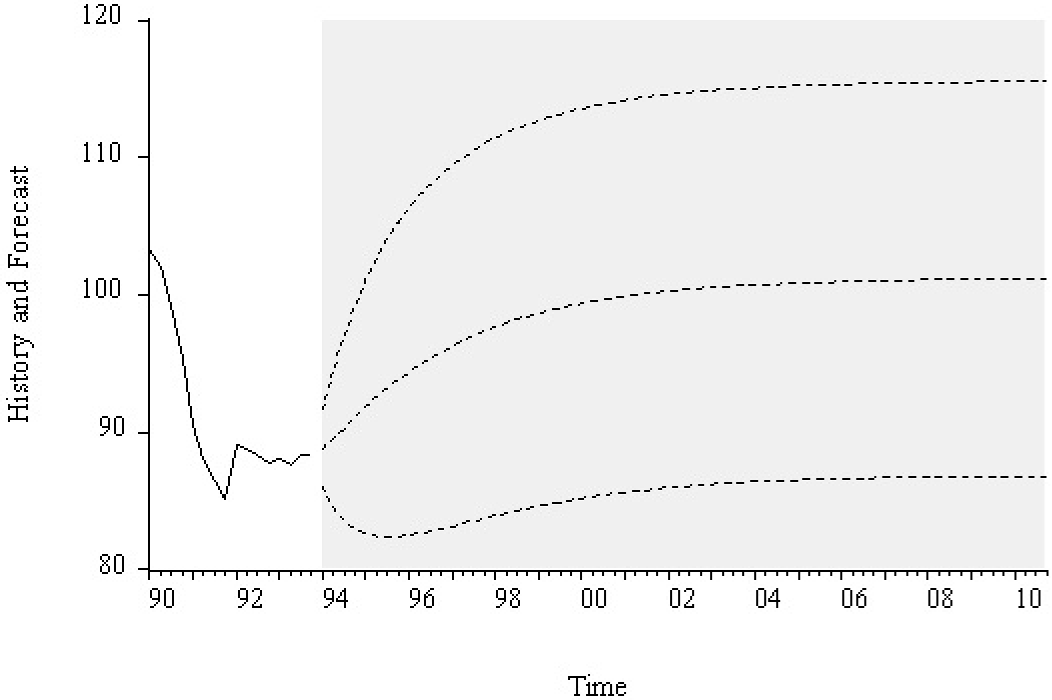
Employment History and Forecast
AR(2) Model

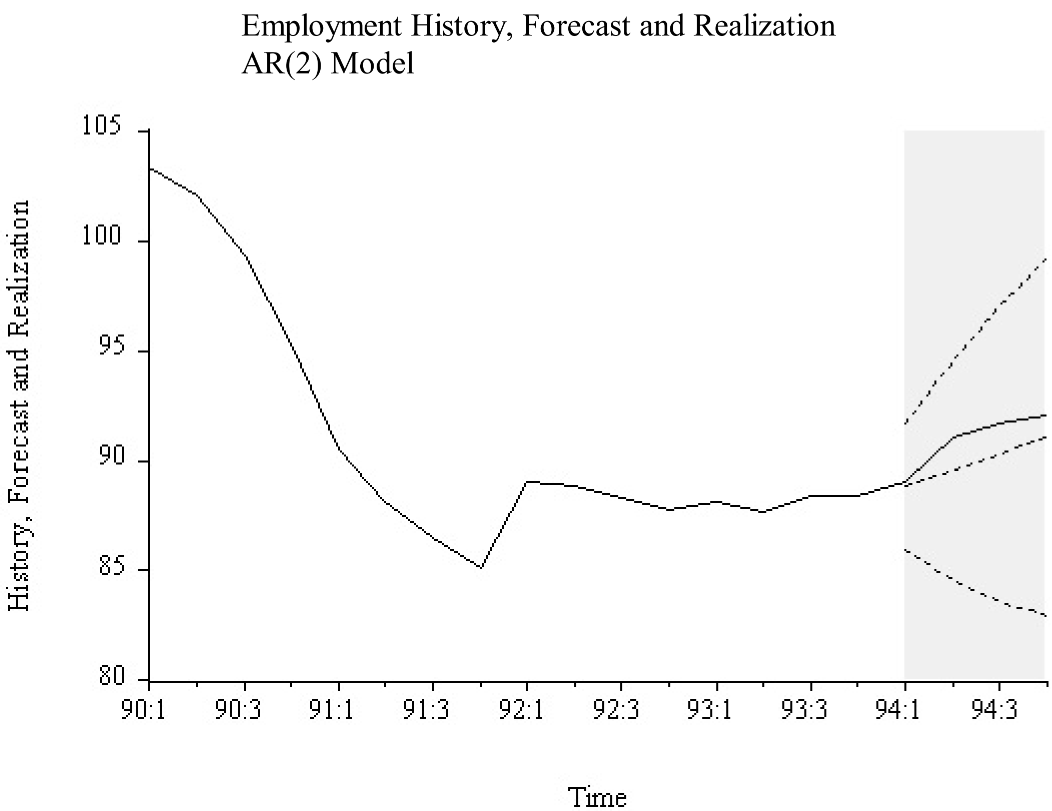




Employment History and Long-Horizon Forecast
AR(2) Model
110
105
100
95
90
85
80
History and Forecast
1990
1991
1992
1993
1994
1995
1996
Time

Employment History and Very Long-Horizon Forecast
AR(2) Model




Employment History, Forecast and Realization
AR(2) Model
105
100
95
90
85
80
History, Forecast and Realization
90:1
90:3
91:1
91:3
92:1
92:3
93:1
93:3
94:1
94:3
Time

Putting it all Together:
A Forecasting Model
with Trend, Seasonal and Cyclical Components

The full model:

$$y_t = T_t(\theta) + \sum_{i=1}^{s} \gamma_i D_{it} + \sum_{i=1}^{v_1} \delta_i^{HD} HDV_{it} + \sum_{i=1}^{v_2} \delta_i^{TD} TDV_{it} + \varepsilon_t$$

$$\Phi(L)\varepsilon_t = \Theta(L)v_t$$

$$\Phi(L) = 1 - \varphi_1 L - \ldots - \varphi_p L^p$$

$$\Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$$

$$v_t \sim WN(0, \sigma^2).$$

Point Forecasting

$$y_{T+h} = T_{T+h}(\theta) + \sum_{i=1}^{s} \gamma_i D_{i,T+h} + \sum_{i=1}^{v_1} \delta_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{v_2} \delta_i^{TD} TDV_{i,T+h} + \varepsilon_{T+h}.$$

$$y_{T+h,T} = T_{T+h}(\theta) + \sum_{i=1}^{s} \gamma_i D_{i,T+h} + \sum_{i=1}^{v_1} \delta_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{v_2} \delta_i^{TD} TDV_{i,T+h} + \varepsilon_{T+h,T}.$$

$$\hat{y}_{T+h,T} = T_{T+h}(\hat{\theta}) + \sum_{i=1}^{s} \hat{\gamma}_i D_{i,T+h} + \sum_{i=1}^{v_1} \hat{\delta}_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{v_2} \hat{\delta}_i^{TD} TDV_{i,T+h} + \hat{\varepsilon}_{T+h,T}.$$

Interval Forecasting

$$\hat{y}_{T+h,T} \pm z_{\alpha/2}\hat{\sigma}_h$$

e.g.: (95% interval) $\hat{y}_{T+h,T} \pm 1.96\hat{\sigma}_h$

Density Forecasting

$$N(\hat{y}_{T+h,T},\ \hat{\sigma}_h^2)$$

Assessing the Stability of Forecasting Models: Recursive Parameter Estimation and Recursive Residuals

At each t, t = k, ... ,T-1, compute:

Recursive parameter est. and forecast: $\hat{y}_{t+1,t} = \sum_{i=1}^{k} \hat{\beta}_{i,t} x_{i,t+1}$

Recursive residual: $\hat{e}_{t+1,t} = y_{t+1} - \hat{y}_{t+1,t}$

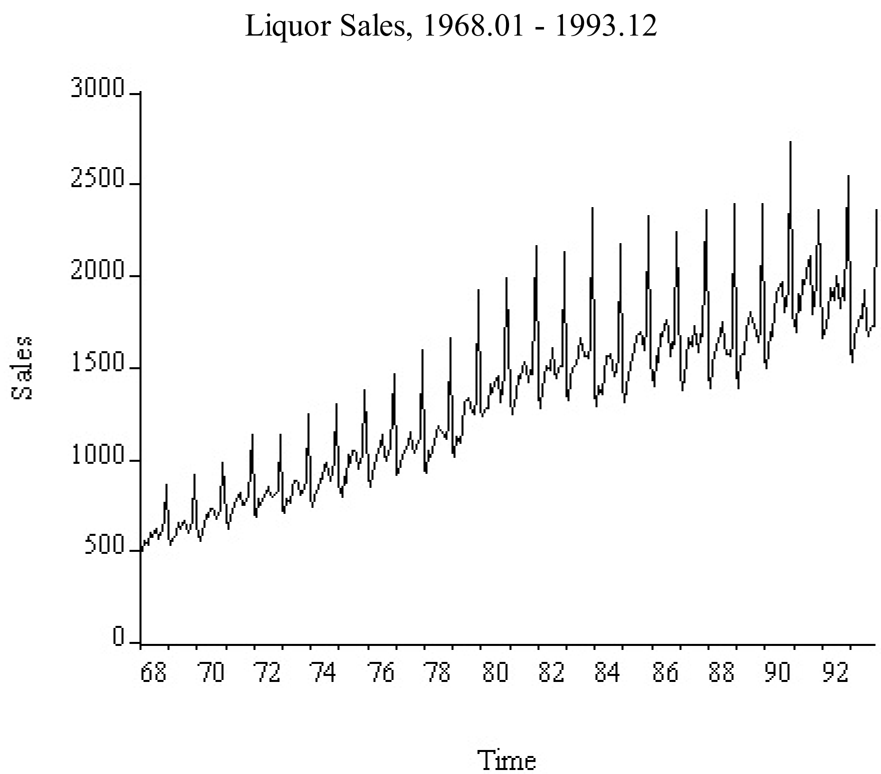
If all is well: $\hat{e}_{t+1,t} \sim N(0, \sigma^2 r_t)$

Sequence of 1-step forecast tests: $\hat{e}_{t+1,t} \pm 2\hat{\sigma}\sqrt{r_t}$

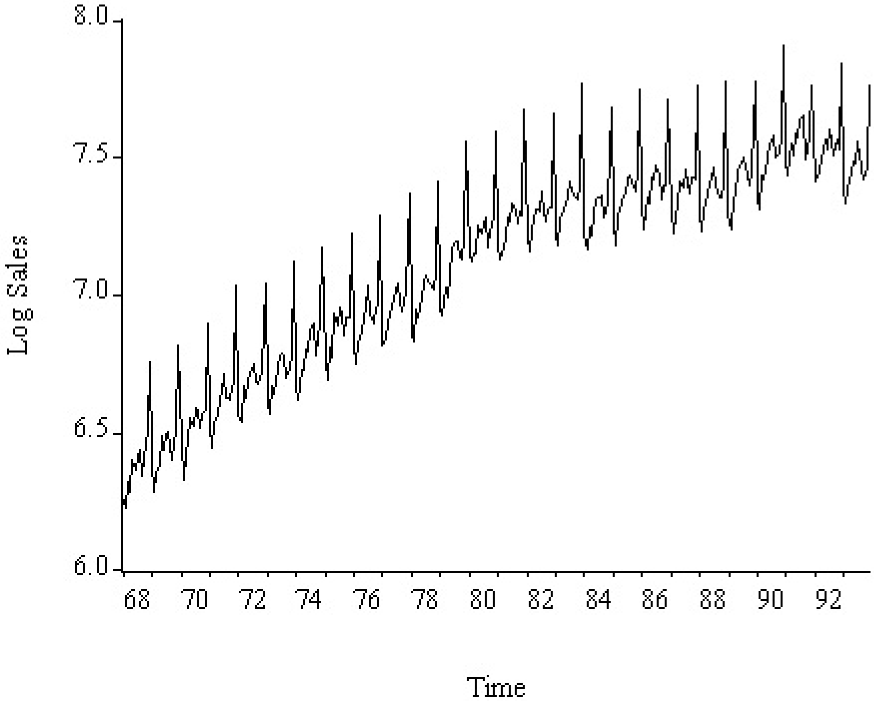
Standardized recursive residuals: $w_{t+1,t} \equiv \frac{\hat{e}_{t+1,t}}{\sigma \sqrt{r_t}}$

If all is well: $w_{t+1,t} \overset{iid}{\sim} N(0, 1)$

$$CUSUM_t \equiv \sum_{\tau=k}^{t} w_{\tau+1,\tau}, \; t = k, ..., T-1$$


Liquor Sales, 1968.01 - 1993.12
3000
2500
2000
1500
1000
500
0
Sales
68
70
72
74
76
78
80
82
84
86
88
90
92
Time

Log Liquor Sales, 1968.01 - 1993.12



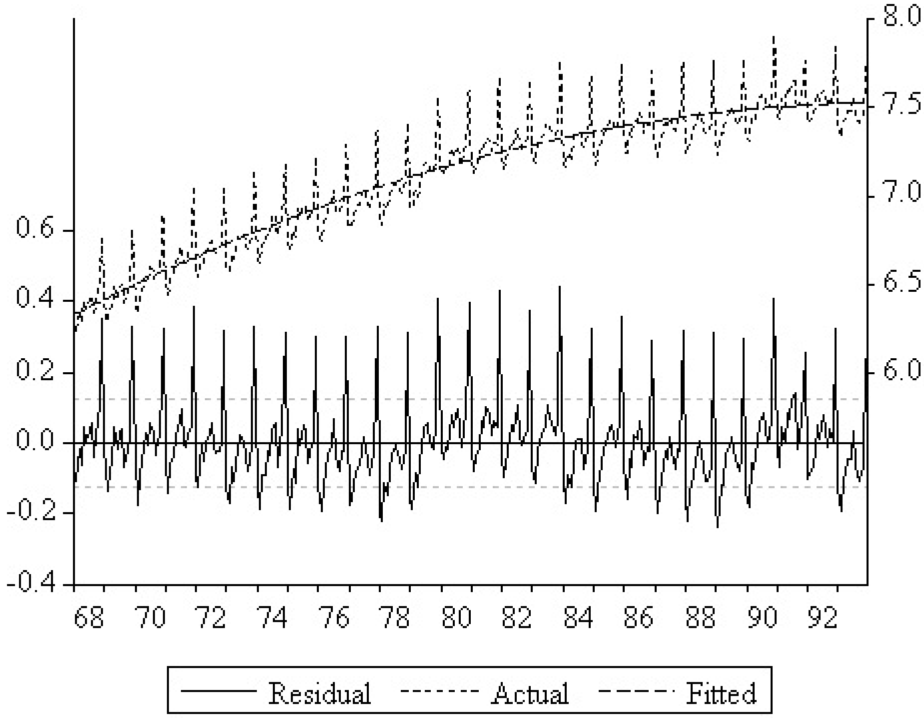
Log Liquor Sales
Quadratic Trend Regression

LS // Dependent Variable is LSALES
Sample: 1968:01 1993:12
Included observations: 312

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.237356	0.024496	254.6267	0.0000
TIME	0.007690	0.000336	22.91552	0.0000
TIME2	-1.14E-05	9.74E-07	-11.72695	0.0000

R-squared	0.892394	Mean dependent var	7.112383
Adjusted R-squared	0.891698	S.D. dependent var	0.379308
S.E. of regression	0.124828	Akaike info criterion	-4.152073
Sum squared resid	4.814823	Schwarz criterion	-4.116083
Log likelihood	208.0146	F-statistic	1281.296
Durbin-Watson stat	1.752858	Prob(F-statistic)	0.000000

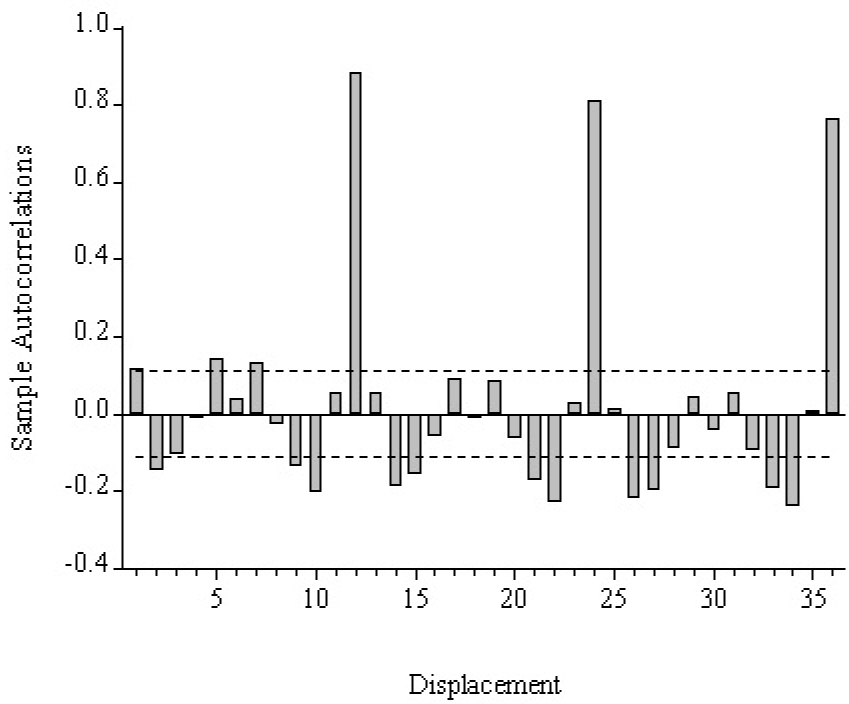
Log Liquor Sales
Quadratic Trend Regression
Residual Plot

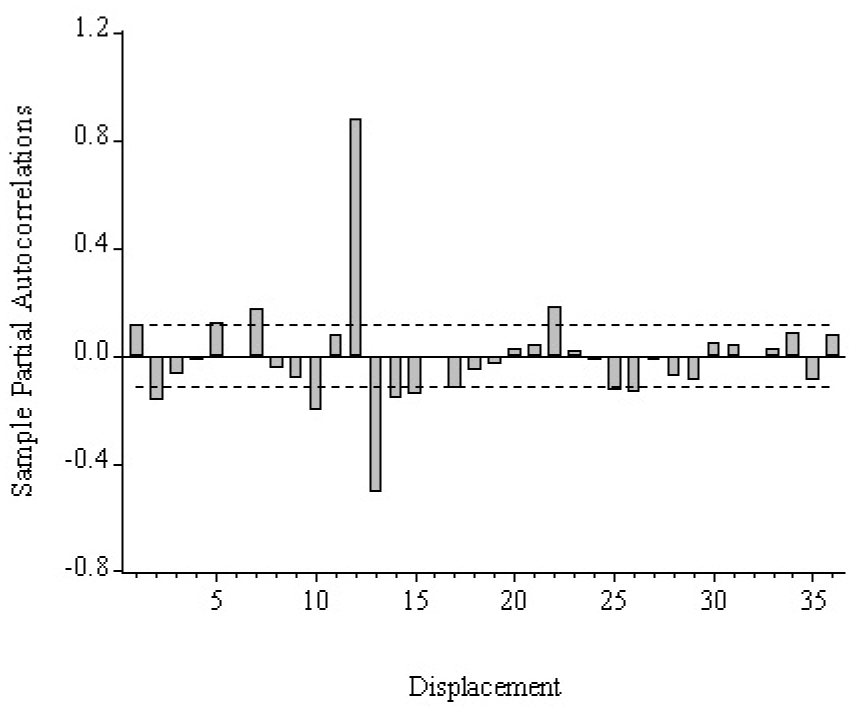


Log Liquor Sales
Quadratic Trend Regression
Residual Correlogram

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.117	0.117	.056	4.3158	0.038
2	-0.149	-0.165	.056	11.365	0.003
3	-0.106	-0.069	.056	14.943	0.002
4	-0.014	-0.017	.056	15.007	0.005
5	0.142	0.125	.056	21.449	0.001
6	0.041	-0.004	.056	21.979	0.001
7	0.134	0.175	.056	27.708	0.000
8	-0.029	-0.046	.056	27.975	0.000
9	-0.136	-0.080	.056	33.944	0.000
10	-0.205	-0.206	.056	47.611	0.000
11	0.056	0.080	.056	48.632	0.000
12	0.888	0.879	.056	306.26	0.000
13	0.055	-0.507	.056	307.25	0.000
14	-0.187	-0.159	.056	318.79	0.000
15	-0.159	-0.144	.056	327.17	0.000
16	-0.059	-0.002	.056	328.32	0.000
17	0.091	-0.118	.056	331.05	0.000
18	-0.010	-0.055	.056	331.08	0.000
19	0.086	-0.032	.056	333.57	0.000
20	-0.066	0.028	.056	335.03	0.000
21	-0.170	0.044	.056	344.71	0.000
22	-0.231	0.180	.056	362.74	0.000
23	0.028	0.016	.056	363.00	0.000
24	0.811	-0.014	.056	586.50	0.000
25	0.013	-0.128	.056	586.56	0.000
26	-0.221	-0.136	.056	603.26	0.000
27	-0.196	-0.017	.056	616.51	0.000
28	-0.092	-0.079	.056	619.42	0.000
29	0.045	-0.094	.056	620.13	0.000
30	-0.043	0.045	.056	620.77	0.000
31	0.057	0.041	.056	621.89	0.000
32	-0.095	-0.002	.056	625.07	0.000
33	-0.195	0.026	.056	638.38	0.000
34	-0.240	0.088	.056	658.74	0.000
35	0.006	-0.089	.056	658.75	0.000
36	0.765	0.076	.056	866.34	0.000

Log Liquor Sales
Quadratic Trend Regression
Residual Sample Autocorrelation and Partial Autocorrelation Functions





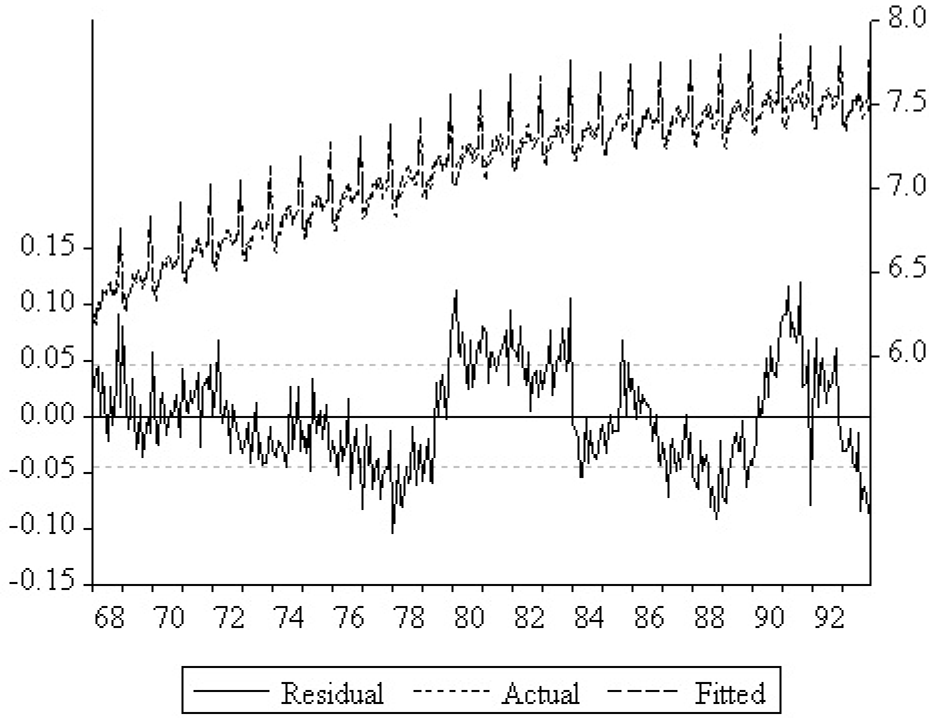
Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies

LS // Dependent Variable is LSALES
Sample: 1968:01 1993:12
Included observations: 312

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TIME	0.007656	0.000123	62.35882	0.0000
TIME2	-1.14E-05	3.56E-07	-32.06823	0.0000
D1	6.147456	0.012340	498.1699	0.0000
D2	6.088653	0.012353	492.8890	0.0000
D3	6.174127	0.012366	499.3008	0.0000
D4	6.175220	0.012378	498.8970	0.0000
D5	6.246086	0.012390	504.1398	0.0000
D6	6.250387	0.012401	504.0194	0.0000
D7	6.295979	0.012412	507.2402	0.0000
D8	6.268043	0.012423	504.5509	0.0000
D9	6.203832	0.012433	498.9630	0.0000
D10	6.229197	0.012444	500.5968	0.0000
D11	6.259770	0.012453	502.6602	0.0000
D12	6.580068	0.012463	527.9819	0.0000

R-squared	0.986111	Mean dependent var	7.112383
Adjusted R-squared	0.985505	S.D. dependent var	0.379308
S.E. of regression	0.045666	Akaike info criterion	-6.128963
Sum squared resid	0.621448	Schwarz criterion	-5.961008
Log likelihood	527.4094	F-statistic	1627.567
Durbin-Watson stat	0.586187	Prob(F-statistic)	0.000000

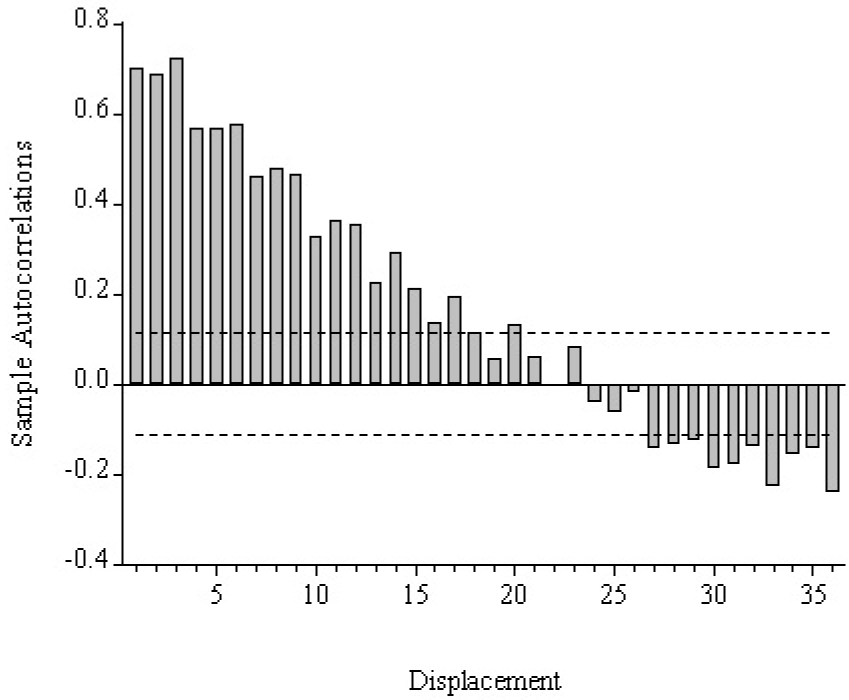
Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies
Residual Plot

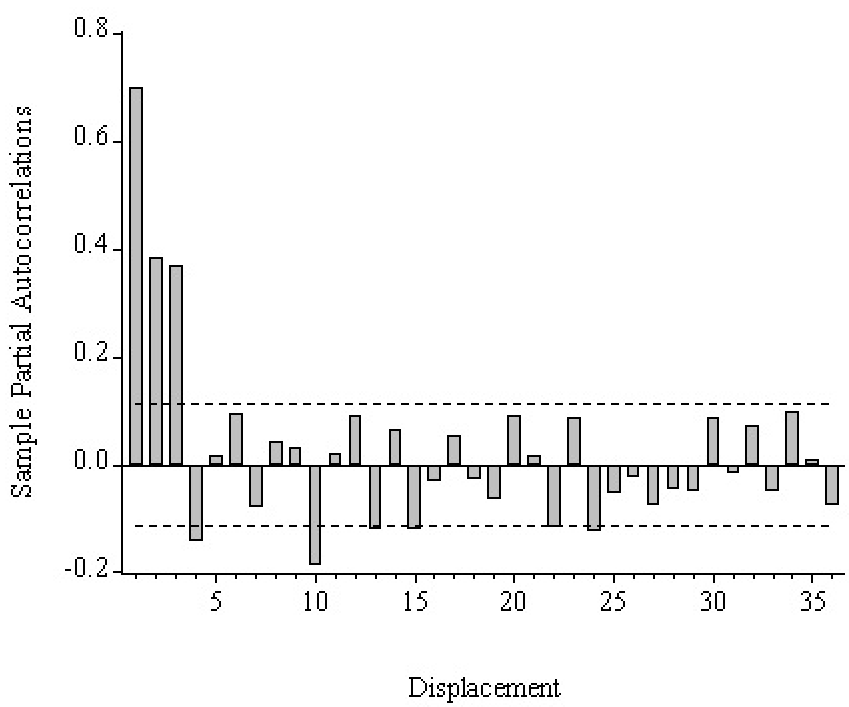


Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies
Residual Correlogram

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.700	0.700	.056	154.34	0.000
2	0.686	0.383	.056	302.86	0.000
3	0.725	0.369	.056	469.36	0.000
4	0.569	-0.141	.056	572.36	0.000
5	0.569	0.017	.056	675.58	0.000
6	0.577	0.093	.056	782.19	0.000
7	0.460	-0.078	.056	850.06	0.000
8	0.480	0.043	.056	924.38	0.000
9	0.466	0.030	.056	994.46	0.000
10	0.327	-0.188	.056	1029.1	0.000
11	0.364	0.019	.056	1072.1	0.000
12	0.355	0.089	.056	1113.3	0.000
13	0.225	-0.119	.056	1129.9	0.000
14	0.291	0.065	.056	1157.8	0.000
15	0.211	-0.119	.056	1172.4	0.000
16	0.138	-0.031	.056	1178.7	0.000
17	0.195	0.053	.056	1191.4	0.000
18	0.114	-0.027	.056	1195.7	0.000
19	0.055	-0.063	.056	1196.7	0.000
20	0.134	0.089	.056	1202.7	0.000
21	0.062	0.018	.056	1204.0	0.000
22	-0.006	-0.115	.056	1204.0	0.000
23	0.084	0.086	.056	1206.4	0.000
24	-0.039	-0.124	.056	1206.9	0.000
25	-0.063	-0.055	.056	1208.3	0.000
26	-0.016	-0.022	.056	1208.4	0.000
27	-0.143	-0.075	.056	1215.4	0.000
28	-0.135	-0.047	.056	1221.7	0.000
29	-0.124	-0.048	.056	1227.0	0.000
30	-0.189	0.086	.056	1239.5	0.000
31	-0.178	-0.017	.056	1250.5	0.000
32	-0.139	0.073	.056	1257.3	0.000
33	-0.226	-0.049	.056	1275.2	0.000
34	-0.155	0.097	.056	1283.7	0.000
35	-0.142	0.008	.056	1290.8	0.000
36	-0.242	-0.074	.056	1311.6	0.000

Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies
Residual Sample Autocorrelation and Partial Autocorrelation Functions





Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies and AR(3) Disturbances

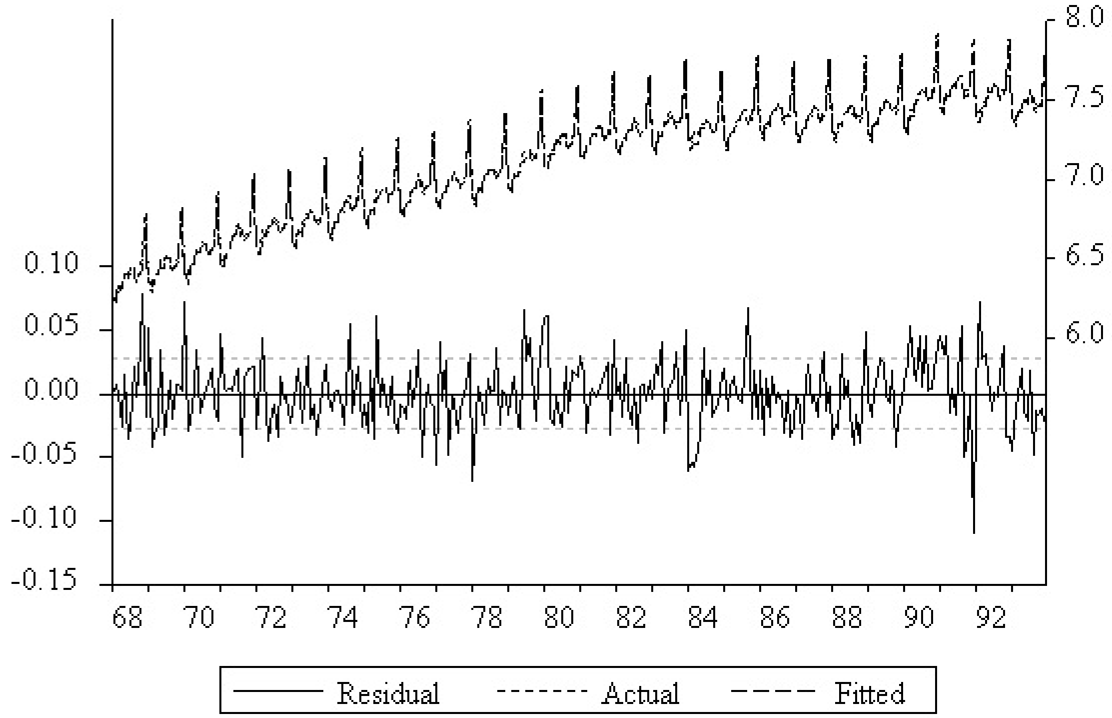
LS // Dependent Variable is LSALES
Sample: 1968:01 1993:12
Included observations: 312
Convergence achieved after 4 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TIME	0.008606	0.000981	8.768212	0.0000
TIME2	-1.41E-05	2.53E-06	-5.556103	0.0000
D1	6.073054	0.083922	72.36584	0.0000
D2	6.013822	0.083942	71.64254	0.0000
D3	6.099208	0.083947	72.65524	0.0000
D4	6.101522	0.083934	72.69393	0.0000
D5	6.172528	0.083946	73.52962	0.0000
D6	6.177129	0.083947	73.58364	0.0000
D7	6.223323	0.083939	74.14071	0.0000
D8	6.195681	0.083943	73.80857	0.0000
D9	6.131818	0.083940	73.04993	0.0000
D10	6.157592	0.083934	73.36197	0.0000
D11	6.188480	0.083932	73.73176	0.0000
D12	6.509106	0.083928	77.55624	0.0000
AR(1)	0.268805	0.052909	5.080488	0.0000
AR(2)	0.239688	0.053697	4.463723	0.0000
AR(3)	0.395880	0.053109	7.454150	0.0000

R-squared	0.995069	Mean dependent var	7.112383
Adjusted R-squared	0.994802	S.D. dependent var	0.379308
S.E. of regression	0.027347	Akaike info criterion	-7.145319
Sum squared resid	0.220625	Schwarz criterion	-6.941373
Log likelihood	688.9610	F-statistic	3720.875
Durbin-Watson stat	1.886119	Prob(F-statistic)	0.000000

Inverted AR Roots	.95	-.34+.55i	-.34 -.55i

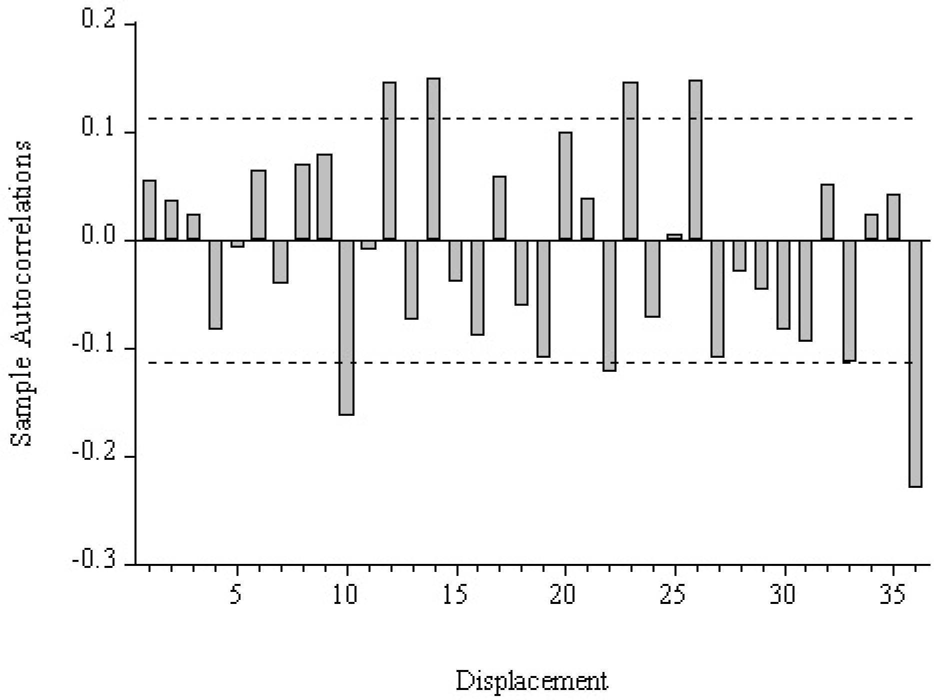
Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies and AR(3) Disturbances
Residual Plot

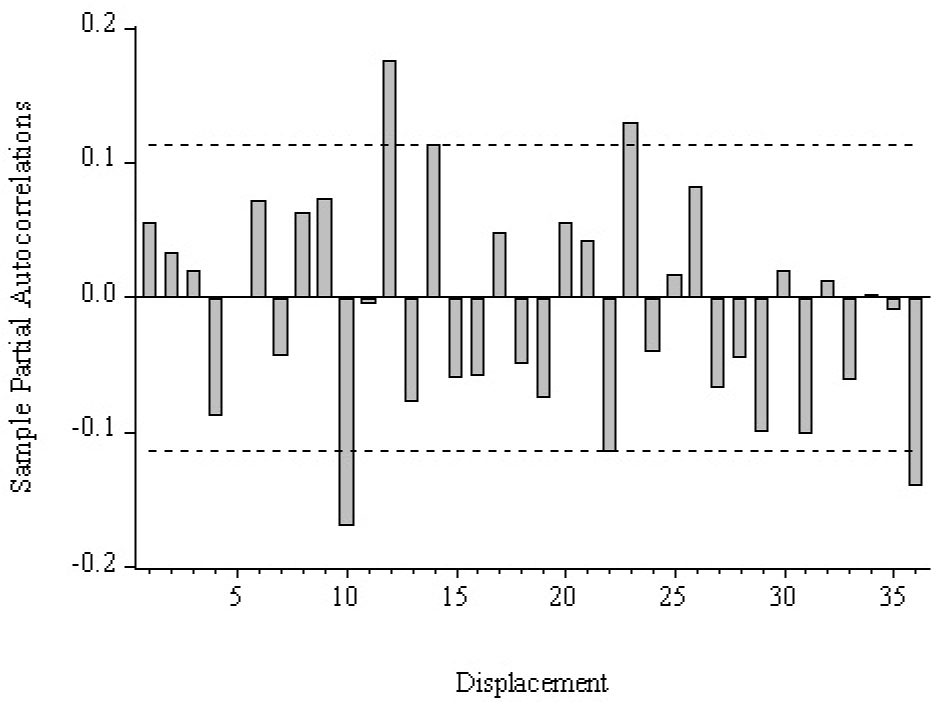


Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies and AR(3) Disturbances
Residual Correlogram

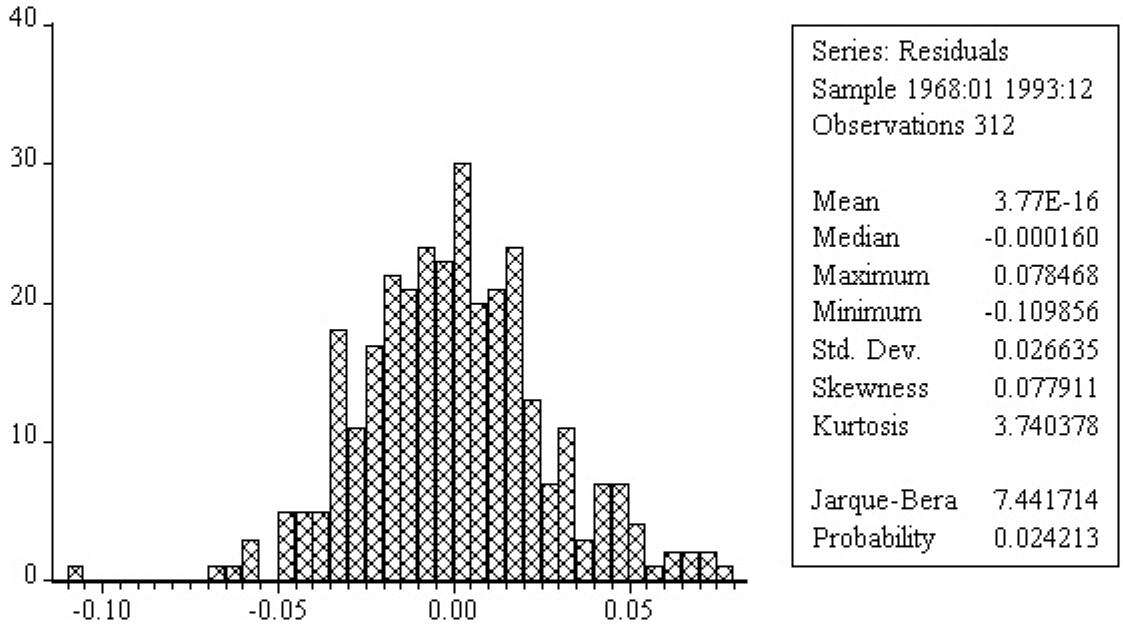
	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.056	0.056	.056	0.9779	0.323
2	0.037	0.034	.056	1.4194	0.492
3	0.024	0.020	.056	1.6032	0.659
4	-0.084	-0.088	.056	3.8256	0.430
5	-0.007	0.001	.056	3.8415	0.572
6	0.065	0.072	.056	5.1985	0.519
7	-0.041	-0.044	.056	5.7288	0.572
8	0.069	0.063	.056	7.2828	0.506
9	0.080	0.074	.056	9.3527	0.405
10	-0.163	-0.169	.056	18.019	0.055
11	-0.009	-0.005	.056	18.045	0.081
12	0.145	0.175	.056	24.938	0.015
13	-0.074	-0.078	.056	26.750	0.013
14	0.149	0.113	.056	34.034	0.002
15	-0.039	-0.060	.056	34.532	0.003
16	-0.089	-0.058	.056	37.126	0.002
17	0.058	0.048	.056	38.262	0.002
18	-0.062	-0.050	.056	39.556	0.002
19	-0.110	-0.074	.056	43.604	0.001
20	0.100	0.056	.056	46.935	0.001
21	0.039	0.042	.056	47.440	0.001
22	-0.122	-0.114	.056	52.501	0.000
23	0.146	0.130	.056	59.729	0.000
24	-0.072	-0.040	.056	61.487	0.000
25	0.006	0.017	.056	61.500	0.000
26	0.148	0.082	.056	69.024	0.000
27	-0.109	-0.067	.056	73.145	0.000
28	-0.029	-0.045	.056	73.436	0.000
29	-0.046	-0.100	.056	74.153	0.000
30	-0.084	0.020	.056	76.620	0.000
31	-0.095	-0.101	.056	79.793	0.000
32	0.051	0.012	.056	80.710	0.000
33	-0.114	-0.061	.056	85.266	0.000
34	0.024	0.002	.056	85.468	0.000
35	0.043	-0.010	.056	86.116	0.000
36	-0.229	-0.140	.056	104.75	0.000

Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies and AR(3) Disturbances
Residual Sample Autocorrelation and Partial Autocorrelation Functions

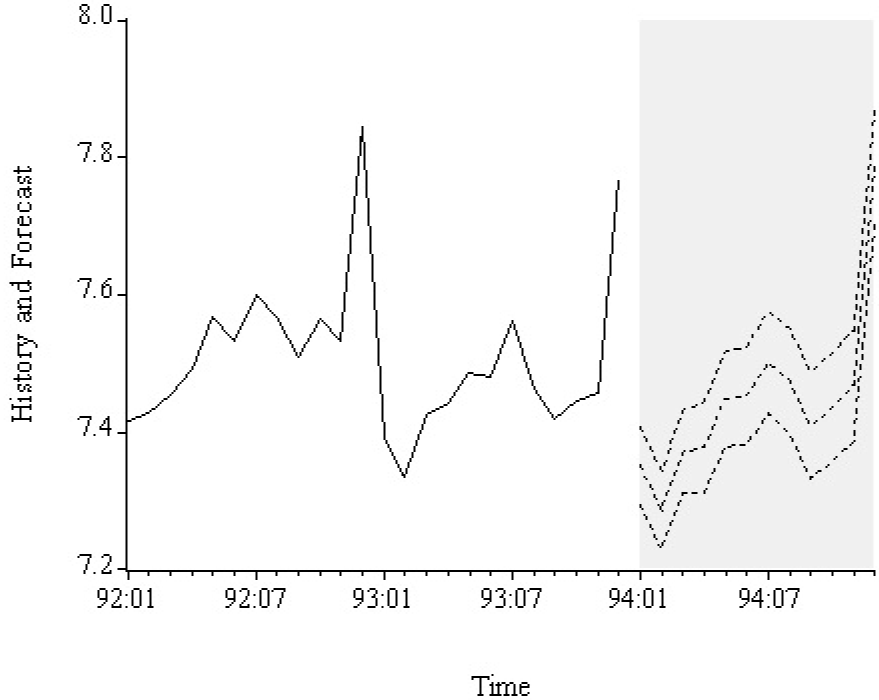




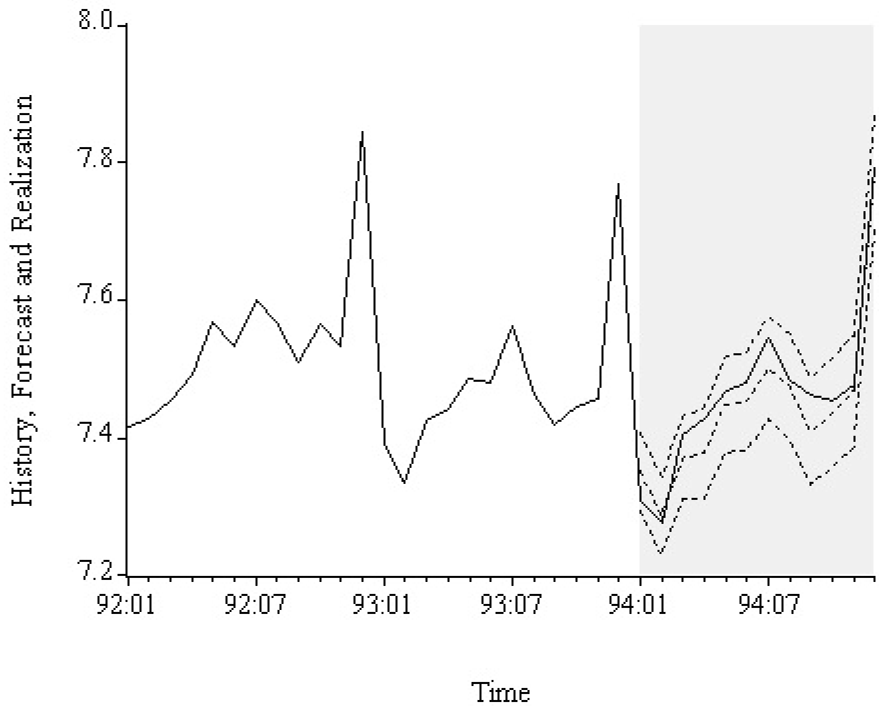
Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies and AR(3) Disturbances
Residual Histogram and Normality Test



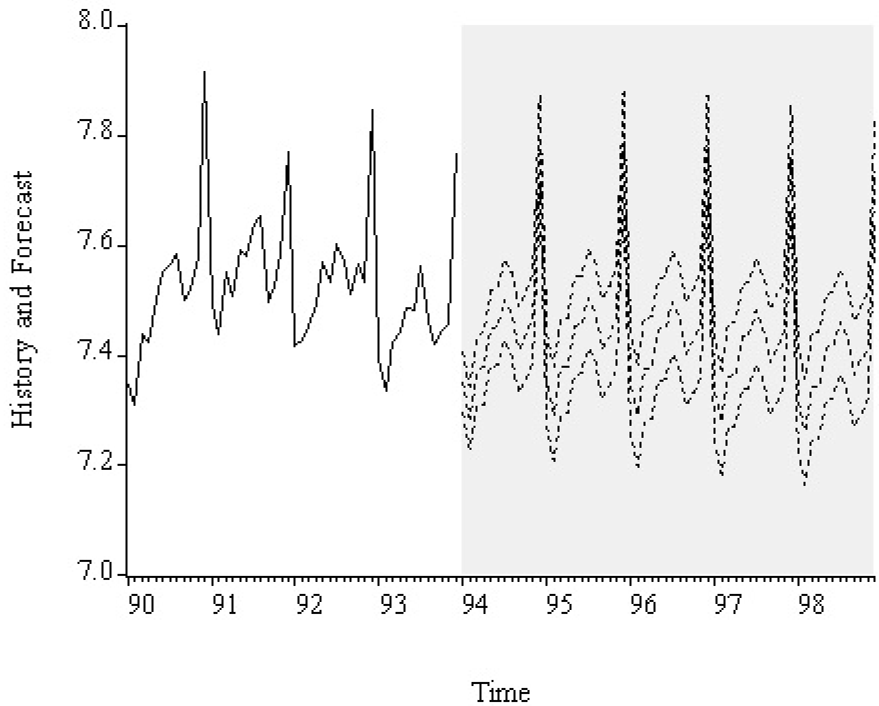
Log Liquor Sales
History and 12-Month-Ahead Forecast



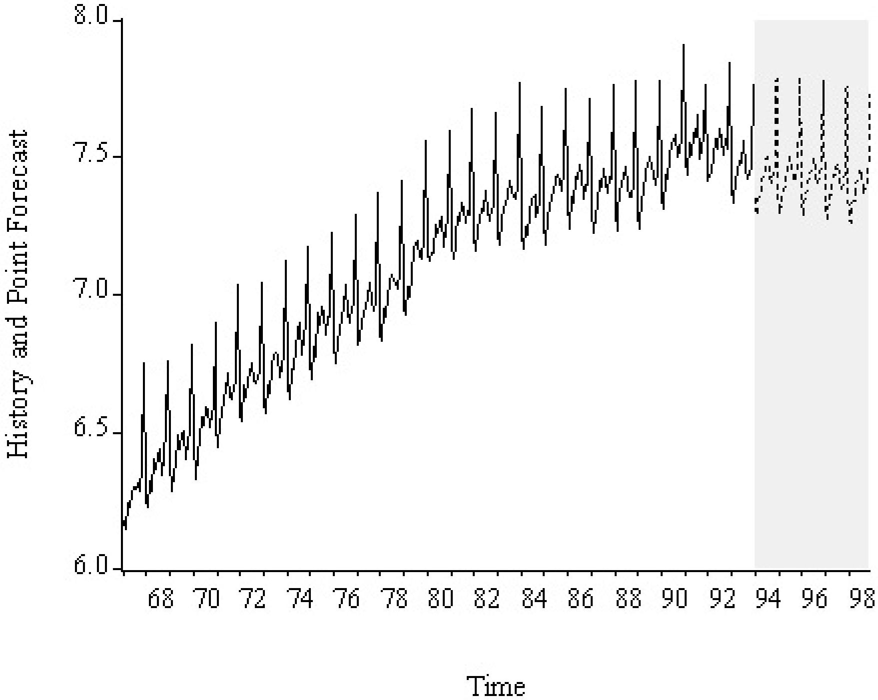
Log Liquor Sales
History, 12-Month-Ahead Forecast, and Realization



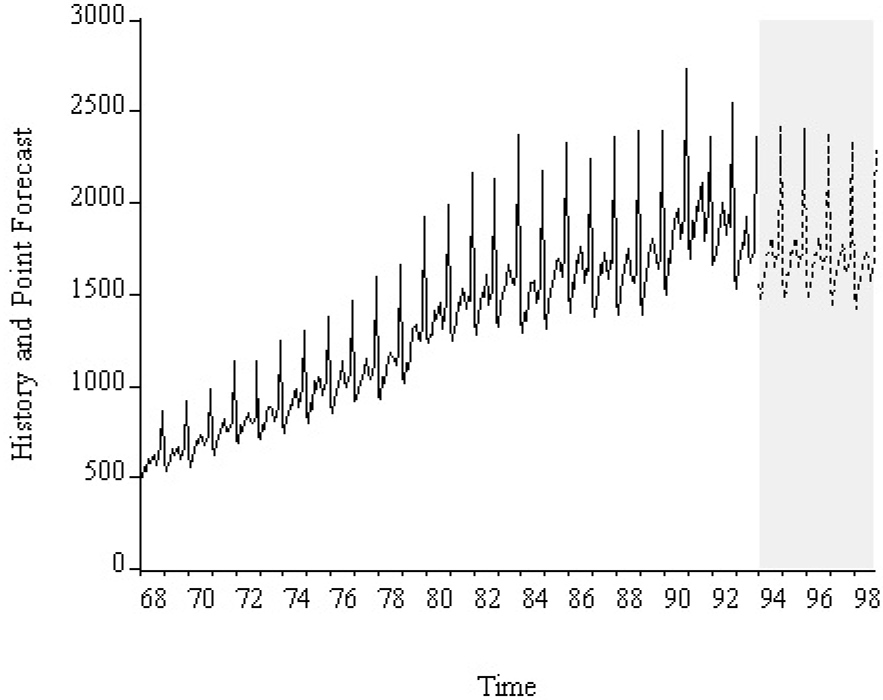
Log Liquor Sales
History and 60-Month-Ahead Forecast



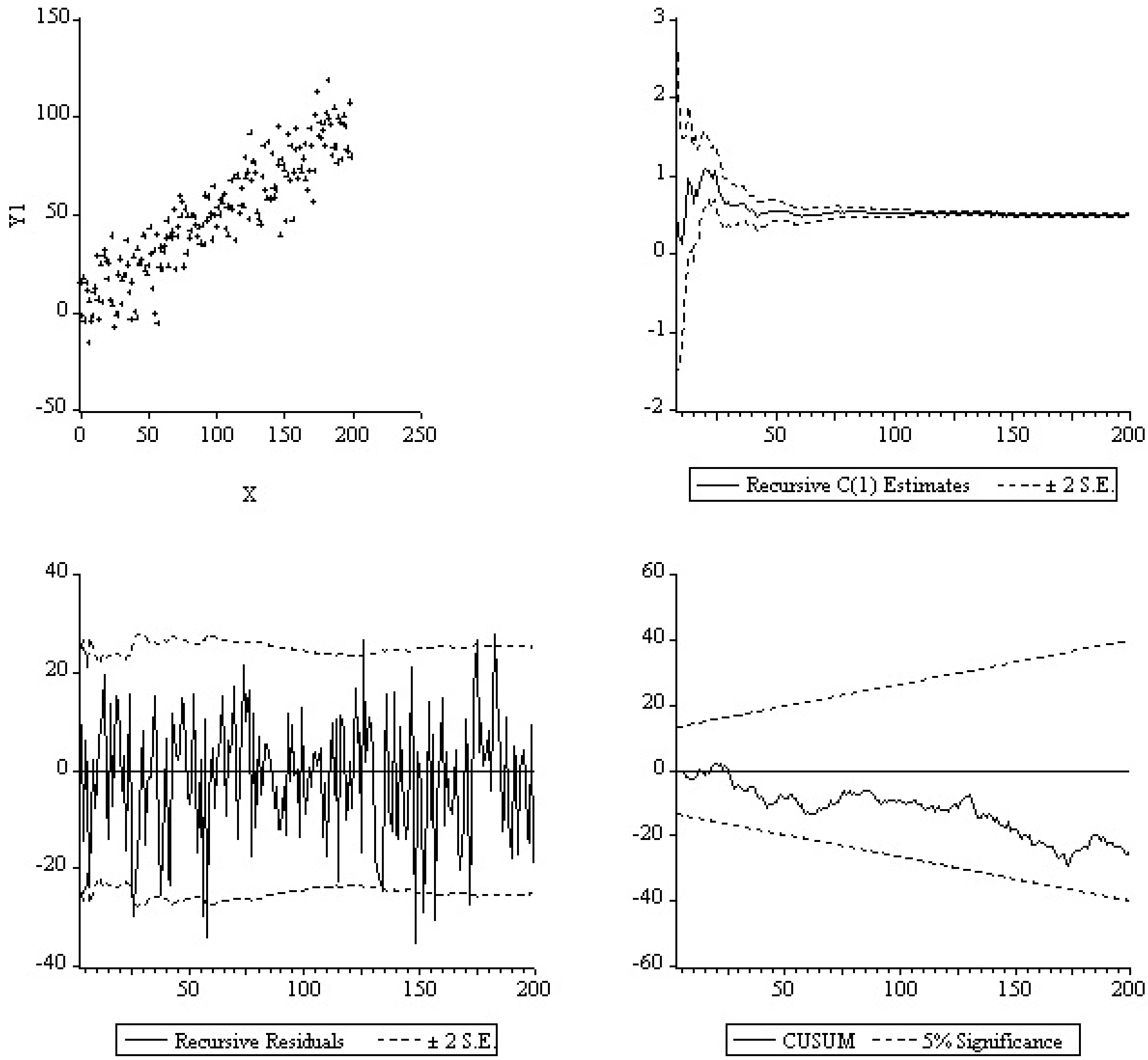
Log Liquor Sales
Long History and 60-Month-Ahead Forecast



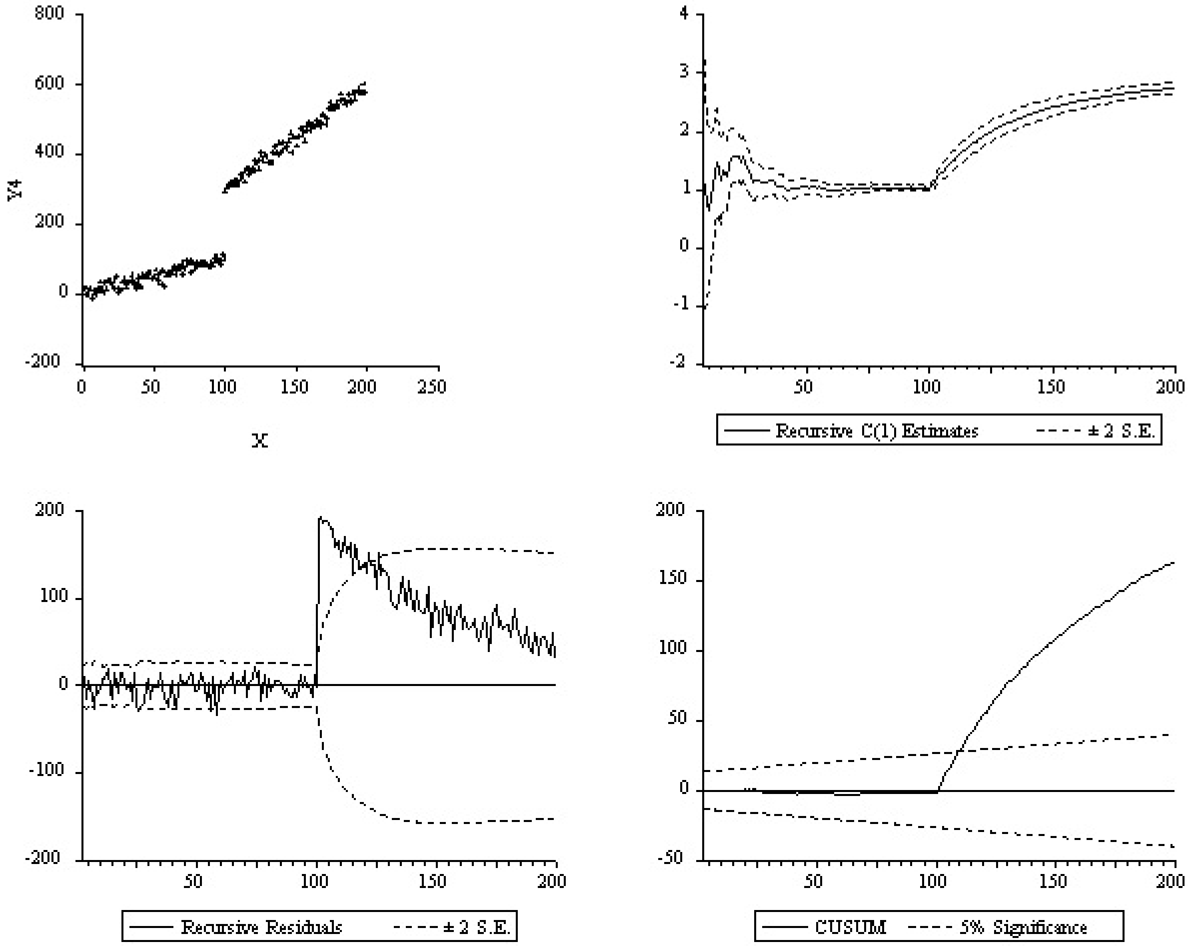
Liquor Sales
Long History and 60-Month-Ahead Forecast



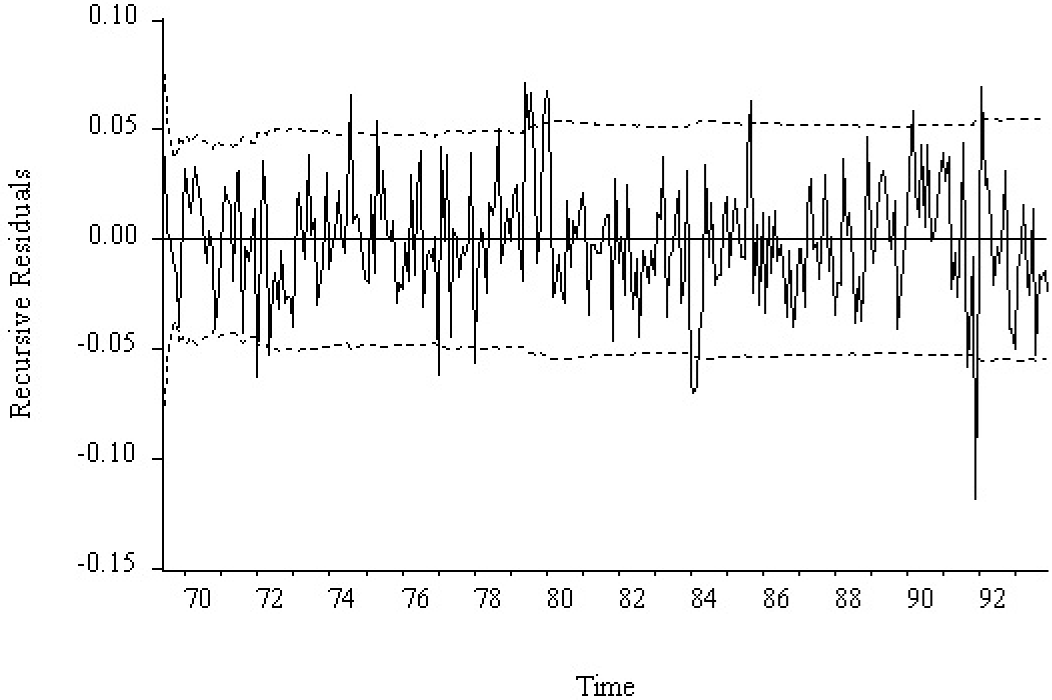
Recursive Analysis
Constant Parameter Model



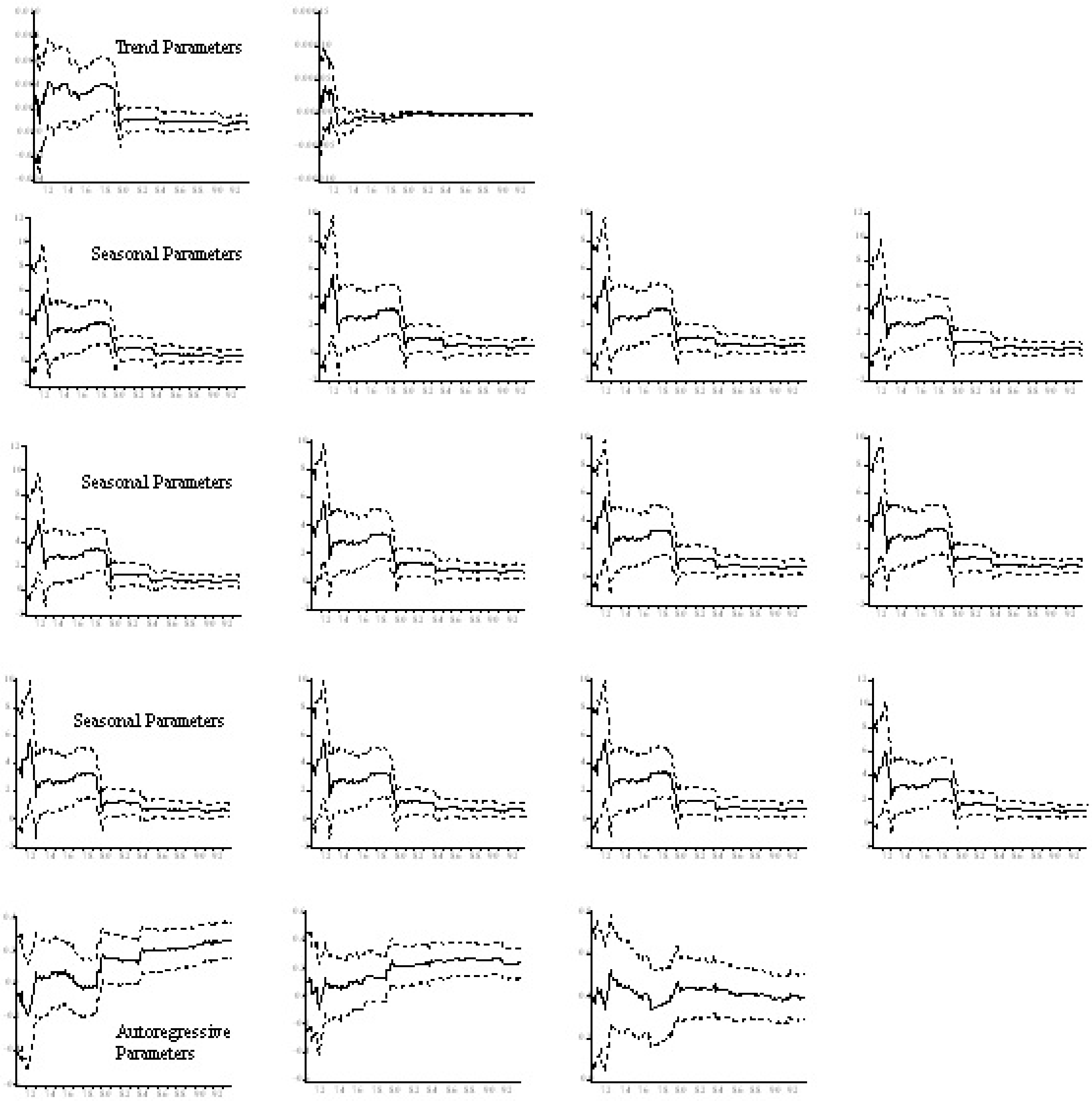
Recursive Analysis
Breaking Parameter Model



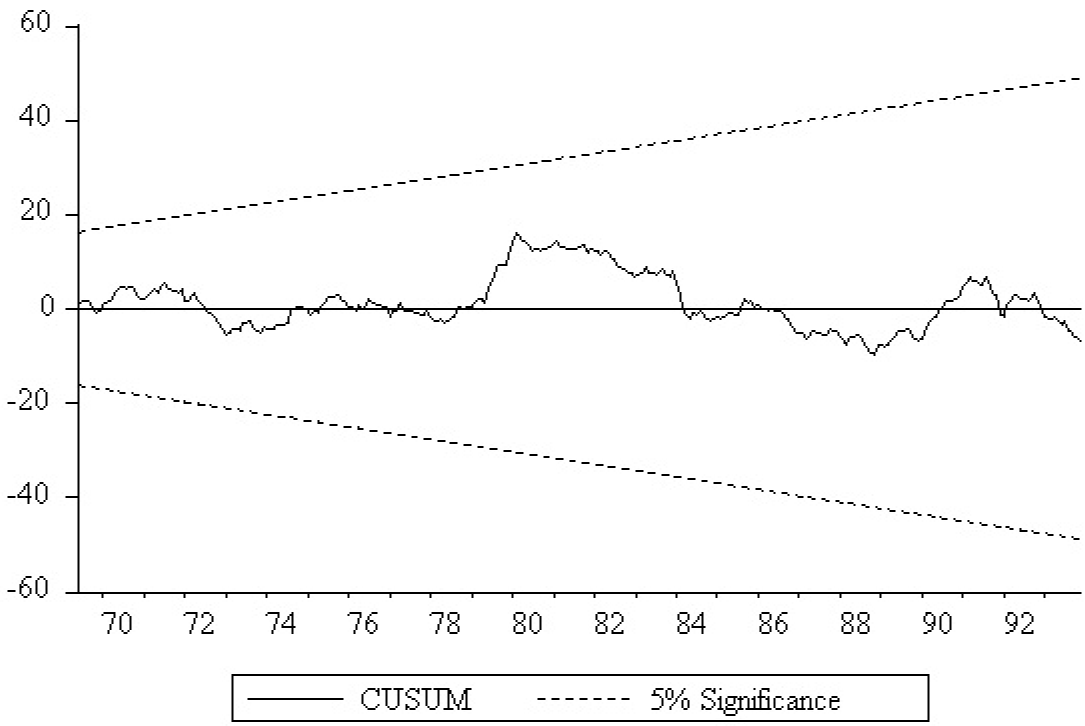
Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies and AR(3) Disturbances
Recursive Residuals and Two Standard Error Bands



Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies and AR(3) Disturbances
Recursive Parameter Estimates



Log Liquor Sales
Quadratic Trend Regression with Seasonal Dummies and AR(3) Disturbances
CUSUM Analysis



Forecasting with Regression Models

Conditional Forecasting Models and Scenario Analysis

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma^2)$$

$$y_{T+h,T} | x^*_{T+h} = \beta_0 + \beta_1 x^*_{T+h}$$

Density forecast: $N(y_{T+h,T} | x^*_{T+h}, \sigma^2)$

- "Scenario analysis," "contingency analysis"
- No "forecasting the RHS variables problem"

Unconditional Forecasting Models

$$y_{T+h,T} = \beta_0 + \beta_1 x_{T+h,T}$$

- "Forecasting the RHS variables problem"
- Could fit a model to x (e.g., an autoregressive model)
- Preferably, regress y on x_{t-h}, $x_{t-h-1, \ldots}$
- No problem in trend and seasonal models

Distributed Lags

Start with unconditional forecasting model:

$$y_t = \beta_0 + \delta x_{t-1} + \varepsilon_t$$

Generalize to

$$y_t = \beta_0 + \sum_{i=1}^{N_x} \delta_i x_{t-i} + \varepsilon_t$$

- "distributed lag model"
- "lag weights"
- "lag distribution"

Polynomial distributed lags

Solve the problem:

$$\min_{\beta_0, \delta_i} \sum_{t = N_x+1}^{T} \left[y_t - \beta_0 - \sum_{i=1}^{N_x} \delta_i x_{t-i} \right]^2$$

subject to

$$\delta_i = P(i) = a + bi + ci^2, \; i = 1, ..., N_x$$

- Lag weights constrained to lie on low-order polynomial
- Additional constraints can be imposed, such as $P(N_x) = 0$
- Smooth lag distribution
- Parsimonious

Rational distributed lags

$$y_t = \frac{A(L)}{B(L)} x_t + \varepsilon_t$$

Equivalently,

$$B(L)y_t = A(L)x_t + B(L)\ \varepsilon_t$$

- Lags of x *and* y included
- Important to allow for lags of y, one way or another

Another way:

distributed lag regression with lagged dependent variables

$$y_t = \beta_0 + \sum_{i=1}^{N_y} \alpha_i y_{t-i} + \sum_{j=1}^{N_x} \delta_j x_{t-j} + \varepsilon_t$$

Another way:

distributed lag regression with ARMA disturbances

$$y_t = \beta_0 + \sum_{i=1}^{N_x} \delta_i x_{t-i} + \varepsilon_t$$

$$\varepsilon_t = \frac{\Theta(L)}{\Phi(L)} v_t$$

$$v_t \sim WN(0, \sigma^2)$$

Another way: the transfer function model and various special cases

Name	Model	Restrictions
Transfer Function	$y_t = \frac{A(L)}{B(L)} x_t + \frac{C(L)}{D(L)} \varepsilon_t$	None
Standard Distributed Lag	$y_t = A(L)\, x_t + \varepsilon_t$	B(L)=C(L)=D(L)=1
Rational Distributed Lag	$y_t = \frac{A(L)}{B(L)} x_t + \varepsilon_t$	C(L)=D(L)=1
Univariate AR	$y_t = \frac{1}{D(L)} \varepsilon_t$	A(L)=0, C(L)=1
Univariate MA	$y_t = C(L)\, \varepsilon_t$	A(L)=0, D(L)=1
Univariate ARMA	$y_t = \frac{C(L)}{D(L)} \varepsilon_t$	A(L)=0
Distributed Lag with Lagged Dep. Variables	$B(L)\, y_t = A(L)\, x_t + \varepsilon_t$, or $y_t = \frac{A(L)}{B(L)} x_t + \frac{1}{B(L)} \varepsilon_t$	C(L)=1, D(L)=B(L)
Distributed Lag with ARMA Disturbances	$y_t = A(L)\, x_t + \frac{C(L)}{D(L)} \varepsilon_t$	B(L)=1
Distributed Lag with AR Disturbances	$y_t = A(L)\, x_t + \frac{1}{D(L)} \varepsilon_t$	B(L)=C(L)=1

Vector Autoregressions

e.g., bivariate VAR(1)

$$y_{1,t} = \varphi_{11} y_{1,t-1} + \varphi_{12} y_{2,t-1} + \varepsilon_{1,t}$$

$$y_{2,t} = \varphi_{21} y_{1,t-1} + \varphi_{22} y_{2,t-1} + \varepsilon_{2,t}$$

$$\varepsilon_{1,t} \sim WN(0, \sigma_1^2)$$

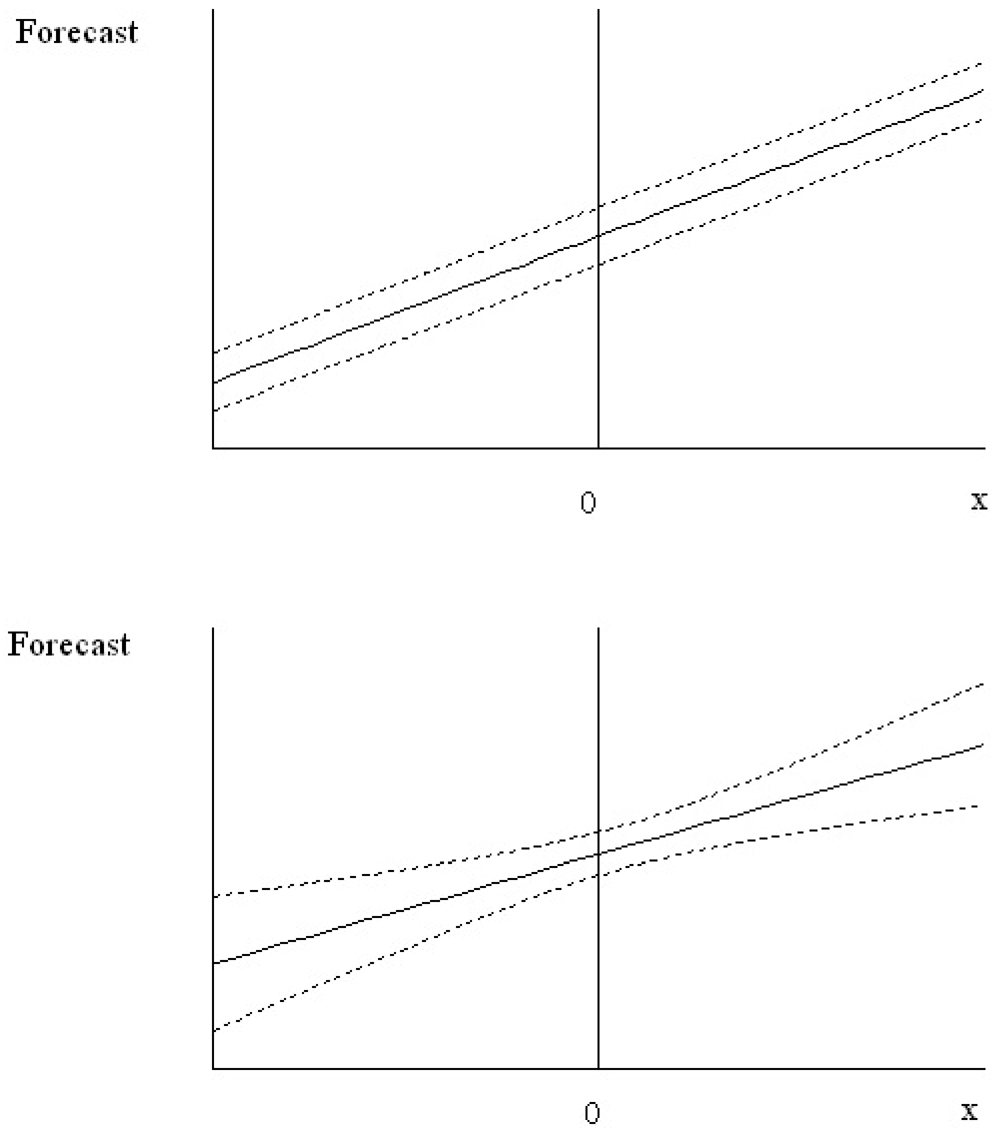
- Estimation by OLS

$$\varepsilon_{2,t} \sim WN(0, \sigma_2^2)$$

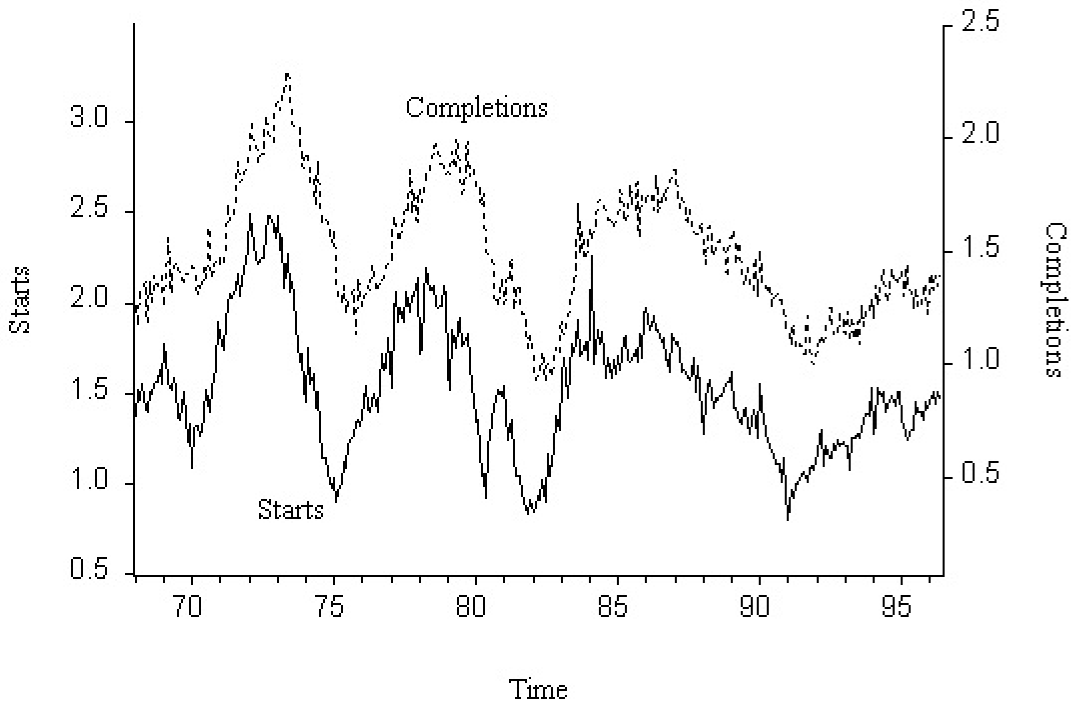
$$\text{cov}(\varepsilon_{1,t}, \varepsilon_{2,t}) = \sigma_{12}$$

- Order selection by information criteria
- Impulse-response functions, variance decompositions, predictive causality
- Forecasts via Wold's chain rule

Point and Interval Forecasts
Top Panel Interval Forecasts *Don't* Acknowledge Parameter Uncertainty
Bottom Panel Interval Forecasts *Do* Acknowledge Parameter Uncertainty



U.S. Housing Starts and Completions, 1968.01 - 1996.06



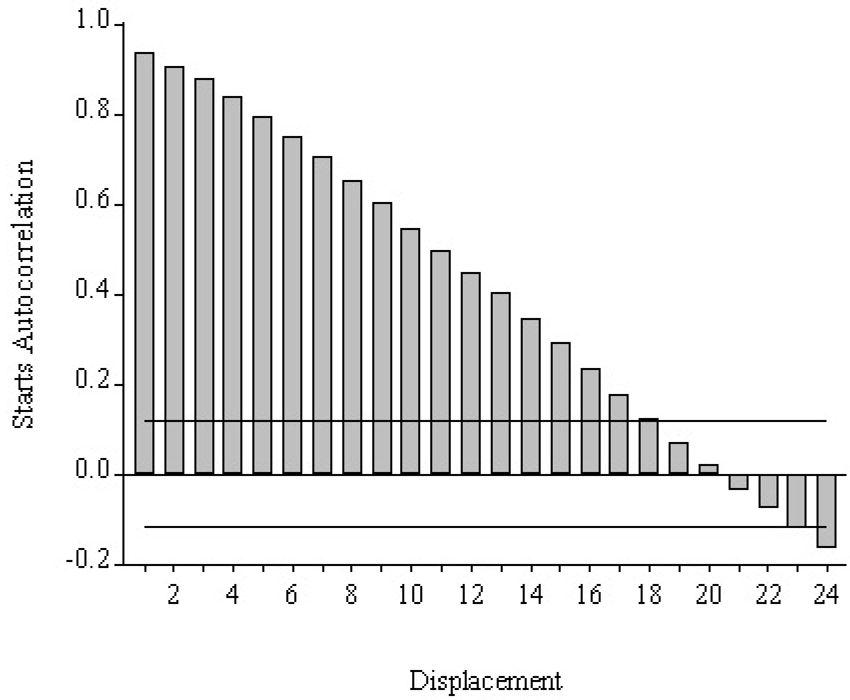
Notes to figure: The left scale is starts, and the right scale is completions.

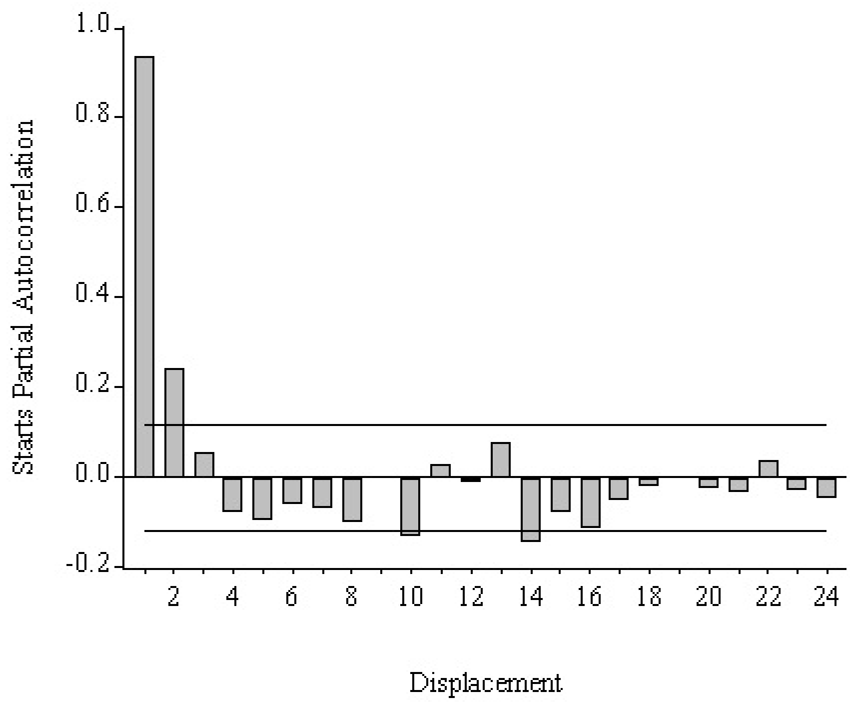
Starts Correlogram

Sample: 1968:01 1991:12
Included observations: 288

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.937	0.937	0.059	255.24	0.000
2	0.907	0.244	0.059	495.53	0.000
3	0.877	0.054	0.059	720.95	0.000
4	0.838	-0.077	0.059	927.39	0.000
5	0.795	-0.096	0.059	1113.7	0.000
6	0.751	-0.058	0.059	1280.9	0.000
7	0.704	-0.067	0.059	1428.2	0.000
8	0.650	-0.098	0.059	1554.4	0.000
9	0.604	0.004	0.059	1663.8	0.000
10	0.544	-0.129	0.059	1752.6	0.000
11	0.496	0.029	0.059	1826.7	0.000
12	0.446	-0.008	0.059	1886.8	0.000
13	0.405	0.076	0.059	1936.8	0.000
14	0.346	-0.144	0.059	1973.3	0.000
15	0.292	-0.079	0.059	1999.4	0.000
16	0.233	-0.111	0.059	2016.1	0.000
17	0.175	-0.050	0.059	2025.6	0.000
18	0.122	-0.018	0.059	2030.2	0.000
19	0.070	0.002	0.059	2031.7	0.000
20	0.019	-0.025	0.059	2031.8	0.000
21	-0.034	-0.032	0.059	2032.2	0.000
22	-0.074	0.036	0.059	2033.9	0.000
23	-0.123	-0.028	0.059	2038.7	0.000
24	-0.167	-0.048	0.059	2047.4	0.000

Starts
Sample Autocorrelations and Partial Autocorrelations



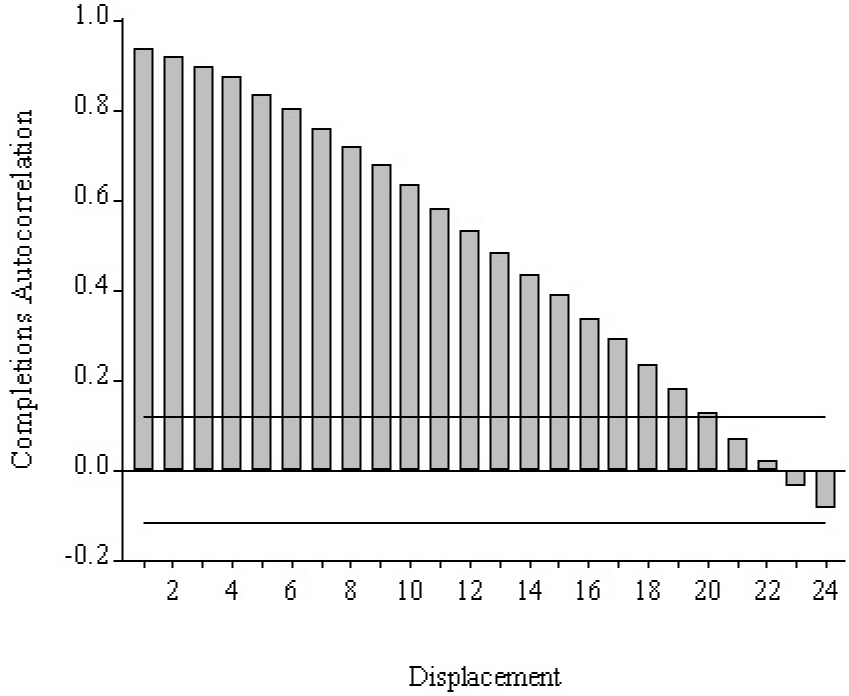


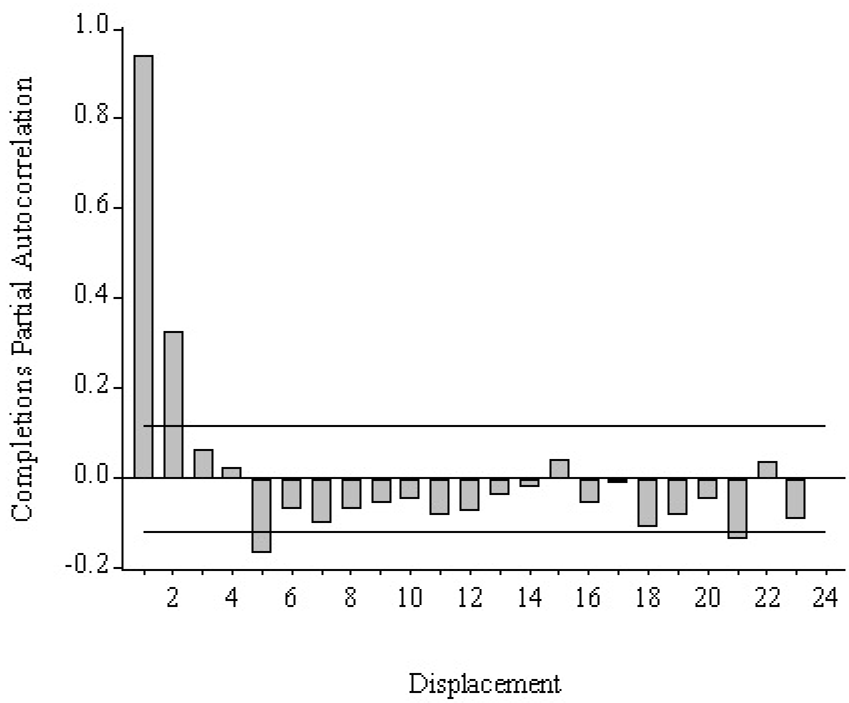
Completions Correlogram

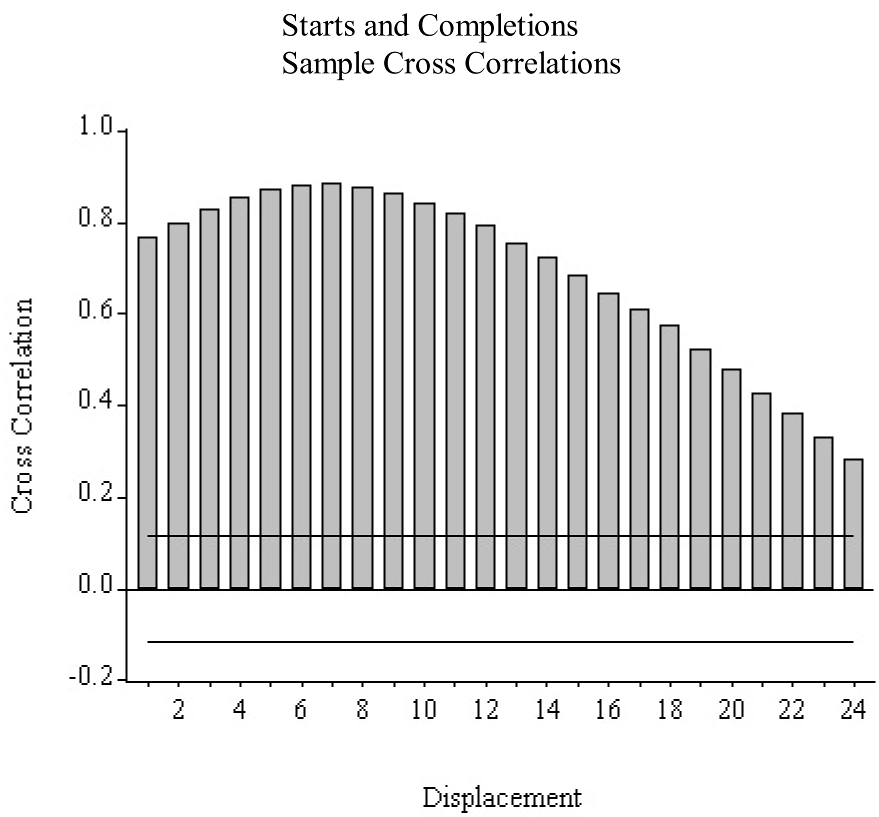
Sample: 1968:01 1991:12
Included observations: 288

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.939	0.939	0.059	256.61	0.000
2	0.920	0.328	0.059	504.05	0.000
3	0.896	0.066	0.059	739.19	0.000
4	0.874	0.023	0.059	963.73	0.000
5	0.834	-0.165	0.059	1168.9	0.000
6	0.802	-0.067	0.059	1359.2	0.000
7	0.761	-0.100	0.059	1531.2	0.000
8	0.721	-0.070	0.059	1686.1	0.000
9	0.677	-0.055	0.059	1823.2	0.000
10	0.633	-0.047	0.059	1943.7	0.000
11	0.583	-0.080	0.059	2046.3	0.000
12	0.533	-0.073	0.059	2132.2	0.000
13	0.483	-0.038	0.059	2203.2	0.000
14	0.434	-0.020	0.059	2260.6	0.000
15	0.390	0.041	0.059	2307.0	0.000
16	0.337	-0.057	0.059	2341.9	0.000
17	0.290	-0.008	0.059	2367.9	0.000
18	0.234	-0.109	0.059	2384.8	0.000
19	0.181	-0.082	0.059	2395.0	0.000
20	0.128	-0.047	0.059	2400.1	0.000
21	0.068	-0.133	0.059	2401.6	0.000
22	0.020	0.037	0.059	2401.7	0.000
23	-0.038	-0.092	0.059	2402.2	0.000
24	-0.087	-0.003	0.059	2404.6	0.000

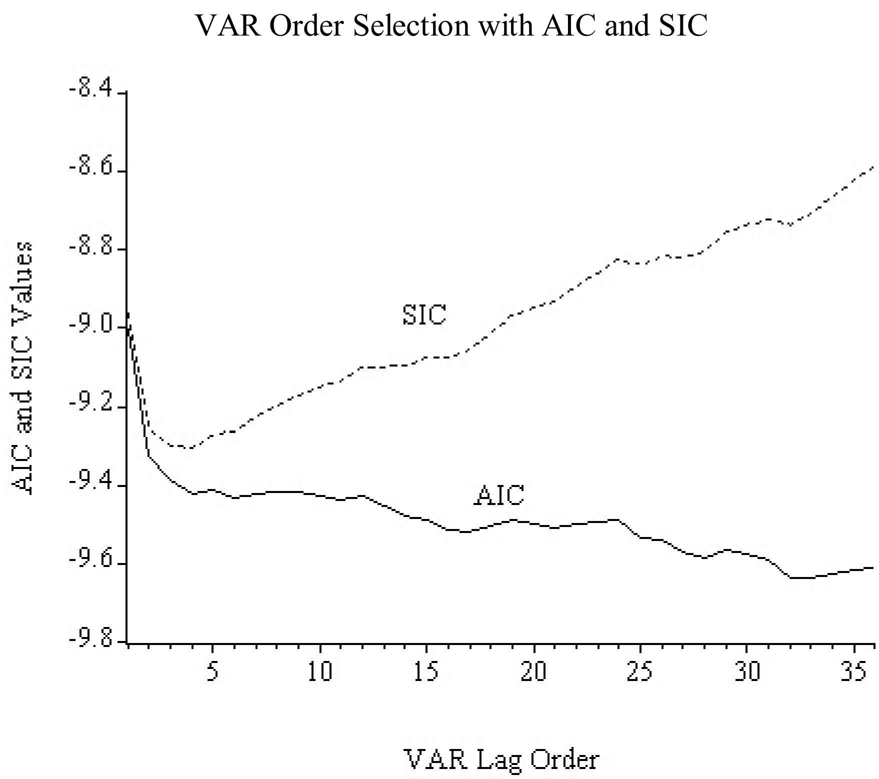
Completions
Sample Autocorrelations and Partial Autocorrelations







Notes to figure: We graph the sample correlation between completions at time t and starts at time t-i, i = 1, 2, ..., 24.


VAR Order Selection with AIC and SIC
AIC and SIC Values
-8.4
-8.6
-8.8
-9.0
-9.2
-9.4
-9.6
-9.8
SIC
AIC
5
10
15
20
25
30
35
VAR Lag Order

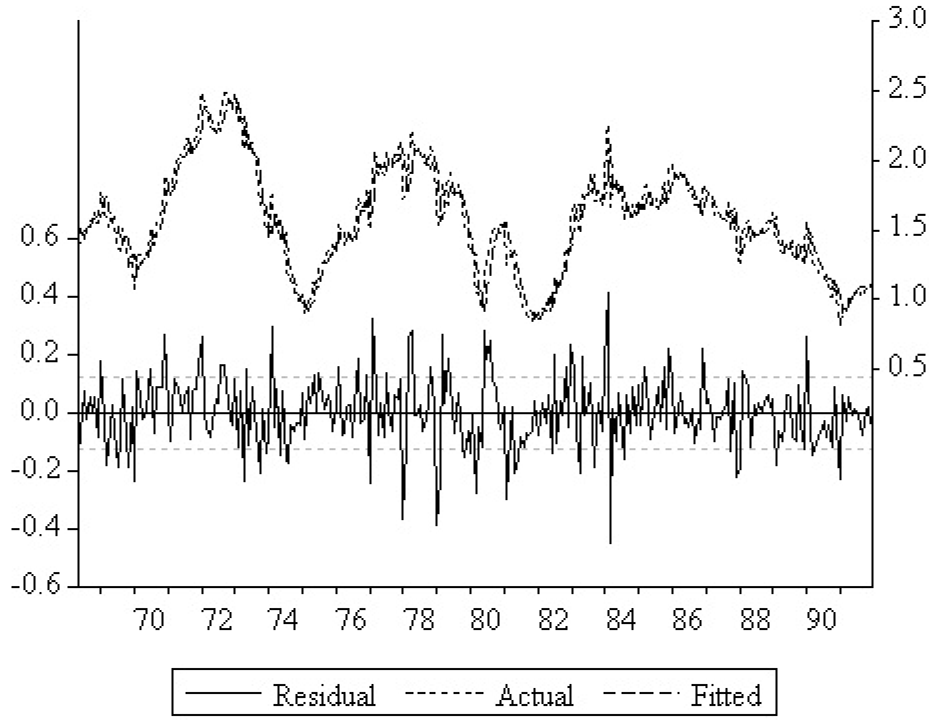
VAR Starts Equation

LS // Dependent Variable is STARTS
Sample(adjusted): 1968:05 1991:12
Included observations: 284 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.146871	0.044235	3.320264	0.0010
STARTS(-1)	0.659939	0.061242	10.77587	0.0000
STARTS(-2)	0.229632	0.072724	3.157587	0.0018
STARTS(-3)	0.142859	0.072655	1.966281	0.0503
STARTS(-4)	0.007806	0.066032	0.118217	0.9060
COMPS(-1)	0.031611	0.102712	0.307759	0.7585
COMPS(-2)	-0.120781	0.103847	-1.163069	0.2458
COMPS(-3)	-0.020601	0.100946	-0.204078	0.8384
COMPS(-4)	-0.027404	0.094569	-0.289779	0.7722

R-squared	0.895566	Mean dependent var	1.574771
Adjusted R-squared	0.892528	S.D. dependent var	0.382362
S.E. of regression	0.125350	Akaike info criterion	-4.122118
Sum squared resid	4.320952	Schwarz criterion	-4.006482
Log likelihood	191.3622	F-statistic	294.7796
Durbin-Watson stat	1.991908	Prob(F-statistic)	0.000000

VAR Starts Equation
Residual Plot

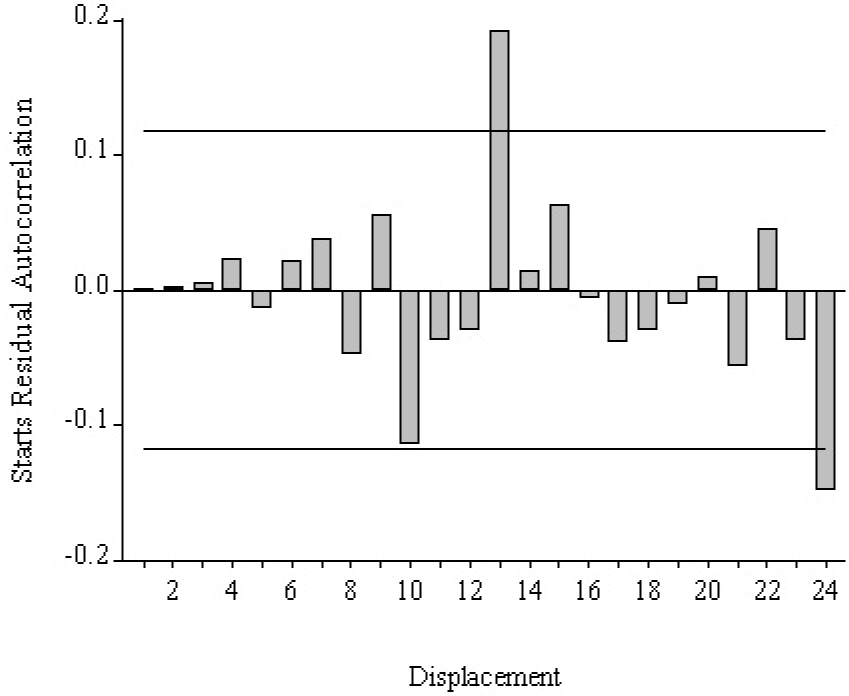


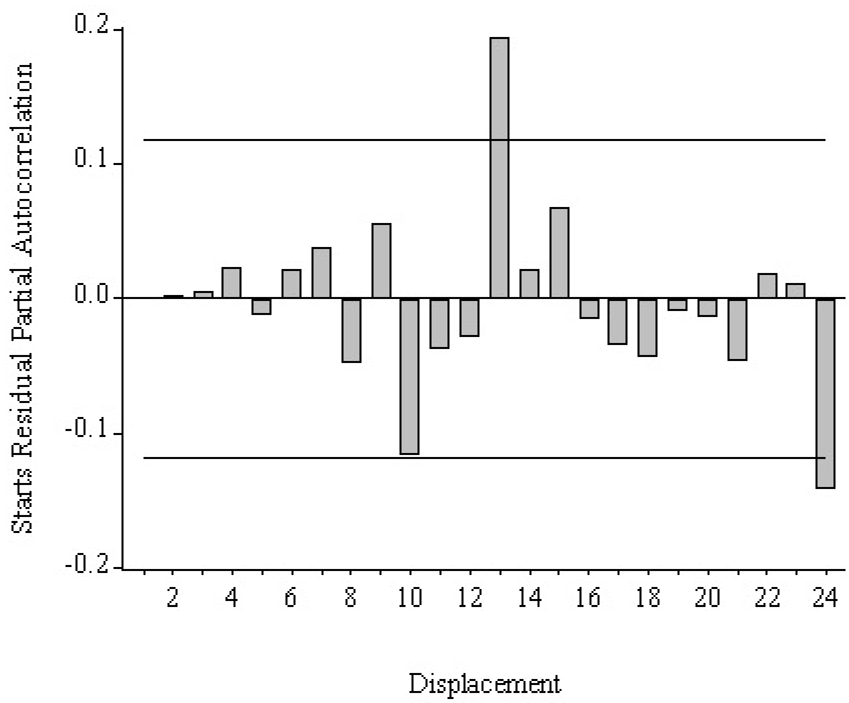
VAR Starts Equation
Residual Correlogram

Sample: 1968:01 1991:12
Included observations: 284

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.001	0.001	0.059	0.0004	0.985
2	0.003	0.003	0.059	0.0029	0.999
3	0.006	0.006	0.059	0.0119	1.000
4	0.023	0.023	0.059	0.1650	0.997
5	-0.013	-0.013	0.059	0.2108	0.999
6	0.022	0.021	0.059	0.3463	0.999
7	0.038	0.038	0.059	0.7646	0.998
8	-0.048	-0.048	0.059	1.4362	0.994
9	0.056	0.056	0.059	2.3528	0.985
10	-0.114	-0.116	0.059	6.1868	0.799
11	-0.038	-0.038	0.059	6.6096	0.830
12	-0.030	-0.028	0.059	6.8763	0.866
13	0.192	0.193	0.059	17.947	0.160
14	0.014	0.021	0.059	18.010	0.206
15	0.063	0.067	0.059	19.199	0.205
16	-0.006	-0.015	0.059	19.208	0.258
17	-0.039	-0.035	0.059	19.664	0.292
18	-0.029	-0.043	0.059	19.927	0.337
19	-0.010	-0.009	0.059	19.959	0.397
20	0.010	-0.014	0.059	19.993	0.458
21	-0.057	-0.047	0.059	21.003	0.459
22	0.045	0.018	0.059	21.644	0.481
23	-0.038	0.011	0.059	22.088	0.515
24	-0.149	-0.141	0.059	29.064	0.218

VAR Starts Equation
Residual Sample Autocorrelations and Partial Autocorrelations





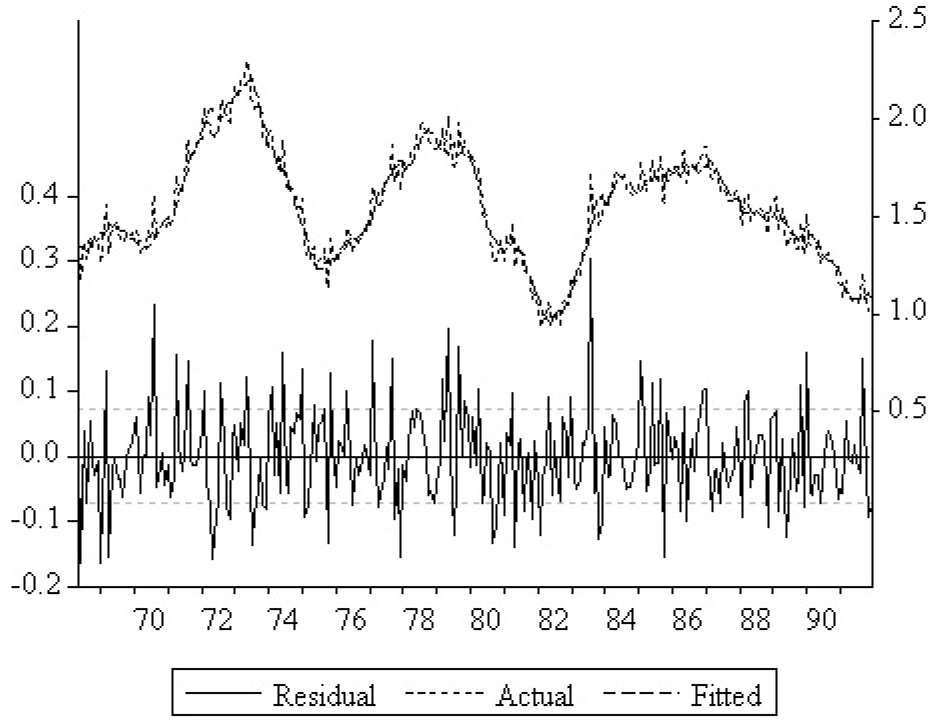
VAR Completions Equation

LS // Dependent Variable is COMPS
Sample(adjusted): 1968:05 1991:12
Included observations: 284 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.045347	0.025794	1.758045	0.0799
STARTS(-1)	0.074724	0.035711	2.092461	0.0373
STARTS(-2)	0.040047	0.042406	0.944377	0.3458
STARTS(-3)	0.047145	0.042366	1.112805	0.2668
STARTS(-4)	0.082331	0.038504	2.138238	0.0334
COMPS(-1)	0.236774	0.059893	3.953313	0.0001
COMPS(-2)	0.206172	0.060554	3.404742	0.0008
COMPS(-3)	0.120998	0.058863	2.055593	0.0408
COMPS(-4)	0.156729	0.055144	2.842160	0.0048

R-squared	0.936835	Mean dependent var	1.547958
Adjusted R-squared	0.934998	S.D. dependent var	0.286689
S.E. of regression	0.073093	Akaike info criterion	-5.200872
Sum squared resid	1.469205	Schwarz criterion	-5.085236
Log likelihood	344.5453	F-statistic	509.8375
Durbin-Watson stat	2.013370	Prob(F-statistic)	0.000000

VAR Completions Equation
Residual Plot

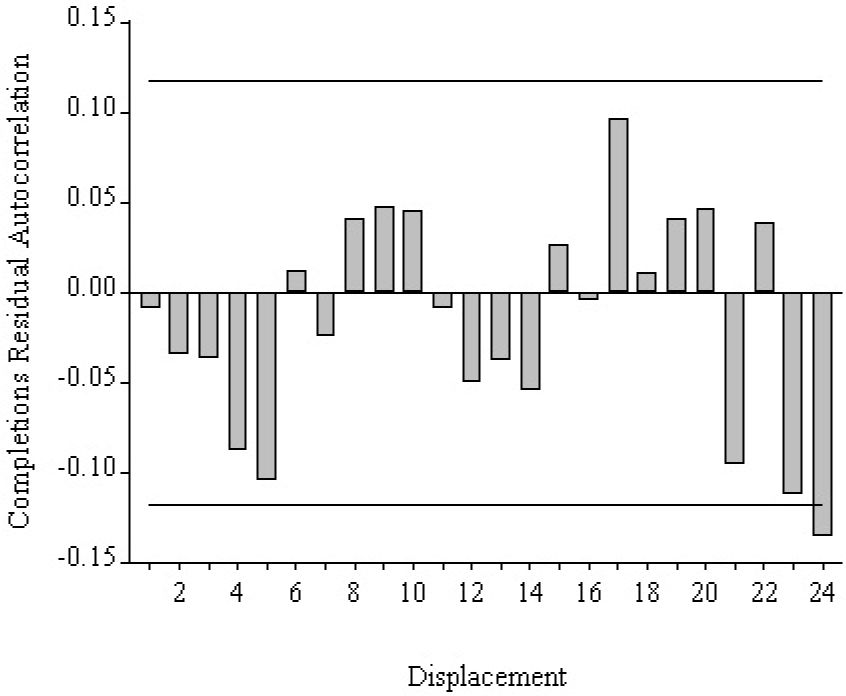


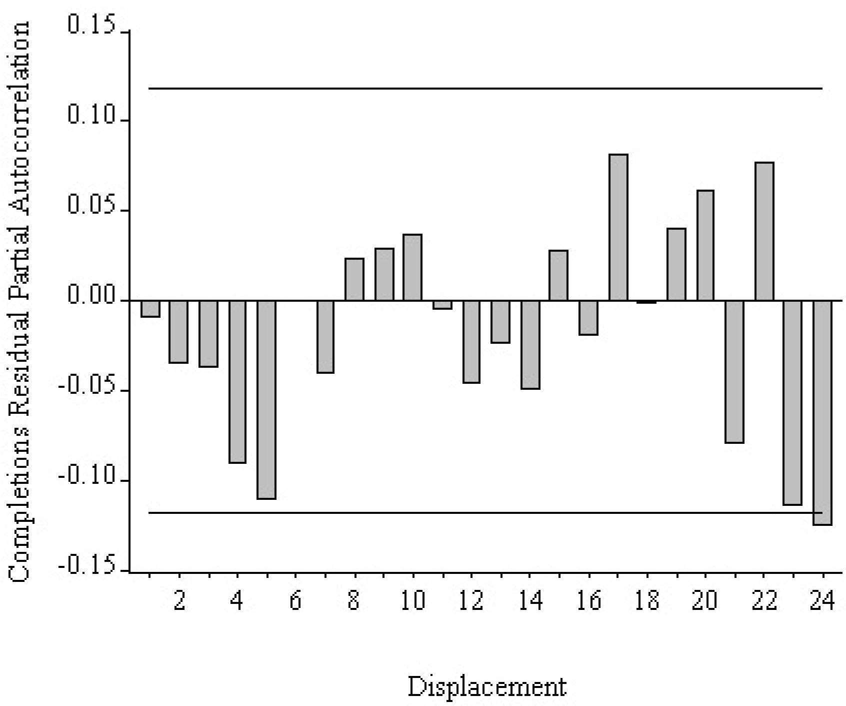
VAR Completions Equation
Residual Correlogram

Sample: 1968:01 1991:12
Included observations: 284

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	-0.009	-0.009	0.059	0.0238	0.877
2	-0.035	-0.035	0.059	0.3744	0.829
3	-0.037	-0.037	0.059	0.7640	0.858
4	-0.088	-0.090	0.059	3.0059	0.557
5	-0.105	-0.111	0.059	6.1873	0.288
6	0.012	0.000	0.059	6.2291	0.398
7	-0.024	-0.041	0.059	6.4047	0.493
8	0.041	0.024	0.059	6.9026	0.547
9	0.048	0.029	0.059	7.5927	0.576
10	0.045	0.037	0.059	8.1918	0.610
11	-0.009	-0.005	0.059	8.2160	0.694
12	-0.050	-0.046	0.059	8.9767	0.705
13	-0.038	-0.024	0.059	9.4057	0.742
14	-0.055	-0.049	0.059	10.318	0.739
15	0.027	0.028	0.059	10.545	0.784
16	-0.005	-0.020	0.059	10.553	0.836
17	0.096	0.082	0.059	13.369	0.711
18	0.011	-0.002	0.059	13.405	0.767
19	0.041	0.040	0.059	13.929	0.788
20	0.046	0.061	0.059	14.569	0.801
21	-0.096	-0.079	0.059	17.402	0.686
22	0.039	0.077	0.059	17.875	0.713
23	-0.113	-0.114	0.059	21.824	0.531
24	-0.136	-0.125	0.059	27.622	0.276

VAR Completions Equation
Residual Sample Autocorrelations and Partial Autocorrelations



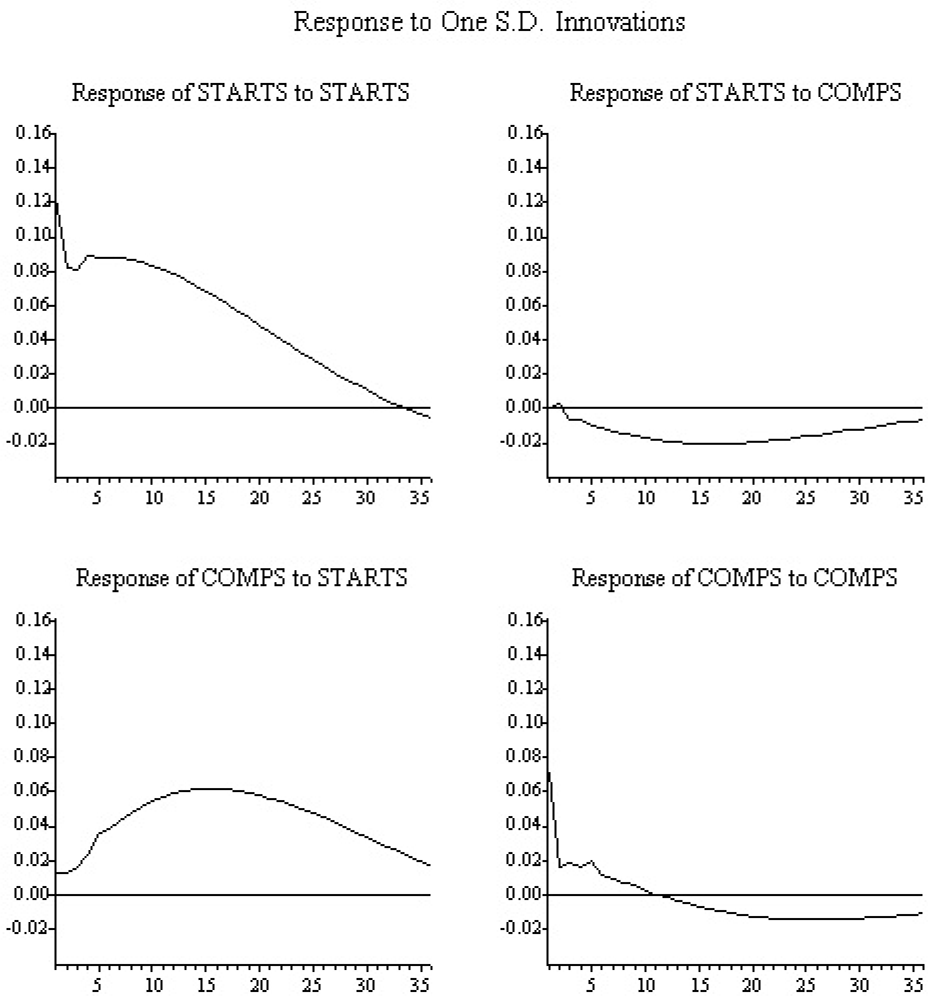


Housing Starts and Completions
Causality Tests

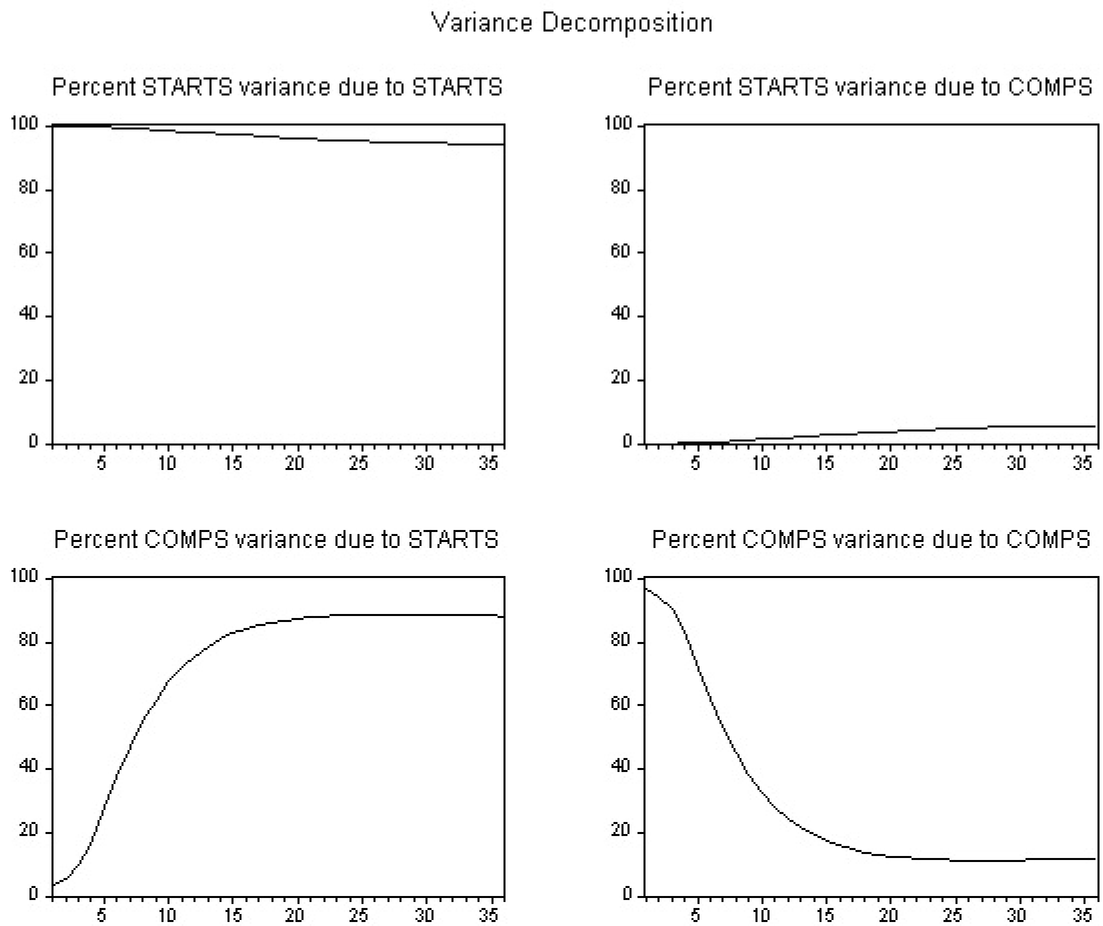
Sample: 1968:01 1991:12
Lags: 4
Obs: 284

Null Hypothesis:	F-Statistic	Probability
STARTS does not Cause COMPS	26.2658	0.00000
COMPS does not Cause STARTS	2.23876	0.06511

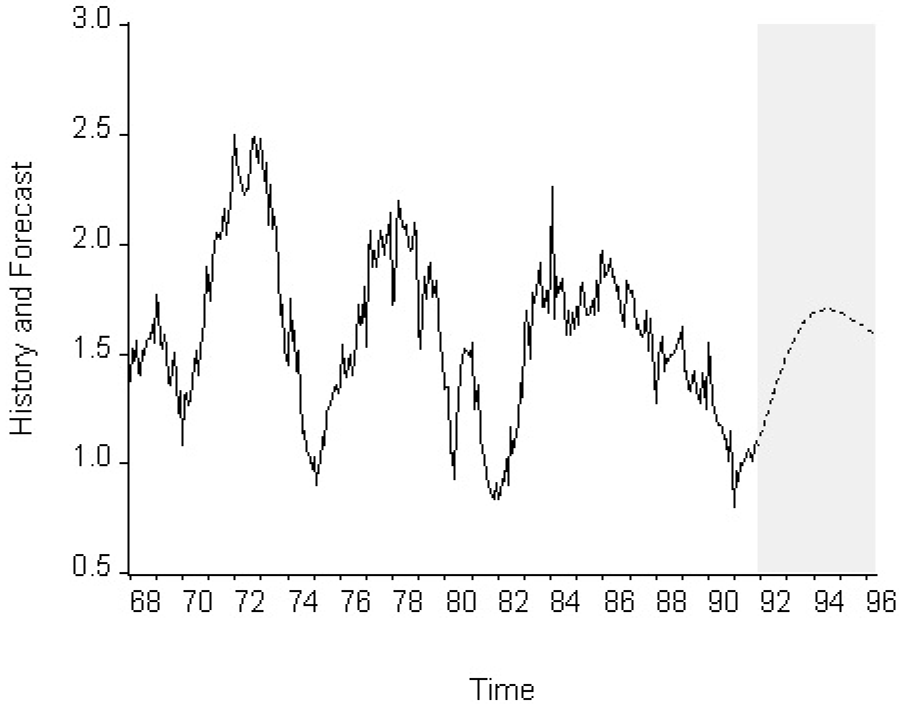
Housing Starts and Completions
VAR Impulse-Response Functions



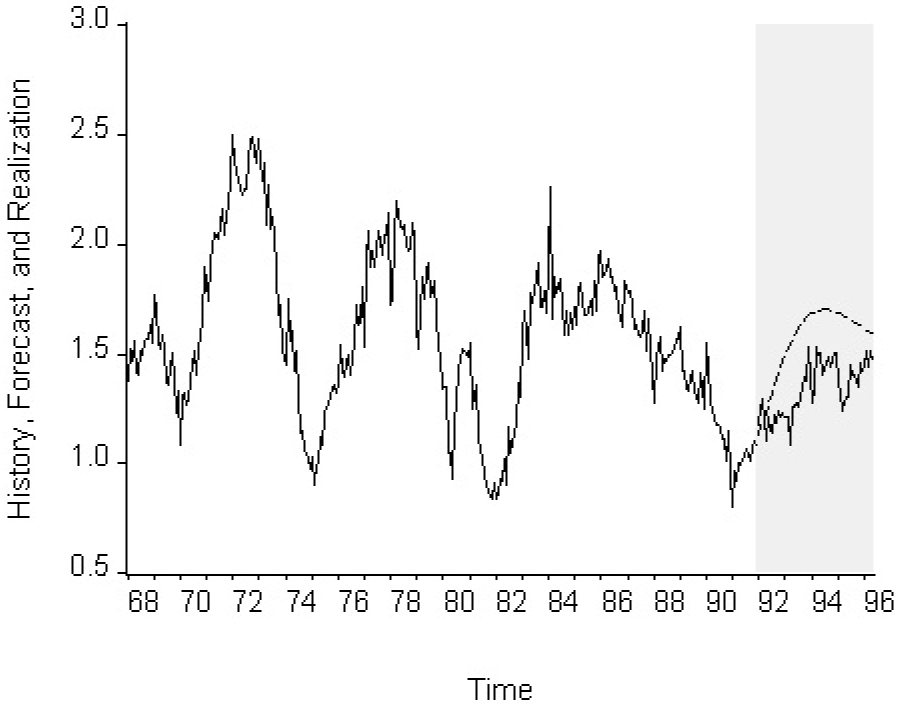
Housing Starts and Completions
VAR Variance Decompositions



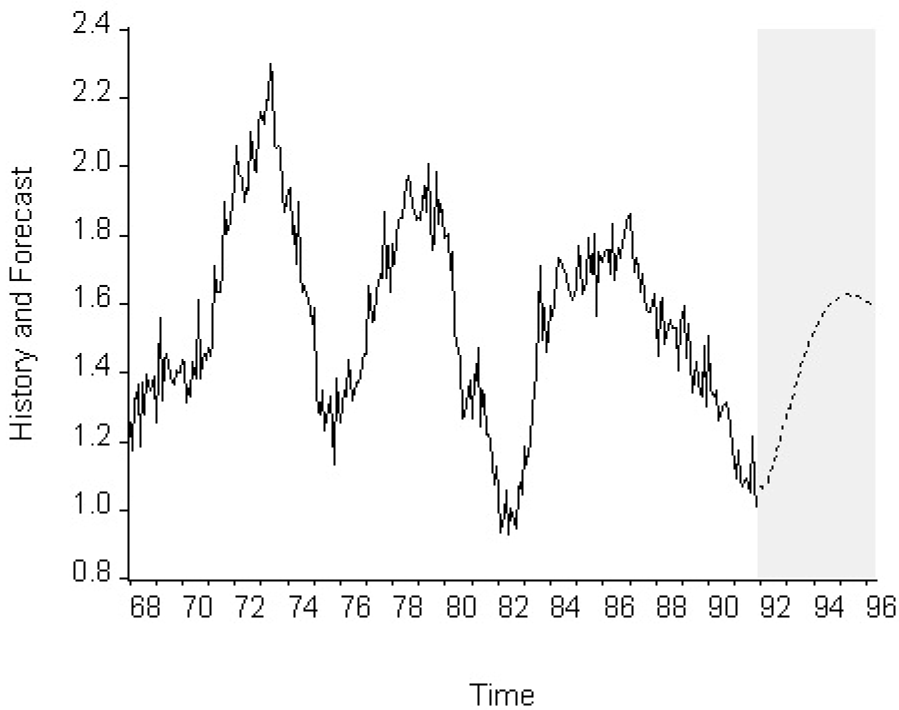
Starts
History, 1968.01-1991.12
Forecast, 1992.01-1996.06



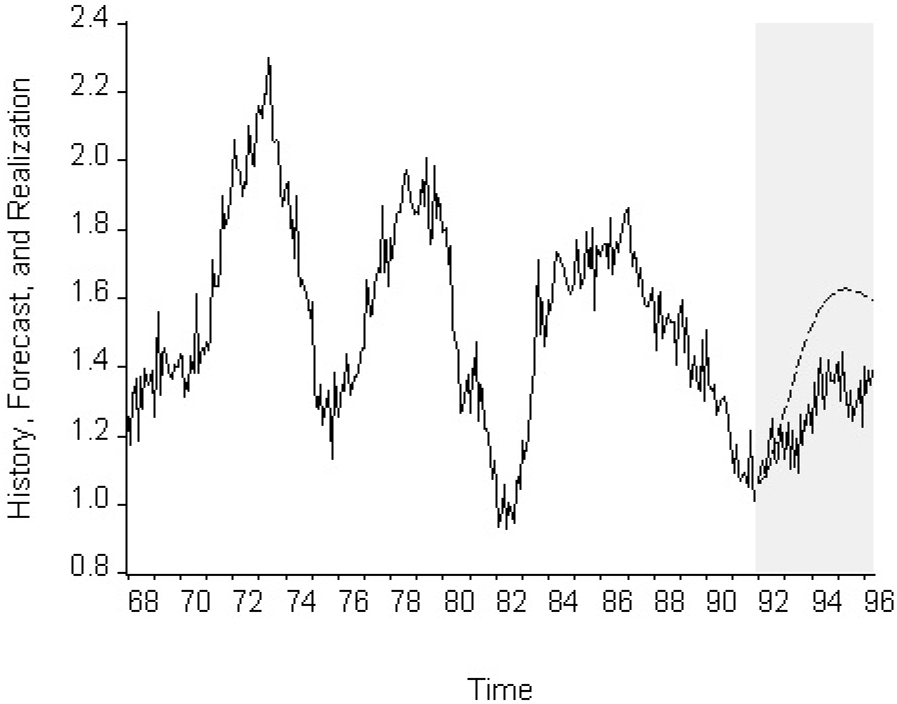
Starts
History, 1968.01-1991.12
Forecast and Realization, 1992.01-1996.06



Completions
History, 1968.01-1991.12
Forecast, 1992.01-1996.06



Completions
History, 1968.01-1991.12
Forecast and Realization, 1992.01-1996.06



Evaluating and Combining Forecasts

Evaluating a single forecast

Process:

$$y_t = \mu + \varepsilon_t + b_1 \varepsilon_{t-1} + b_2 \varepsilon_{t-2} + \ldots$$

$$\varepsilon_t \sim WN(0,\sigma^2)$$

h-step-ahead linear least-squares forecast:

$$y_{t+h,t} = \mu + b_h \varepsilon_t + b_{h+1} \varepsilon_{t-1} + \ldots$$

Corresponding h-step-ahead forecast error:

$$e_{t+h,t} = y_{t+h} - y_{t+h,t} = \varepsilon_{t+h} + b_1 \varepsilon_{t+h-1} + \ldots + b_{h-1} \varepsilon_{t+1}$$

with variance

$$\sigma_h^2 = \sigma^2 \left(1 + \sum_{i=1}^{h-1} b_i^2 \right)$$

So, four key properties of optimal forecasts:

a. Optimal forecasts are unbiased
b. Optimal forecasts have 1-step-ahead errors that are white noise
c. Optimal forecasts have h-step-ahead errors that are at most MA(h-1)
d. Optimal forecasts have h-step-ahead errors with variances that are non-decreasing in h and that converge to the unconditional variance of the process

- All are easily checked. How?

Assessing optimality with respect to an information set

Unforecastability principle: The errors from good forecasts are not be forecastable!

Regression:

$$e_{t+h,t} = \alpha_0 + \sum_{i=1}^{k-1} \alpha_i x_{it} + u_t$$

- Test whether $\alpha_0, \ldots, \alpha_{k-1}$ are 0

Important case:

$$e_{t+h,t} = \alpha_0 + \alpha_1 y_{t+h,t} + u_t$$

- Test whether $(\alpha_0, \alpha_1) = (0, 0)$

Equivalently,

$$y_{t+h} = \beta_0 + \beta_1 y_{t+h,t} + u_t$$

- Test whether $(\beta_0, \beta_1) = (0, 1)$

Evaluating multiple forecasts: comparing forecast accuracy

Forecast errors, $e_{t+h,t} = y_{t+h} - y_{t+h,t}$

Forecast percent errors, $p_{t+h,t} = (y_{t+h} - y_{t+h,t})/y_{t+h}$

$$ME = \frac{1}{T}\sum_{t=1}^{T} e_{t+h,t}$$

$$EV = \frac{1}{T}\sum_{t=1}^{T} (e_{t+h,t} - ME)^2$$

$$MSE = \frac{1}{T}\sum_{t=1}^{T} e_{t+h,t}^2$$

$$MSPE = \frac{1}{T}\sum_{t=1}^{T} p_{t+h,t}^2$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} e_{t+h,t}^2}$$

$$RMSPE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} p_{t+h,t}^2}$$

$$\text{MAE} = \frac{1}{T}\sum_{t=1}^{T} |e_{t+h,t}|$$

$$\text{MAPE} = \frac{1}{T}\sum_{t=1}^{T} |p_{t+h,t}|$$

Forecast encompassing

$$y_{t+h} = \beta_a y^a_{t+h,t} + \beta_b y^b_{t+h,t} + \varepsilon_{t+h,t}$$

- If $(\beta_a, \beta_b) = (1,0)$, model a forecast-encompasses model b
- If $(\beta_a, \beta_b) = (0,1)$, model b forecast-encompasses model a
- Otherwise, neither model encompasses the other

Alternative approach:

$$(y_{t+h} - y_t) = \beta_a \left(y^a_{t+h,t} - y_t \right) + \beta_b \left(y^b_{t+h,t} - y_t \right) + \varepsilon_{t+h,t}$$

- Useful in I(1) situations

Variance-covariance forecast combination

Composite formed from two unbiased forecasts:

$$y^{c}_{t+h,t} = \omega y^{a}_{t+h,t} + (1-\omega) y^{b}_{t+h,t}$$

$$e^{c}_{t+h,t} = \omega e^{a}_{t+h,t} + (1-\omega) e^{b}_{t+h,t}$$

$$\sigma^{2}_{c} = \omega^{2}\sigma^{2}_{aa} + (1-\omega)^{2}\sigma^{2}_{bb} + 2\omega(1-\omega)\sigma^{2}_{ab}$$

$$\omega^{*} = \frac{\sigma^{2}_{bb} - \sigma^{2}_{ab}}{\sigma^{2}_{bb} + \sigma^{2}_{aa} - 2\sigma^{2}_{ab}}$$

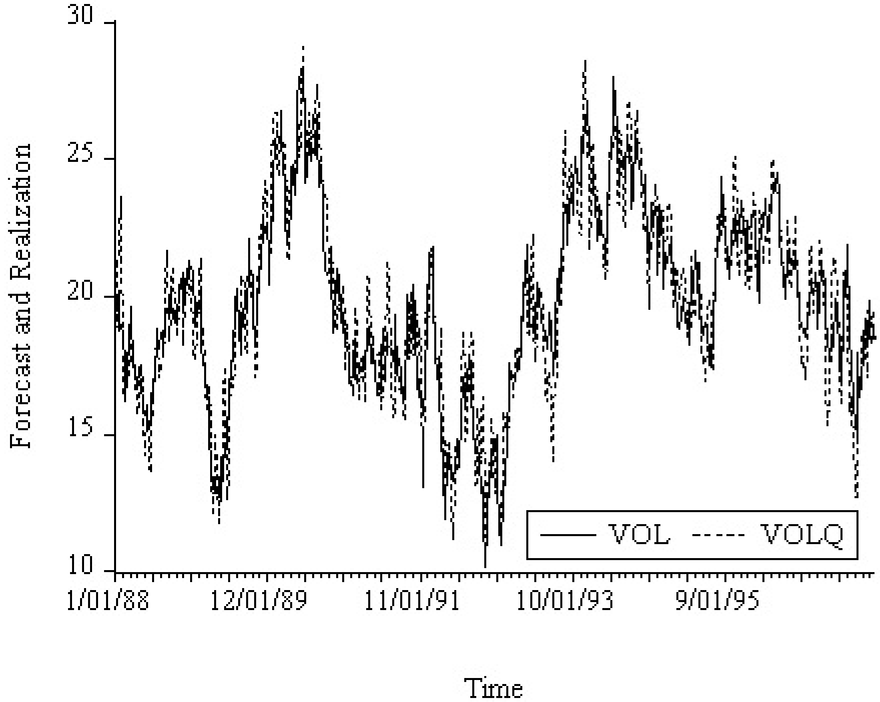
$$\hat{\omega}^{*} = \frac{\hat{\sigma}^{2}_{bb} - \hat{\sigma}^{2}_{ab}}{\hat{\sigma}^{2}_{bb} + \hat{\sigma}^{2}_{aa} - 2\hat{\sigma}^{2}_{ab}}$$

Regression-based forecast combination

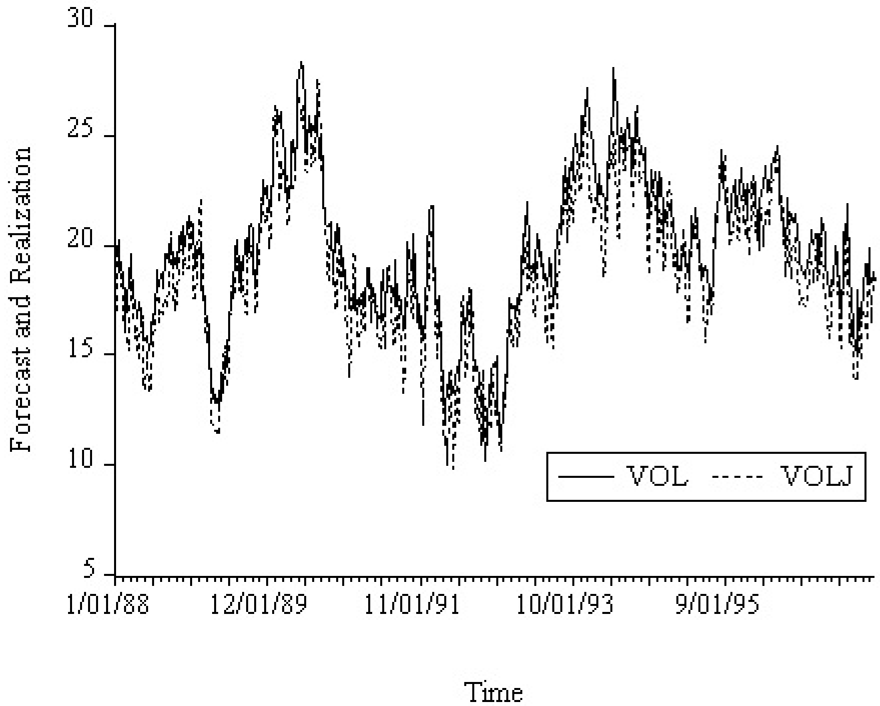
$$y_{t+h} = \beta_0 + \beta_1 y^a_{t+h,t} + \beta_2 y^b_{t+h,t} + \varepsilon_{t+h,t}$$

- Equivalent to variance-covariance combination if weights sum to unity and intercept is excluded
- Easy extension to include more than two forecasts
- Time-varying combining weights
- Dynamic combining regressions
- Shrinkage of combining weights toward equality
- Nonlinear combining regressions

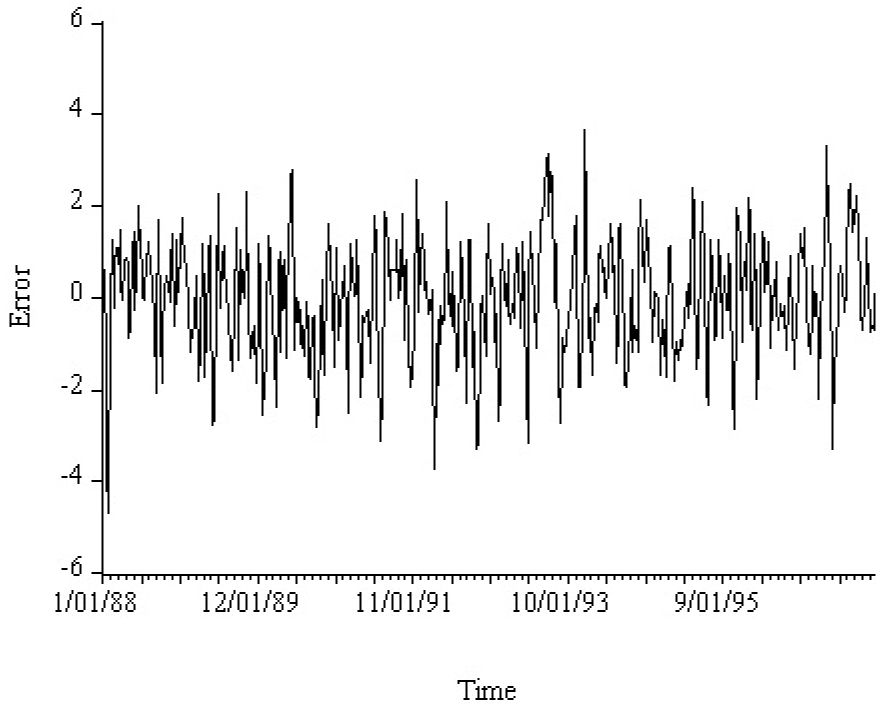
Shipping Volume
Quantitative Forecast and Realization



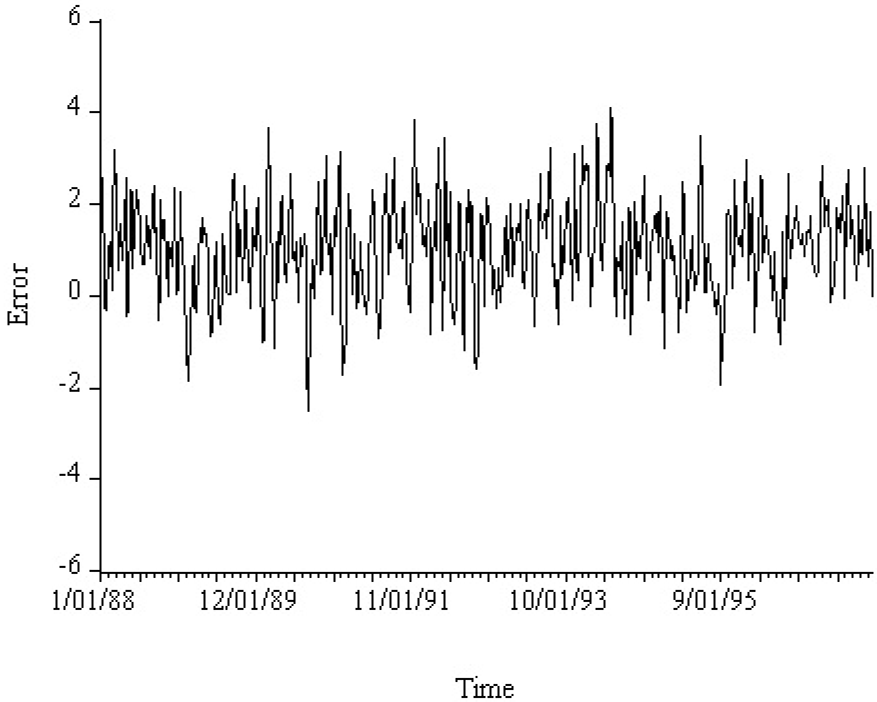
Shipping Volume
Judgmental Forecast and Realization



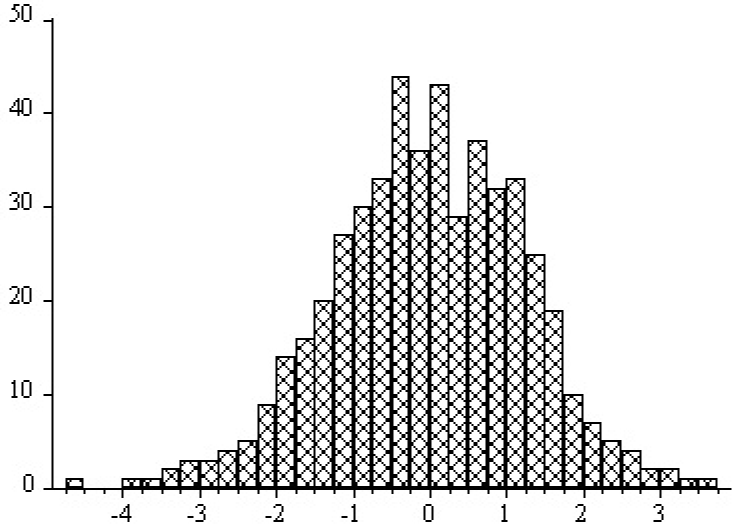
Quantitative Forecast Error



Judgmental Forecast Error

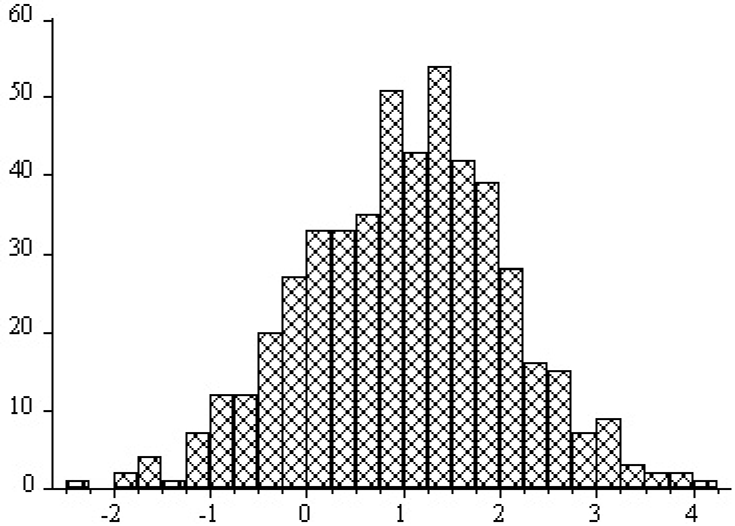


Histogram and Related Statistics
Quantitative Forecast Error



Series: EQ	
Sample 1/01/1988 7/18/1997	
Observations 499	
Mean	-0.026572
Median	0.002625
Maximum	3.681641
Minimum	-4.663387
Std. Dev.	1.262817
Skewness	-0.199902
Kurtosis	3.187625
Jarque-Bera	4.055331
Probability	0.131642

Histogram and Related Statistics
Judgmental Forecast Error



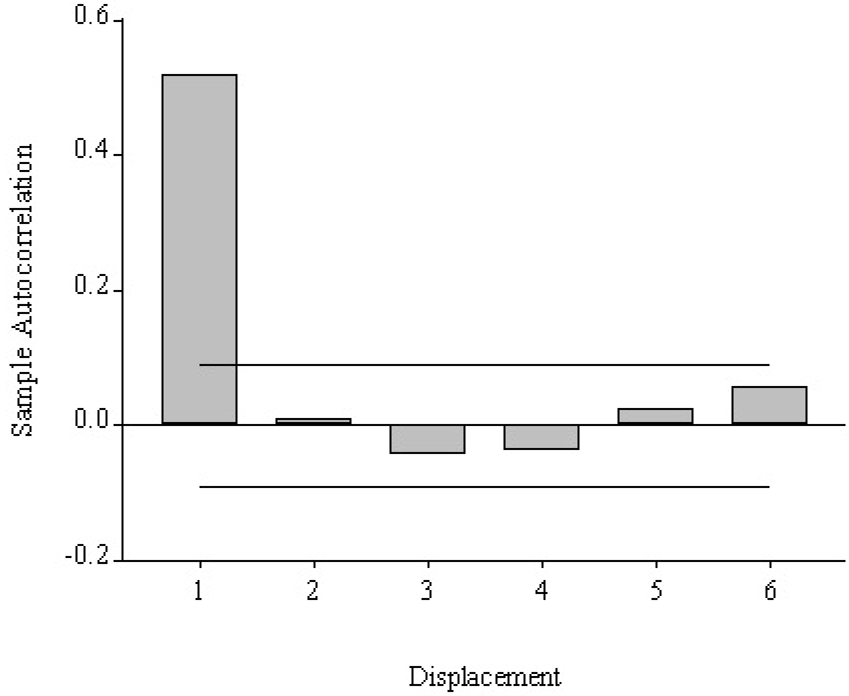
Series: EJ	
Sample 1/01/1988 7/18/1997	
Observations 499	
Mean	1.023744
Median	1.060523
Maximum	4.100623
Minimum	-2.481030
Std. Dev.	1.063681
Skewness	-0.106148
Kurtosis	3.078680
Jarque-Bera	1.065789
Probability	0.586904

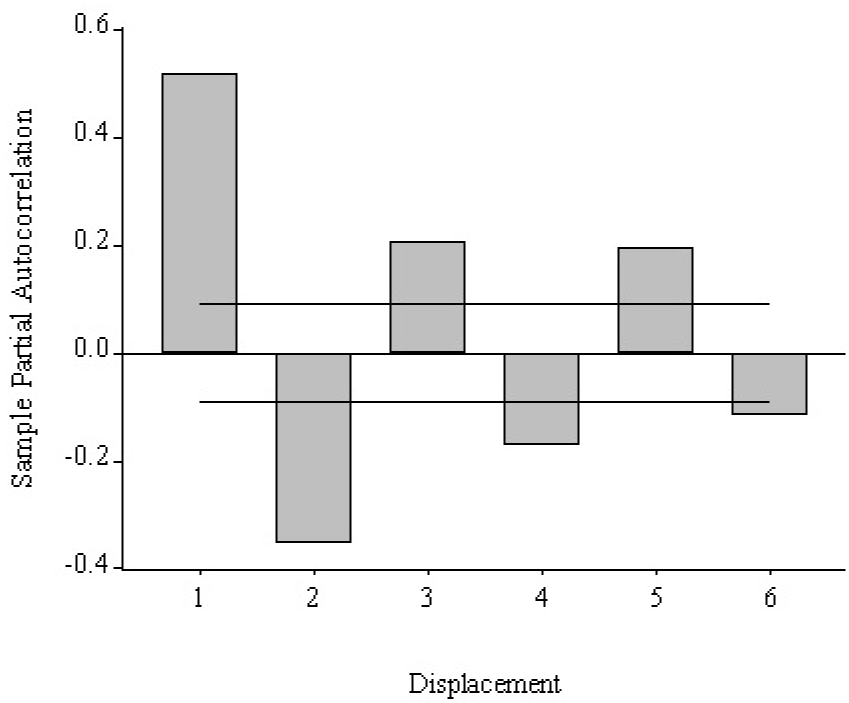
Correlogram, Quantitative Forecast Error

Sample: 1/01/1988 7/18/1997
Included observations: 499

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.518	0.518	.045	134.62	0.000
2	0.010	-0.353	.045	134.67	0.000
3	-0.044	0.205	.045	135.65	0.000
4	-0.039	-0.172	.045	136.40	0.000
5	0.025	0.195	.045	136.73	0.000
6	0.057	-0.117	.045	138.36	0.000

Sample Autocorrelations and Partial Autocorrelations
Quantitative Forecast Error



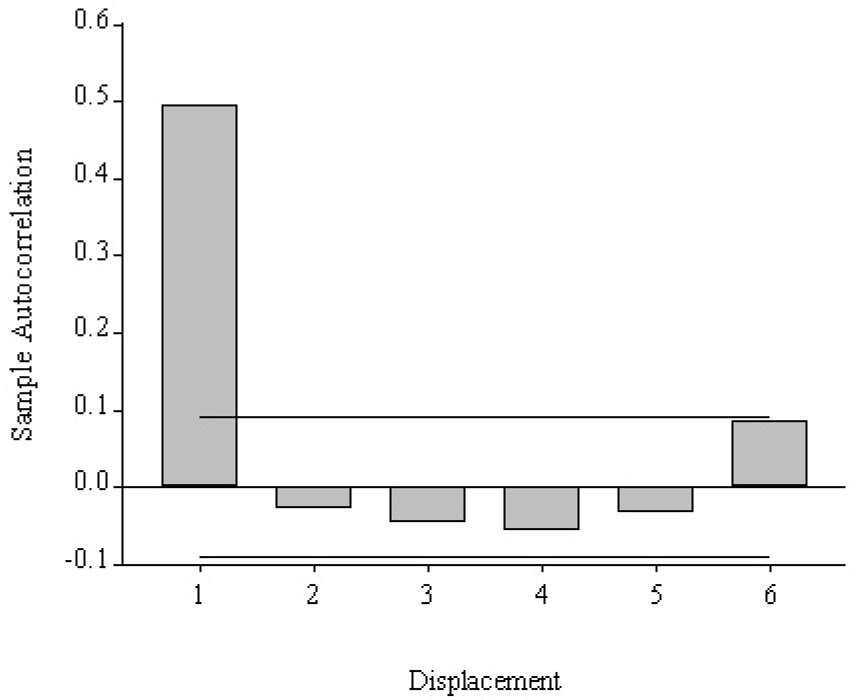


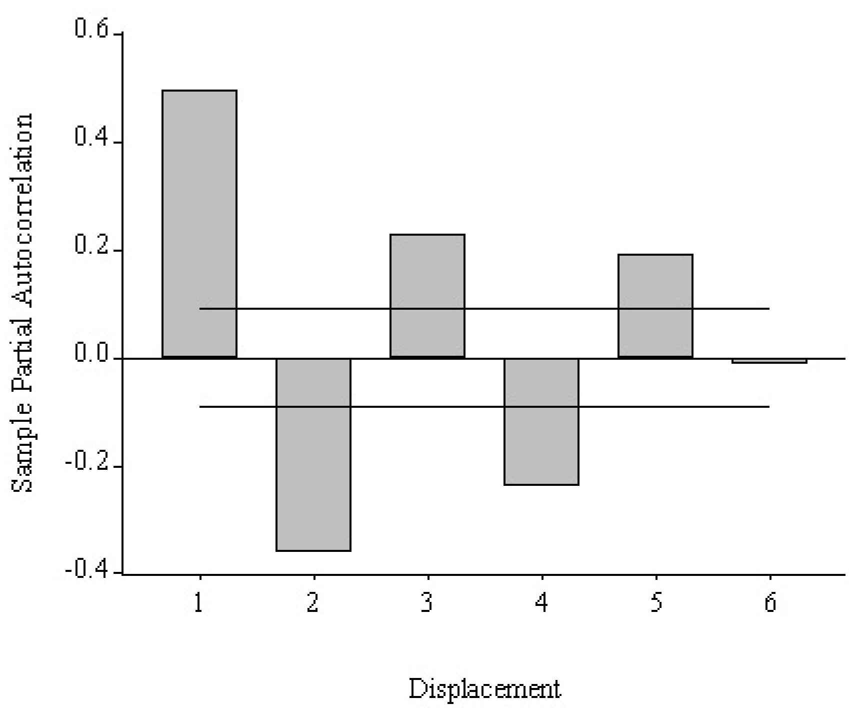
Correlogram, Judgmental Forecast Error

Sample: 1/01/1988 7/18/1997
Included observations: 499

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.495	0.495	.045	122.90	0.000
2	-0.027	-0.360	.045	123.26	0.000
3	-0.045	0.229	.045	124.30	0.000
4	-0.056	-0.238	.045	125.87	0.000
5	-0.033	0.191	.045	126.41	0.000
6	0.087	-0.011	.045	130.22	0.000

Sample Autocorrelations and Partial Autocorrelations
Judgmental Forecast Error





Quantitative Forecast Error
Regression on Intercept, MA(1) Disturbances

LS // Dependent Variable is EQ
Sample: 1/01/1988 7/18/1997
Included observations: 499
Convergence achieved after 6 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.024770	0.079851	-0.310200	0.7565
MA(1)	0.935393	0.015850	59.01554	0.0000

R-squared	0.468347	Mean dependent var	-0.026572
Adjusted R-squared	0.467277	S.D. dependent var	1.262817
S.E. of regression	0.921703	Akaike info criterion	-0.159064
Sum squared resid	422.2198	Schwarz criterion	-0.142180
Log likelihood	-666.3639	F-statistic	437.8201
Durbin-Watson stat	1.988237	Prob(F-statistic)	0.000000

Inverted MA Roots -.94

Judgmental Forecast Error
Regression on Intercept, MA(1) Disturbances

LS // Dependent Variable is EJ
Sample: 1/01/1988 7/18/1997
Included observations: 499
Convergence achieved after 7 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.026372	0.067191	15.27535	0.0000
MA(1)	0.961524	0.012470	77.10450	0.0000

R-squared	0.483514	Mean dependent var	1.023744
Adjusted R-squared	0.482475	S.D. dependent var	1.063681
S.E. of regression	0.765204	Akaike info criterion	-0.531226
Sum squared resid	291.0118	Schwarz criterion	-0.514342
Log likelihood	-573.5094	F-statistic	465.2721
Durbin-Watson stat	1.968750	Prob(F-statistic)	0.000000

Inverted MA Roots -.96

Mincer-Zarnowitz Regression
Quantitative Forecast Error

LS // Dependent Variable is VOL
Sample: 1/01/1988 7/18/1997
Included observations: 499
Convergence achieved after 10 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.958191	0.341841	8.653696	0.0000
VOLQ	0.849559	0.016839	50.45317	0.0000
MA(1)	0.912559	0.018638	48.96181	0.0000

R-squared	0.936972	Mean dependent var	19.80609
Adjusted R-squared	0.936718	S.D. dependent var	3.403283
S.E. of regression	0.856125	Akaike info criterion	-0.304685
Sum squared resid	363.5429	Schwarz criterion	-0.279358
Log likelihood	-629.0315	F-statistic	3686.790
Durbin-Watson stat	1.815577	Prob(F-statistic)	0.000000

Inverted MA Roots -.91

Wald Test:

Null Hypothesis:	C(1)=0	C(2)=1	
F-statistic	39.96862	Probability	0.000000
Chi-square	79.93723	Probability	0.000000

Mincer-Zarnowitz Regression
Judgmental Forecast Error

LS // Dependent Variable is VOL
Sample: 1/01/1988 7/18/1997
Included observations: 499
Convergence achieved after 11 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.592648	0.271740	9.540928	0.0000
VOLJ	0.916576	0.014058	65.20021	0.0000
MA(1)	0.949690	0.014621	64.95242	0.0000

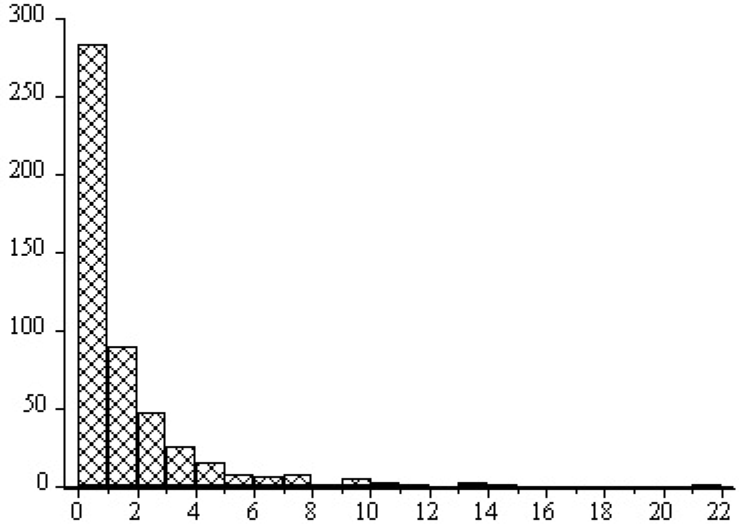
R-squared	0.952896	Mean dependent var	19.80609
Adjusted R-squared	0.952706	S.D. dependent var	3.403283
S.E. of regression	0.740114	Akaike info criterion	-0.595907
Sum squared resid	271.6936	Schwarz criterion	-0.570581
Log likelihood	-556.3715	F-statistic	5016.993
Durbin-Watson stat	1.917179	Prob(F-statistic)	0.000000

Inverted MA Roots -.95

Wald Test:
Null Hypothesis: C(1)=0 C(2)=1

F-statistic	143.8323	Probability	0.000000
Chi-square	287.6647	Probability	0.000000

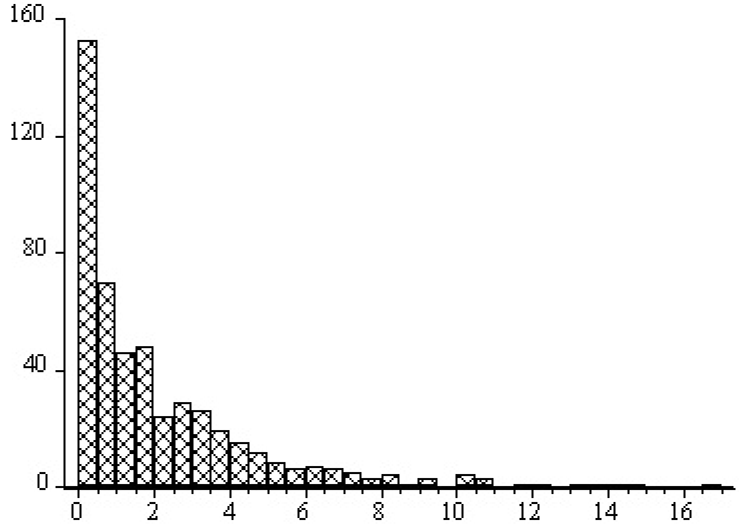
Histogram and Related Statistics
Squared Quantitative Forecast Error



Series: EQSQ
Sample 1/01/1988 7/18/1997
Observations 499

Mean	1.592217
Median	0.763750
Maximum	21.74718
Minimum	5.61E-06
Std. Dev.	2.369751
Skewness	3.293315
Kurtosis	18.88079
Jarque-Bera	6145.666
Probability	0.000000

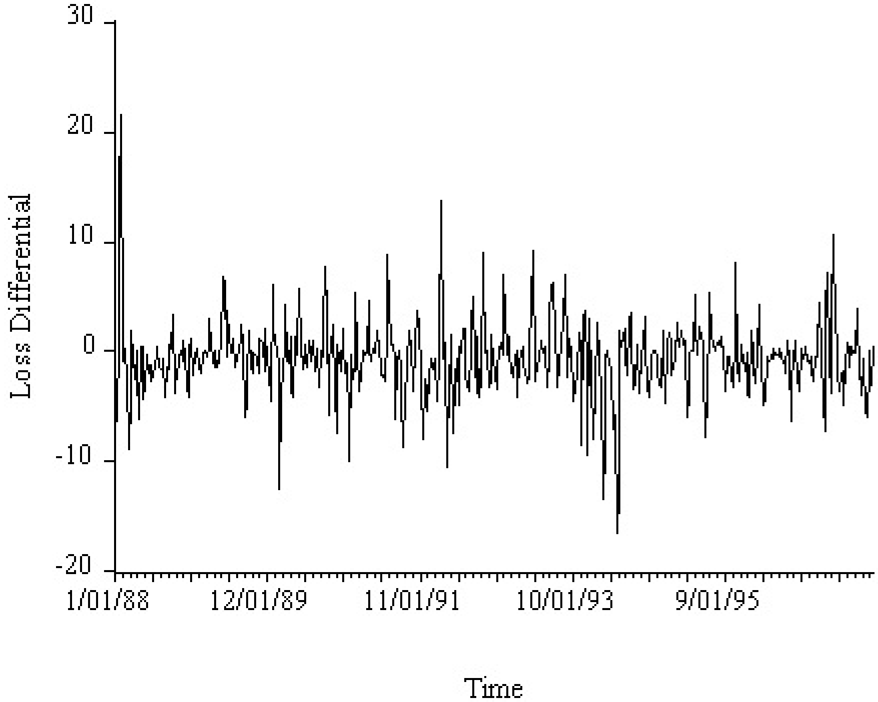
Histogram and Related Statistics
Squared Judgmental Forecast Error



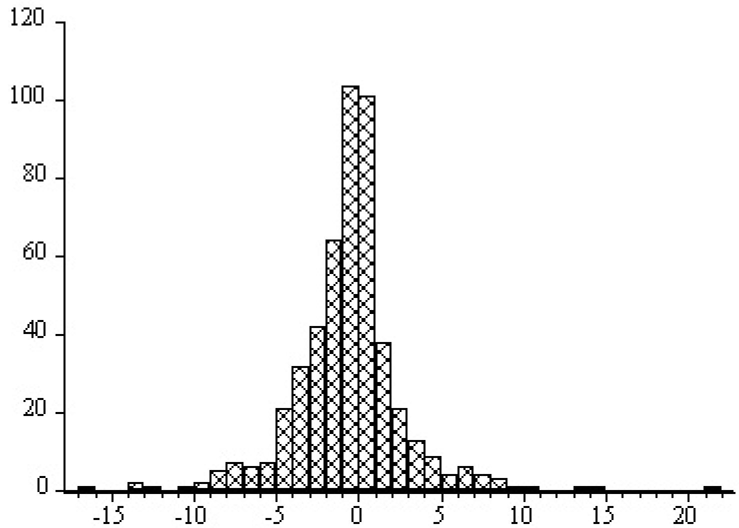
Series: EJSQ
Sample 1/01/1988 7/18/1997
Observations 499

Mean	2.177201
Median	1.308296
Maximum	16.81511
Minimum	4.63E-05
Std. Dev.	2.623644
Skewness	2.134551
Kurtosis	8.646748
Jarque-Bera	1041.891
Probability	0.000000

Loss Differential



Histogram and Related Statistics
Loss Differential



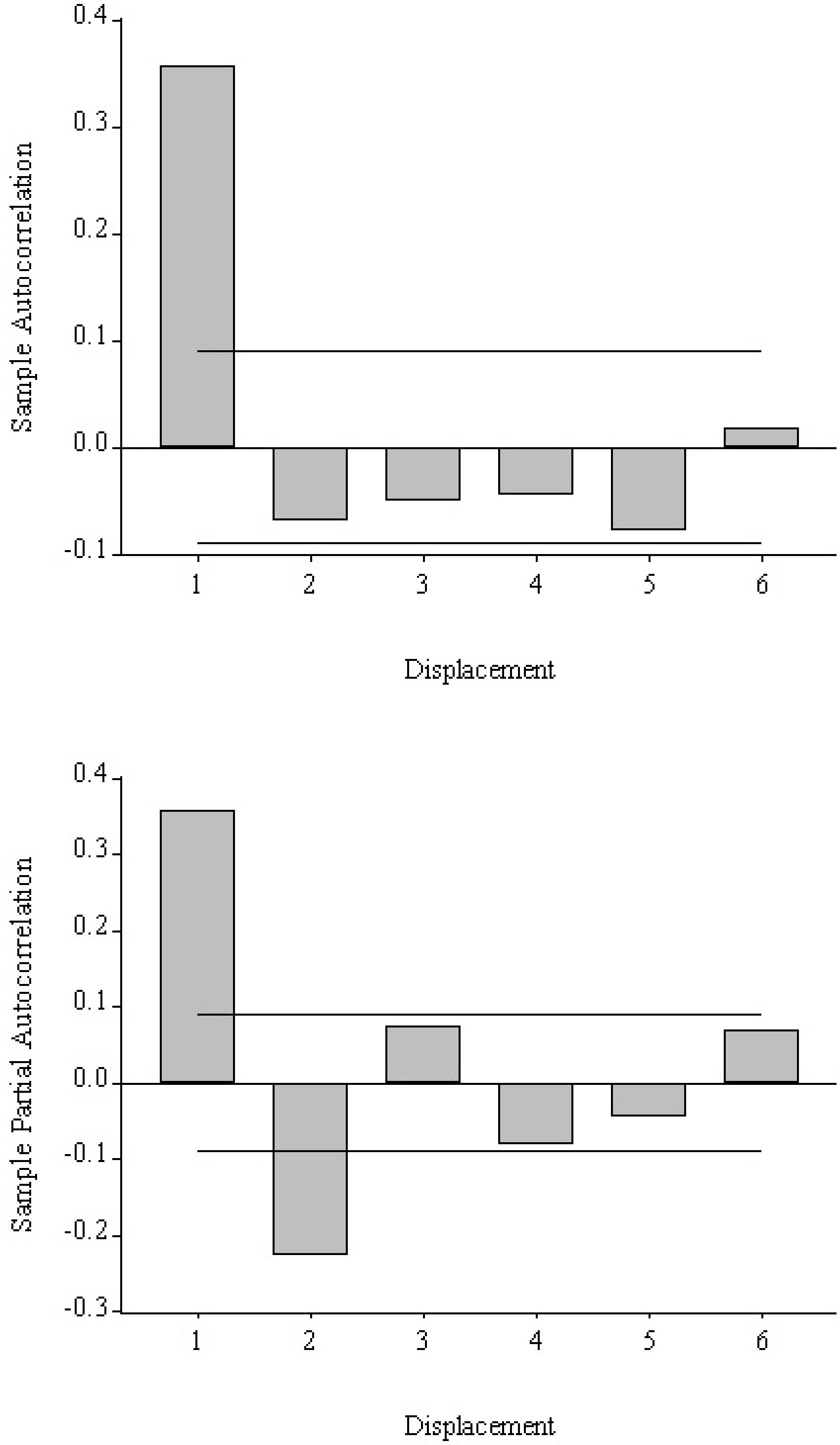
Series: DD	
Sample 1/01/1988 7/18/1997	
Observations 499	
Mean	-0.584984
Median	-0.395646
Maximum	21.65003
Minimum	-16.50010
Std. Dev.	3.416190
Skewness	0.421513
Kurtosis	9.472586
Jarque-Bera	885.8303
Probability	0.000000

Loss Differential Correlogram

Sample: 1/01/1988 7/18/1997
Included observations: 499

	Acorr.	P. Acorr.	Std. Error	Ljung-Box	p-value
1	0.357	0.357	.045	64.113	0.000
2	-0.069	-0.226	.045	66.519	0.000
3	-0.050	0.074	.045	67.761	0.000
4	-0.044	-0.080	.045	68.746	0.000
5	-0.078	-0.043	.045	71.840	0.000
6	0.017	0.070	.045	71.989	0.000

Sample Autocorrelations and Partial Autocorrelations
Loss Differential



Loss Differential
Regression on Intercept with MA(1) Disturbances

LS // Dependent Variable is DD
Sample: 1/01/1988 7/18/1997
Included observations: 499
Convergence achieved after 4 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.585333	0.204737	-2.858945	0.0044
MA(1)	0.472901	0.039526	11.96433	0.0000

R-squared	0.174750	Mean dependent var	-0.584984
Adjusted R-squared	0.173089	S.D. dependent var	3.416190
S.E. of regression	3.106500	Akaike info criterion	2.270994
Sum squared resid	4796.222	Schwarz criterion	2.287878
Log likelihood	-1272.663	F-statistic	105.2414
Durbin-Watson stat	2.023606	Prob(F-statistic)	0.000000

Inverted MA Roots -.47

Shipping Volume Combining Regression

LS // Dependent Variable is VOL
Sample: 1/01/1988 7/18/1997
Included observations: 499
Convergence achieved after 11 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.181977	0.259774	8.399524	0.0000
VOLQ	0.291577	0.038346	7.603919	0.0000
VOLJ	0.630551	0.039935	15.78944	0.0000
MA(1)	0.951107	0.014174	67.10327	0.0000

R-squared	0.957823	Mean dependent var	19.80609
Adjusted R-squared	0.957567	S.D. dependent var	3.403283
S.E. of regression	0.701049	Akaike info criterion	-0.702371
Sum squared resid	243.2776	Schwarz criterion	-0.668603
Log likelihood	-528.8088	F-statistic	3747.077
Durbin-Watson stat	1.925091	Prob(F-statistic)	0.000000

Inverted MA Roots -.95

Unit Roots, Stochastic Trends,
ARIMA Forecasting Models, and Smoothing

1. Stochastic Trends and Forecasting

$$\Phi(L)y_t = \Theta(L)\varepsilon_t$$

$$\Phi(L) = \Phi'(L)(1-L)$$

$$\Phi'(L)\ (1-L)y_t = \Theta(L)\varepsilon_t$$

$$\Phi'(L)\ \Delta y_t = \Theta(L)\varepsilon_t$$

I(0) vs I(1) processes

Random walk:

$$y_t = y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

Random walk with drift:

$$y_t = \delta + y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

Stochastic trend vs deterministic trend

Properties of random walks

$$y_t = y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

With time 0 value y_0:

$$y_t = y_0 + \sum_{i=1}^{t} \varepsilon_i$$

$$E(y_t) = y_0$$

$$var(y_t) = t\sigma^2$$

$$\lim_{t \to \infty} var(y_t) = \infty$$

Random walk with drift

$$y_t = \delta + y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

Assuming time 0 value y_0:

$$y_t = t\delta + y_0 + \sum_{i=1}^{t} \varepsilon_i$$

$$E(y_t) = y_0 + t\delta$$

$$var(y_t) = t\sigma^2$$

$$\lim_{t \to \infty} var(y_t) = \infty$$

ARIMA(p,1,q) model

$$\Phi(L)\ (1-L)y_t \ =\ c\ +\ \Theta(L)\varepsilon_t$$

or

$$(1-L)y_t \ =\ c\Phi^{-1}(1)\ +\ \Phi^{-1}(L)\Theta(L)\varepsilon_t,$$

where

$$\Phi(L)\ =\ 1\ -\ \Phi_1 L\ -\ \ldots\ -\ \Phi_p L^{p}$$

$$\Theta(L)\ =\ 1\ -\ \Theta_1 L\ -\ \ldots\ -\ \Theta_q L^{q}$$

and all the roots of both lag operator polynomials are outside the unit circle.

ARIMA(p,d,q) model

$$\Phi(L)\ (1-L)^d y_t = c + \Theta(L)\varepsilon_t$$

or

$$(1-L)^d y_t = c\Phi^{-1}(1) + \Phi^{-1}(L)\Theta(L)\varepsilon_t$$

where

$$\Phi(L) = 1 - \Phi_1 L - \ldots - \Phi_p L^p$$

$$\Theta(L) = 1 - \Theta_1 L - \ldots - \Theta_q L^q$$

and all the roots of both lag operator polynomials are outside the unit circle.

Properties of ARIMA(p,1,q) processes

- Appropriately made stationary by differencing

- Shocks have permanent effects
  -- Forecasts don't revert to a mean

- Variance grows without bound as time progresses
  -- Interval forecasts widen without bound as horizon grows

Random walk example

<u>Point forecast</u>

Recall that for the AR(1) process,

$$y_t = \varphi y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim WN(0, \sigma^2)$$

the optimal forecast is

$$y_{T+h,T} = \varphi^h y_T$$

Thus in the random walk case,

$$y_{T+h,T} = y_T, \text{ for all } h$$

Interval and density forecasts

Recall error associated with optimal AR(1) forecast:

$$e_{T+h,T} = (y_{T+h} - y_{T+h,T}) = \varepsilon_{T+h} + \varphi\varepsilon_{T+h-1} + \ldots + \varphi^{h-1}\varepsilon_{T+1}$$

with variance

$$\sigma_h^2 = \sigma^2 \sum_{i=0}^{h-1} \varphi^{2i}$$

Thus in the random walk case,

$$e_{T+h,T} = \sum_{i=0}^{h-1} \varepsilon_{T+h-i}$$

$$\sigma_h^2 = h\sigma^2$$

h-step-ahead 95% interval forecast: $y_T \pm 1.96\sigma\sqrt{h}$

h-step-ahead density forecast: $N(y_T,\ h\sigma^2)$

Effects of Unit Roots

- Sample autocorrelation function "fails to damp"

- Sample partial autocorrelation function near 1 for $\tau=1$, and then damps quickly

- Properties of estimators change

e.g., least-squares autoregression with unit roots

True process:

$$y_t = y_{t-1} + \varepsilon_t$$

Estimated model:

$$y_t = \varphi y_{t-1} + \varepsilon_t$$

Superconsistency: $T(\hat{\varphi}_{LS} - 1)$ stabilizes as sample size grows

Bias: $E(\hat{\varphi}_{LS}) < 1$

-- Ofsetting effects of bias and superconsistency

Unit Root Tests

$$y_t = \varphi y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$$

$$\hat{\tau} = \frac{\hat{\varphi} - 1}{s\sqrt{\dfrac{1}{\sum_{t=2}^{T} y_{t-1}^2}}}$$

"Dickey-Fuller $\hat{\tau}$ distribution"

Trick regression:

$$y_t - y_{t-1} = (\varphi - 1)\, y_{t-1} + \varepsilon_t$$

Allowing for nonzero mean under the alternative

Basic model:

$$(y_t - \mu) = \varphi(y_{t-1} - \mu) + \varepsilon_t$$

which we rewrite as

$$y_t = \alpha + \varphi y_{t-1} + \varepsilon_t$$

where

$$\alpha = \mu(1 - \varphi)$$

- α vanishes when $\varphi = 1$ (null)
- α is nevertheless present under the alternative,
  so we include an intercept in the regression

Dickey-Fuller $\hat{\tau}_\mu$ distribution

Allowing for deterministic linear trend under the alternative

Basic model:

$$(y_t - a - b\ TIME_t) = \varphi(y_{t-1} - a - b\ TIME_{t-1}) + \varepsilon_t$$

or

$$y_t = \alpha + \beta TIME_t + \varphi y_{t-1} + \varepsilon_t,$$

where $\alpha = a(1-\varphi) + b\varphi$ and $\beta = b(1-\varphi)$.

- Under the null hypothesis we have a random walk with drift,

$$y_t = b + y_{t-1} + \varepsilon_t$$

- Under the deterministic-trend alternative hypothesis both the intercept and the trend enter and so are included in the regression.

Allowing for higher-order autoregressive dynamics

AR(p) process:

$$y_t + \sum_{j=1}^{p} \varphi_j y_{t-j} = \varepsilon_t$$

Rewrite:

$$y_t = \rho_1 y_{t-1} + \sum_{j=2}^{p} \rho_j (y_{t-j+1} - y_{t-j}) + \varepsilon_t$$

where $p \geq 2$, $\rho_1 = -\sum_{j=1}^{p} \varphi_j$, and $\rho_i = \sum_{j=i}^{p} \varphi_j$, $i = 2,\ldots,p$.

Unit root: $\rho_1 = 1$ (AR(p-1) in first differences)

$\hat{\tau}$ distribution holds asymptotically

Allowing for a nonzero mean in the AR(p) case

$$(y_t - \mu) + \sum_{j=1}^{p} \varphi_j \, (y_{t-j} - \mu) = \varepsilon_t$$

or

$$y_t = \alpha + \rho_1 y_{t-1} + \sum_{j=2}^{p} \rho_j \, (y_{t-j+1} - y_{t-j}) + \varepsilon_t$$

where $\alpha = \mu(1 + \sum_{j=1}^{p} \varphi_j)$, and the other parameters are as above. In the unit root case, the intercept vanishes, because $\sum_{j=1}^{p} \varphi_j = -1$. $\hat{\tau}_\mu$ distribution holds asymptotically.

Allowing for trend under the alternative

$$(y_t - a - bTIME_t) + \sum_{j=1}^{p} \varphi_j(y_{t-j} - a - bTIME_{t-j}) = \varepsilon_t$$

or

$$y_t = k_1 + k_2TIME_t + \rho_1 y_{t-1} + \sum_{j=2}^{p} \rho_j(y_{t-j+1} - y_{t-j}) + \varepsilon_t,$$

where

$$k_1 = a(1 + \sum_{i=1}^{p} \varphi_i) - b\sum_{i=1}^{p} i\varphi_i$$

and

$$k_2 = b\ TIME_t\ (1 + \sum_{i=1}^{p} \varphi_i).$$

In the unit root case, $k_1 = -b\sum_{i=1}^{p} i\varphi_i$ and $k_2 = 0$.

$\hat{\tau}_\tau$ distribution holds asymptotically.

General ARMA representations: augmented Dickey-Fuller tests

$$y_t = \rho_1 y_{t-1} + \sum_{j=2}^{k-1} \rho_j (y_{t-j+1} - y_{t-j}) + \varepsilon_t$$

$$y_t = \alpha + \rho_1 y_{t-1} + \sum_{j=2}^{k-1} \rho_j (y_{t-j+1} - y_{t-j}) + \varepsilon_t$$

$$y_t = k_1 + k_2 TIME_t + \rho_1 y_{t-1} + \sum_{j=2}^{k-1} \rho_j (y_{t-j+1} - y_{t-j}) + \varepsilon_t$$

- k-1 augmentation lags have been included

- $\hat{\tau}$, $\hat{\tau}_\mu$, and $\hat{\tau}_\tau$ hold asymptotically under the null

Simple moving average smoothing

Original data: $\{y_t\}_{t=1}^{T}$

Smoothed data: $\{\bar{y}_t\}$

Two-sided moving average is $\bar{y}_t = (2m+1)^{-1} \sum_{i=-m}^{m} y_{t-i}$

One-sided moving average is $\bar{y}_t = (m+1)^{-1} \sum_{i=0}^{m} y_{t-i}$

One-sided weighted moving average is $\bar{y}_t = \sum_{i=0}^{m} w_i y_{t-i}$

- Must choose smoothing parameter, m

Exponential Smoothing

Local level model:

$$y_t = c_{0t} + \varepsilon_t$$

$$c_{0t} = c_{0,t-1} + \eta_t$$

$$\eta_t \sim WN(0, \sigma_\eta^2)$$

- Exponential smoothing can construct the optimal estimate of c_0 -- and hence the optimal forecast of any future value of y -- on the basis of current and past y

- What if the model is misspecified?

Exponential smoothing algorithm

Observed series, $\{y_t\}_{t=1}^{T}$

Smoothed series, $\{\bar{y}_t\}_{t=1}^{T}$ (estimate of the local level)

Forecasts, $\hat{y}_{T+h,T}$

(1) Initialize at t=1: $\bar{y}_1 = y_1$

(2) Update: $\bar{y}_t = \alpha y_t + (1-\alpha)\bar{y}_{t-1}$, $t = 2, ..., T$

(3) Forecast: $\hat{y}_{T+h,T} = \bar{y}_T$

- Smoothing parameter $\alpha \in [0,1]$.

Demonstration that the weights are exponential

Start:

$$\bar{y}_t = \alpha y_t + (1-\alpha)\bar{y}_{t-1}$$

Substitute backward for $\bar{y}$:

$$\bar{y}_t = \sum_{j=0}^{t-1} w_j y_{t-j}$$

where

$$w_j = \alpha(1-\alpha)^j$$

- Exponential weighting, as claimed

- Convenient recursive structure

Holt-Winters Smoothing

$$y_t = c_{0t} + c_{1t}TIME_t + \varepsilon_t$$

$$c_{0t} = c_{0,t-1} + \eta_t$$

$$c_{1t} = c_{1,t-1} + \nu_t$$

- Local level and slope model

- Holt-Winters smoothing can construct optimal estimates of c_0 and c_1 -- and hence the optimal forecast of any future value of y by extrapolating the trend -- on the basis of current and past y

Holt-Winters smoothing algorithm

(1) Initialize at t=2:

$$\bar{y}_2 = y_2$$

$$F_2 = y_2 - y_1$$

(2) Update:

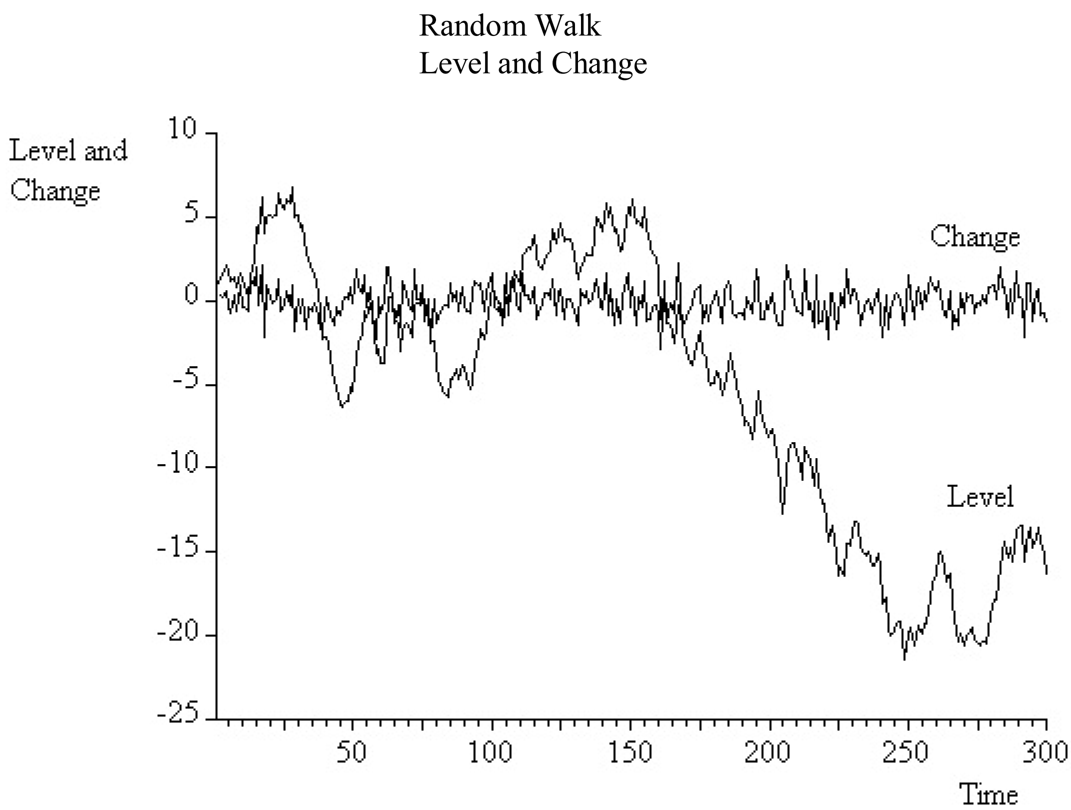
$$\bar{y}_t = \alpha y_t + (1-\alpha)(\bar{y}_{t-1} + F_{t-1}), \quad 0<\alpha<1$$

$$F_t = \beta(\bar{y}_t - \bar{y}_{t-1}) + (1-\beta)F_{t-1}, \quad 0<\beta<1$$

$t = 3, 4, ..., T.$

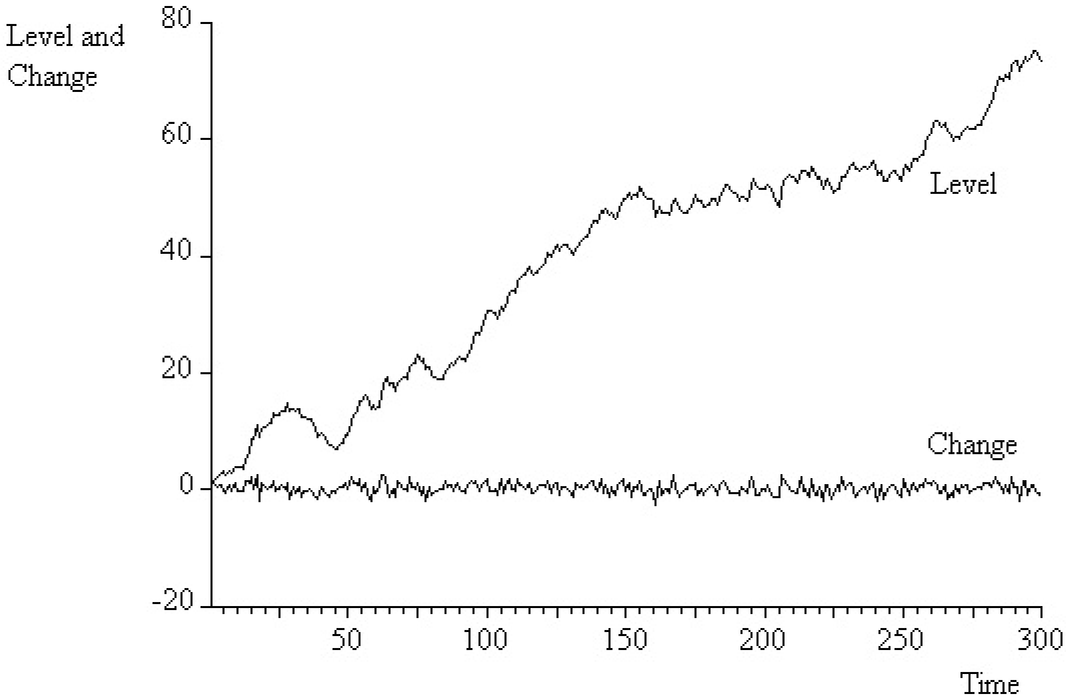
(3) Forecast: $\hat{y}_{T+h,T} = \bar{y}_T + hF_T$

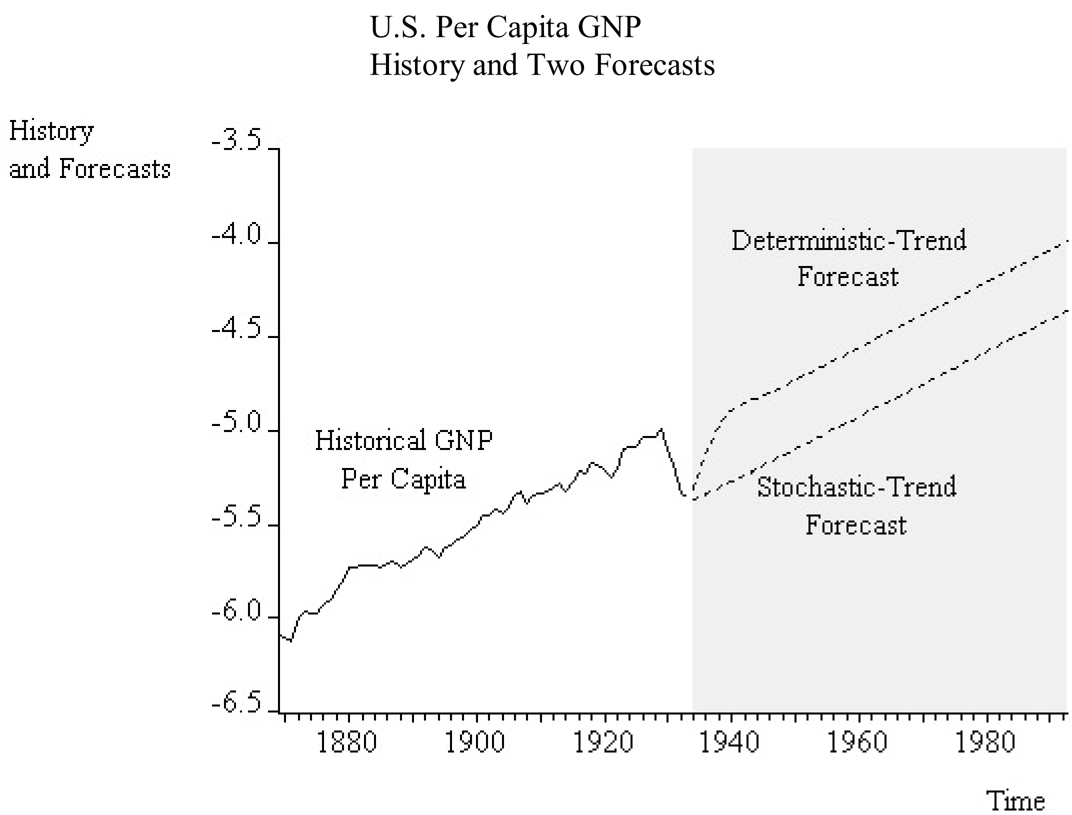
- $\bar{y}_t$ is the estimated level at time t
- F_t is the estimated slope at time t



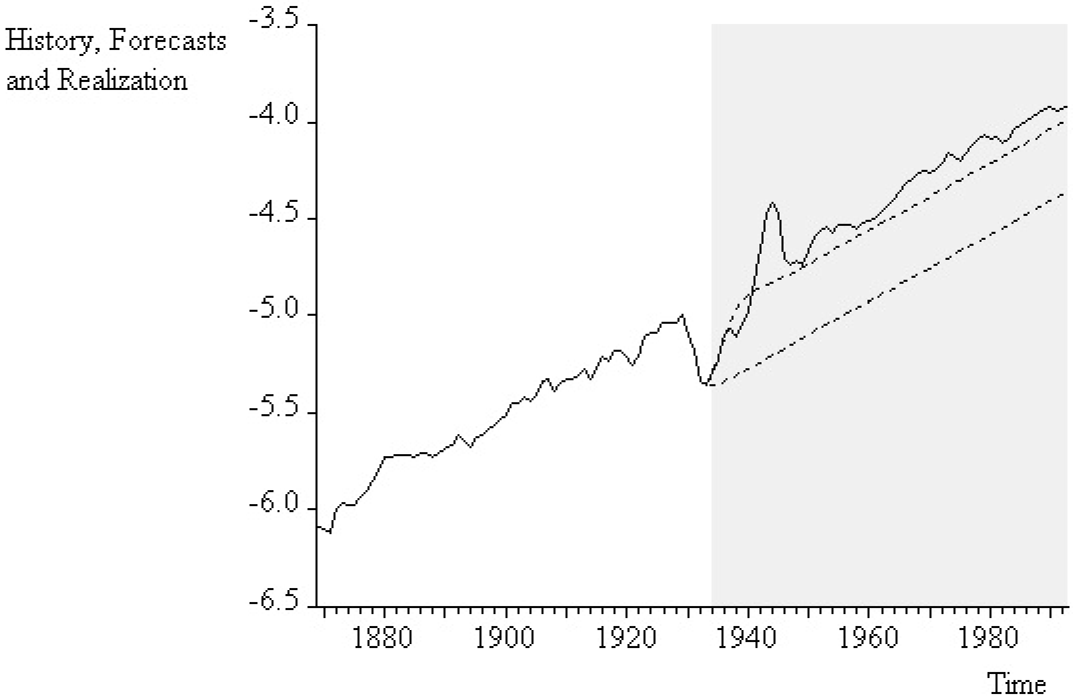
Random Walk
Level and Change
Level and
Change
10
5
0
-5
-10
-15
-20
-25
Change
Level
50
100
150
200
250
300
Time

Random Walk With Drift
Level and Change

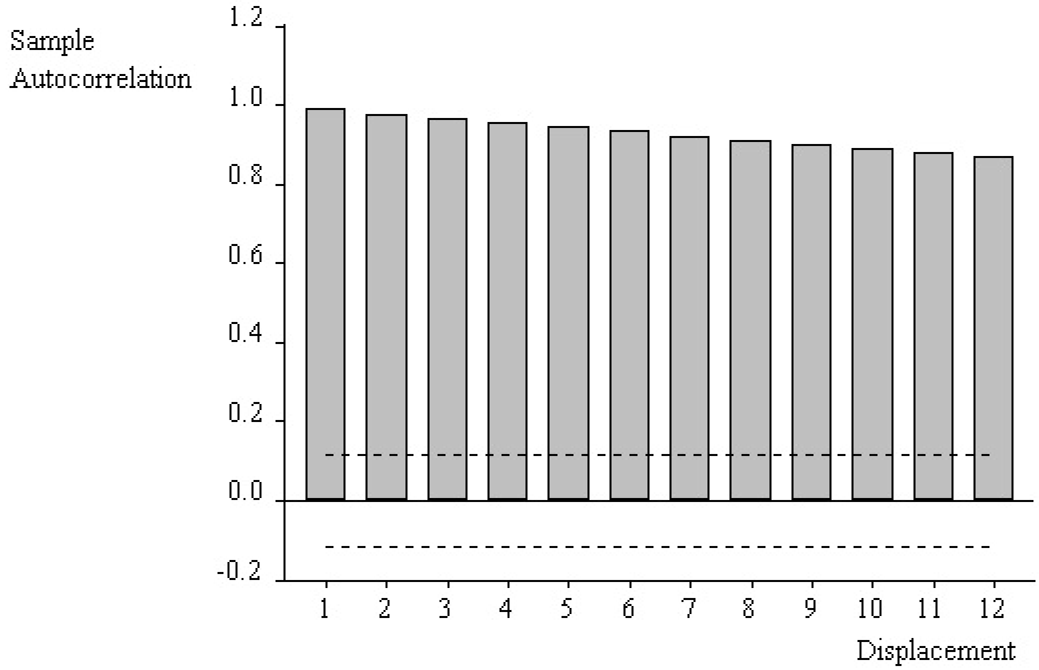


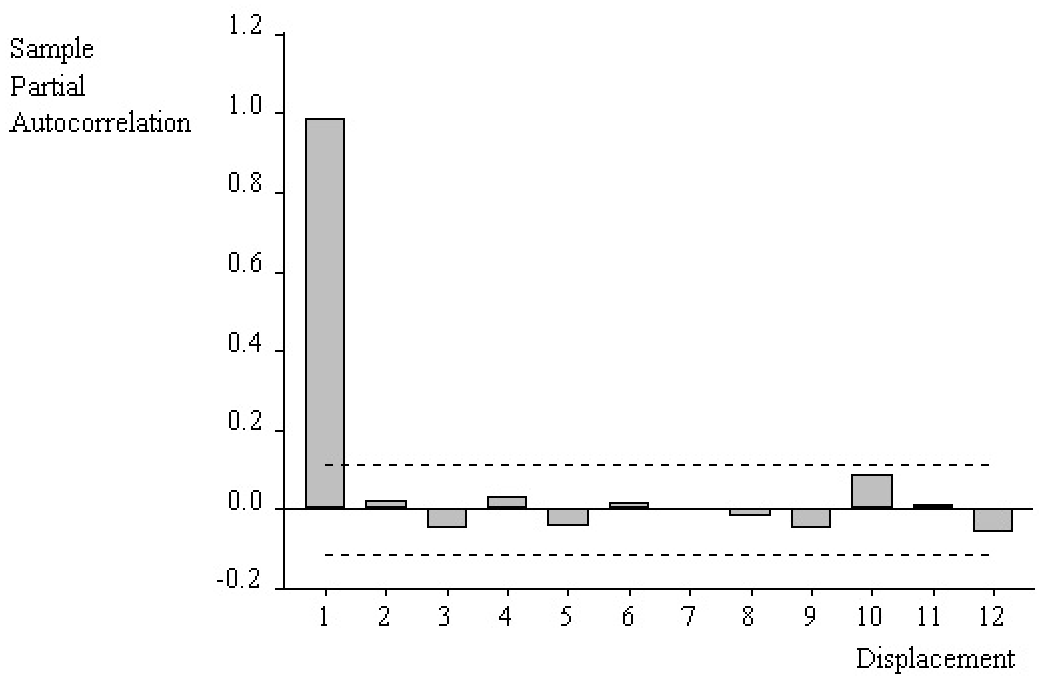

U.S. Per Capita GNP
History and Two Forecasts
History
and Forecasts
-3.5
-4.0
-4.5
-5.0
-5.5
-6.0
-6.5
Deterministic-Trend
Forecast
Historical GNP
Per Capita
Stochastic-Trend
Forecast
1880
1900
1920
1940
1960
1980
Time

U.S. Per Capita GNP
History, Two Forecasts, and Realization

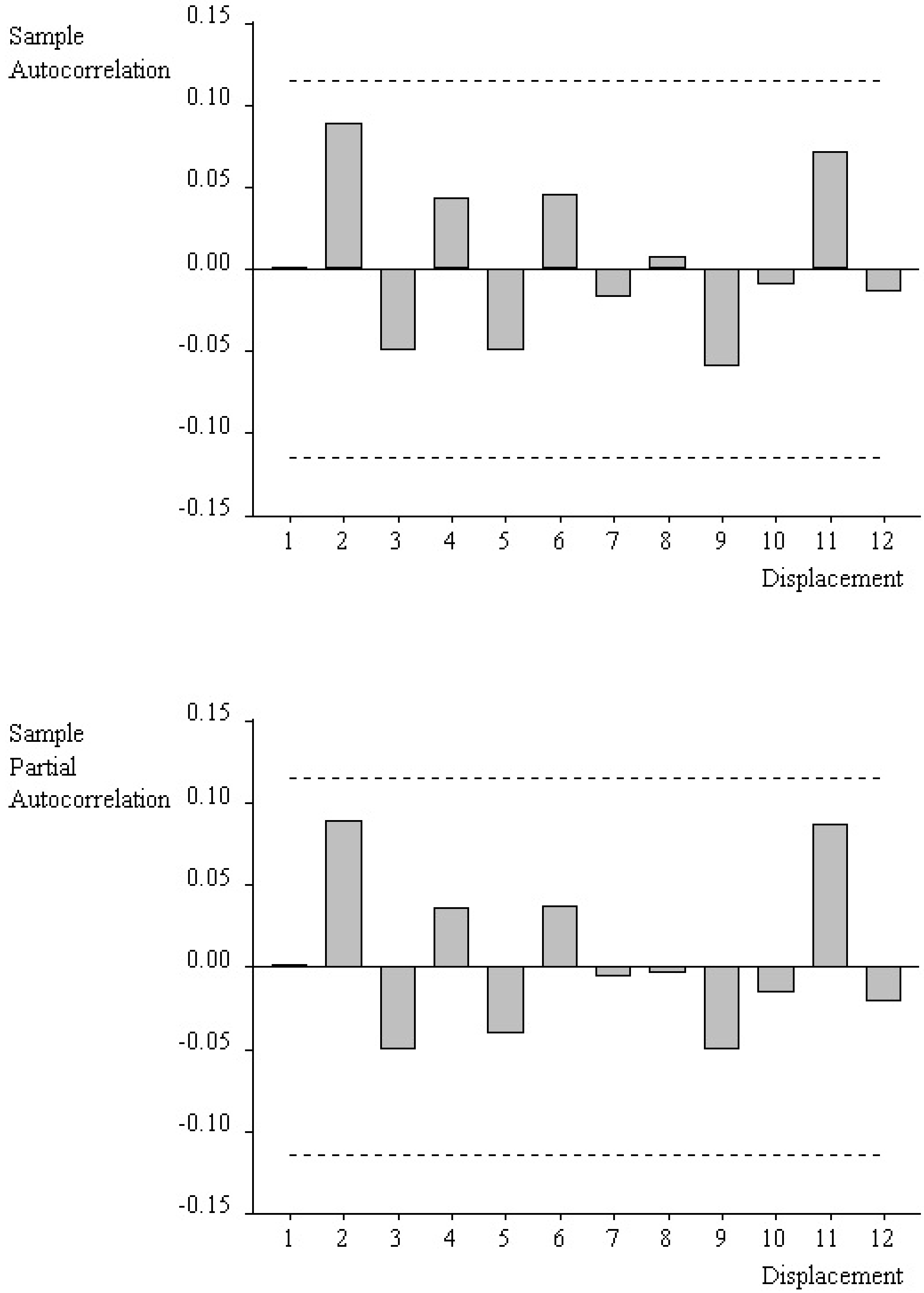


Random Walk, Levels
Sample Autocorrelation Function (Top Panel)
Sample Partial Autocorrelation Function (Bottom Panel)

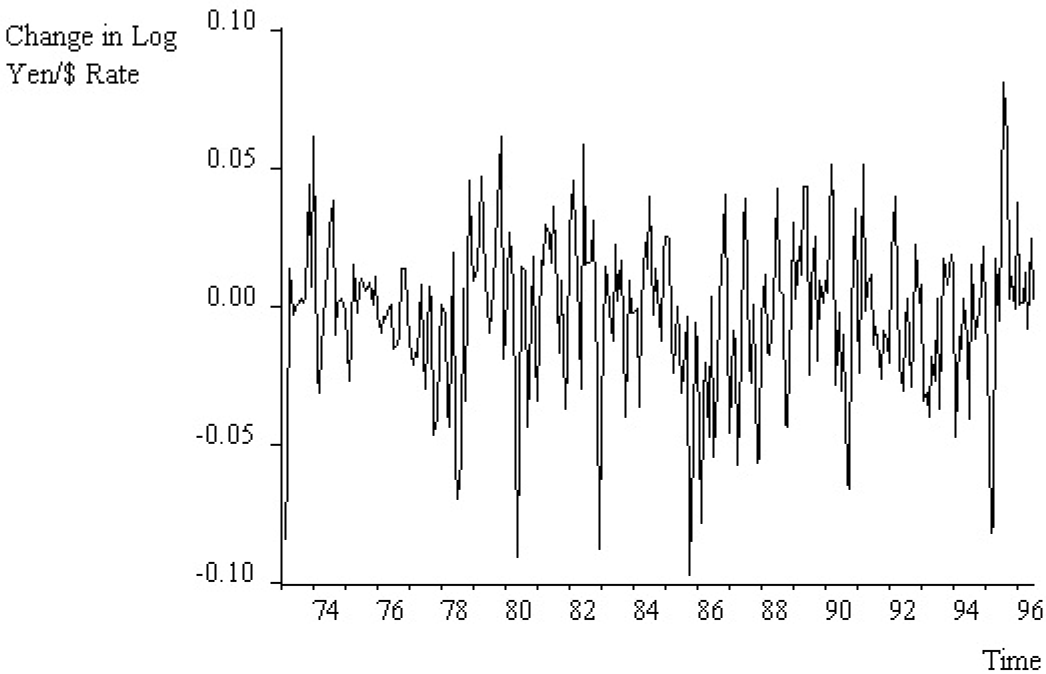




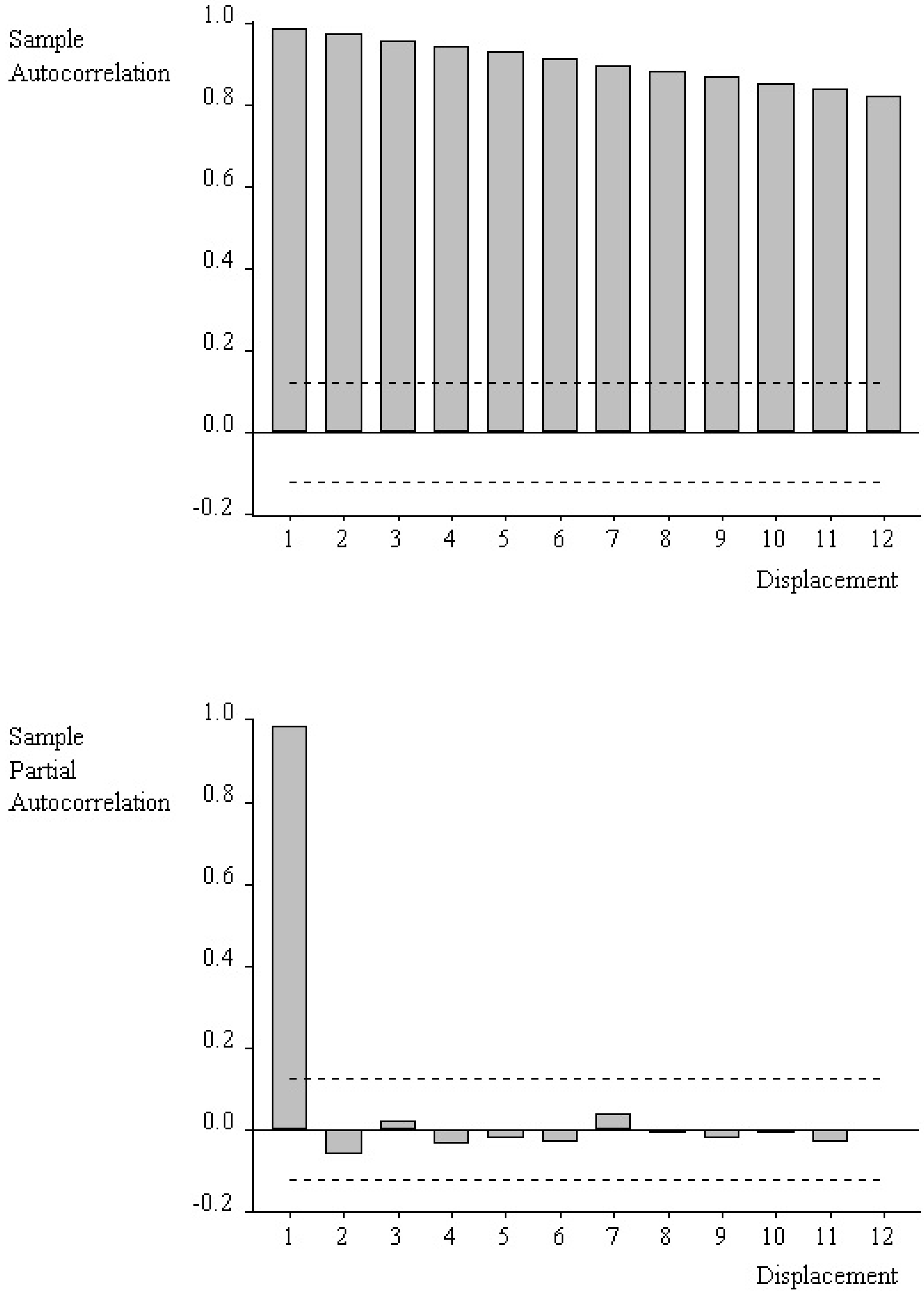
Random Walk, First Differences
Sample Autocorrelation Function (Top Panel)
Sample Partial Autocorrelation Function (Bottom Panel)



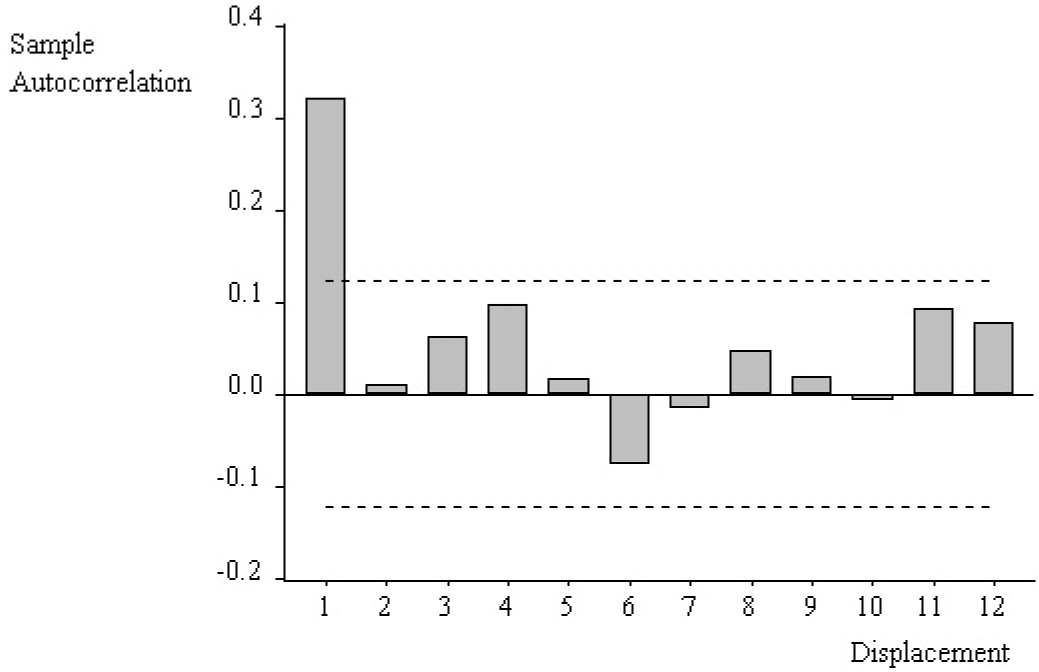
Log Yen / Dollar Exchange Rate (Top Panel)
Change in Log Yen / Dollar Exchange Rate (Bottom Panel)

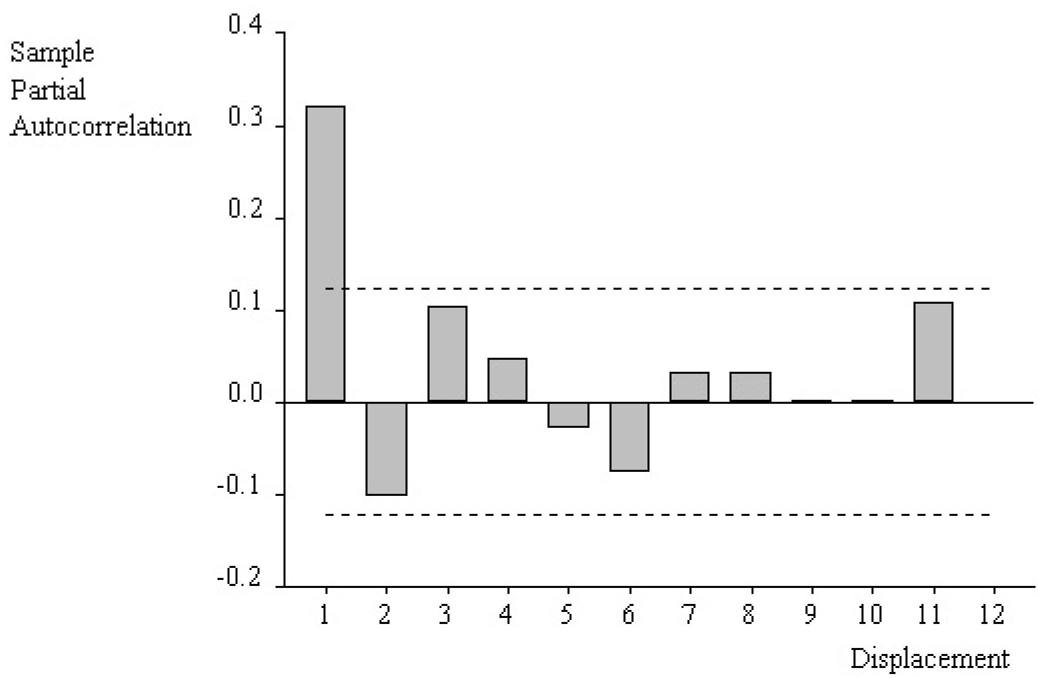


Log Yen / Dollar Exchange Rate
Sample Autocorrelations (Top Panel)
Sample Partial Autocorrelations (Bottom Panel)



Log Yen / Dollar Exchange Rate, First Differences
Sample Autocorrelations (Top Panel)
Sample Partial Autocorrelations (Bottom Panel)





Log Yen / Dollar Rate, Levels
AIC Values
Various ARMA Models

			MA Order		
		0	1	2	3
	0		-5.171	-5.953	-6.428
AR Order	1	-7.171	-7.300	-7.293	-7.287
	2	-7.319	-7.314	-7.320	-7.317
	3	-7.322	-7.323	-7.316	-7.308

Log Yen / Dollar Rate, Levels
SIC Values
Various ARMA Models

			MA Order		
		0	1	2	3
	0		-5.130	-5.899	-6.360
AR Order	1	-7.131	-7.211	-7.225	-7.205
	2	-7.265	-7.246	-7.238	-7.221
	3	-7.253	-7.241	-7.220	-7.199

Log Yen / Dollar Exchange Rate
Best-Fitting Deterministic-Trend Model

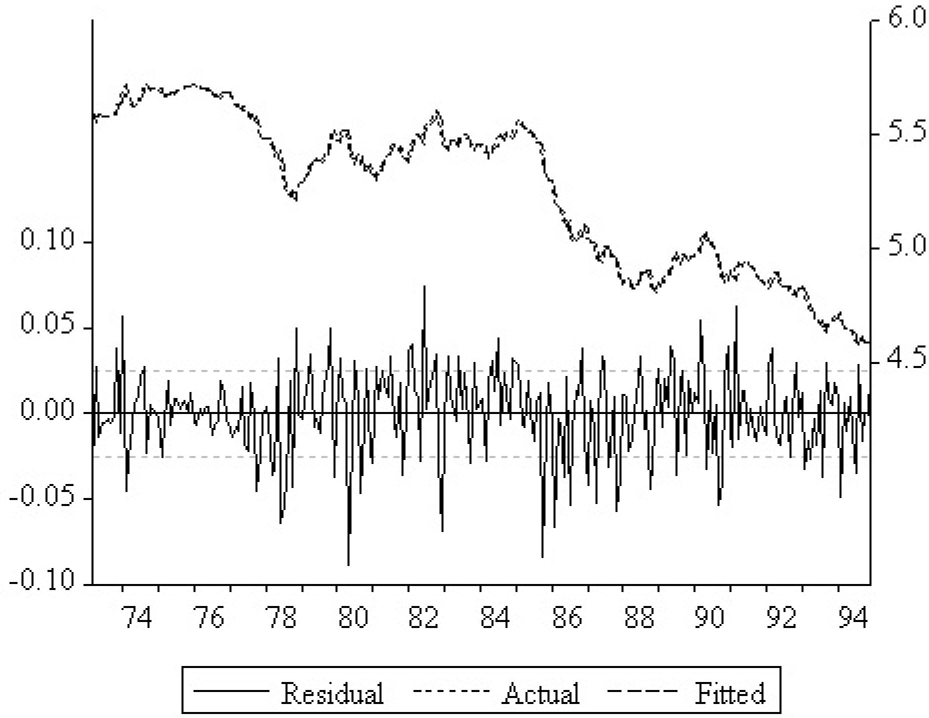
LS // Dependent Variable is LYEN
Sample(adjusted): 1973:03 1994:12
Included observations: 262 after adjusting endpoints
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.904705	0.136665	43.20570	0.0000
TIME	-0.004732	0.000781	-6.057722	0.0000
AR(1)	1.305829	0.057587	22.67561	0.0000
AR(2)	-0.334210	0.057656	-5.796676	0.0000

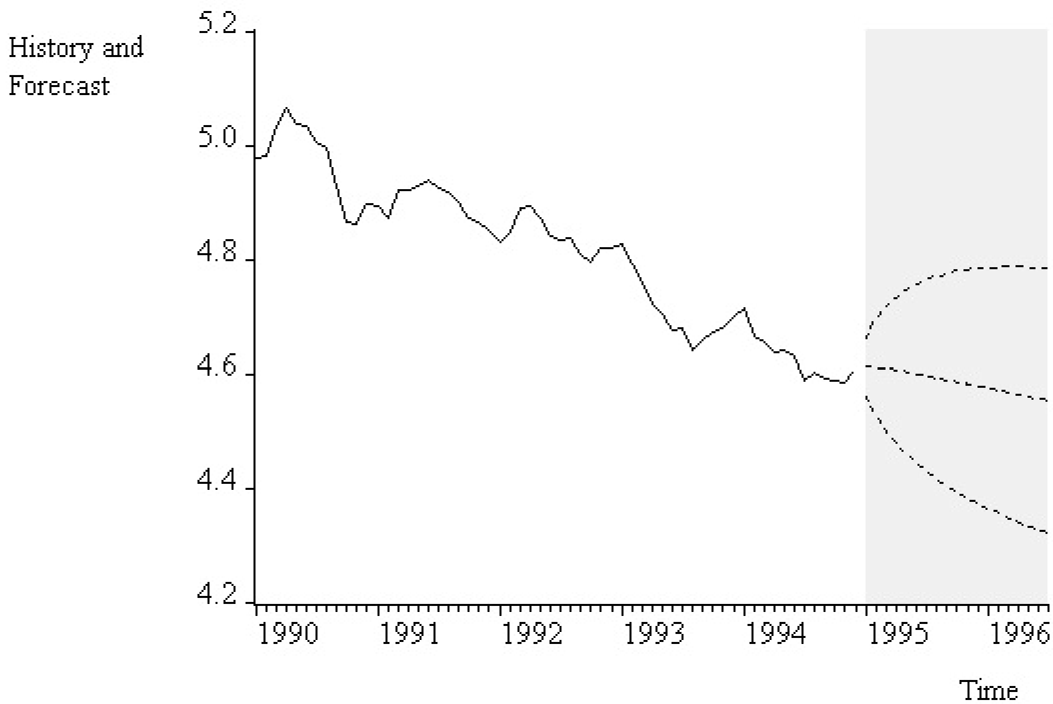
R-squared	0.994468	Mean dependent var	5.253984
Adjusted R-squared	0.994404	S.D. dependent var	0.341563
S.E. of regression	0.025551	Akaike info criterion	-7.319015
Sum squared resid	0.168435	Schwarz criterion	-7.264536
Log likelihood	591.0291	F-statistic	15461.07
Durbin-Watson stat	1.964687	Prob(F-statistic)	0.000000

Inverted AR Roots	.96	.35

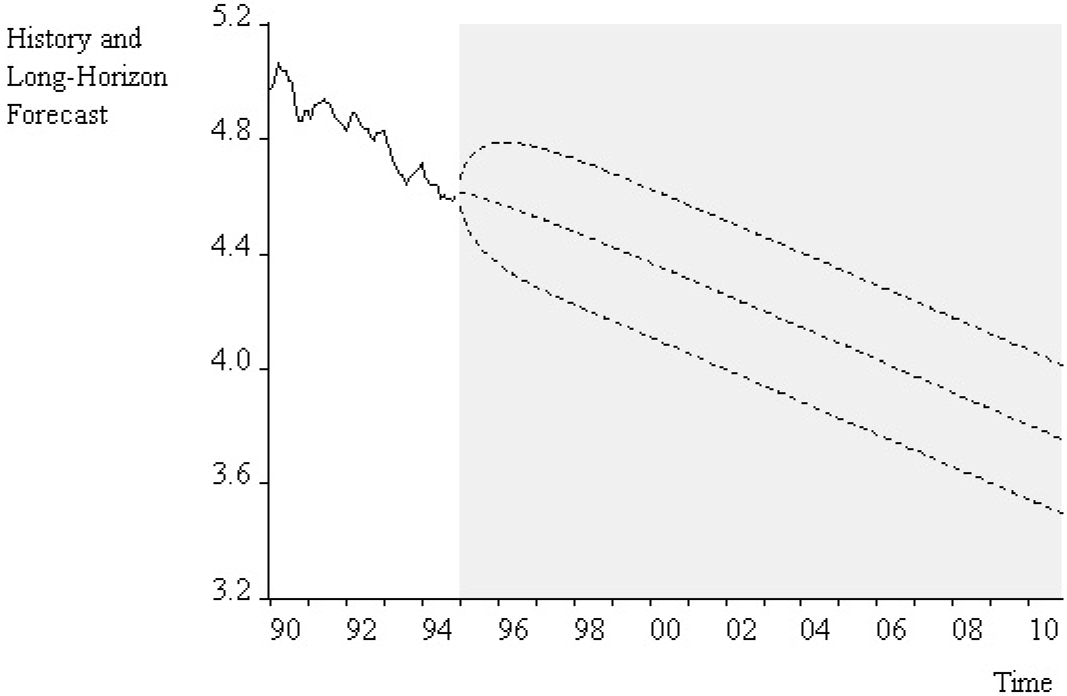
Log Yen / Dollar Exchange Rate
Best-Fitting Deterministic-Trend Model
Residual Plot



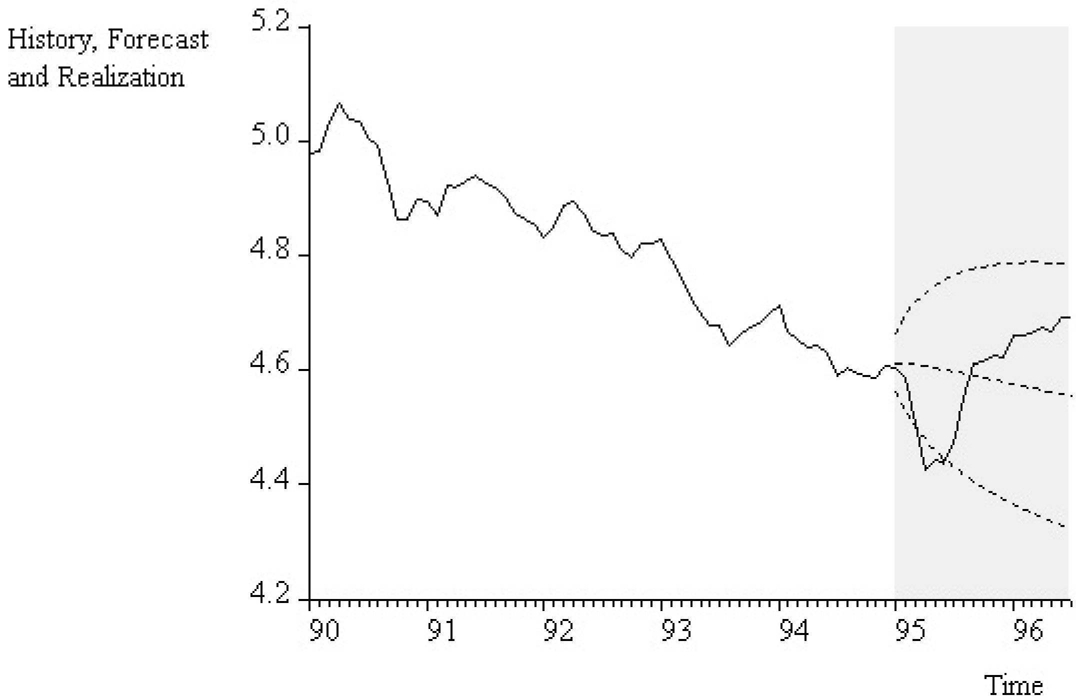
Log Yen / Dollar Rate
History and Forecast
AR(2) in Levels with Linear Trend



Log Yen / Dollar Rate
History and Long-Horizon Forecast
AR(2) in Levels with Linear Trend



Log Yen / Dollar Rate
History, Forecast and Realization
AR(2) in Levels with Linear Trend



Log Yen / Dollar Exchange Rate
Augmented Dickey-Fuller Unit Root Test

Augmented Dickey-Fuller Test Statistic	-2.498863	1% Critical Value	-3.9966
		5% Critical Value	-3.4284
		10% Critical Value	-3.1373

Augmented Dickey-Fuller Test Equation
LS // Dependent Variable is D(LYEN)
Sample(adjusted): 1973:05 1994:12
Included observations: 260 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LYEN(-1)	-0.029423	0.011775	-2.498863	0.0131
D(LYEN(-1))	0.362319	0.061785	5.864226	0.0000
D(LYEN(-2))	-0.114269	0.064897	-1.760781	0.0795
D(LYEN(-3))	0.118386	0.061020	1.940116	0.0535
C	0.170875	0.068474	2.495486	0.0132
@TREND(1973:01)	-0.000139	5.27E-05	-2.639758	0.0088

R-squared	0.142362	Mean dependent var	-0.003749
Adjusted R-squared	0.125479	S.D. dependent var	0.027103
S.E. of regression	0.025345	Akaike info criterion	-7.327517
Sum squared resid	0.163166	Schwarz criterion	-7.245348
Log likelihood	589.6532	F-statistic	8.432417
Durbin-Watson stat	2.010829	Prob(F-statistic)	0.000000

Log Yen / Dollar Rate, Changes
AIC Values
Various ARMA Models

			MA Order		
		0	1	2	3
	0		-7.298	-7.290	-7.283
AR Order	1	-7.308	-7.307	-7.307	-7.302
	2	-7.312	-7.314	-7.307	-7.299
	3	-7.316	-7.309	-7.340	-7.336

Log Yen / Dollar Rate, Changes
SIC Values
Various ARMA Models

			MA Order		
		0	1	2	3
	0		-7.270	-7.249	-7.228
AR Order	1	-7.281	-7.266	-7.252	-7.234
	2	-7.271	-7.259	-7.238	-7.217
	3	-7.261	-7.241	-7.258	-7.240

Log Yen / Dollar Exchange Rate
Best-Fitting Stochastic-Trend Model

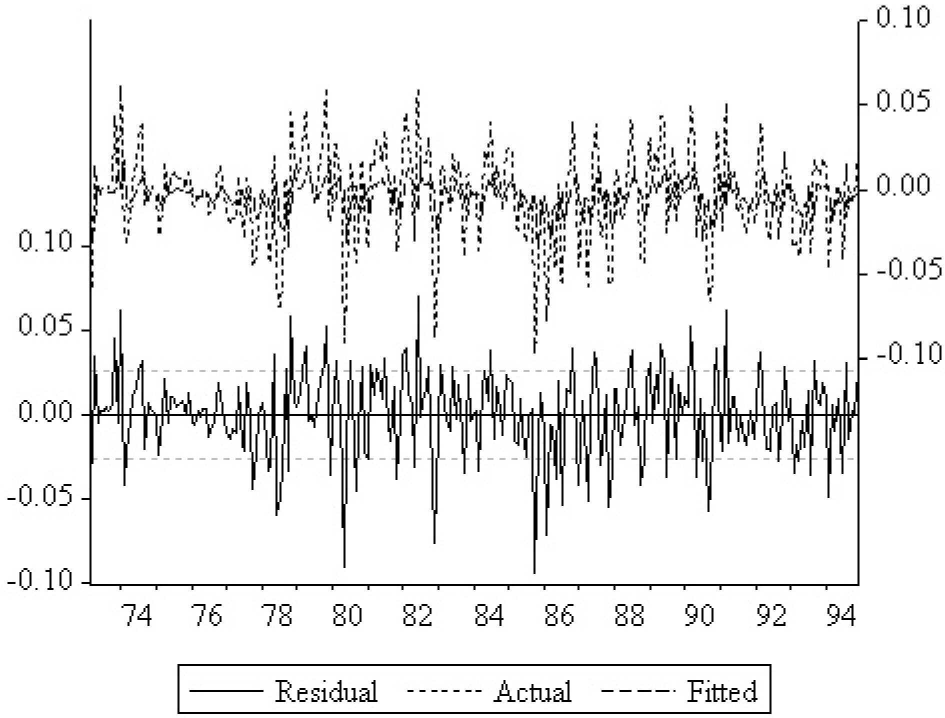
LS // Dependent Variable is DLYEN
Sample(adjusted): 1973:03 1994:12
Included observations: 262 after adjusting endpoints
Convergence achieved after 3 iterations

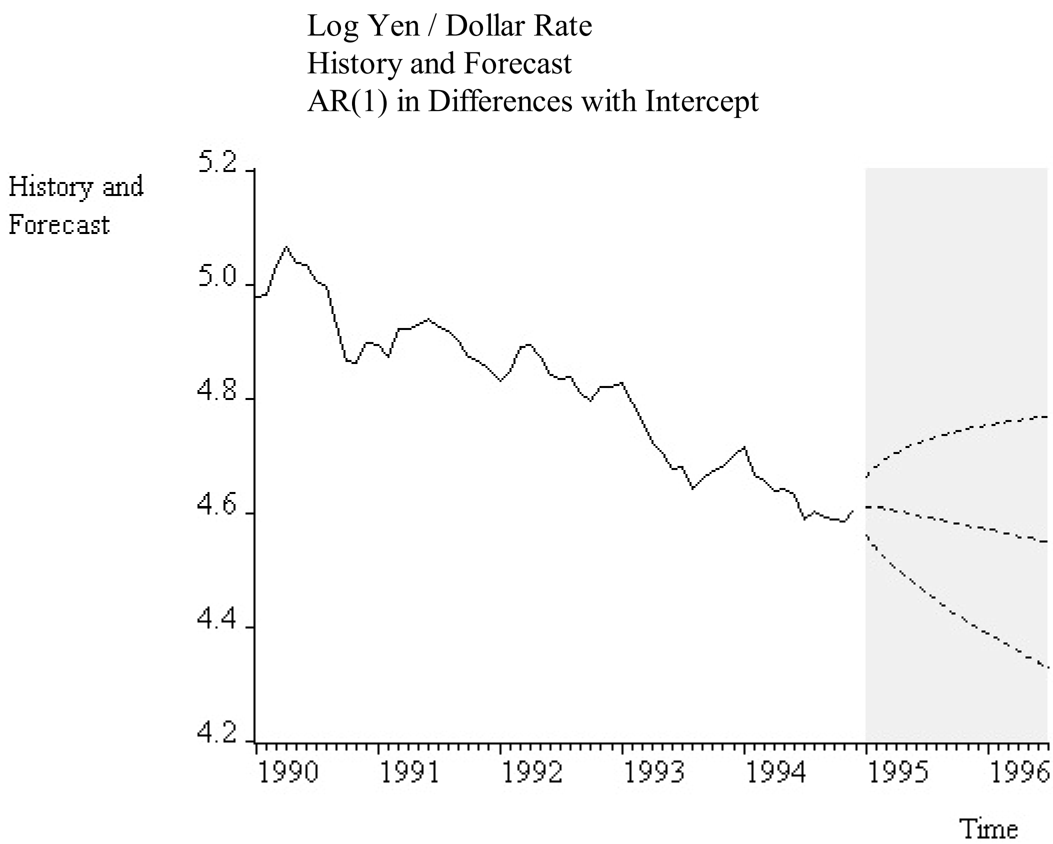
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.003697	0.002350	-1.573440	0.1168
AR(1)	0.321870	0.057767	5.571863	0.0000

R-squared	0.106669	Mean dependent var	-0.003888
Adjusted R-squared	0.103233	S.D. dependent var	0.027227
S.E. of regression	0.025784	Akaike info criterion	-7.308418
Sum squared resid	0.172848	Schwarz criterion	-7.281179
Log likelihood	587.6409	F-statistic	31.04566
Durbin-Watson stat	1.948933	Prob(F-statistic)	0.000000

Inverted AR Roots .32

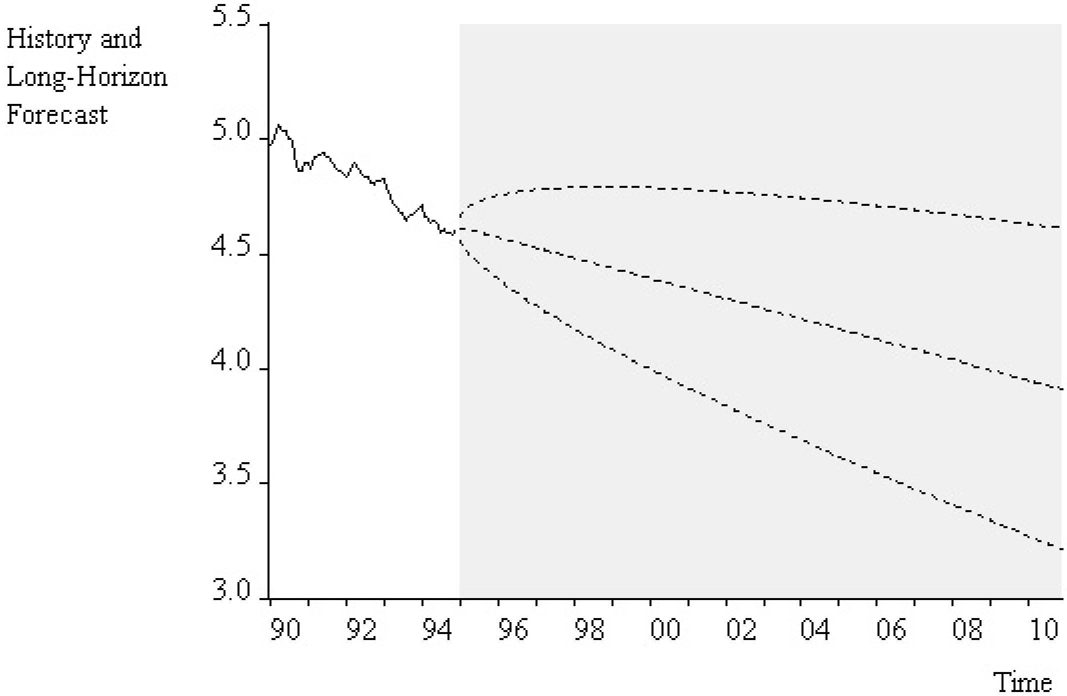
Log Yen / Dollar Exchange Rate
Best-Fitting Stochastic-Trend Model
Residual Plot



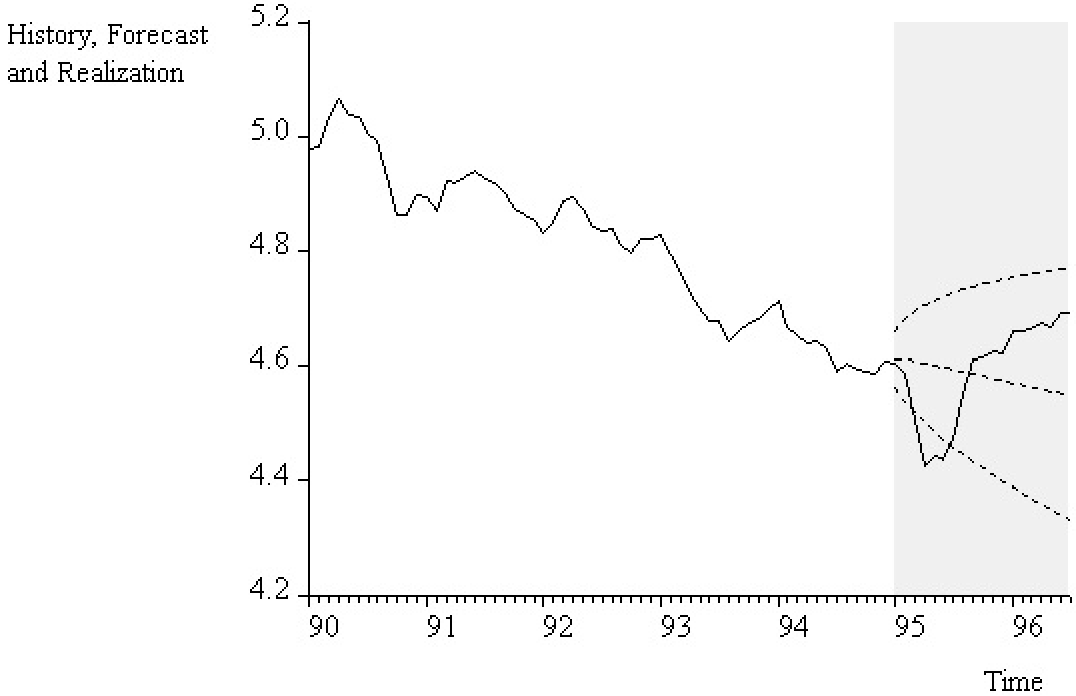


Log Yen / Dollar Rate
History and Forecast
AR(1) in Differences with Intercept
History and Forecast
5.2
5.0
4.8
4.6
4.4
4.2
1990
1991
1992
1993
1994
1995
1996
Time

Log Yen / Dollar Rate
History and Long-Horizon Forecast
AR(1) in Differences with Intercept



Log Yen / Dollar Rate
History, Forecast and Realization
AR(1) in Differences with Intercept



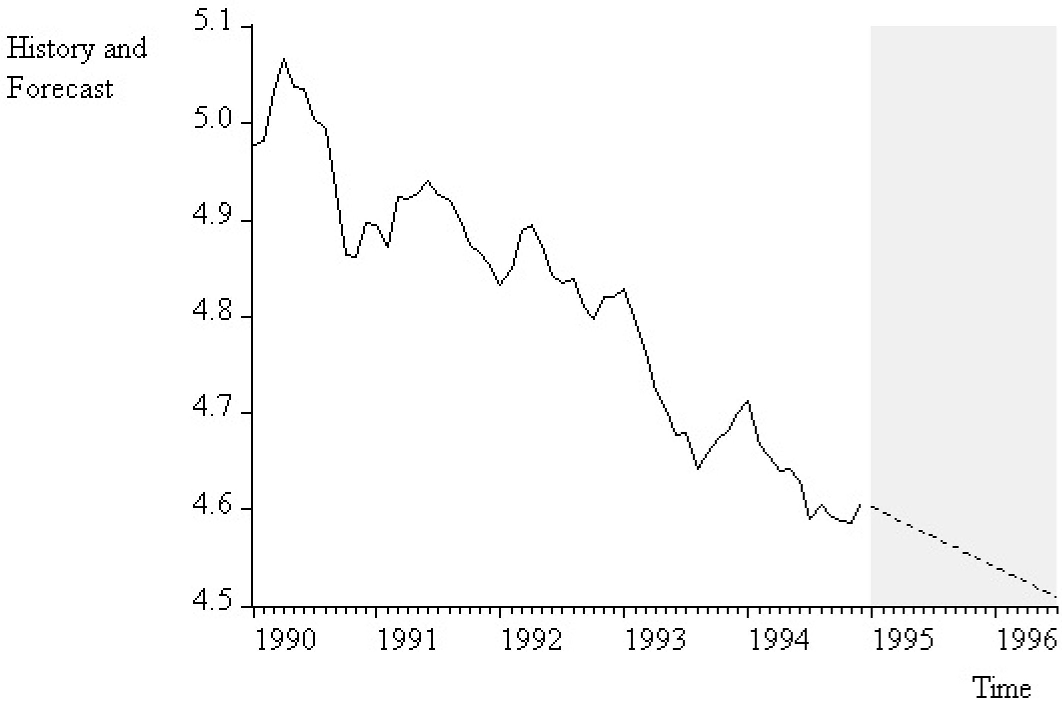
Log Yen / Dollar Exchange Rate
Holt-Winters Smoothing

Sample: 1973:01 1994:12
Included observations: 264
Method: Holt-Winters, No Seasonal
Original Series: LYEN
Forecast Series: LYENSM

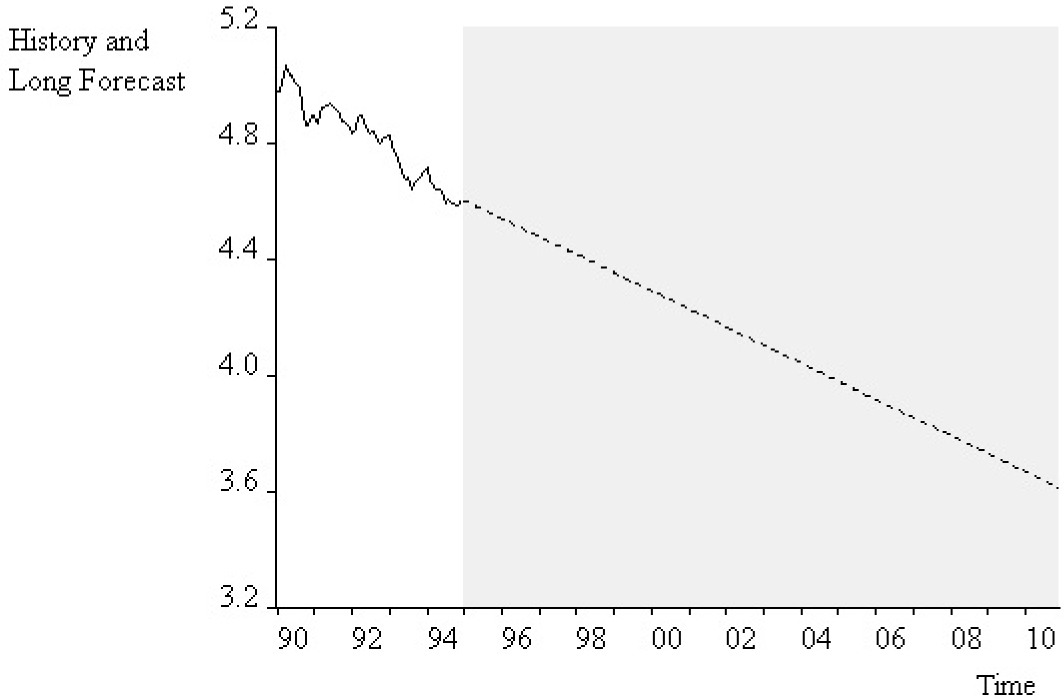
Parameters:	Alpha	1.000000
	Beta	0.090000
Sum of Squared Residuals		0.202421
Root Mean Squared Error		0.027690

End of Period Levels:	Mean	4.606969
	Trend	-0.005193

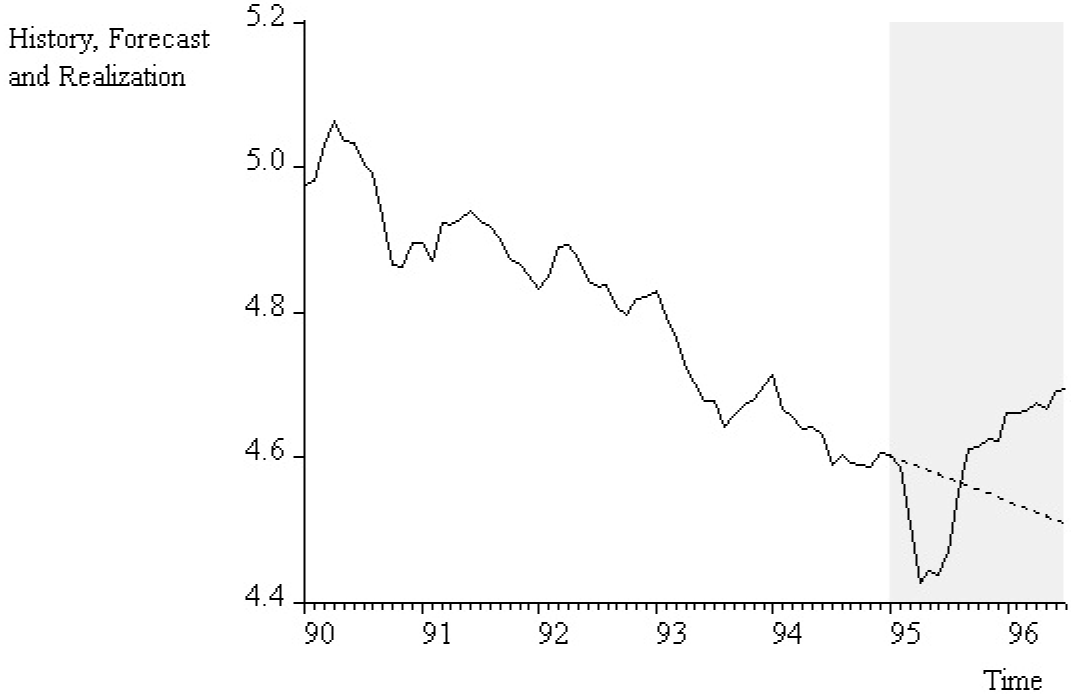
Log Yen / Dollar Rate
History and Forecast
Holt-Winters Smoothing



Log Yen / Dollar Rate
History and Long-Horizon Forecast
Holt-Winters Smoothing



Log Yen / Dollar Rate
History, Forecast and Realization
Holt-Winters Smoothing



Volatility Measurement, Modeling and Forecasting

The main idea:

$$\varepsilon_t \mid \Omega_{t-1} \sim (0, \sigma_t^2)$$

$$\Omega_{t-1} = \{\varepsilon_{t-1}, \varepsilon_{t-2}, ...\}$$

We'll look at:

Basic structure and properties

Time variation in volatility and prediction-error variance

ARMA representation in squares

GARCH(1,1) and exponential smoothing

Unconditional symmetry and leptokurtosis

Convergence to normality under temporal aggregation

Estimation and testing

Basic Structure and Properties

Standard models (e.g., ARMA):

Unconditional mean: constant
Unconditional variance: constant

Conditional mean: varies
Conditional variance: constant (unfortunately)

k-step-ahead forecast error variance: depends only on k, not on Ω_t (again unfortunately)

1. The Basic ARCH Process

$$y_t = B(L)\,\varepsilon_t$$

$$B(L) = \sum_{i=0}^{\infty} b_i L^i \qquad \sum_{i=0}^{\infty} b_i^2 < \infty \qquad b_0 = 1$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0,\ \sigma_t^2)$$

$$\sigma_t^2 = \omega + \gamma(L)\,\varepsilon_t^2$$

$$\omega>0 \qquad \gamma(L) = \sum_{i=1}^{p} \gamma_i L^i \qquad \gamma_i \geq 0 \text{ for all } i \qquad \sum \gamma_i < 1\ .$$

ARCH(1) process:

$$r_t|\Omega_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2$$

Unconditional mean: $E(r_t) = 0$

Unconditional variance: $E(r_t - E(r_t))^2 = \dfrac{\omega}{1-\alpha}$

Conditional mean: $E(r_t \mid \Omega_{t-1}) = 0$

Conditional variance: $E([r_t - E(r_t|\Omega_{t-1})]^2|\Omega_{t-1}) = \omega + \alpha r_{t-1}^2$

2. The GARCH Process

$$y_t = \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N\left(0, \sigma_t^2\right)$$

$$\sigma_t^2 = \omega + \alpha(L)\,\varepsilon_t^2 + \beta(L)\,\sigma_t^2$$

$$\alpha(L) = \sum_{i=1}^{p} \alpha_i L^i, \quad \beta(L) = \sum_{i=1}^{q} \beta_i L^i$$

$$\omega > 0, \quad \alpha_i \geq 0, \quad \beta_i \geq 0, \quad \sum \alpha_i + \sum \beta_i < 1 \ .$$

Time Variation in Volatility and Prediction Error Variance

Prediction error variance depends on Ω_{t-1}

e.g., 1-step-ahead prediction error variance is now

$$\sigma_t^2 = \omega + \alpha\, r_{t-1}^2 + \beta\, \sigma_{t-1}^2$$

Conditional variance is a serially correlated RV

Again, follows immediately from

$$\sigma_t^2 = \omega + \alpha\, r_{t-1}^2 + \beta\, \sigma_{t-1}^2$$

ARMA Representation in Squares

r_t^2 has the ARMA(1,1) representation:

$$r_t^2 = \omega + (\alpha + \beta) r_{t-1}^2 - \beta \nu_{t-1} + \nu_t,$$

where $\nu_t = r_t^2 - \sigma_t^2$.

Important result:

The above equation is simply

$$r_t^2 = \left(\omega + (\alpha + \beta) r_{t-1}^2 - \beta \nu_{t-1}\right) + \nu_t,$$

$$= \sigma_t^2 + \nu_t$$

Thus r_t^2 is a *noisy* indicator of σ_t^2.

GARCH(1,1) and Exponential Smoothing

Exponential smoothing recursion:

$$\overline{r}_t^2 = \gamma r_t^2 + (1-\gamma)\overline{r}_{t-1}^2$$

Back substitution yields:

$$\overline{r}_t^2 = \sum w_j r_{t-j}^2$$

where

$$w_j = \gamma(1-\gamma)^j$$

GARCH(1,1):

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2$$

Back substitution yields:

$$\sigma_t^2 = \frac{\omega}{1-\beta} + \alpha\sum \beta^{j-1} r_{t-j}^2$$

Unconditional Symmetry and Leptokurtosis

- Volatility clustering produces unconditional leptokurtosis
- Conditional symmetry translates into unconditional symmetry

Unexpected agreement with the facts!

Convergence to Normality under Temporal Aggregation

- Temporal aggregation of covariance stationary GARCH processes produces convergence to normality.

Again, unexpected agreement with the facts!

Estimation and Testing

Estimation: easy!

Maximum Likelihood Estimation

$$L(\theta;\ r_1,\ \ldots,\ r_T)\ =\ f(r_T \mid \Omega_{T-1};\ \theta)\ f(r_{T-1} \mid \Omega_{T-2};\ \theta) \ldots$$

If the conditional densities are Gaussian,

$$f(r_t \mid \Omega_{t-1}; \theta)\ =\ \frac{1}{\sqrt{2\pi}}\, \sigma_t^2(\theta)^{-1/2} \exp\left(-\frac{1}{2} \frac{r_t^2}{\sigma_t^2(\theta)} \right).$$

We can ignore the $f(r_p, \ldots, r_1; \theta)$ term, yielding the likelihood:

$$-\frac{T-p}{2}\ \ln(2\pi)\ -\ \frac{1}{2}\sum_{t=p+1}^{T} \ln\ \sigma_t^2(\theta)\ -\ \frac{1}{2}\sum_{t=p+1}^{T} \frac{r_t^2}{\sigma_t^2(\theta)}\ .$$

Testing: likelihood ratio tests

Graphical diagnostics: Correlogram of squares, correlogram of squared standardized residuals

Variations on Volatility Models

We will look at:

- Asymmetric response and the leverage effect
- Exogenous variables
- GARCH-M and time-varying risk premia

Asymmetric Response and the Leverage Effect: TGARCH and EGARCH

Asymmetric response I: TARCH

Standard GARCH:

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2$$

TARCH:

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \gamma r_{t-1}^2 D_{t-1} + \beta\sigma_{t-1}^2$$

where

$$D_t = \begin{cases} 1, \text{ if } r_t<0 \\ 0 \text{ otherwise} \end{cases}$$

positive return (good news): α effect on volatility

negative return (bad news): $\alpha+\gamma$ effect on volatility

$\gamma \neq 0$: Asymetric news response

$\gamma > 0$: "Leverage effect"

Asymmetric Response II: E-GARCH

$$\ln(\sigma_t^2) = \omega + \alpha \left| \frac{r_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{r_{t-1}}{\sigma_{t-1}} + \beta \ln(\sigma_{t-1}^2)$$

– Log specification ensures that the conditional variance is positive.

– Volatility driven by both size and sign of shocks

– Leverage effect when $\gamma<0$

Introducing Exogenous Variables

$$r_t|\Omega_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma X_t$$

where:

γ is a parameter vector

X is a set of positive exogenous variables.

Component GARCH

Standard GARCH:

$$(\sigma_t^2 - \bar{\omega}) = \alpha(r_{t-1}^2 - \bar{\omega}) + \beta(\sigma_{t-1}^2 - \bar{\omega}),$$

for constant long-run volatility $\bar{\omega}$.

Component GARCH:

$$(\sigma_t^2 - q_t) = \alpha(r_{t-1}^2 - q_{t-1}) + \beta(\sigma_{t-1}^2 - q_{t-1}),$$

for time-varying long-run volatility q_t, where

$$q_t = \omega + \rho(q_{t-1} - \omega) + \varphi(r_{t-1}^2 - \sigma_{t-1}^2)$$

– Transitory dynamics governed by $\alpha+\beta$

– Persistent dynamics governed by ρ

– Equivalent to nonlinearly restricted GARCH(2,2)

– Exogenous variables and asymmetry can be allowed:

$$(\sigma_t^2 - q_t) = \alpha(r_{t-1}^2 - q_{t-1}) + \gamma(r_{t-1}^2 - q_{t-1})D_{t-1} + \beta(\sigma_{t-1}^2 - q_{t-1}) + \theta X_t$$

Regression with GARCH Disturbances

$$y_t = x_t'\beta + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, \sigma_t^2)$$

GARCH-M and Time-Varying Risk Premia

Standard GARCH regression model:

$$y_t = x_t' \beta + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, \sigma_t^2)$$

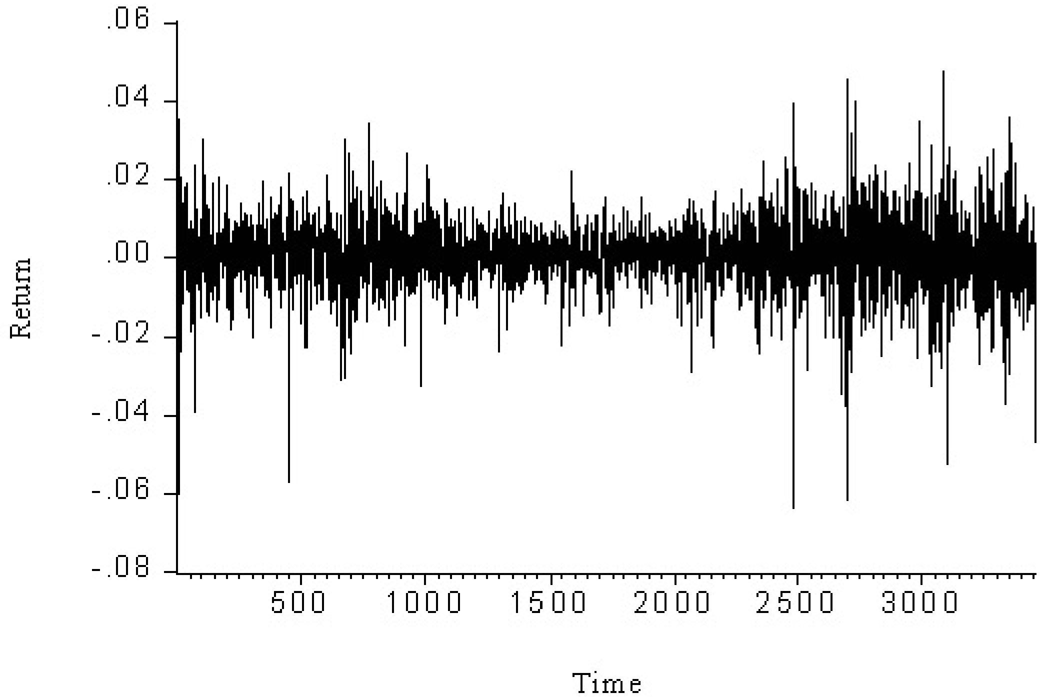
GARCH-M model is a special case:

$$y_t = x_t' \beta + \gamma \sigma_t^2 + \varepsilon_t$$

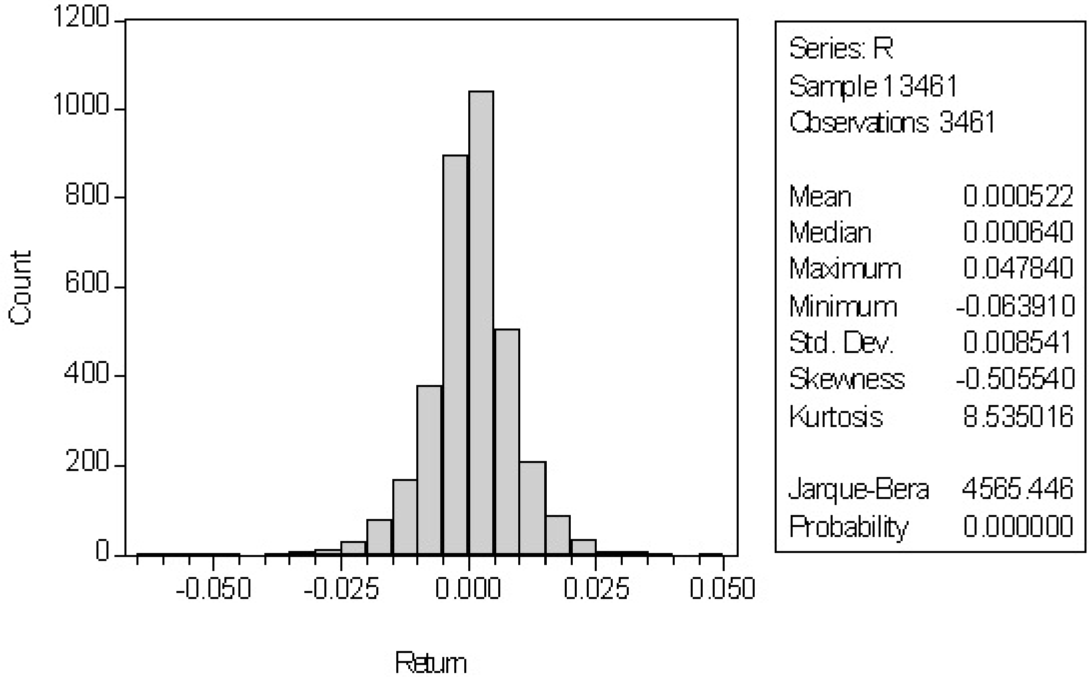
$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, \sigma_t^2)$$

– Time-varying risk premia in excess returns

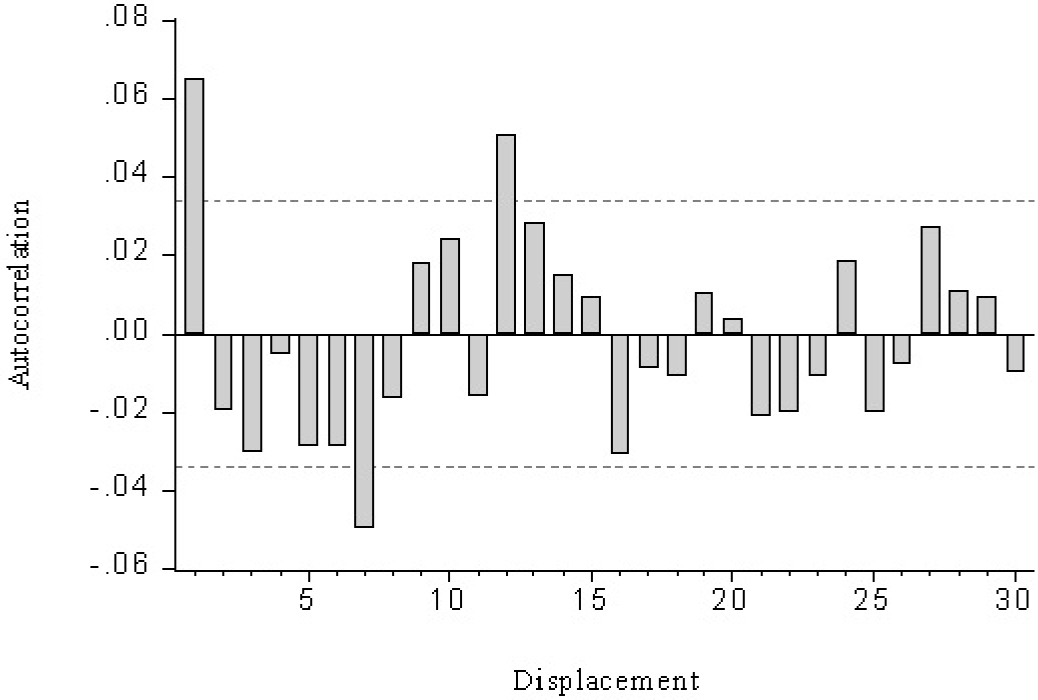
Time Series Plot
NYSE Returns



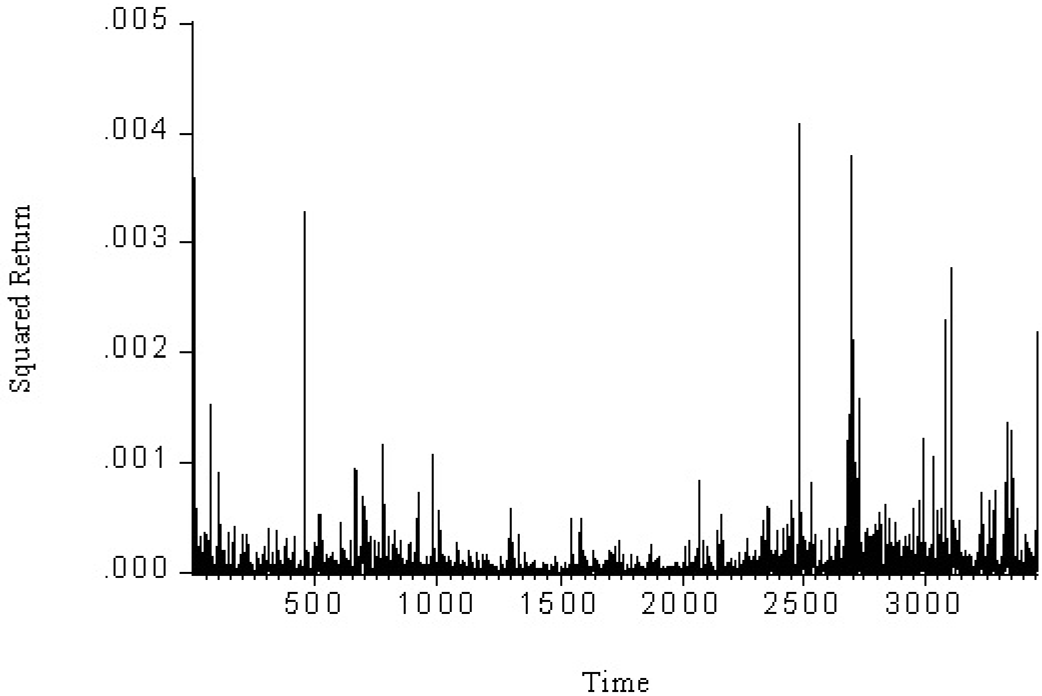
Histogram and Related Diagnostic Statistics
NYSE Returns



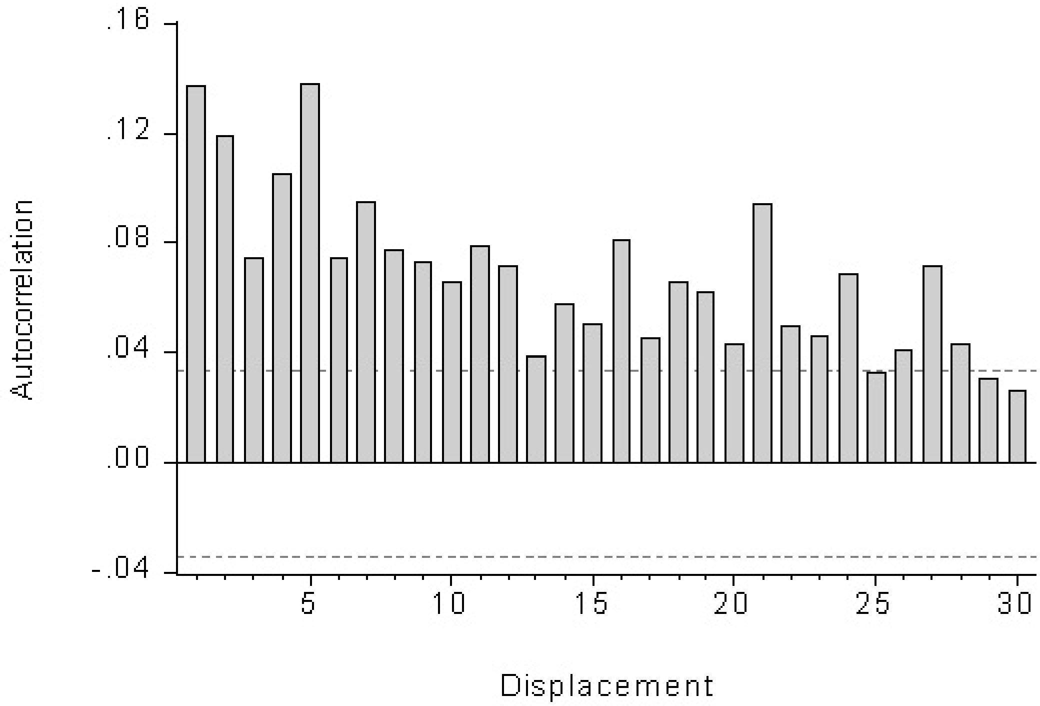
Correlogram
NYSE Returns



Time Series Plot
Squared NYSE Returns



Correlogram
Squared NYSE Returns



AR(5) Model
Squared NYSE Returns

Dependent Variable: R2
Method: Least Squares

Sample(adjusted): 6 3461
Included observations: 3456 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.40E-05	3.78E-06	11.62473	0.0000
R2(-1)	0.107900	0.016137	6.686547	0.0000
R2(-2)	0.091840	0.016186	5.674167	0.0000
R2(-3)	0.028981	0.016250	1.783389	0.0746
R2(-4)	0.039312	0.016481	2.385241	0.0171
R2(-5)	0.116436	0.016338	7.126828	0.0000

R-squared	0.052268	Mean dependent var	7.19E-05
Adjusted R-squared	0.050894	S.D. dependent var	0.000189
S.E. of regression	0.000184	Akaike info criterion	-14.36434
Sum squared resid	0.000116	Schwarz criterion	-14.35366
Log likelihood	24827.58	F-statistic	38.05372
Durbin-Watson stat	1.975672	Prob(F-statistic)	0.000000

ARCH(5) Model
NYSE Returns

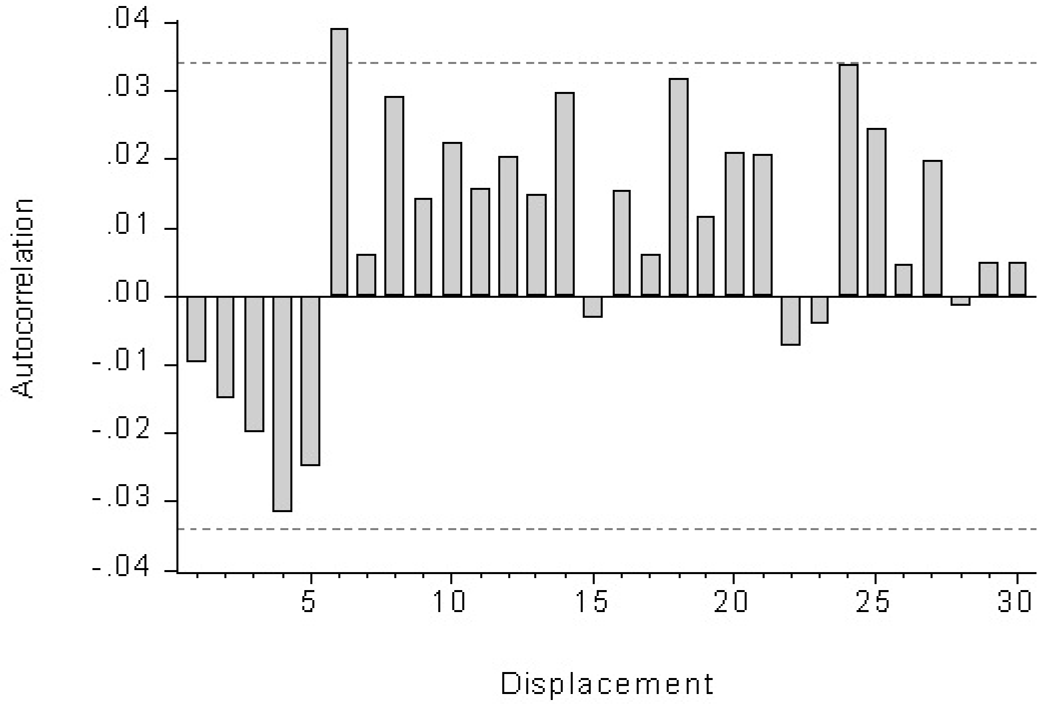
Dependent Variable: R
Method: ML - ARCH (Marquardt)

Sample: 1 3461
Included observations: 3461
Convergence achieved after 13 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000689	0.000127	5.437097	0.0000
Variance Equation				
C	3.16E-05	1.08E-06	29.28536	0.0000
ARCH(1)	0.128948	0.013847	9.312344	0.0000
ARCH(2)	0.166852	0.015055	11.08281	0.0000
ARCH(3)	0.072551	0.014345	5.057526	0.0000
ARCH(4)	0.143778	0.015363	9.358870	0.0000
ARCH(5)	0.089254	0.018480	4.829789	0.0000

R-squared	-0.000381	Mean dependent var	0.000522
Adjusted R-squared	-0.002118	S.D. dependent var	0.008541
S.E. of regression	0.008550	Akaike info criterion	-6.821461
Sum squared resid	0.252519	Schwarz criterion	-6.809024
Log likelihood	11811.54	Durbin-Watson stat	1.861036

Correlogram
Standardized ARCH(5) Residuals
NYSE Returns



GARCH(1,1) Model
NYSE Returns

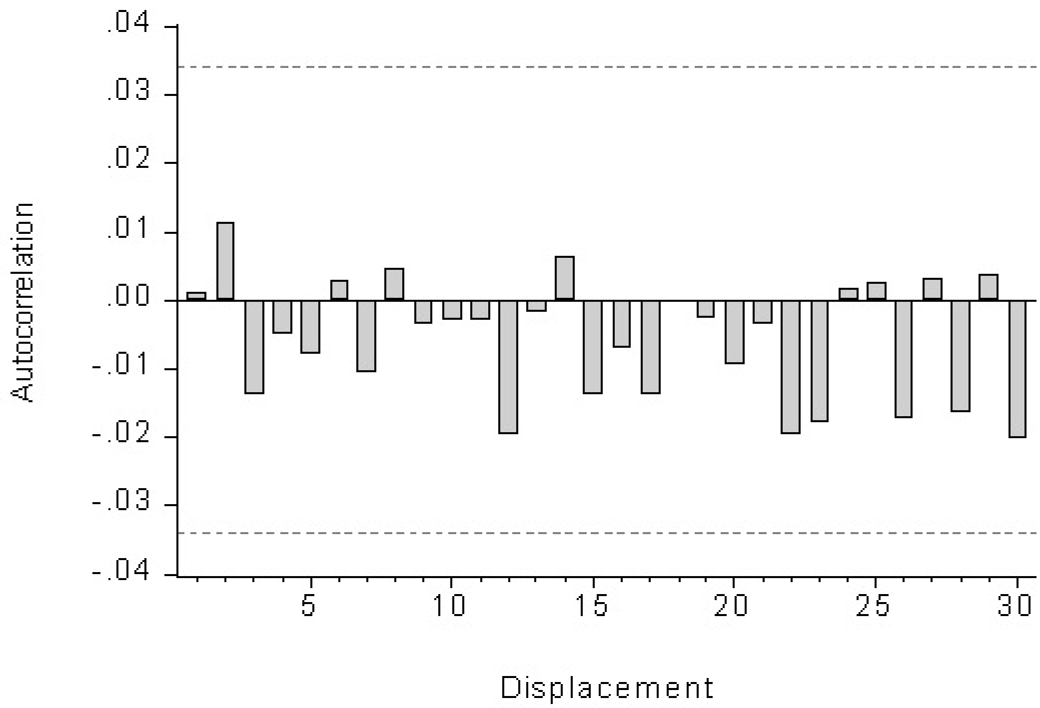
Dependent Variable: R
Method: ML - ARCH (Marquardt)

Sample: 1 3461
Included observations: 3461
Convergence achieved after 19 iterations
Variance backcast: ON

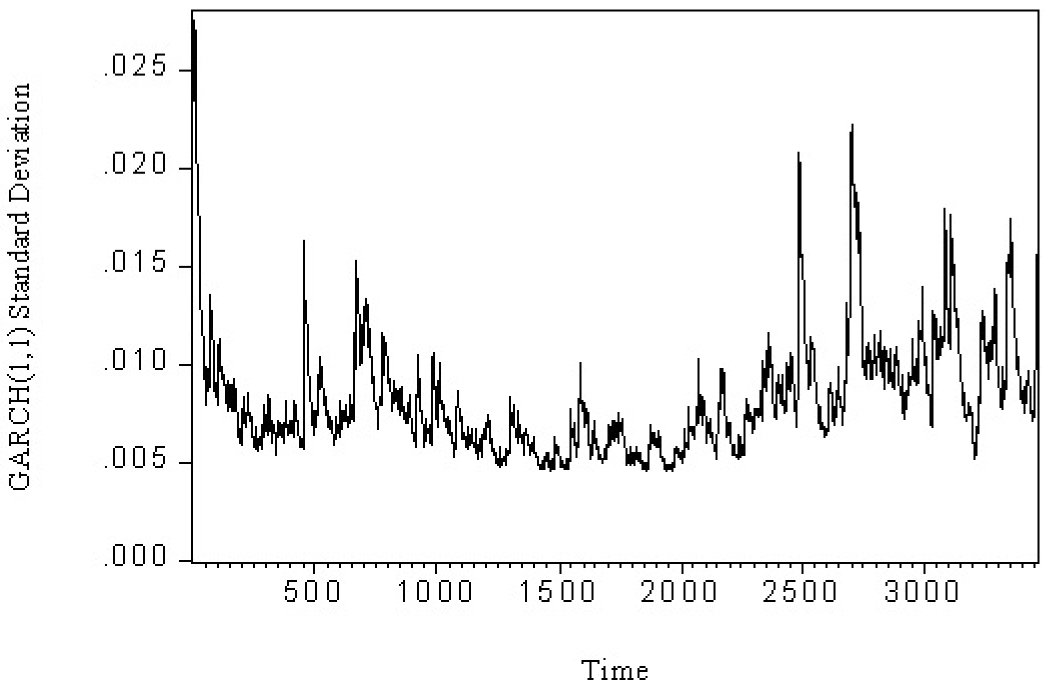
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000640	0.000127	5.036942	0.0000
Variance Equation				
C	1.06E-06	1.49E-07	7.136840	0.0000
ARCH(1)	0.067410	0.004955	13.60315	0.0000
GARCH(1)	0.919714	0.006122	150.2195	0.0000

R-squared	-0.000191	Mean dependent var	0.000522
Adjusted R-squared	-0.001059	S.D. dependent var	0.008541
S.E. of regression	0.008546	Akaike info criterion	-6.868008
Sum squared resid	0.252471	Schwarz criterion	-6.860901
Log likelihood	11889.09	Durbin-Watson stat	1.861389

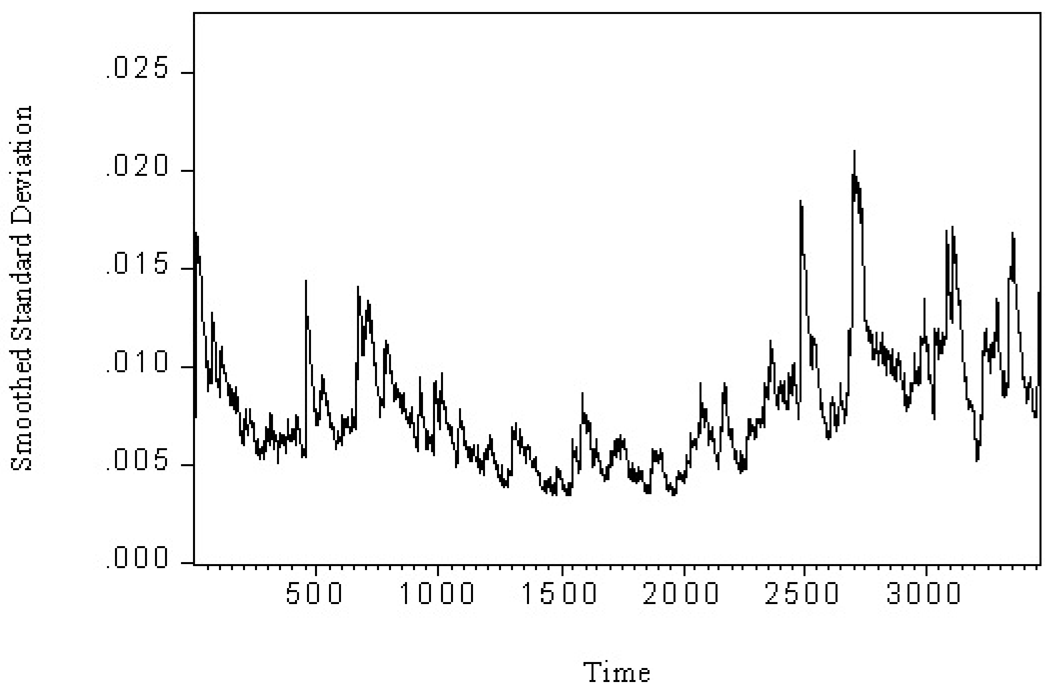
Correlogram
Standardized GARCH(1,1) Residuals
NYSE Returns



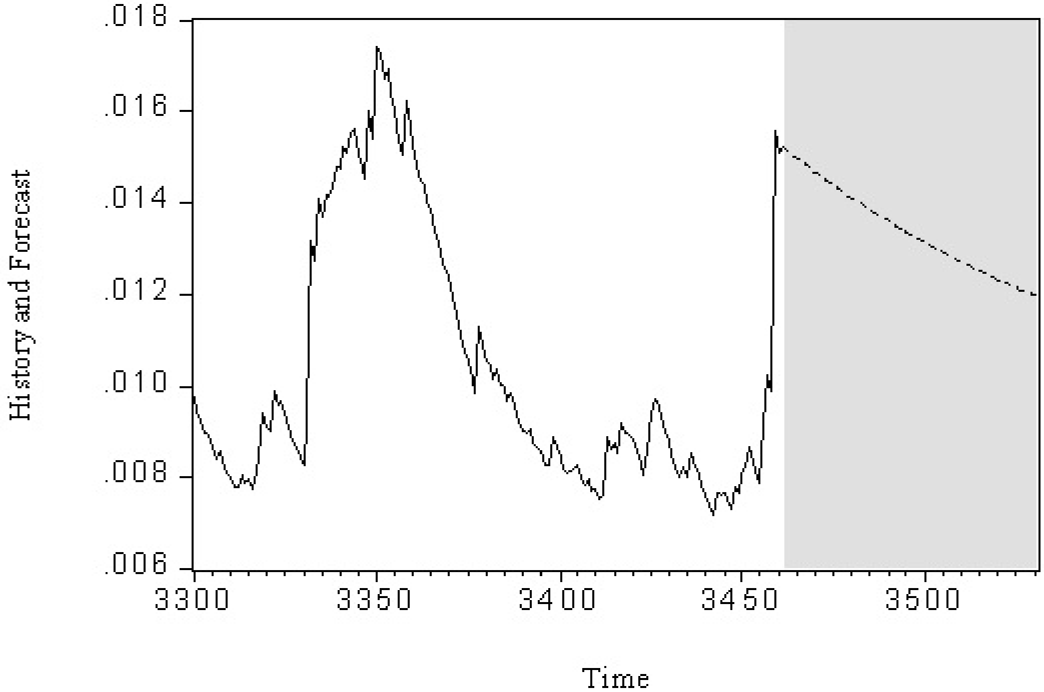
Estimated Conditional Standard Deviation
GARCH(1,1) Model
NYSE Returns



Estimated Conditional Standard Deviation
Exponential Smoothing
NYSE Returns



Conditional Standard Deviation
History and Forecast
GARCH(1,1) Model



Conditional Standard Deviation
Extended History and Extended Forecast
GARCH(1,1) Model